A Course on Biodiversity

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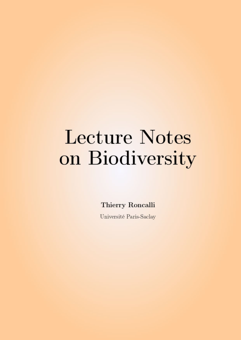
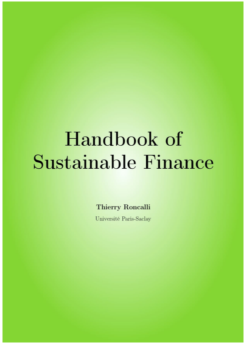
[1]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

1. **Overview**
   This course aims to explore the principles of biodiversity to support asset owners and managers in developing biodiversity-focused investments
2. **Textbook**





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Part 1. Definition

Part 2. Ecosystem functions and services

Part 3a. Biodiversity threats and risks

Part 3b. Biodiversity threats and risks

Part 4. Biodiversity measurement

Part 5. Biodiversity governance and regulation

Part 6. Investment approaches

Exercise 1. Calculating the prevalence of undernourishment

Exercise 2. Calculating the species-area relationship using the theory of island biogeography

Exercise 3. Species abundance models

Exercise 4. Valuation of life and health

A Course on Biodiversity
Lecture 1. Definition

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Biodiversity & ESG

An important issue in the ESG financial community:

- SFDR's mandatory **principal adverse impact** indicator on biodiversity (PAI 7)
- Sixth economic objective of the **EU green taxonomy** (protection and restoration of biodiversity and ecosystems)
- ESRS E4 category of the **CSRD** (biodiversity and ecosystems)

Definition

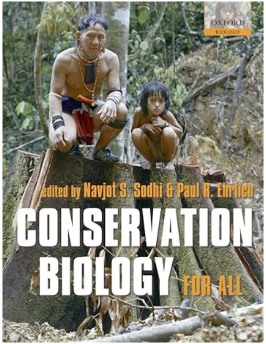
Definition

Biodiversity, or biological **diversity**, refers to the variety and variability of life on Earth in all its many manifestations

- Biodiversity is a broad, unifying concept that encompasses all forms, levels, and combinations of natural variation at all levels of biological organization (Gaston and Spicer, 2004)
- It includes genetic diversity within species, the diversity of species in different habitats, and the diversity of ecosystems themselves
- Biodiversity encompasses all living organisms, from the smallest bacteria to the largest mammals, and the complex relationships and interactions among them

Definition

Figure 1: Biodiversity ≈ Conservation Biology



Sodhi, N. S. and Ehrlich, P. R. (2010). *Conservation Biology for All*. Oxford University Press, 351 pages.

Key components of biodiversity

Genetic diversity

- Variety of genes within a species (*e.g.*, rice)
- Essential for the survival and adaptability of species
- Evolution in response to changing conditions

Organismal diversity

- Species diversity (species richness)
- Relative abundance of each species (species evenness)
- Variation in the distribution of species in space (beta diversity or species density)

Ecological diversity

- Ecosystem diversity
- Habitats, biological communities, and ecological processes
- forests, grasslands, wetlands, deserts, marine environments, etc.

Key components of biodiversity

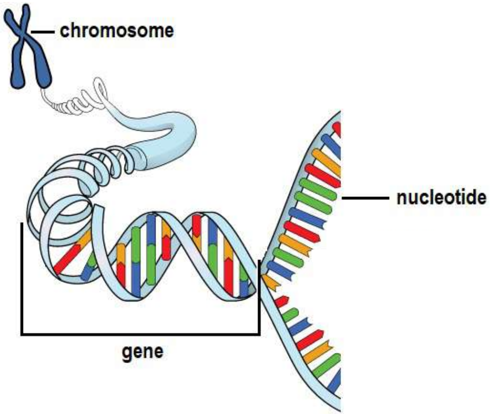
Table 1: Elements of biodiversity

Ecological diversity	Genetic diversity	Organismal diversity
Biogeographic realms		Domains or Kingdoms
Biomes		Phyla
Provinces		Families
Ecoregions		Genera
Ecosystems		Species
Habitats		Subspecies
Populations	Populations	Populations
	Individuals	Individuals
	Chromosomes	
	Genes	
	Nucleotides	

Source: Gaston (2010).

Genetic diversity

Figure 2: Nucleotides, genes, and chromosomes



Source: blog.myheritage.com/2018/02/dna-basics-chapter-3-dna-expression.

Genetic diversity

- Genetic (or genomic) diversity can be assessed at different structural levels: nucleotides, genes, or chromosomes
- Nucleotide differences: adenine (A), cytosine (C), guanine (G), and thymine (T)
- Allelic diversity (average number of alleles per locus)
- Gene diversity or polymorphism (proportion of polymorphic loci across the genome)
- Heterozygosity

Genetic diversity

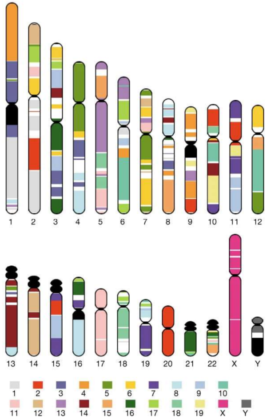
Genetic diversity measurement

- Number of genes
- Genome size (or C-value)
- The **C-value** is the amount of DNA contained in a haploid set of chromosomes
- It is typically measured in picograms (pg) or base pairs (bp)
- A **base pair** is the basic unit of DNA sequence and corresponds to two nucleotides that combine to form the DNA double helix
- The conversion between C-value and base pairs uses the following correspondence: 1 picogram is equal to 978 Mbp (million base pairs)

Genetic diversity

Table 2: Human genome sequencing

Statistics	GRCH38
Base pairs (Gbp)	2.92
Number of genes	**60 090**
Number of protein-coding genes	19 890
% of repeats	51.89
Statistics	**T2T-CHM13**
Base pairs (Gbp)	3.05
Number of genes	**63 494**
Number of protein-coding genes	19 969
% of repeats	53.94



Source: https://www.nature.com/articles/35057062.

Genetic diversity

Table 3: Genetic diversity of some organisms

Organism	C-value (in pg)	Base pairs (in Mbp)	Genes ($\times 10^3$)	Chromosomes ($2n$ or kn)
Mycoplasma (bacterium)		0.580	0.45 – 0.70	1*
Haemophilus influenzae (bacterium)		1.8	1.750	1*
Escherichia coli (bacterium)		4.6	4 – 5	1*
Drosophila melanogaster (fruit fly)	0.17	180	13 – 17	8
Arabidopsis thaliana (mustard plant)	0.14	135	27	10
Caenorhabditis elegans (nematode)	0.10	100	21	12
Saccharomyces cerevisiae (yeast)	0.02	12	6	16
Zea mays (corn)	2.30	2 300	32 – 40	20
Oryza sativa (rice)	0.40	430	32 – 50	24
Musmusculus (mouse)	2.60	2 700	20 – 25	40
Rattus norvegicus (brown rat)	2.75	2 700	20 – 25	42
Homo sapiens (human)	3.20	3 050	20	46
Solanum tuberosum (tetraploid potato)	3.50	3 400	39 – 45	48*
Fragaria ananassa (octoploid strawberry)	2.50	2 500	35 – 45	56*
Canis lupus familiaris (dog)	2.80	2 800	20	78
Agrodiaetus shahrami (butterfly)	0.75	750	12 – 14	100 – 268*
Ophioglossum reticulatum (polyploid fern)	6.25	6 200	30 – 50	1 440*

Genetic diversity

- The number of chromosomes in butterflies can vary greatly from species to species
- Most butterfly species have between 28 and 100 chromosomes
- Common butterflies such as the Monarch (*Danaus plexippus*) have 30 chromosomes
- The *Agrodiaetus* butterfly has 268 chromosomes

Figure 3: Blue Morpho butterfly

~~Butterflies form a species~~

Species diversity

Definition

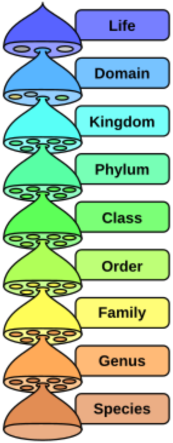
A biological species is a group of organisms that can reproduce with one another in nature and produce fertile offspring. Species are defined by the fact that they are reproductively isolated from other groups, meaning that organisms within one species cannot successfully reproduce with those of another species.

Source: `https://www.nature.com/scitable/definition/species-312`.

Species diversity

- Domain
- Kingdom
- phylum (or division)
- Class
- Order
- Family
- Genus
- Species

- Magn(order)
- Super(order)
- Grand(order)
- Mir(order)
- Order
- Sub(order)
- Infra(order)
- Parv(order)



- Domain: *Eukarya*
- Kingdom: *Animalia*
- Phylum: *Chordata*
- Class: *Mammalia*
- Order: *Primates*
- Family: *Hominidae* (great apes)
- Genus: *Homo*
- Species: ***Homo sapiens***, *Homo neanderthalensis*, *Homo erectus*, and *Homo habilis*

Species diversity

Robert May (1988): ***How Many Species are There on Earth?***

"If some alien version of the Starship Enterprise visited Earth, what might be the visitors' first question? I think it would be: How many distinct life forms — species — does your planet have? Embarrassingly, our best-guess answer would be in the range of 5 to 10 million eukaryotes (never mind the viruses and bacteria), but we could defend numbers exceeding ***100 million, or as low as 3 million****." (May, 2010).*

"In 2010, Robert May pointed out an embarrassing truth about modern science. Even as we invest huge amounts of time, money, and effort to find life on other planets, we still do not know how much life (i.e., how many species) is on our own. Although 'do not know' might sound like hyperbole, estimates have ranged wildly, ***from 2 million to 3 trillion****." (Wiens, 2023).*

Species diversity

Table 4: Currently catalogued and predicted total number of species on Earth and in the ocean

Species	Earth			Ocean		
	Catalogued	Predicted	± SE	Catalogued	Predicted	± SE
Eukaryotes	1 233 500	8 740 000	1 300 000	193 756	2 210 000	182 000
Animalia	953 434	7 770 000	958 000	171 082	2 150 000	145 000
Chromista	13 033	27 500	30 500	4 859	7 400	9 640
Fungi	43 271	611 000	297 000	1 097	5 320	11 100
Plantae	215 644	298 000	8 200	8 600	16 600	9 130
Protozoa	8 118	36 400	6 690	8 118	36 400	6 690
Prokaryotes	10 860	10 100	3 630	653	1 320	436
Archaea	502	455	160	1	1	0
Bacteria	10 358	9 680	3 470	652	1 320	436
Total	1 244 360	**8 750 000**	1 300 000	194 409	2 210 000	182 000

Source: Mora *et al.* (2011).

Species diversity

- `www.ncbi.nlm.nih.gov/taxonomy`
- `www.fws.gov/explore-taxonomic-tree`

Try the following species: *Apis mellifera*, *Homo sapiens*, *Rosa*, *Solanum tuberosum* and *Zea mays*

Ecological diversity

- Biogeographic realm
- Biome
- Province
- Ecoregion
- Ecosystem
- Habitat
- Population

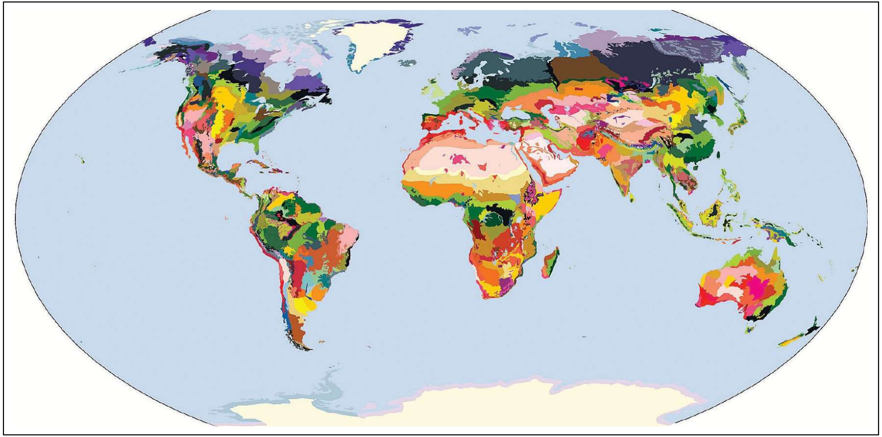
- Terrestrial biomes
  - Boreal forests (taiga)
  - Chaparral (Mediterranean climate)
  - Deserts
  - Savannas
  - Temperate forests
  - Temperate grasslands
  - Tropical rainforests
  - Tundra
- Aquatic biomes
  - Freshwater biomes (wetlands)
  - Marine biomes (oceans, coral reefs and mangroves)

8 realms:

- Australasia
- Antarctic
- Afrotropic
- Indo-Malaya
- Nearctic
- Neotropic
- Oceania
- Palearctic

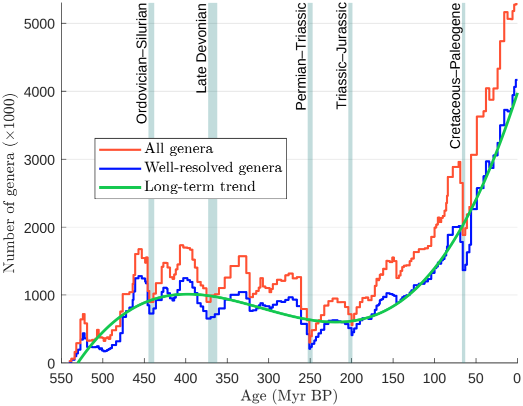
Ecological diversity

Figure 4: The 867 terrestrial ecoregions of Olson *et al.* (2001)



Biodiversity loss (and gain)

Figure 5: Genus diversity during the Phanerozoic era (Sepkoski curve)



Source: Rohde and Muller (2005) & Author's calculations.

Speciation, extinction and the birth-death model

"Like all species, plants, mammals, and birds have been subject to extinction as a fundamental part of evolution. Indeed, only about 2–4% of all the species that have ever lived during the 600 million years of the fossil record still survive today. Looking at the fossil record, it can be said that invertebrate species and mammals have had an average life span of 5–10 and 1–2 million years, respectively." (Mace, 1998).

$\Rightarrow$ An example with donkeys, horses and zebras (their common ancestor lived 2 millions years ago)

Speciation, extinction and the birth-death model

- The number of species $N(t)$ at time $t+1$ can be expressed as:

$$N(t+1) = N(t) + \Delta N^{+}(t+1) - \Delta N^{-}(t+1)$$

  where $\Delta N^{+}(t+1)$ and $\Delta N^{-}(t+1)$ are the number of new species and extinct species between t and $t+1$

- In continuous time, this equation becomes:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \frac{\mathrm{d}N^{+}(t)}{\mathrm{d}t} - \frac{\mathrm{d}N^{-}(t)}{\mathrm{d}t}$$

- Dividing both sides by $N(t)$ gives:

$$\underbrace{\frac{\mathrm{d}N(t)}{N(t)\,\mathrm{d}t}}_{\delta(t)} = \underbrace{\frac{\mathrm{d}N^{+}(t)}{N(t)\,\mathrm{d}t}}_{\lambda(t)} - \underbrace{\frac{\mathrm{d}N^{-}(t)}{N(t)\,\mathrm{d}t}}_{\mu(t)}$$

Speciation, extinction and the birth-death model

- The growth rate $\delta(t)$ is the difference between the origination (or speciation) rate $\lambda(t)$ and the extinction rate $\mu(t)$:
$$\delta(t) = \lambda(t) - \mu(t)$$
- $\delta(t)$ is called the **net diversification rate**
- The average net diversification rate is equal to:

$$\bar{\delta}(t_1, t_2) = \frac{\delta(t_1, t_2)}{t_2 - t_1} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \delta(t) \, \mathrm{d}t$$

Speciation*, and extinction during the Phanerozoic era

Figure 6: Rates of origination and extinction

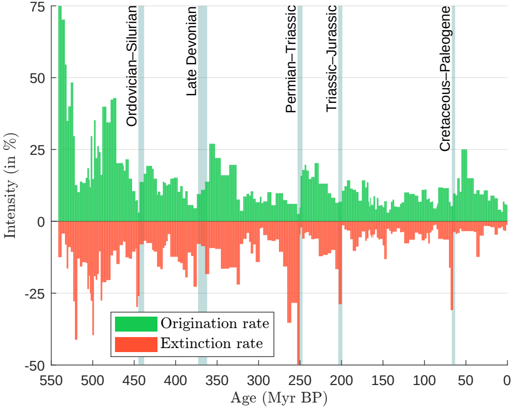
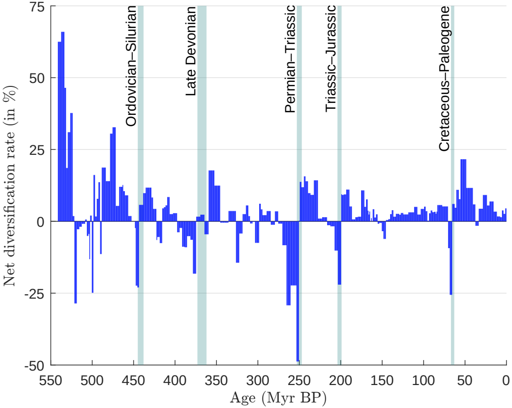


Figure 7: Net diversification rate



Source: Rohde and Muller (2005) & Author's calculations.

Calculation of the extinction rate

- If the extinction rate μ equals 0.1% per millennium, we have:

$$\mu = \frac{0.1\%}{1 \text{ millenium}} = \frac{0.1\%}{1000 \text{ years}} = 0.0001\% \text{ per year}$$

- The lifespan (or average lifetime) of species is the inverse of the extinction rate:

$$\tau = \frac{1}{\mu} = \frac{1000 \text{ years}}{0.1\%} = 10^6 \text{ years}$$

- η measures the number of extinctions (E) per million species per year (MSY) or E/MSY. The relationship between η, μ and τ is then:

$$\eta = 10^6 \mu = \frac{10^6}{\tau}$$

- If there are 1 million species, the number of extinctions per year would be one:

$$\eta = \frac{0.1\%}{10^3 \text{ years}} \times 10^6 = \frac{10^6}{10^6 \text{ years}} = 1\,\text{E/MSY}$$

Calculation of the extinction rate

Example #1

We consider three datasets with different species:

Species	N_0	ΔN^+	ΔN^-	Δt
Birds	5 000	7	5	10 years
Insects	75 000	25	50	3 centuries
Plants	10^6	30×10^3	15×10^3	1 millennium

where N_0 is the number of species at the beginning of the period, and ΔN^+ and ΔN^- are the number of new and dead species during the period Δt

Calculation of the extinction rate

Species	λ	μ (in % per year)	δ	λ	μ (in % per millenium)	δ
Birds	0.01400	0.01000	0.00400	14.00	10.00	4.00
Insects	0.00011	0.00022	−0.00011	0.11	0.22	−0.11
Plants	0.00300	0.00150	0.00150	3.00	1.50	1.50

Species	$\mu(0, \Delta t)$ (in %)	μ (in % per year)	$\mu^\star$	τ (in years)	$\tau^\star$	η (in E/MSY)	$\eta^\star$
Birds	0.10000	0.01000	0.01401	10 000	7 138	100.0	140.1
Insects	0.06667	0.00022	0.00011	450 000	899 850	2.2	1.1
Plants	1.50000	0.00150	0.00305	66 667	32 831	15.0	30.5

*Logarithm calculation: $\mu^\star$, $\tau^\star$ and $\eta^\star$

Background extinction rate

Definition

The background extension rate $\bar{\eta}$ is the normal or typical extension rate that has occurred over the past 500 million years. By normal, we mean the long-term rate at which species would go extinct in the absence of human presence.

$$0.1\,\mathrm{E/MSY} \leq \bar{\eta} \leq 1\,\mathrm{E/MSY}$$

Estimates of the background extension rate $\bar{\eta}$

Taxonomy	τ (in myr)	η (in E/MSY)	Source
All species	1 – 10	0.10 – 1.00	Pimm *et al.* (1995)
All species	1.0	0.10	De Vos *et al.* (2015)
All fossil groups	0.5 – 5	0.20 – 2.00	Simpson (1952)
Marine fossil groups	7.4 – 20	0.05 – 0.13	Raup and Sepkoski (1982)
Marine invertebrates	5 – 10	0.10 – 0.20	Valentine (1970)
Cetacea (genus)	3.61	0.277	Van Valen (1973)
Devonian & Cenozoic bivalves	6.5 – 9.7	0.10 – 0.15	Valentine (1970)
Silurian graptolites	2.0 – 3.0	0.33 – 0.50	Rickards (1977)
Diatoms	8.02	0.125	Van Valen (1973)
Dinoflagellata	13.12	0.076	Van Valen (1973)
Foraminifera (planktonic)	7.21	0.139	Van Valen (1973)
Foraminifera (genus)	24.04	0.042	Van Valen (1973)
Foraminifera (family)	72.13	0.014	Van Valen (1973)
Arthropods	1.07 – 11.11	0.090 – 0.934	De Vos *et al.* (2015)
Chordates	1.71 – 15.63	0.064 – 0.586	De Vos *et al.* (2015)
Mammals	0.56	1.800	Barnosky *et al.* (2011)
Mammals & birds	0.55 – 4.80	0.208 – 1.818	Loehle and Eschenbach (2012)
Mammals	9.80 – 43.48	0.023 – 0.102	De Vos *et al.* (2015)
Mammals	0.50	2.000	Ceballos *et al.* (2015)
Mollusca	0.60 – 7.41	0.135 – 1.672	De Vos *et al.* (2015)
Primates (genus)	3.28	0.305	Van Valen (1973)
Reptilia (family)	24.05	0.042	Van Valen (1973)
Plants	2.84 – 18.87	0.053 – 0.352	De Vos *et al.* (2015)
Plants	7.69 – 20.00	0.050 – 0.130	Gray (2019)

Mass extinction

Definition

- A mass extinction is a widespread and rapid decline in Earth's biodiversity (genetic or species diversity), during which a substantial proportion of the planet's species disappear over a relatively short period of time — typically thousands to millions of years, which is short on the geologic time scale
- The characterization of an extinction event is then determined using calculated extinction rates:

$$[t_1, t_2] \text{ is an extinction period} \Leftrightarrow \mu(t_1, t_2) \geq \mu^\star \text{ and } \bar{\eta}(t_1, t_2) \gg \bar{\eta}$$

- Small extinction events
- Pulse events
- '*Big Five*' extinctions

Mass extinction

1. Ordovician-Silurian mass extinction — LOME (445–443 Myr BP)
   About 27% of all families, 57% of all genera and 85% of all species became extinct
2. Late Devonian mass extinction — LDME (372–359 Myr BP)
   About 19% of all families, 35-50% of all genera and 75% of all species became extinct
3. Permian-Triassic extinction or '*The Great Dying*' — EPME (252–251 Myr BP)
   About 57% of marine families, 84% of marine genera, 81% of all marine species and 90% of terrestrial vertebrate species became extinct
4. Triassic-Jurassic extinction — ETME (200–201 Myr BP)
   About 23% of all families, 48% of all genera (20% of marine families and 55% of marine genera) and 70-75% of all species became extinct
5. **Cretaceous-Paleogene extinction** — ECME (66 Myr BP)
   About 17% of all families, 47% of all genera and 75% of all species became extinct

Mass extinction

Holocene extinction, Anthropocene extinction or sixth mass extinction?

IUCN Red List

IUCN Red List of Threatened Species

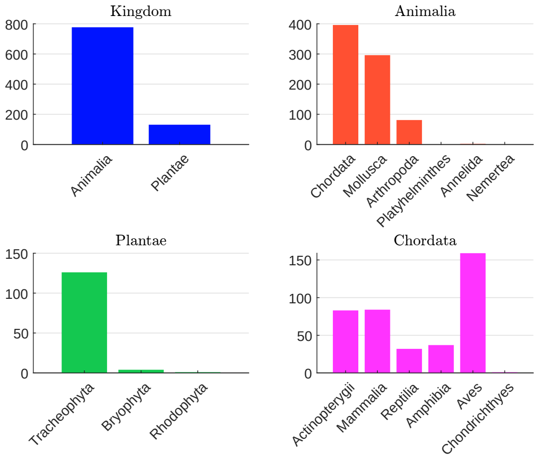
- Not Evaluated (NE) & Data Deficient (DD)
- Least Concern (LC) & Near Threatened (NT)
- Vulnerable (VU), Endangered (EN) & Critically Endangered (CR)
- Extinct in the Wild (EW)
- Extinct (EX)

RED LIST©

https://www.iucnredlist.org/search

IUCN Red List

Figure 8: Number of extinct species since 1500 AD



IUCN Red List

Table 5: Statistics of the IUCN Red List database

Kingdom	Animalia	Chromistra	Fungi	Plantae	Total
Extinct	777			131	**908**
Extinct in the Wild	36			45	**81**
Critically Endangered	4 067	4	45	5 915	**10 031**
Endangered	6 426	1	105	11 477	18 009
Vulnerable	7 165	1	178	9 937	17 281
Conservation Dependent	18			114	132
Near Threatened	5 149		66	4 203	9 418
Least Concern	51 689		240	33 373	85 302
Data Deficient	15 895	12	160	5 811	21 878
Total	91 222	18	794	71 006	**163 040**

Source: IUCN (2024), `www.iucnredlist.org` & Author's calculations.

IUCN Red List

Table 6: Number of species assessed and number of threatened species by major group of organisms

Taxon	Clade	Number of species	Evaluated species #	Evaluated species %	Threatened species #	Threatened species %
Vertebrates	Mammals	6 701	5 983	89.3%	1 338	22.4%
	Birds	11 195	11 195	100.0%	1 354	12.1%
	Reptiles	12 162	10 309	84.8%	1 844	17.9%
	Amphibians	8 744	8 011	91.6%	2 873	**35.9%**
	Fishes	36 863	27 972	75.9%	3 927	14.0%
	Subtotal	75 665	63 470	83.9%	11 336	17.9%
Plants	Mosses	21 925	327	1.5%	181	**55.4%**
	Ferns and Allies	11 800	821	7.0%	321	39.1%
	Gymnosperms	1 113	1 059	95.1%	451	42.6%
	Flowering Plants	369 000	68 704	18.6%	26 367	38.4%
	Green Algae	13 960	17	0.1%	0	0.0%
	Red Algae	7 523	78	1.0%	9	11.5%
	Subtotal	425 321	71 006	16.7%	27 329	38.5%

Source: IUCN (2024), `www.iucnredlist.org` & Author's calculations.

IUCN Red List

Table 7: Number of species assessed and number of threatened species by major group of organisms

Taxon	Clade	Number of species	Evaluated species #	Evaluated species %	Threatened species #	Threatened species %
Invertebrates	Insects	1 053 578	12 718	1.2%	2 415	19.0%
	Molluscs	86 859	9 111	10.5%	2 451	26.9%
	Crustaceans	90 531	3 213	3.5%	747	23.2%
	Corals	5 623	831	14.8%	252	30.3%
	Arachnids	95 894	774	0.8%	272	35.1%
	Velvet Worms	222	11	5.0%	9	**81.8%**
	Horseshoe Crabs	4	4	100.0%	2	50.0%
	Others	157 543	1 090	0.7%	174	16.0%
	Subtotal	1 490 254	27 752	1.9%	6 322	22.8%
Fungi	Mushrooms, etc.	156 313	794	0.5%	328	**41.3%**
	Brown Algae	4 683	18	0.4%	6	33.3%
	Subtotal	160 996	812	0.5%	334	41.1%
	Total	2 152 236	163 040	7.6%	45 321	**27.8%**

Source: IUCN (2024), www.iucnredlist.org & Author's calculations.

IUCN Red List — Threatened categories (CR, EN, and VU)

Figure 9: Number of species

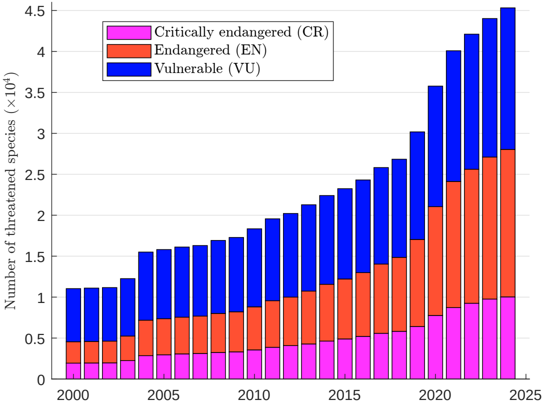
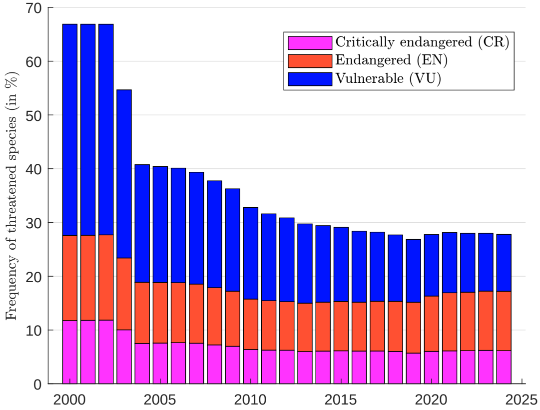


Figure 10: Percentage of species



Source: IUCN (2024), `www.iucnredlist.org` & Author's calculations.

Extinction debt

"The idea that species can initially survive habitat change but later become extinct without any further habitat modification has a long history. It was first conceptualized in island biogeography (MacArthur and Wilson, 1967) and further elaborated by Jared Diamond, who introduced the term relaxation time as the delay of expected extinctions after habitat loss. According to theoretical predictions and supporting empirical data, the relaxation time increases with increasing patch area and with decreasing isolation. A second root stems from metapopulation modeling. Tilman et al. (1994) introduced the term ***extinction debt*** *and considered the order of extinctions in relation to competitive dominance [...] The concept of extinction debt is related to* ***relaxation time*** *but specifies the number or proportion of extant species predicted to become extinct as the species community reaches a new equilibrium after an environmental perturbation." (Kuussaari et al., 2009).*

Extinction debt

- Halley *et al.* (2016) is assumed that the remaining habitat area is reduced from A_0 to A
- The species richness $S(t)$ is given by:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \lambda(t) - \mu(t) S(t)$$

- The remaining habitat contains a (constant) number $N(t)$ of individuals, which is proportional to the area A and the density ρ of individuals per unit area: $N(t) = \rho A$
- $n(t) = N(t)/S(t)$ is the average population size per species
- At time $t = 0$, we have $n = N_0/S_0 = \rho A/S_0$ where $S(0) = S_0$ is the initial species richness
- The extinction rate is described by:

$$\mu(t) = kn(t)^{-\alpha} = k\left(\frac{S(t)}{N(t)}\right)^{\alpha}$$

Extinction debt

- We deduce that:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \lambda(t) - k\left(\frac{S(t)}{\rho A}\right)^{\alpha} S(t) = \lambda(t) - \frac{k}{n^{\alpha} S_0^{\alpha}} S(t)^{\alpha+1}$$

- If we assume that $\lambda(t) = 0$, the solution is:

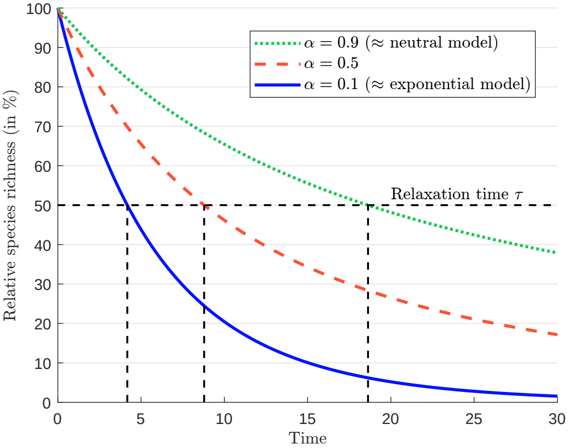
$$S(t) = S_0 \left(1 + \frac{k\alpha}{n^{\alpha}} t\right)^{-1/\alpha}$$

- Extinction debt is quantified by the relaxation time τ, which represents the time required for species richness to decrease by half:

$$S_0 \left(1 + \frac{k\alpha}{n^{\alpha}} \tau\right)^{-1/\alpha} = \frac{S_0}{2} \Leftrightarrow \tau = (2^{\alpha} - 1) \frac{n^{\alpha}}{k\alpha} \propto n^{\alpha}$$

Extinction debt

Figure 11: Relative species richness and relaxation time



Extinction debt

- We assume a constant origination rate: $\lambda(t) = \lambda$
- The equilibrium state $\bar{S}$ is reached when the rate of change in species richness becomes zero:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = 0 \Leftrightarrow \lambda - \frac{k}{n^{\alpha} S_0^{\alpha}} \bar{S}^{\alpha+1} = 0 \Leftrightarrow \bar{S} = \left(\frac{\lambda n^{\alpha} S_0^{\alpha}}{k}\right)^{1/(\alpha+1)}$$

  This is the value of the steady state after the reduction of the area to A

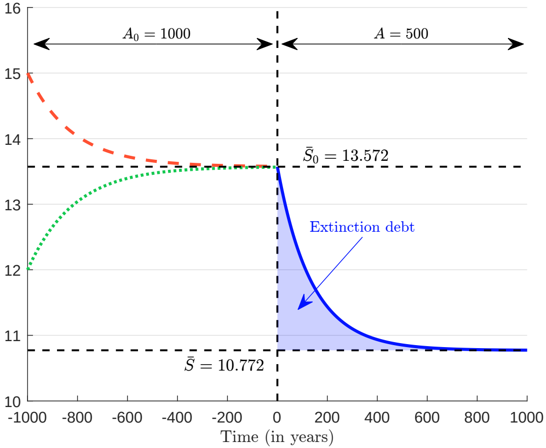
- Before reducing the area, the steady state $\bar{S}_0$ satisfies the following equation:

$$\lambda - \frac{k}{n_0^{\alpha} \bar{S}_0^{\alpha}} \bar{S}_0^{\alpha+1} = 0 \Leftrightarrow \bar{S}_0 = \left(\frac{\lambda(\rho A_0)^{\alpha}}{k}\right)^{1/(\alpha+1)}$$

  because the original habitat area was A_0 and $n_0 = \rho A_0 / \bar{S}_0$

Extinction debt

Figure 12: Extinction debt and steady state



Biodiversity hotspot

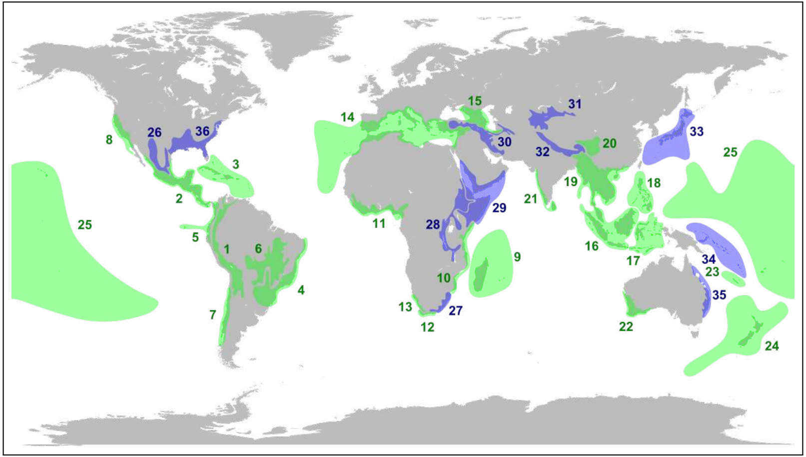
Definition

A biodiversity hotspot is a region of the world that is both rich in plant and animal species and highly threatened by human activities. Specifically, it is characterized by the following two criteria:

- **Exceptional levels of endemism**: The region must have at least 1 500 species of vascular plants that are endemic, meaning that they are found nowhere else on Earth;
- **High levels of habitat loss**: The region must have lost at least 70% of its original natural vegetation, typically due to human activities such as deforestation, agriculture, or urbanization.

Biodiversity hotspot

Figure 13: The 36 biodiversity hotspots



Biodiversity hotspot

The 36 regions are (1) Tropical Andes, (2) Mesoamerica, (3) Caribbean Islands, (4) Atlantic Forest, (5) Tumbes-Chocó-Magdalena, (6) Cerrado, (7) Chilean Winter Rainfall-Valdivian Forests, (8) California Floristic Province, (9) Madagascar and the Indian Ocean Islands, (10) Coastal Forests of Eastern Africa, (11) Guinean Forests of West Africa, (12) Cape Floristic Region, (13) Succulent Karoo, (14) Mediterranean Basin, (15) Caucasus, (16) Sundaland, Indonesia and Nicobar islands of India, (17) Wallacea of Indonesia, (18) Philippines, (19) Indo-Burma, Bangladesh, India and Myanmar, (20) Mountains of Southwest China, (21) Western Ghats and Sri Lanka, (22) Southwest Australia, (23) New Caledonia, (24) New Zealand, (25) Polynesia-Micronesia, (26) Madrean pine-oak woodlands, (27) Maputaland-Pondoland-Albany, (28) Eastern Afromontane, (29) Horn of Africa, (30) Irano-Anatolian, (31) Mountains of Central Asia, (32) Eastern Himalaya, (33) Japan, (34) East Melanesian Islands, (35) Eastern Australian temperate forests, and (36) North American Coastal Plain

A Course on Biodiversity
Lecture 2. Ecosystem functions and services

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[3]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Ecosystem functions and services

*"In our increasingly technological society, people give little thought to how dependent they are on the proper functioning of ecosystems and the crucial services for humanity that flow from them. Ecosystem services are "**the conditions and processes through which natural ecosystems, and the species that make them up, sustain and fulfill human life**"; in other words, "**the set of ecosystem functions that are useful to humans**". Although people have been long aware that natural ecosystems help support human societies, the explicit recognition of "ecosystem services" is relatively recent." (Sekercioglu, 2010).*

Ecosystem functions and services

Sekercioglu (2010) classified ecosystem services into 6 categories:

1. Climate and biogeochemical cycles (climate stability, air purification, UV protection)
2. Regulation of the hydrological cycle (drought mitigation, flood mitigation, water purification)
3. Soils and erosion (detoxification and decomposition of wastes, soil formation and soil fertility)
4. Biodiversity and ecosystem function (ecosystem goods)
5. Mobile linkages (pollination, seed dispersal)
6. Nature's remedies for emerging diseases (medicine, pest control)

$\Rightarrow$ Millennium Ecosystem Assessment, IPBES, ENCORE, TNFD, etc.

Classification of ecosystem functions and services

1. Aesthetic and cultural services

- Aesthetic, symbolic and spiritual values (nature inspires creativity, provides spiritual connections, and contributes to cultural identity)
- Cultural and spiritual significance (many ecosystems have deep cultural, historical or spiritual significance for local communities and indigenous peoples)
- Educational, scientific and research services (biodiversity provides opportunities for scientific study and learning)
- Recreational opportunities and tourism (forests, parks and other natural areas provide opportunities for recreation and tourism)
- Visual amenity services (non-material benefits that contribute to well-being, emotional satisfaction, and cultural enrichment)

Classification of ecosystem functions and services

2. Provisioning services

- Energy (natural processes by which ecosystems produce energy such as biomass, solar energy capture, and fossil fuels)
- Food and feed (agriculture, biomass supply, fisheries, plants, animals, seafood, and livestock)
- Genetic resources (biodiversity provides genetic material essential for breeding crops and livestock, and developing new technologies)
- Medicinal and biochemical resources (many medicines and pharmaceutical products are derived from natural compounds found in biodiversity)
- Raw materials (timber, fuel wood, minerals, fibers, and other natural resources)
- Water supply (clean water for drinking, irrigation, and industrial use)

Classification of ecosystem functions and services

3. Regulating services

- Air quality regulation (ecosystems filter pollutants from the air, improving air quality)
- Climate regulation (forests absorb carbon dioxide and help regulate global temperatures)
- Waste detoxification and decomposition (natural decomposition of organic matter, natural ability to detoxify chemicals and pollutants)
- Erosion control (vegetation helps stabilize soils, reducing erosion and preventing landslides)
- Hazard and extreme event regulation (flood, storm, rainfall)
- Pest and disease control (natural predators and parasites help regulate populations of harmful organisms)
- Pollination and seed dispersal (bees, birds, insects, and other pollinators allow many plants to reproduce)
- Water purification (freshwater, wetlands, & forests filter pollutants)

Classification of ecosystem functions and services

4. Supporting services

- Habitat creation and maintenance (habitats are the natural environments in which organisms live, grow, and reproduce; they form the basis of ecosystems by providing the resources necessary for species to thrive)
- Nutrient cycling (the movement of nutrients through ecosystems, essential for plant growth and productivity)
- Photosynthesis (plants convert solar energy into chemical energy, producing oxygen and forming the base of the food chain)
- Primary production (the production of organic material by plants and algae forms the foundation of ecosystems)
- Soil formation and fertility (the breakdown of rocks and organic matter to create soil)
- Water cycle regulation (ecosystems play an important role in regulating the water cycle, from evaporation to precipitation)

Natural capital

- Ecosystem services are derived from natural capital, which can be defined as the world's stock of natural assets, including geology, soil, air, water, and all living things
- The concept of natural capital in biodiversity is generally attributed to David Pearce

Natural capital

Table 8: Total value of annual ecosystem services in 1997 (1995 price levels)

Biome	Area (in ha $\times 10^6$)	Value (in \$/ha/yr)	Total value (in \$ bn/yr)	Breakdown (in %)
Marine	36 302	577	20 949	63.0
Open ocean	33 200	252	8 381	25.2
Coastal	3 102	4 052	12 568	37.8
Terrestrial	15 323	804	12 319	37.0
Forest	4 855	969	4 706	14.1
Grassland & meadow	3 898	232	906	2.7
Wetland	330	14 785	4 879	14.7
Lake & river	200	8 498	1 700	5.1
Desert	1 925			
Tundra	743			
Ice & rock	1 640			
Cropland	1 400	92	128	0.4
Urban	332			
Total	51 625		**33 268**	100.0

Source: Costanza *et al.* (1997) & Author's calculations.

Natural capital

According to the World Economic Forum (2020) **$44 trillion of economic value creation** — more than half of global GDP — **is moderately or highly dependent on nature**

The three largest industries most dependent on nature are:

1. Construction ($4 tn)
2. Agriculture ($2.5 tn)
3. Food and beverages ($1.4 tn)

Pollination service

There are two main types of pollination:

1. Self-pollination
2. Cross-pollination

We also distinguish three methods of cross-pollination:

1. **Abiotic pollination** involves natural transport phenomena such as wind, water, and rain
2. **Biotic pollination** requires living pollinators to move pollen from one flower to another
3. **Hand pollination** (also known as mechanical or human pollination) is a technique in which humans manually transfer pollen from the male to the female plant

The story of the original vanilla bean

The Chinantla Forest is considered the birthplace of vanilla, and the vanilla plant, or vine, is native to Mexico. There, the vines grew and flourished without the help of humans. Wild vanilla is naturally pollinated by melipona bees and small hummingbirds found only in Mexico. Before 1850, all vanilla beans came from the forests of Mexico, and France was the number one importer of the '*black flower*'. The Aztecs, and the Mayans before them, believed that the scent of vanilla could help them communicate with the gods and had long mastered the fermentation techniques needed to cure the beans. They cultivated '*tlilxotchitl*' or black flowers so that the flavors could be combined with cocoa and coffee. In 1521, Cortés was the first European to bring the dark pods or beans back to Charles Quint. Vanilla beans first arrived in France in 1664. Later, Louis XIV fell in love with the taste of vanilla and wanted vanilla beans to be grown on the island of Réunion, then known as Bourbon Island. But until the mid-19th century, vanilla beans were still only made in Mexico. Although the technique of curing the beans was known, pollination of the flower was not. In 1836 and 1841, Charles Morren, a Belgian botanist, and Edmond Albius, a slave on Réunion Island discovered how to bypass bee pollination by manually pollinating vanilla flowers. Soon after, vanilla plants were exported by the French to plantations in Tahiti, Madagascar, Mauritius, Réunion Island, and the Comoros.

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Pollination service

- Klein *et al.* (2007) identified 124 major crops, representing 99% of total world food production

Pollination	Dependent	Independent	Not evaluated	Total
Number of crops	87	28	9	124
(in %)	**70%**	**23%**	7%	100%
Production (2024)	**35%**	**60%**	5%	100%

- Aizen *et al.* estimated that the direct reduction in total agricultural production in the absence of animal pollination would be 5% for developed regions and 8% for developing regions

Why?

Pollination service

Table 9: How dependent are foods on pollinator insects?

No dependency: yields are not affected by pollinators
Cereals: barley, maize, millet, oats, rice, rye, sorghum, wheat
Roots and tubers: carrots, cassava, potatoes, sweet potatoes
Legumes including chickpeas, lentils, peas
Fruit and veg including bananas, grapes, lettuce, pepper, pineapples
Sugar crops: sugar beet, sugar cane
Also includes areca nuts, asparagus, broccoli, cabbages, castor oil seed, cauliflower, chicory roots, dates, garlic, hazelnuts, jojoba seeds, leeks, olives, onions, pistachios, quinoa, spinach, taro, triticale, walnuts, yams

Source: Klein *et al* (2007) & Aizen *et al.* (2019).

Pollination service

Table 10: How dependent are foods on pollinator insects?

Little dependency: yield reduction of 0% to 10% without pollinators	
	Fruits and veg including lemons, limes, oranges, papayas, tomatoes
	Oilcrops including linseed, palm oil, poppy seed, safflower seed
	Legumes including beans (dry & green), cow peas, pigeon peas
	Groundnuts
	Also includes bambara beans, chillies, clementines, grapefruit, mandarins, persimmons, string beans, tangerines

Source: Klein *et al* (2007) & Aizen *et al.* (2019).

Pollination service

Table 11: How dependent are foods on pollinator insects?

Modest dependency: yield reduction of 10% to 40% without pollinators
Oilcrops including mustard seed, rapeseed, sesame, sunflower seed
Soybeans
Fruits including currants, egglant, figs, gooseberries, strawberries
Coconuts and okra
Coffee beans
Also includes broad beans, chestnut, karite nuts, seed cotton

Source: Klein *et al* (2007) & Aizen *et al.* (2019).

Pollination service

Table 12: How dependent are foods on pollinator insects?

High dependency: Yield reduction of 40% to 90% without pollinators	
	Fruits including apples, apricots, blueberries, cherries, cranberries, guavas, mangoes, nectarines, peaches, plums, pears, raspberries
	Nuts including almonds, cashew nuts, kola nuts
	Avocados
	Also includes anise, badian, buckwheat, coriander, cucumber, fennel, nutmeg

Source: Klein *et al* (2007) & Aizen *et al.* (2019).

Pollination service

Table 13: How dependent are foods on pollinator insects?

Essential: yield reduction greater than 90% without pollinators	
	Fruits including kiwi, melons, pumpkins, watermelons
	Cocoa beans
	Brazil nuts
	Also includes quinces, vanilla

Source: Klein *et al* (2007) & Aizen *et al.* (2019).

Pollination service

- Let $V_{j,t}^{(\mathrm{nutritional})}$ be the total amount of nutrient j in year t

$$V_{j,t}^{(\mathrm{nutritional})} = \sum_{i=1}^{n} V_{i,j}^{(\mathrm{nutritional})} P_{i,t} \left(1 - R_i\right)$$

  where $V_{i,j}^{(\mathrm{nutritional})}$ is the amount of nutrient j in a metric tonne of crop i, $P_{i,t}$ is the volume production in tonnes of crop i in year t, and R_i is the proportion of crop i that is not consumed by humans due to inedible parts, such as pits, stems, or shells
- $V_{j,t}^{(1)} = \sum_{i=1}^{n} \mathbb{1}\left\{\delta_i = 0\right\} \cdot V_{i,j}^{(\mathrm{nutritional})} P_{i,t} \left(1 - R_i\right)$ is **the nutritional value of pollinator-independent crops**
- $V_{j,t}^{(2)} = \sum_{i=1}^{n} \mathbb{1}\left\{\delta_i > 0\right\} \cdot \left(1 - \delta_i\right) V_{i,j}^{(\mathrm{nutritional})} P_{i,t} \left(1 - R_i\right)$ is **the nutritional value of pollinator-dependent crops due to abiotic and self-pollination**
- $V_{j,t}^{(3)} = \sum_{i=1}^{n} \mathbb{1}\left\{\delta_i > 0\right\} \cdot \delta_i V_{i,j}^{(\mathrm{nutritional})} P_{i,t} \left(1 - R_i\right)$ is **the nutritional value of pollinator-dependent crops attributed to animal pollination alone**

Pollination service

Table 14: Proportion in % of nutrients derived from pollinator-independent and pollinator-dependent crops

Nutrient		$V_{j,t}^{(1)}$	$V_{j,t}^{(2)}$	$V_{j,t}^{(3)}$
	Energy	78.83	18.59	2.58
Macro-nutrients	Protein	83.43	13.57	3.00
	Fat	26.02	66.98	7.00

Source: Eilers *et al.* (2011).

Pollination service

Table 15: Proportion in % of nutrients derived from pollinator-independent and pollinator-dependent crops

Nutrient		$V_{j,t}^{(1)}$	$V_{j,t}^{(2)}$	$V_{j,t}^{(3)}$
Vitamins	A	28.71	30.26	**41.03**
	β-carotene	27.44	34.19	**38.37**
	α-carotene	32.25	29.83	**37.92**
	E (α-tocopherol)	63.73	28.94	7.33
	E (β-tocopherol)	0.63	72.50	**26.87**
	E (γ-tocopherol)	32.92	52.66	14.42
	K	71.55	19.28	9.17
	C	6.99	73.37	19.64
	B1 (Thiamin)	95.29	4.00	0.71
	B2 (Riboflavin)	97.66	1.92	0.42
	B3 (Niacin)	89.46	8.93	1.61

Source: Eilers *et al.* (2011).

Pollination service

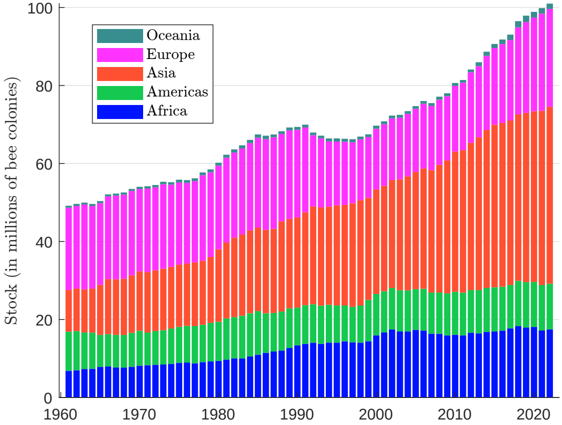
Table 16: Proportion in % of nutrients derived from pollinator-independent and pollinator-dependent crops

Nutrient		$V_{j,t}^{(1)}$	$V_{j,t}^{(2)}$	$V_{j,t}^{(3)}$
	Calcium	42.40	48.49	9.11
	Iron	70.66	23.14	6.20
	Magnesium	88.50	9.06	2.44
	Phosphorus	89.06	8.72	2.22
	Potassium	72.74	20.93	6.33
Minerals	Sodium	87.18	8.63	4.19
	Zinc	91.80	6.54	1.66
	Copper	80.92	15.21	3.87
	Mangan	93.87	4.94	1.19
	Selenium	97.46	1.97	0.57
	Fluoride	45.57	34.60	**19.83**

Source: Eilers *et al.* (2011).

Pollination service

Figure 14: Trend in the number of bee colonies (1961–2022)



Source: www.fao.org/faostat/en/#data/QCL & Author's calculations.

Pollination service

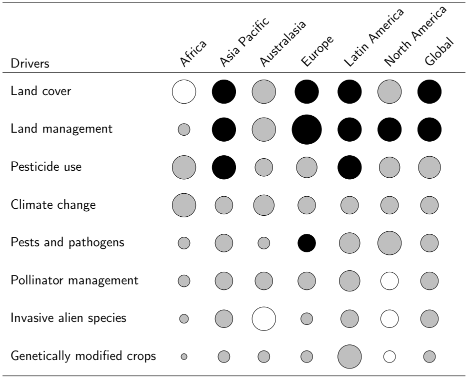
Table 17: Regional distribution of managed honey bee colonies (in millions)

Region	Stock (in million colonies)				Growth (in %)	
	1961	1980	2000	2022	1961–2022	2000–2022
Europe	21.10	21.42	15.55	25.12	19.1	61.6
Western Europe	3.76	3.35	2.45	3.55	−5.5	45.3
Northern Europe	0.44	0.40	0.27	0.64	45.6	138.3
Eastern Europe	14.02	13.71	7.36	10.66	−24.0	44.8
Southern Europe	2.87	3.95	5.47	10.27	257.3	87.6
Americas	10.02	10.03	10.62	11.71	16.9	10.2
Northern America	5.85	4.75	3.22	3.40	−41.9	5.5
Central America	2.26	2.80	2.19	2.68	18.5	22.3
Caribbean	0.23	0.32	0.28	0.40	72.5	45.8
South America	1.67	2.16	4.94	5.23	212.4	6.0
Africa	6.85	9.37	15.92	17.46	155.1	9.7
Asia	10.70	18.61	26.82	45.34	323.6	69.1
Oceania	0.51	0.76	0.80	1.36	168.7	70.9
World	49.17	60.20	69.71	101.00	105.4	44.9

Source: `www.fao.org/faostat/en/#data/QCL` & Author's calculations.

Pollination service

Table 18: Assessment of the importance of the top eight drivers of pollinator decline (Dicks *et al.*, 2021)



Food and feed service

- **Production** perspective
- **Consumption** perspective

Food and feed service

Table 19: World production of primary crops

Crop	Production (in billion tonnes)					1961 breakdown		2022 breakdown		Yield (in t/ha)	
	1961	1980	2000	2020	2022	(in %)	(% cum.)	(in %)	(% cum.)	1961	2022
Primary crops	2.54	4.02	6.14	9.38	9.61	100.00		100.00		**2.61**	**6.49**
Cereals	0.88	1.55	2.06	3.00	3.06	34.58		31.84		1.35	4.18
Fruit	0.22	0.40	0.68	1.07	1.10	8.87		11.44		7.62	14.09
Oil crops	0.07	0.15	0.32	0.66	0.68	2.90		7.06		0.33	1.01
Pulses, Roots and Tubers	0.50	0.56	0.75	0.96	1.00	19.56		10.44		4.45	6.02
Sugar crops	0.61	1.00	1.50	2.13	2.18	24.02		22.73		38.16	71.61
Vegetables	0.20	0.29	0.69	1.15	1.17	7.79		12.21		9.34	20.13
Other	0.06	0.06	0.15	0.41	0.41	2.28		4.29			

Source: www.fao.org/faostat/en/#data/QCL & Author's calculations.

Food and feed service

Table 20: World production of primary crops

Crop	Production (in billion tonnes)					1961 breakdown		2022 breakdown		Yield (in t/ha)	
	1961	1980	2000	2020	2022	(in %)	(% cum.)	(in %)	(% cum.)	1961	2022
Sugar cane	0.45	0.73	1.25	1.88	1.92	17.66	17.66	20.00	20.00	50.27	73.67
Maize (corn)	0.21	0.40	0.59	1.16	1.16	**8.09**	25.75	**12.11**	32.11	1.94	5.72
Wheat	0.22	0.44	0.59	0.76	0.81	8.77	34.52	8.41	40.52	1.09	3.69
Rice	0.22	0.40	0.60	0.77	0.78	8.50	43.02	8.08	48.60	1.87	4.70
Oil palm fruit	0.01	0.03	0.12	0.42	0.42	0.54	43.56	4.42	53.02	3.77	14.15
Potatoes	0.27	0.24	0.32	0.37	0.37	**10.67**	54.23	**3.90**	56.92	12.22	21.07
Soya beans	0.03	0.08	0.16	0.36	0.35	1.06	55.29	3.63	60.55	1.13	2.61
Cassava, fresh	0.07	0.12	0.18	0.31	0.33	2.81	58.10	3.44	63.99	7.40	10.31
Other vegetables	0.06	0.09	0.21	0.29	0.30	2.46	60.55	3.10	67.09	8.42	14.53
Sugar beet	0.16	0.27	0.25	0.25	0.26	6.33	66.88	2.72	69.81	23.17	60.77
Tomatoes	0.03	0.05	0.11	0.19	0.19	1.09	67.97	1.94	71.74	16.43	37.84
Barley	0.07	0.16	0.13	0.16	0.15	2.86	70.83	1.61	73.36	1.33	3.29
Bananas	0.02	0.04	0.07	0.13	0.14	0.88	71.71	1.41	74.76	10.65	22.75
Onions and shallots	0.01	0.02	0.05	0.11	0.11	0.55	72.26	1.15	75.91	11.68	18.54
Watermelons	0.02	0.03	0.08	0.10	0.10	0.70	72.96	1.04	76.95	9.13	34.27
Apples	0.02	0.03	0.06	0.09	0.10	0.67	73.64	1.00	77.95	9.91	19.86
Cucumbers and gherkins	0.01	0.01	0.04	0.09	0.09	0.38	74.01	0.99	78.94	9.43	43.56
Yams	0.01	0.01	0.04	0.08	0.09	0.33	74.34	0.92	79.85	7.23	8.49
Rape or colza seed	0.00	0.01	0.04	0.07	0.09	0.14	74.48	0.91	80.76	0.57	2.18
Sweet potatoes	0.10	0.14	0.14	0.09	0.09	3.87	78.36	0.90	81.66	7.35	11.92

Source: `www.fao.org/faostat/en/#data/QCL` & Author's calculations.

Food and feed service

Figure 15: Lorenz curve of world crop production

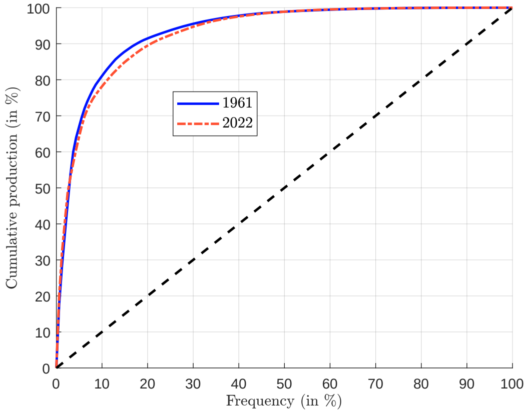
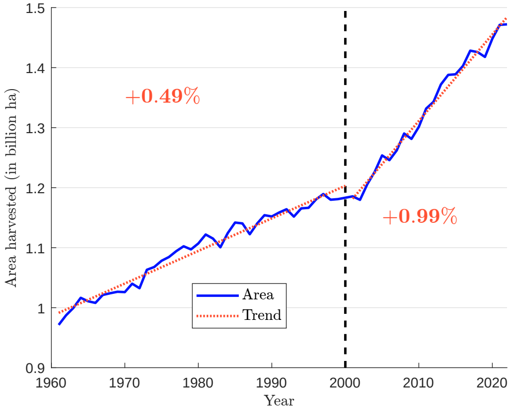


Figure 16: Harvested area for crop production



Source: www.fao.org/faostat/en/#data/QCL & Author's calculations.

Food and feed service

Table 21: World production of primary livestock (in million tonnes)

Livestock	1961	1970	1980	1990	2000	2010	2020	2021	2022	Growth
Eggs	15	20	27	37	55	69	93	93	93	517%
Milk	344	392	466	542	579	725	921	941	930	170%
Meat	71	101	137	180	232	294	339	355	361	405%
Beef & Buffalo	29	40	47	55	58	67	74	75	76	165%
Pork	25	36	53	70	89	108	108	121	123	395%
Poultry	9	15	26	41	69	99	135	136	139	**1 456%**
Sheep & Goat	6	7	7	10	11	14	16	16	17	176%
Other	3	3	4	4	4	6	6	6	6	106%

Source: `www.fao.org/faostat/en/#data/QCL` & Author's calculations.

Food and feed service

Table 22: Agricultural use of inputs (fertilizers and pesticides)

Input	1961	1980	2000	2020	2022	1961	1980	2000	2020	2022
	Agricultural use (in million tonnes)					Use par area of cropland (in kg/ha)				
Fertilizer	31.0	116.6	135.2	201.7	185.4	20.8	76.8	85.9	123.5	113.1
Nitrogen	11.5	60.6	81.0	114.7	108.1	7.6	39.6	51.3	69.6	65.4
Phosphate	10.9	31.8	32.5	47.8	41.9	7.5	21.4	21.0	29.8	26.0
Potash	8.6	24.2	21.7	39.3	35.5	5.7	15.8	13.7	24.1	21.7
Pesticide			2.2	3.4	3.7			1.5	2.2	2.4

Source: `www.fao.org/faostat/en/#data/RFN`, `www.fao.org/faostat/en/#data/RP` & Author's calculations.

Food and feed service

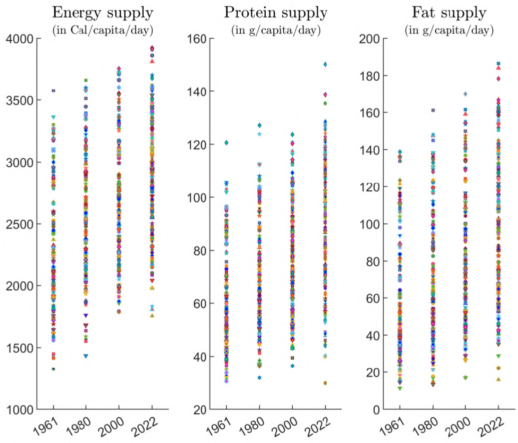
Table 23: Food supply per capital per day (energy, protein & fat)

Region	Energy (in Cal/capita/day)			Protein (in g/capita/day)			Fat (in g/capita/day)		
	1961	1990	2022	1961	1990	2022	1961	1990	2022
Africa	1 993	2 291	2 567	53	59	66	40	47	56
Eastern Africa	1 989	1 925	**2 263**	56	50	**59**	29	32	**47**
Southern Africa	2 603	2 755	2 713	70	73	79	58	64	91
Americas	2 559	2 953	3 392	77	82	104	78	97	135
Northern America	2 873	3 447	**3 881**	95	107	122	110	138	**177**
Caribbean	1 992	2 390	2 828	47	57	75	42	65	78
Asia	1 805	2 414	2 944	47	61	93	25	49	78
South-eastern Asia	1 836	2 178	2 880	40	48	80	27	41	69
Western Asia	2 501	3 273	3 128	76	93	93	57	84	101
Europe	3 041	3 367	3 471	90	104	112	89	125	140
Eastern Europe	3 100	3 360	3 375	95	105	109	73	108	121
Northern Europe	3 176	3 214	3 402	91	96	113	131	134	141
Oceania	3 021	3 139	3 101	100	105	101	108	126	128
Australasia	3 060	3 188	3 417	103	109	**115**	111	129	152
Melanesia	2 534	2 547	2 314	54	65	66	60	86	66
World	2 196	2 621	2 985	61	70	92	48	67	87

Source: `www.fao.org/faostat/en/#data/FBS` & Author's calculations.

Food and feed service

Figure 17: Country dispersion of food supply



Food and feed service

Table 24: Split of food supply between vegetal and animal products

Food Origin	1961 Energy	1961 Protein	1961 Fat	2022 Energy	2022 Protein	2022 Fat
Vegetal	1 858	41.80	22.80	2 460	53.45	51.33
Animal	338	19.66	24.72	525	38.08	35.98
Total	2 196	61.46	47.52	2 945	91.52	87.31

Source: `www.fao.org/faostat/en/#data/FBS`, `www.fao.org/faostat/en/#data/FBSH` & Author's calculations.

Food and feed service

- Irish potato famine (1845–1852)
- Coffee leaf rust outbreak in Sri Lanka (1860–1890)
- Panama disease in bananas (1950s)
- Southern corn leaf blight in the United States (1970)
- Wheat stem rust Ug99 in Africa (1998–present)
- Citrus greening disease in Florida (2005–present)

Food and feed service

Table 25: Share of world crop production exported (in %)

Crop	1961	2021	Crop	1961	2021
Apples	9.4	8.8	**Olive oil**	14.9	66.2
Apricots	5.0	8.6	Onions and shallots	2.0	9.1
Avocados	0.2	36.3	Oranges	16.3	10.0
Bananas	16.6	19.3	Peaches and nectarines	6.0	7.0
Barley	9.9	30.0	Peas	4.0	48.7
Blueberries	36.9	39.7	Persimmons	0.4	14.0
Cauliflowers	6.4	6.1	Pineapples	2.8	12.7
Cherries	2.9	35.2	Pomelos	10.3	11.0
Coconut oil	21.1	80.6	Potatoes	1.0	3.8
Cranberries	0.0	50.9	**Quinoa**	0.0	68.8
Cucumbers	1.5	3.5	Sesame seeds	11.0	32.2
Dates	14.0	19.1	**Soybeans**	15.5	43.2
Eggplants	0.2	1.1	Spinach	0.3	1.1
Kiwi fruit	0.0	36.4	Strawberries	5.4	11.1
Lentils	6.5	67.5	Tomatoes	3.9	4.4
Maize	6.8	16.2	**Vanilla**	73.8	91.6
Mustard seeds	19.3	53.8	Watermelons	0.9	4.7
Natural honey	11.0	42.3	Wheat	17.8	25.9

Source: `www.fao.org/faostat/en/#data/TCL` & Author's calculations.

Food and feed service

Table 26: Share of world crop production exported (in %)

Commodity	1990	2023	Commodity	1990	2023
Beef meat	9.2	18.2	Pork meat	1.9	8.7
Butter	10.0	7.8	Poultry meat	8.5	11.2
Cheese	4.0	13.6	Pulses	10.5	19.8
Cotton	24.2	36.5	Rice	3.4	10.1
Edible fish meals	44.7	67.6	Roots and tubers	7.3	7.8
Eggs	2.6	1.6	Sheep meat	12.2	9.0
Fish	15.4	23.0	Skim milk powder	26.4	57.9
Fresh dairy products	0.0	0.1	Soybeans	26.6	45.1
Maize	12.8	15.3	Sugar	9.8	36.8
Oilseed meals	24.2	24.5	Vegetable oils	24.3	37.2
Other coarse grains	7.7	14.0	Wheat	19.4	24.1
Other oilseeds	15.2	13.6	Whole milk powder	45.9	49.9

Source: `https://data-explorer.oecd.org` & Author's calculations.

The case of maize

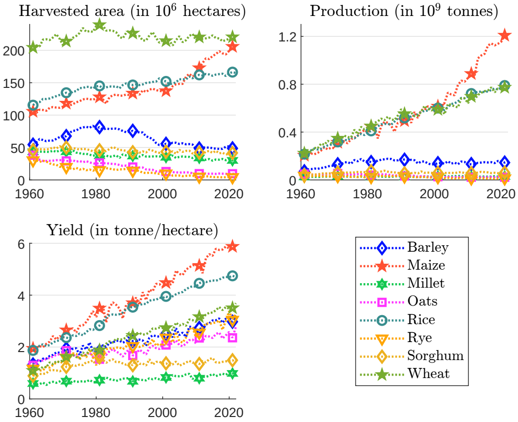
Table 27: Breakdown of maize use

Year	Animal feed	Human food	Losses	Seed	Processing	Other uses
2010	**55.3%**	13.6%	3.5%	0.7%	5.7%	21.2%
2022	**60.4%**	11.4%	5.2%	0.7%	5.8%	16.5%

Source: `www.fao.org/faostat/en/#data/QCL` & Author's calculations.

The case of maize

Figure 18: Area harvested, production and yield of cereal crops



Source: www.fao.org/faostat/en/#data/QCL & Author's calculations.

Ecological footprint

Figure 19: Global ecological footprint and biocapacity (in global hectares per capita)

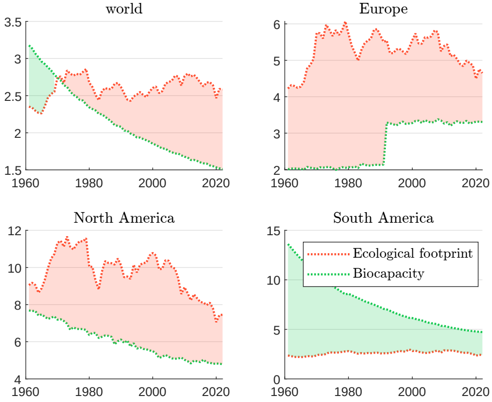
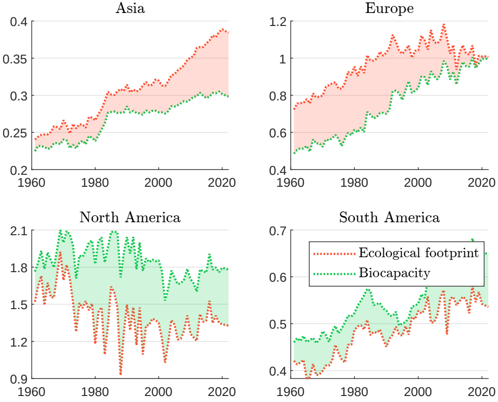


Figure 20: Cropland ecological footprint and biocapacity (in global hectares per capita)



Source: `https://data.footprintnetwork.org` & Author's calculations.

Food security

- Let X and R be the random variables for energy intake and energy requirement, respectively, with a joint probability distribution $\mathbf{F}(x, r)$
- The prevalence of undernourishment is equal to:

$$\mathrm{PoU} = \Pr\{X < R\} = \iint \mathbb{1}\{x < r\} \cdot \mathrm{d}\mathbf{F}(x, r) = \iint_{x<r} f(x, r)\,\mathrm{d}x\,\mathrm{d}r$$

  where $f(x, r)$ is the bivariate density function of (X, R)
- The non-parametric estimator of PoU is the empirical frequency:

$$\mathrm{PoU} = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\{x_i < r_i\}$$

  where n denotes the population size and (x_i, r_i) are the observed intake and requirement values for individual i
- Another approach assumes a parametric density function $f(x, r; \theta)$, estimates the vector of parameters θ, and calculates $\mathrm{PoU} = \iint_{x<r} f\left(x, r; \hat{\theta}\right)\,\mathrm{d}x\,\mathrm{d}r$

Food security

- The FAO approximates the prevalence of undernourishment as follows:

$$\mathrm{PoU} = \Pr\left\{X < r_L\right\} = \int_{x<r_L} f_x\left(x\right)\,\mathrm{d}x = \mathbf{F}_x\left(r_L\right)$$

  where:

  - $\mathbf{F}_x\left(x\right)$ is the cumulative distribution function of energy intake, often called dietary energy consumption (DEC)
  - r_L is a cut-off point representing the minimum requirement, also known as the minimum dietary energy requirement (MDER)

- Assuming $X \sim \mathcal{LN}\left(\mu_x, \sigma_x^2\right)$, we get:

$$\left\{\begin{array}{l} \mu_x = \ln\mu\left(X\right) - \dfrac{1}{2}\ln\left(\mathrm{CV}^2\left(X\right)+1\right) \\ \sigma_x = \sqrt{\ln\left(\mathrm{CV}^2\left(X\right)+1\right)} \end{array}\right.$$

Dietary energy consumption and prevalence of undernourishment

Figure 21: DEC (India, 2022)

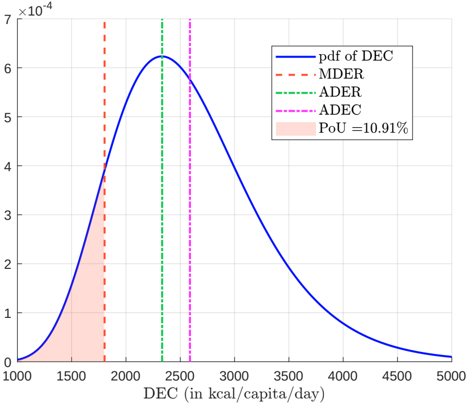
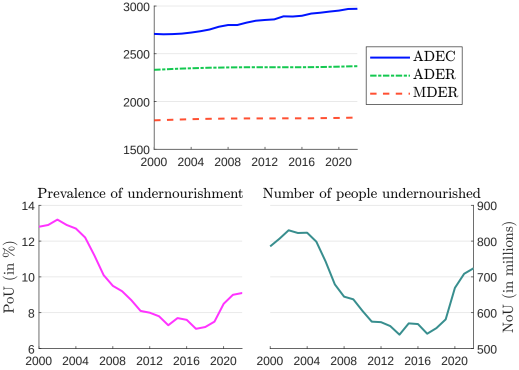


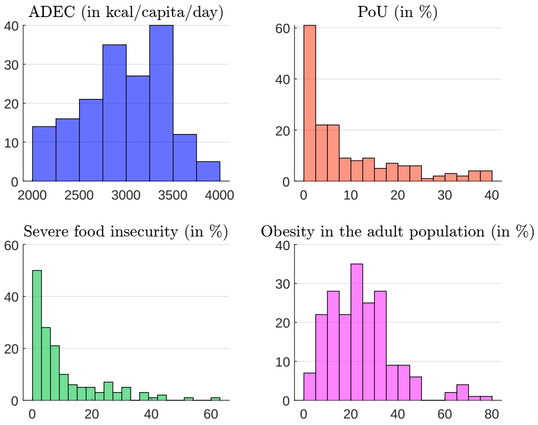
Figure 22: NoU (World)



Source: `www.fao.org/faostat/en/#data/FS` & Author's calculations.

Food security

Figure 23: Histogram of food insecurity indicators (country level, 2022)



Source: `www.fao.org/faostat/en/#data/FS` & Author's calculations.

Food security

Table 28: Food security indicators by region (2022)

Indicator	Africa	Asia	Europe	North America	Oceania	South America	World
MDER (kcal/capita/day)	1 736	1 831.0	1 931	1 962	1 871	1 856	**1 832**
ADER (kcal/capita/day)	2 237	2 369.0	2 505	2 554	2 424	2 403	2 370
ADEC (kcal/capita/day)	2 578	2 917.0	3 467	3 882	3 104	3 104	**2 971**
Prevalence of undernourishment	19.9	8.2			7.1	6.6	**9.1**
People undernourished (million)	284.1	386.5			3.2	43.9	723.8

All statistics are expressed in %, except those whose units are indicated.

Source: `www.fao.org/faostat/en/#data/FS` & Author's calculations.

Food security

Table 29: Food security indicators by region (2022)

Indicator	Africa	Asia	Europe	North America	Oceania	South America	World
Severe food insecurity							
Total population	21.7	9.7	1.8	0.9	9.3	11.0	**10.8**
Rural adult population	23.5	10.4	1.7	0.8	2.8	13.5	12.2
Town adult population	22.2	10.9	1.9	0.7	4.0	12.9	11.5
Urban adult population	19.8	8.3	1.8	1.2	3.0	9.6	9.3
Male adult population	20.8	8.6	1.7	0.7	8.5	9.6	9.1
Female adult population	21.3	9.9	1.9	1.2	8.3	12.0	10.2
Total population (million)	309.0	459.2	13.3	3.5	4.2	72.5	**861.7**
Male adults (million)	87.9	157.2	6.4	1.0	1.5	23.7	277.7
Female adults (million)	92.0	177.6	7.8	1.9	1.4	31.2	311.9

All statistics are expressed in %, except those whose units are indicated.

Source: `www.fao.org/faostat/en/#data/FS` & Author's calculations.

Food security

Table 30: Food security indicators by region (2022)

Indicator	Africa	Asia	Europe	North America	Oceania	South America	World
Water services							
Safely managed drinking water	33.0	76.0	93.0	97.0		75.0	73.0
Basic drinking water	66.0	95.0	98.0	100.0		98.0	91.0
Sanitation services							
Safely managed sanitation	26.0	59.0	79.0	96.0	73.0	49.0	57.0
Basic sanitation	36.0	86.0	97.0	100.0	80.0	90.0	81.0

All statistics are expressed in %, except those whose units are indicated.

Source: www.fao.org/faostat/en/#data/FS & Author's calculations.

Food security

Table 31: Food security indicators by region (2022)

Indicator	Africa	Asia	Europe	North America	Oceania	South America	World
Children under 5 years							
Affected by wasting	5.8	9.3		0.2		1.4	6.8
Who are stunted	30.0	22.3	4.0	3.6	22.0	11.5	22.3
Who are overweight	4.9	5.1	7.3	8.2	16.8	8.6	5.6
Affected by wasting (million)	12.2	31.6				0.7	45.0
Who are stunted (million)	63.1	76.6	1.4	0.7	0.8	5.7	148.1
Who are overweight (million)	10.2	17.7	2.6	1.7	0.6	4.2	37.0
Obesity							
Adult population	16.2	10.4	21.4	40.3	29.5	29.9	15.8
Adult population (million)	123.9	353.9	129.0	119.2	9.6	141.4	880.7

All statistics are expressed in %, except those whose units are indicated.

Source: `www.fao.org/faostat/en/#data/FS` & Author's calculations.

A Course on Biodiversity
Lecture 3a. Biodiversity Threats and Risks

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March 2025

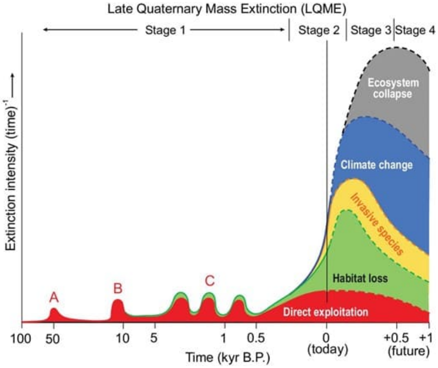
[4] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Biodiversity threats and risks

- **Biodiversity threats** are the specific drivers or causes of biodiversity loss. They are external factors that contribute directly or indirectly to biodiversity loss by impacting species, habitats, and ecosystems. For example, invasive species can severely disrupt an ecosystem and therefore constitute a threat to biodiversity.
- **Biodiversity risks** refer to the potential consequences of biodiversity loss due to these threats. Specifically, biodiversity risks encompass the likelihood and severity of negative impacts on human life and ecosystem functions. For example, the emergence of new diseases as a result of ecosystem disturbances is a biodiversity risk.

Biodiversity threats and risks

Figure 24: Stages of the Late Quaternary mass extinction



Source: Algeo and Shen (2024, Figure 5, page 11).

- **Stage 1** (from $\sim$ 50 to 0.25 ka), characterized by direct exploitation of species, comprised megafaunal extinctions in (A) Australasia, (B) the Americas and (C) the Indo-Pacific region.
- **Stage 2** (from $\sim$ 0.25 ka to the near future) is dominated by extinctions due to habitat loss.
- **Stages 3 and 4** (future, timeline speculative) will be marked by climate change and ecosystem collapse, respectively, as the dominant proximate causes of extinction, while invasive species will play a supporting role during Stages 2 to 4.

New Nature Economy Report II (World Economic Forum, 2020)

The WEF identifies 15 major pressing business-related threats to nature:

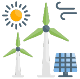
- Food, land and ocean use: (1) annual and perennial non-timber crops, (2) logging and wood harvesting, (3) livestock farming and ranching, (4) invasive non-native/alien species/diseases, (5) fire and fire suppression, (6) agricultural and forestry effluents, (7a) water management/use*, (8) fishing and aquatic resources;
- Infrastructure and the built environment: (9) housing and urban areas, (10) tourism and recreational areas, (11) domestic and urban wastewater, (12) roads and railroads, (13) commercial and industrial areas, (14) industrial and military effluents;
- Energy and extractives: (15) mining and quarrying, (7b) dams*.

New Nature Economy Report II (World Economic Forum, 2020)

The WEF identities 15 key socio-economic transitions needed to tackle the nature crisis:

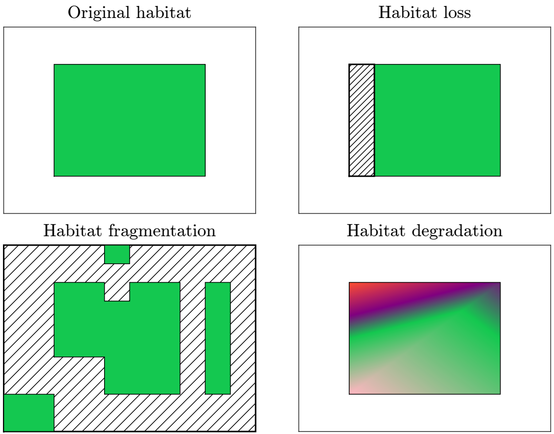
- Food, land and ocean use: (1) ecosystem restoration and avoided expansion, (2) productive and regenerative agriculture, (3) healthy and productive ocean, (4) sustainable management of forests, (5) planet-compatible consumption, (6) transparent and sustainable supply chains;
- Infrastructure and the built environment: (7) densification of the urban environment, (8) nature-positive built environment design, (9) planet-compatible urban utilities, (10) nature as infrastructure, (11) nature-positive connecting infrastructure;
- Energy and extractives: (12) circular and resource efficient models, (13) nature-positive metals and minerals extraction, (14) sustainable materials supply chains, (15) nature-positive energy transition.

New Nature Economy Report II (World Economic Forum, 2020)

System	Description	Share of threatened species impacted	Threats	Total business opportunities by 2030	Total jobs by 2030	Annualized investment costs (2020–2030)
	Food, land and ocean use	72%	8	\$3 565 bn	191 mn	\$440 bn
	Infrastructure and the built environment	29%	6	\$3 015 bn	117 mn	\$1 430 bn
	Energy and extractives	18%	2	\$3 530 bn	87 mn	\$840 bn
	Total[†] of the three socio-economic systems	79%	15	\$10 110 bn	395 mn	**2 710 bn**

Habitat loss, fragmentation and degradation

Figure 25: Habitat loss vs. fragmentation vs. degradation



Habitat loss, fragmentation and degradation

- Habitat loss occurs when a natural habitat is completely removed, destroyed, or converted to another use, resulting in the disappearance of species that previously lived there (*e.g.*, **deforestation**)
- Habitat fragmentation occurs when a large, continuous habitat is divided into smaller, isolated patches by human-made structures (*e.g.*, **building highways through a forest**)
- Habitat degradation is the process by which the quality of a habitat is damaged or reduced, making it less suitable for the species that live there (*e.g.*, **water pollution**)

Habitat loss, fragmentation and degradation

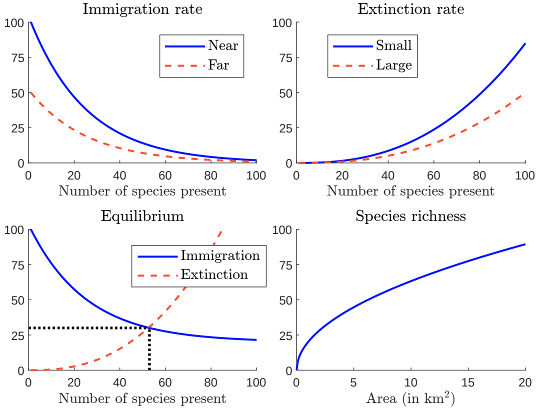
"I found 118 studies reporting 381 significant responses to habitat fragmentation independent of habitat amount. Of these responses, 76% were positive. Most significant fragmentation effects were positive, irrespective of how the authors controlled for habitat amount, the measure of fragmentation, the taxonomic group, the type of response variable, or the degree of specialization or conservation status of the species or species group. [...] Thus, although 24% of significant responses to habitat fragmentation were negative, I found no conditions in which most responses were negative. Authors suggest a wide range of possible explanations for significant positive responses to habitat fragmentation: increased functional connectivity, habitat diversity, positive edge effects, stability of predator-prey/host-parasitoid systems, reduced competition, spreading of risk, and landscape complementation." (Fahring, 2017, page 1).

Habitat loss, fragmentation and degradation

"We conducted an analysis of global forest cover to reveal that 70% of remaining forest is within 1 km of the forest's edge, subject to the degrading effects of fragmentation. A synthesis of fragmentation experiments spanning multiple biomes and scales, five continents, and 35 years demonstrates that habitat fragmentation reduces biodiversity by 13 to 75% and impairs key ecosystem functions by decreasing biomass and altering nutrient cycles." (Haddad et al., 2015, page 1).

Theory of island biogeography

Figure 26: Theory of island biogeography



- MacArthur and Wilson (1967)
- What factors influence **species richness** on islands?
  - Size of the island (resources)
  - Distance from the mainland (immigration)

Species-area relationship

Ecological version of the law of diminishing returns

We have:

$$S = cA^z$$

where:

- S is species richness
- c is a constant
- A is the area of the island
- z is the slope of the log-log A–S curve

Species-area relationship

- A_0 is the area of the original habitat
- A is the area of the current habitat
- It follows:

$$\frac{S}{S_0} = \left(\frac{A}{A_0}\right)^z$$

- The species loss due to habitat loss is:

$$\mathcal{L}oss_{species} = 1 - \left(1 - \mathcal{L}oss_{habitat}\right)^z$$

Table 32: Species loss $\mathcal{L}oss_{species}$

z	$\mathcal{L}oss_{habitat}$					
	0.00%	25.00%	50.00%	75.00%	90.00%	100.00%
0.05	0.00%	1.43%	3.41%	6.70%	10.87%	100.00
0.10	0.00%	2.84%	6.70%	12.94%	20.57%	100.00%
0.25	0.00%	6.94%	15.91%	29.29%	43.77%	100.00%
0.35	0.00%	9.58%	21.54%	38.44%	55.33%	100.00%

Species-area relationship

Figure 27: Analyzing the species-area relationship in Mediterranean islands

(a) Tyrrhenian islands

(b) Aegean islands

Source: Fattorini *et al.* (2017, Figure 1), created with `www.paintmaps.com`.

Species-area relationship

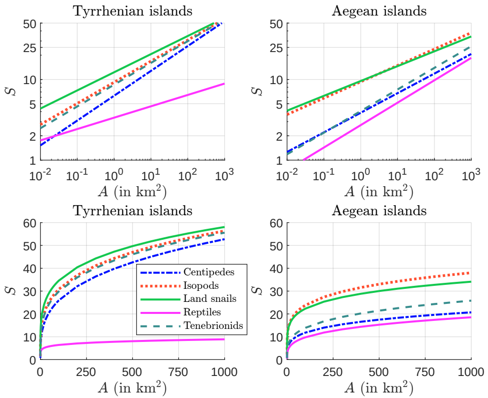
Table 33: Estimated values of c, z, and S for different area values A

Islands	Species	c	z	Area A (in km^2) 0.01	0.10	1	10	100	1 000
Tyrrhenian	Centipedes	6.281	0.308	1.52	3.09	6.28	12.77	25.94	52.73
	Isopods	9.226	0.262	2.76	5.05	9.23	16.87	30.83	56.36
	Land snails	12.274	0.225	4.35	7.31	12.27	20.61	34.59	58.08
	Reptiles	3.357	0.141	1.75	2.43	3.36	4.64	6.43	8.89
	Tenebrionids	8.610	0.270	2.48	4.62	8.61	16.03	29.85	55.59
Aegean	Centipedes	3.864	0.243	1.26	2.21	3.86	6.76	11.83	20.70
	Isopods	9.354	0.203	3.67	5.86	9.35	14.93	23.82	38.02
	Land snails	9.572	0.184	4.10	6.27	9.57	14.62	22.34	34.12
	Reptiles	2.716	0.278	0.75	1.43	2.72	5.15	9.77	18.53
	Tenebrionids	4.055	0.268	1.18	2.19	4.05	7.52	13.93	25.82

Source: Fattorini *et al.* (2017, Table 1).

Species-area relationship

Figure 28: SAR curves in Tyrrhenian and Aegean islands



Source: Fattorini *et al.* (2017) & Author's calculations.

Species distribution, sampling and endemics-area curve

Some issues to count the number of species:

- Species evenness
- Species abundance
- Spatial distribution of species
- Sampling effects

Species abundance models

Some definitions

- We consider a region or community with S species
- For each species i, the number of individuals is denoted by n_i
- The species abundance can be described by a traditional frequency distribution table:

Species	1	2	$\cdots$	i	$\cdots$	S
Frequency	n_1	n_2		n_i		n_S

- The **rank-abundance distribution** (RAD) is obtained by sorting n_i (in ascending or descending order)

Species abundance models

Species abundance distribution (SAD)

- The SAD summarizes the number of species by their abundance:

Number of individuals	1	2	$\cdots$	j	$\cdots$	n^{+}
Number of species	$s(1)$	$s(2)$		$s(j)$		$s(n^{+})$

  where:
  - $s(j)$ is the number of species with j individuals
  - $n^{+} = \max n_i$ is the maximum number of individuals found in any single species
- Mathematically, we have $s(j) = \sum_{i=1}^{S} \mathbb{1}\{n_i = j\}$
- We have the following property:

$$\sum_{j=1}^{\infty} s(j) \cdot j = \sum_{i=1}^{S} n_i = n$$

Species abundance models

Example #1

We consider a community consisting of 25 species and 407 individuals, distributed across a region of $2\,\mathrm{km}^2$. The abundances of the 25 species are as follows: $2, 10, 13, 2, 1, 5, 25, 17, 1, 4, 28, 117, 23, 10, 13, 1, 4, 3, 10, 5, 7, 70, 10, 25, 1$.

Species abundance models

There are four species with only one individual, so $s(1) = 4$. Similarly, $s(2) = 2$ because there are two species with two individuals each. For $s(3)$, we have $s(3) = 1$, as only one species has three individuals, and so on. Finally, we get the resulting species abundance distribution:

j	1	2	3	4	5	7	10	13	17	23	25	28	70	117
$s(j)$	4	2	1	2	2	1	4	2	1	1	2	1	1	1

In practice, we group species whose number of individuals is low[5]:

$\mathcal{C}_c$	1	2	3	4	5	6–10	11–25	26–50	51–100	100–117
$s(\mathcal{C}_c)$	4	2	1	2	2	5	6	1	1	1

[5] We have $s(\mathcal{C}_c) = \sum_{j \in (\mathcal{C}_c)} s(j)$ where $\mathcal{C}_c$ represents the c^{th} grouping class.

Species abundance models

Rank-abundance diagram (RAD)

This is a graphical representation used in ecology to provide insights into species richness and species evenness. On the x-axis, species are ranked according to their abundance, from most abundant (rank 1) to least abundant (rank S), while on the y-axis, we plot relative abundance, that is the proportion of individuals belonging to each species.

Looking at Example #1, we get the following rank-abundance distribution:

k	1	2	3	4, 5	6	7	8, 9	$\cdots$	19	20, 21	22–25
i	12	22	11	7, 24	13	8	3, 15		18	1,4	5, 9, 16, 25
n_i	117	70	28	25	23	17	13	$\cdots$	3	2	1
f_k (in %)	28.75	17.20	6.88	6.14	5.65	4.18	3.19		0.74	0.49	0.25

Species 12 ranks first and represents 28.75% of the total abundance of the community. It is followed by species 22 and 11, whose relative abundance is 17.20% and 6.88%, respectively.

Species abundance models

Figure 29: Species abundance curve (Preston plot)

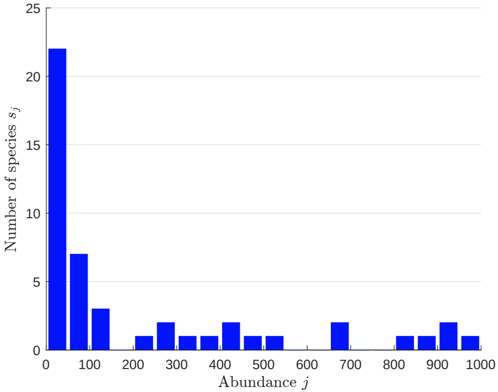
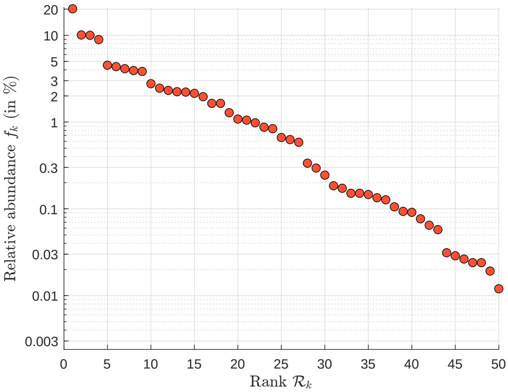


Figure 30: Rank-abundance curve (Whittaker plot)



Species abundance models

Other methods include the species rarefaction curve, the empirical cumulative distribution function (ECDF), the k-dominance plot, the Robbins curve, etc.

One of the famous plots was proposed by Preston (1948), who popularized the use of a frequency histogram with $\log_2$-based classes along the x-axis. In this approach, the number of species is grouped into intervals of 2^k (*e.g.*, 1, 2, 4, 8, 16, 32, 64, etc.), called **octaves**.

Using Example #1, we obtain the following results:

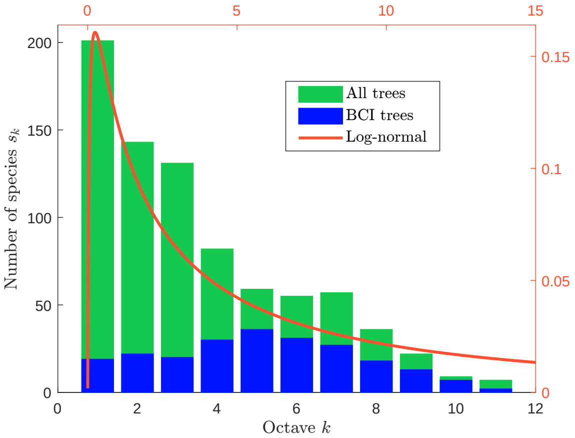
Octave k	1	2	3	4	5	6	7
$\mathcal{C}_k$	1	2–3	4–7	8–15	16–31	32–63	64–127
$s(k)$	4	3	5	6	5	0	2

Species abundance models

Figure 31: Barro Colorado Island (Panama)

Species abundance models

Figure 32: Species abundance distribution of tropical forest trees



Species abundance models

Geometric rank-abundance model

- Developed by Motomura (1932)
- We assume that one species preempts a fraction $\kappa \in (0, 1)$ of the resource, a second species the same fraction κ of the remaining resource $(1 - \kappa)$, and so on
- We obtain the **geometric rank-abundance** model:

$$n_i = \frac{n\kappa \left(1 - \kappa\right)^{i-1}}{1 - \left(1 - \kappa\right)^{S}}$$

where S is the number of species and n is the total number of individuals

Species abundance models

Log-series model

- Developed by Fisher *et al.* (1943)
- The number of species is derived from the limiting form of the negative binomial distribution $\mathcal{NB}(r, p)$, excluding zero observations
- The expected number of species with exactly j individuals is given by:

$$s(j) = \alpha \frac{x^j}{j}$$

  where $x = (1 - p)^{-1} p \in (0, 1)$, p is the success probability of the negative binomial distribution, and α is a scaling factor that depends on the parameters r and p
- The total expected number of species is $S = \sum_{j=1}^{\infty} s(j) = -\alpha \ln(1 - x)$
- The total expected number of individuals is $n = \sum_{j=1}^{\infty} s(j) \cdot j = \sum_{j=1}^{\infty} \alpha x^j = \alpha \frac{x}{1 - x}$

Species abundance models

Log-normal model

- Developed by Preston (1948)
- The probability that a species has j individuals follows a log-normal distribution $\mathcal{LN}\left(\mu,\sigma^2\right)$
- We have:

$$s_j = S\frac{\Phi\left(\sigma^{-1}\left(\ln\left(j+\frac{1}{2}\right)-\mu\right)\right)-\Phi\left(\sigma^{-1}\left(\ln\left(j-\frac{1}{2}\right)-\mu\right)\right)}{1-\Phi\left(\sigma^{-1}\left(\ln\left(\frac{1}{2}\right)-\mu\right)\right)}$$

Species abundance models

Broken-stick model

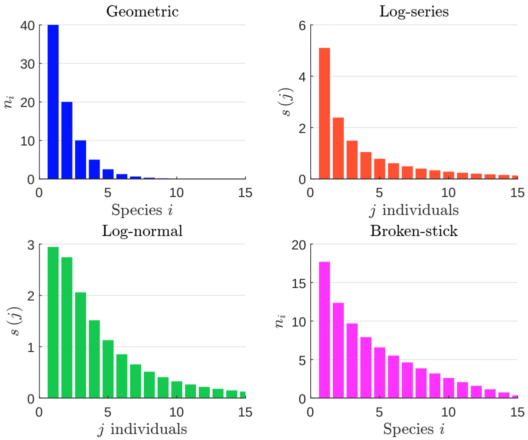
- Proposed by MacArthur (1957) and May (1975)
- The total resources (or individuals) available in a community are divided randomly among species
- The community is represented as a stick of fixed length, the stick is broken into S segments at $S-1$ randomly chosen points, and the lengths of the resulting segments are proportional to the abundances of the S species — they follow a uniform distribution
- The i^{th} largest segment corresponds to a specific harmonic expectation based on the number of breaks
- The abundance of the i^{th} species is then given by $n_i = \dfrac{n}{S}\sum_{k=i}^{S}\dfrac{1}{k}$
- We get:

$$s(j) = \frac{S}{n}(S-1)\left(1-\frac{j}{n}\right)^{S-2}$$

where $j \in [0, S]$

Species abundance models

Figure 33: Species abundance models



Sampling

- The **species accumulation curve** (SAC) is a graphical representation that illustrates how the number of observed species in a particular environment increases with additional sampling effort
- Let $n_i(s)$ be the number of individuals of species i recorded in the s^{th} sample
- The SAC function is defined as:

$$\mathrm{SAC}(m) = \sum_{i=1}^{S} \mathbb{1}\left\{\sum_{s=1}^{m} n_i(s) \geq 1\right\}$$

  where $m \leq m_s$ is the number of samples, and m_s is the total number of samples available
- $\mathrm{SAC}(m)$ counts the number of species present in the m samples

Sampling

- When a sample represents an individual, $\mathrm{SAC}(m)$ is the expected number of species among m individuals selected at random
- We have

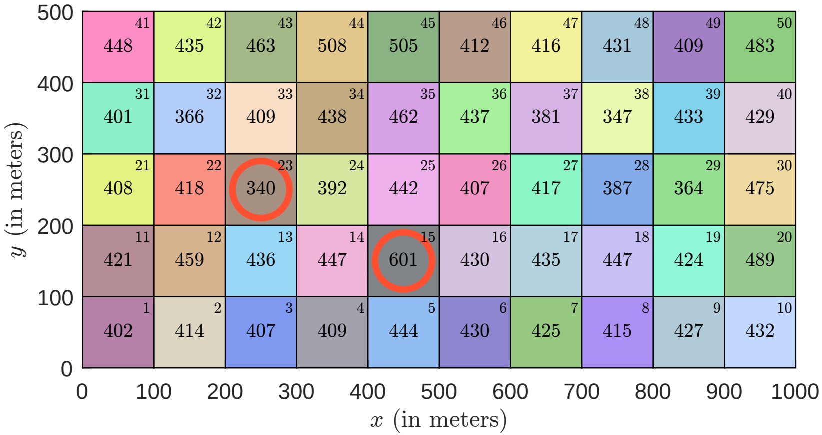
$$\mathrm{SAC}(m) = \sum_{i=1}^{S} \left(1 - \frac{\binom{n - n_i}{m}}{\binom{n}{m}} \right)$$

where n_i is the abundance (number of individuals) of the i^{th} species and n is the total abundance of the community

- In the last case, the species accumulation curve is often referred to as the species rarefaction curve

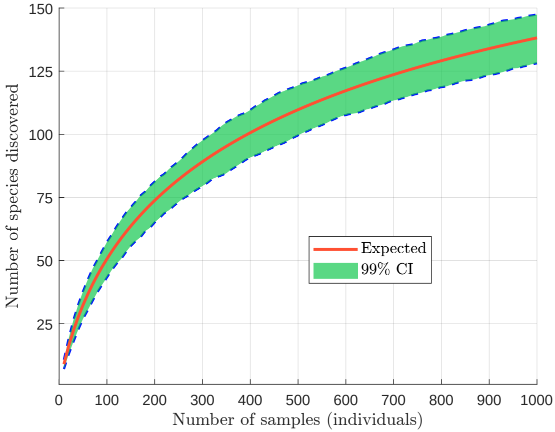
Sampling

Figure 34: Number of individuals per plot (BCI trees)



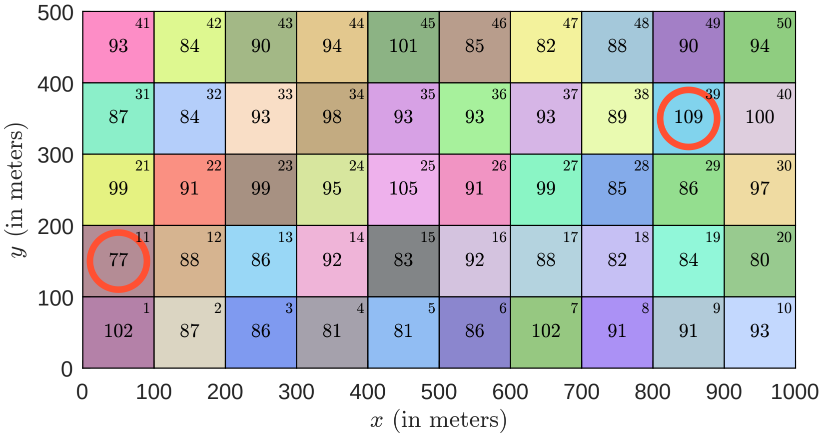
Sampling

Figure 35: Species accumulation curve (BCI trees)



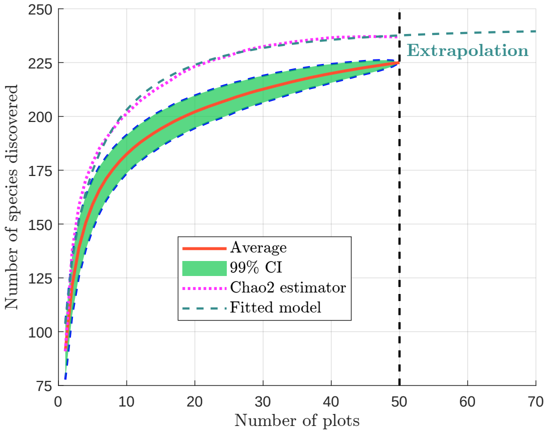
Sampling

Figure 36: Number of species per plot (BCI trees)



Sampling

Figure 37: Species accumulation curve (BCI trees)



Endemism

- Region A with S species and n_i individuals for species $i \Rightarrow \mathrm{SAR}(A) = S$
- S_a is the expected number of species of a subregion (a
- Assuming random placement of individuals within the area, we get:

$$\mathrm{SAR}(a) = S_a = S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i}$$

- Two types of endemism:
  - **Global endemism**: A species is globally endemic to A if it is found exclusively in region A and nowhere else
  - **Local endemism**: A species is locally endemic to a if it occurs exclusively in subarea a and not in the complementary area $A - a$

Endemism

Endemics-area relationship

Under the assumption of random placement, the endemics-area relationship is:

$$\mathrm{EAR}(a) = E_a = \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$$

where E_a is the expected number of species endemic to the subregion a

$\Rightarrow$ The expected number of species lost due to the loss of area a is:

$$\mathcal{L}oss\,(a \mid A) = S_A - S_{A-a} = S - \left(S - \sum_{i=1}^{S} \left(1 - \frac{A-a}{A}\right)^{n_i}\right) = \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i} = E_a$$

E_a can be interpreted as the number of species lost if habitat area a is destroyed

Species-area relationship vs. endemics-area relationship

Trivial result

The number of species in region A is equal to the sum of the species found in subregion $A - a$ and the species locally endemic in subregion a:

$$\mathrm{SAR}\left(A\right) = \mathrm{SAR}\left(A - a\right) + \mathrm{EAR}\left(a\right)$$

Species-area and endemics-area relationships

n_i is the number of individuals of species i, s_j is the number of species with j individuals, and κ is the resource preemption parameter

Model	Specification	SAR (a)	EAR (a)
Random placement	n_i is known	$S_a = S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i}$	$E_a = \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$
Most even	$n_i = \frac{n}{S}$	$S_a = S \left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n}{S}}\right)$	$E_a = S \left(\frac{a}{A}\right)^{\frac{n}{S}}$
Most uneven	$n_{i<S} = 1, n_S = n - S + 1$	$S_a = 1 + (S-1)\frac{a}{A} - \left(1 - \frac{a}{A}\right)^{n-S+1}$	$E_a = (S-1)\frac{a}{A} + \left(\frac{a}{A}\right)^{n-S+1}$
Geometric	$n_i = \frac{n\kappa (1-\kappa)^{i-1}}{1-(1-\kappa)^S}$		
Mixed even-uneven	$n_{i \leq s} = 1,\ n_{i>s} = \frac{n-s}{S-s}$	$S_a = s\frac{a}{A} + (S-s) \cdot \left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n-s}{S-s}}\right)$	$E_a = S - s\left(1 - \frac{a}{A}\right) - (S-s)\left(1 - \left(\frac{a}{A}\right)^{\frac{n-s}{S-s}}\right)$

Species-area and endemics-area relationships

n_i is the number of individuals of species i, s_j is the number of species with j individuals, α and x are the parameters of the log-series model, and γ and ϕ are the shape and scale parameters of the truncated negative binomial distribution (TNBD) model

Model	Specification	SAR (a)	EAR (a)
Random placement	$s(j)$ is known	$S_a = S - \sum_j s_j \left(1 - \frac{a}{A}\right)^j$	$E_a = \sum_j s_j \left(\frac{a}{A}\right)^j$
Log-series	$s(j) = \alpha \frac{x^j}{j}$	$S_a = \alpha \ln \left(1 + \frac{x}{1-x} \frac{a}{A}\right)$	$E_a = -\alpha \ln \left(1 - x \frac{a}{A}\right)$
Broken-stick	$s(j) = \frac{S}{n}(S-1)\left(1 - \frac{j}{n}\right)^{S-2}$	$S_a = \frac{S \ln \left(1 - \frac{a}{A}\right)}{\ln \left(1 - \frac{a}{A}\right) - \frac{S}{n}}$	$E_a = -\frac{S^2}{n \ln \left(\frac{a}{A}\right) - S}$
TNBD	$s(j) = \frac{\Gamma(\gamma + j)}{\Gamma(j+1)\Gamma(\gamma)} \cdot \frac{\phi^j}{(1+\phi)^{\gamma+j} - (1+\phi)}$	$S_a = \frac{S\left(1 - \left(1 + \phi \frac{a}{A}\right)^{-\gamma}\right)}{1 - (1+\phi)^{-\gamma}}$	$E_a = S \frac{\left(1 + \phi\left(1 - \frac{a}{A}\right)\right)^{-\gamma} - (1+\phi)^{-\gamma}}{1 - (1+\phi)^{-\gamma}}$

Species-area and endemics-area relationships

Figure 38: Species-area relationship (Pasoh Forest Reserve)

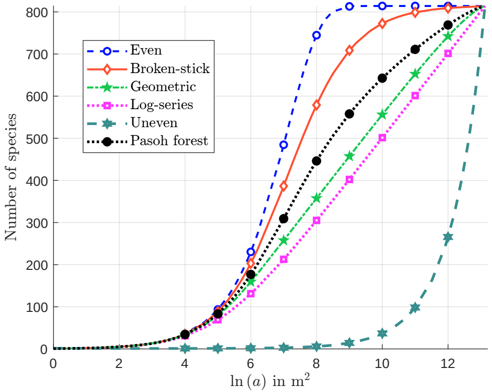
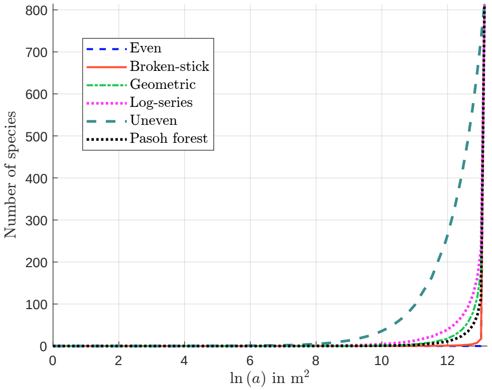


Figure 39: Endemics-area relationship (Pasoh Forest Reserve)

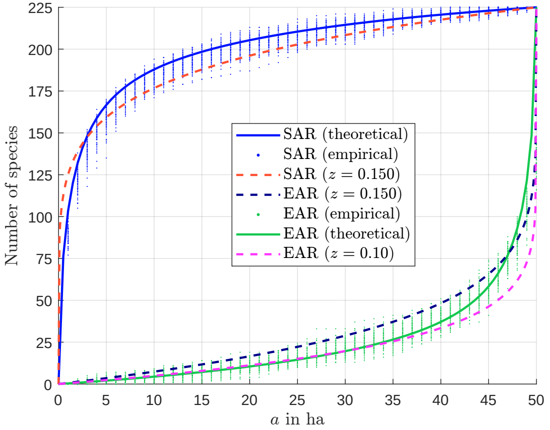


Species-area and endemics-area relationships

"Here we show that extinction rates estimated from the SAR are all overestimates. [...] These overestimates are due to the false assumption that the sampling problem for extinction is simply the reverse of the sampling problem for the SAR. ***The area that must be added to find the first individual of a species is in general much smaller than the area that must be removed to eliminate the last individual of a species****. Therefore, on average, it takes a much greater loss of area to cause the extinction of a species than it takes to add the species on first encounter, except in the degenerate case of a species having a single individual. [...] Only in a very special and biologically unrealistic case, when all species are randomly and independently distributed in space, is it possible to derive the EAR from the SAR." (He and Hubbell, 2011, page 368).*

Species-area and endemics-area relationships

Figure 40: Comparison of species-area and endemics-area curves (Barro Colorado Island)



Source: He and Hubbell (2011, Figure S1) & Author's calculations.

Forest loss

Figure 41: Forest on Borneo in Indonesia, cut down for an oil palm plantation

Source: https://alert-conservation.org & https://forestsnews.cifor.org.

Forest definitions adopted by major international environmental and forestry organizations

- United Nations Food and Agriculture Organization (2000)
  Land with tree crown cover (or equivalent stocking level) of more than 10% and an area of more than 0.5 ha. Trees should be able to reach a minimum height of 5 m at maturity *in situ*.
- United Nations Framework Convention on Climate Change (2002)
  Minimum area of 0.05–1.0 ha of land with tree crown cover (or equivalent stocking level) of more than 10–30% with trees that have the potential to reach a minimum height of 2–5 m at maturity *in situ*.
- United Nations Convention on Biological Diversity
  Land area of more than 0.5 ha, with a tree canopy cover of more than 10%, which is not primarily under agriculture or other specific non-forest land use.
- United Nations Convention to Combat Desertification (2000)
  Dense canopy with multi-layered structure including large trees in the upper story.
- International Union of Forest Research Organizations (2002)
  Land area with a minimum 10% tree crown coverage (or equivalent stocking level), or formerly having such tree cover and that is being naturally or artificially regenerated or that is being afforested.

Forest loss

2025 FRA definition (FAO, 2023)

"*Land spanning more than 0.5 hectares with trees higher than 5 meters and a canopy cover of more than 10 percent, or trees able to reach these thresholds in situ. It does not include land that is predominantly under agricultural or urban land use.*"

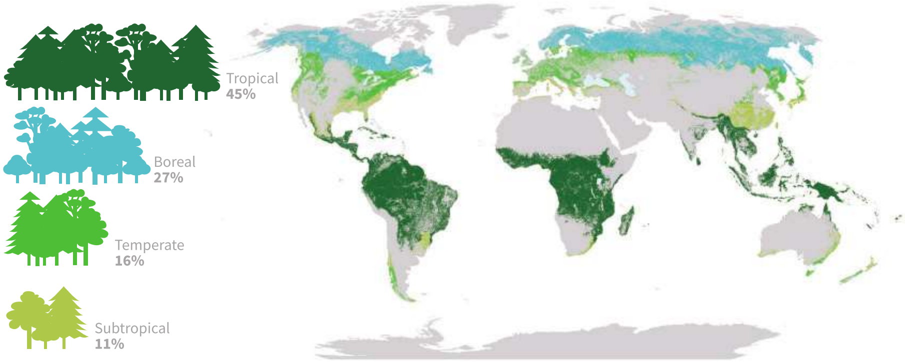
Forest loss

Sources of information:

- Global Forest Resources Assessments — `https://www.fao.org/forest-resources-assessment`
- The Global Forest Review (GFR) — `https://research.wri.org/gfr/global-forest-review`
- Global Forest Watch (GFW) — `www.globalforestwatch.org`

Forest loss

Figure 42: Proportion and distribution of global forest area by climatic domain in 2020



Source: FAO (2020, page 1).

Forest loss

Table 34: Forest area (top 20 countries and six world regions)

Country/region	Value (in million ha)				Variation by decade (in %)			Geographical distribution		Primary forest
	1990	2000	2010	2020	2000	2010	2020	in %	Cumulated	in %
Russia	809	809	815	815	0.04	0.72	0.02	20.09	20.09	31.30
Brazil	**589**	551	512	**497**	**−6.42**	**−7.17**	−2.92	12.24	32.32	43.53
Canada	348	348	347	347	−0.14	−0.14	−0.11	8.55	40.87	59.13
USA	302	304	309	310	0.36	1.71	0.35	7.63	48.50	24.31
China	**157**	177	201	**220**	**12.64**	**13.34**	**9.65**	5.42	53.92	5.21
Australia	134	132	130	134	−1.54	−1.72	3.44	3.30	57.22	
Congo (DRC)	**151**	144	137	**126**	−4.47	−4.68	**−8.03**	3.11	60.33	65.59
Indonesia	119	101	100	92	**−14.56**	−1.60	−7.55	2.27	62.60	48.56
Peru	76	75	74	72	−1.51	−1.66	−2.32	1.78	64.38	
India	64	68	69	72	5.71	2.82	3.83	1.78	66.16	21.76
Angola	79	78	72	67	−1.96	**−7.14**	**−7.69**	1.64	67.80	40.15
Mexico	71	68	67	66	−3.13	−2.10	−1.87	1.62	69.42	48.77
Colombia	65	63	61	59	−3.42	−3.07	−2.74	1.46	70.88	
Bolivia	58	55	53	51	−4.68	−3.66	−4.24	1.25	72.13	

Forest loss

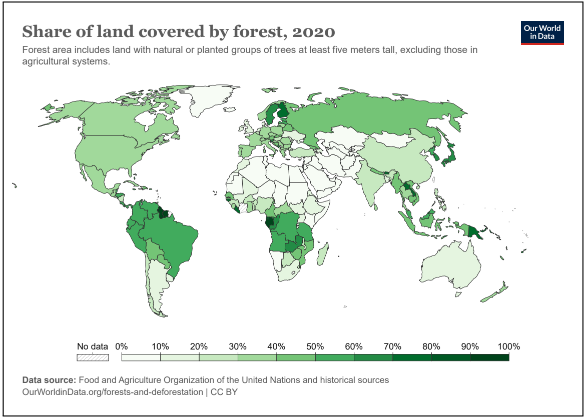
Table 35: Forest area (top 20 countries and six world regions)

Country/region	Value (in million ha)				Variation by decade (in %)			Geographical distribution		Primary forest
	1990	2000	2010	2020	2000	2010	2020	in %	Cumulated	in %
Venezuela	52	49	48	46	−5.53	−3.35	−2.68	1.14	73.27	97.06
Tanzania	57	54	50	46	−6.48	−6.93	−8.42	1.13	74.40	62.32
Zambia	47	47	47	45	−0.76	−0.76	−4.03	1.10	75.50	
Mozambique	43	41	39	37	−5.05	−5.38	−5.72	0.91	76.40	
Papua New Guinea	36	36	36	36	−0.33	−0.27	−0.89	0.88	77.29	
Argentina	35	33	30	29	−5.19	−9.48	−5.43	0.70	77.99	
Africa	743	710	676	637	−4.41	−4.79	−5.82	15.68	15.68	19.30
Asia	585	587	611	623	0.34	4.01	1.92	15.34	31.03	13.79
Europe	994	1 002	1 014	1 017	0.80	1.17	0.34	25.07	56.09	25.22
North America	755	752	754	753	−0.39	0.24	−0.20	18.54	74.64	41.62
Oceania	185	183	181	185	−0.89	−1.26	2.34	4.56	79.20	1.41
South America	974	923	870	844	−5.24	−5.69	−2.98	20.80	100.00	35.38
World	4 236	4 158	4 106	4 059	−1.85	−1.24	−1.15	100.00	200.00	26.61

Source: FAO (2020), `https://fra-data.fao.org/assessments/fra/2020` & Author's calculations.

Forest loss

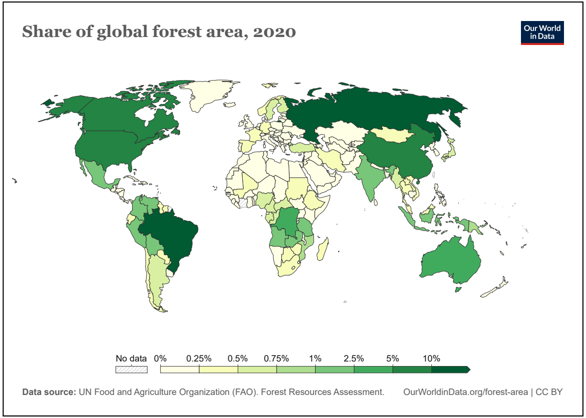
Figure 43: Percentage of land area covered by forest, by country (2020)



Source: Our World in Data, `https://ourworldindata.org/forest-area`.

Forest loss

Figure 44: Distribution of global forest area (2020)



Source: Our World in Data, `https://ourworldindata.org/forest-area`.

Forest loss

Table 36: Forest characteristics, ownership and annual change (top 10 countries and six world regions)

Country/region	Forest area (10^6 ha)	Protected (%)	Regenerative (%)	Planted (%)	Private ownership (%)	Public ownership (%)	Expansion (10^3 ha/year)	Deforestation (10^3 ha/year)	Reforestation (10^3 ha/year)	Coverage (%)	Net change (10^3 ha/year)
Russia	815	2.3	97.7	2.3	0.0	100.0	76	0	945	100.0	76
Brazil	497	30.1	97.7	2.3	44.2	55.8	243	1 696	257	100.0	−1 453
Canada	347	8.5	94.8	5.2	8.2	91.4	0	38	427	100.0	−38
USA	310	10.2	91.1	8.9	58.1	41.9			691	0.0	−60
China	220	13.8	61.5	38.5	41.3	58.7	2 070	133	273	100.0	1 937
Australia	134	18.0	98.2	1.8	32.3	67.0				0.0	182
Congo (DRC)	126	19.3	100.0	0.0	0.0	100.0				0.0	−1 101
Indonesia	92	56.2	95.1	4.9	1.1	91.2	71	650	7	100.0	−579
Peru	72	31.6	98.5	1.5	16.0	84.0	6	179		100.0	−173
India	72	19.4	81.6	18.4	18.5	81.5	935	668		100.0	266

Forest loss

Table 37: Forest characteristics, ownership and annual change (top 10 countries and six world regions)

Country/region	Forest area (10^6 ha)	Protected (%)	Regenerative (%)	Planted (%)	Private ownership (%)	Public ownership (%)	Expansion (10^3 ha/year)	Deforestation (10^3 ha/year)	Reforestation (10^3 ha/year)	Coverage (%)	Net change (10^3 ha/year)
Africa	637	24.8	98.2	1.8	5.5	71.4	404	2 149	353	44.0	−3 969
Asia	623	21.9	78.2	21.7	21.6	76.8	3 477	1 922	1 221	118.2	1 316
Europe	1 017	4.9	91.3	7.4	9.1	88.2	255	69	1 593	56.0	331
North America	753	9.5	93.7	6.2	35.2	60.5	79	372	1 322	84.2	−348
Oceania	185	15.7	97.1	2.6	47.2	52.2	16	40	56	−15.0	163
South America	844	30.5	97.6	2.4	31.9	61.6	460	2 962	947	100.9	−2 481
World	4 059	17.3	92.4	7.2	21.7	71.5	4 691	7 514	5 492	56.6	−4 987

Source: FAO (2020), `https://fra-data.fao.org/assessments/fra/2020` & Author's calculations.

Forest loss

Table 38: Distribution of habitable land on Earth (excluding glaciers and deserts)

Time	Forests	Cropland	Grazing land	Wild grassland and shrubs	Urban and built-up land
10 000 years ago	57%			42%	
5 000 years ago	55%		1%	44%	
1700	52%	3%	6%	38%	
1900	48%	8%	16%	27%	
1950	44%	12%	31%	12%	1%
2020	37%	16%	31%	14%	2%

Source: Our World in Data, https://ourworldindata.org/forest-area & Author's calculations

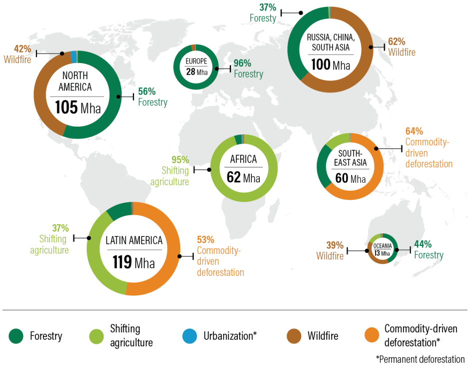
Drivers of forest loss

- Forestry
  Large-scale forestry operations in managed forests or tree plantations where future regrowth is likely. Regrowth may occur through natural regeneration or tree planting.
- Commodity-driven deforestation
  Long-term permanent conversion of forest and shrubland to non-forest land for commodity production, including agriculture, mining, or oil and gas production.
- Wildfire
  Burning of vegetation without visible human conversion or agricultural activity afterward. Some of these fires occur naturally, but others are set by humans. In humid tropical forests, fires are not natural to the ecosystem and are almost always set by humans, usually to clear land for agriculture.
- Shifting agriculture
  Agricultural practices in which forests are cleared, used for agricultural production for a few years, and then temporarily abandoned to allow trees to regrow. Shifting agriculture involves many different types of smallholder farming practices.
- Urbanization
  Permanent conversion of forests to human settlements for the expansion and intensification of existing urban centers.

Source: Curtis *et al.* (2018, pages 1108-1109)

Forest loss

Figure 45: Drivers of tree cover loss by region (2001–2023)



Source: `https://research.wri.org/gfr/global-forest-review`.

Forest degradation

"Forest degradation is broadly defined as a reduction in the capacity of a forest to produce ecosystem services such as carbon storage and wood products as a result of anthropogenic and environmental changes. [...] There is, however, no generally recognized way to identify a degraded forest because perceptions of forest degradation vary depending on the cause, the particular goods or services of interest, and the temporal and spatial scales considered. [...] the types of degradation can be represented using five criteria that relate to the drivers of degradation, loss of ecosystem services and sustainable management, including: ***productivity****,* ***biodiversity****,* ***unusual disturbances****,* ***protective functions****, and* ***carbon storage****." (Thompson et al., 2013, page 1).*

Forest degradation

The Global Forest Review has selected three approaches to measure forest degradation and forest disturbance:

1. Forest area experiencing a partial (more than 20% and less than 90%) loss of tree canopy cover;
2. Tree cover extent experiencing tree cover loss due to fire;
3. Intact forest landscapes that can no longer be considered intact due to evidence of human disturbance.

Forest degradation

- Between 2001 and 2012, 185 Mha of forest experienced a partial reduction in tree canopy cover, representing 5% of the global forest area, with 85% of this occurring in tropical forests
- Additionally, 113 Mha of tree cover loss was associated with fire between 2001 and 2023, accounting for 2.8% of the global forest area
- 155 Mha of forest area classified as intact in 2000 could no longer be considered intact by 2020, representing 4% of the global forest area

Invasive species

Definition

An invasive species is a non-native (or alien) species (plants, animals, or microorganisms) that is intentionally or accidentally introduced into a new environment and poses a threat to native species and biodiversity

Invasive species

Main characteristics of invasive species

- Non-native
  Invasive species are plants and animals that live in areas where they do not naturally exist. However, not all non-native species are invasive. For instance, corn is not native to Europe but is not considered an invasive species.
- Rapid spread
  Invasive species tend to reproduce and grow very quickly because they lack natural predators in the new environment.
- Harmful effects
  Invasive species often outcompete native species for resources, take over habitats, and disrupt native ecosystems.

Invasive species

In the United States

- National Invasive Species Information Center (NISIC)
- www.invasivespeciesinfo.gov
- **194 invasive species** in the United States (December 2024)
- In alphabetical order, the first invasive species listed is the African clawed frog (*Xenopus laevis*), which is native to Africa. It was introduced to California in 1968 and imported for laboratory research and the pet trade. This species negatively impacts native amphibian and fish populations
- The 107$^{\text{th}}$ invasive species on the list is kudzu (*Pueraria montana*), which is native to Asia. It was introduced to the USA in the late 1800s as an ornamental plant and for erosion control. Kudzu vine outcompetes native species and disrupts ecosystems.

Invasive species

Figure 46: Invasive kudzu overtakes trees and shrubs on a hillside in Blount County, Tennessee

Source: www.nature.org & Katie Ashdown via Flickr.

Invasive species

In the European Union

- List of **88 regulated invasive alien species**
- 47 animal species
- 41 plant species
- Some examples:
  Egyptian goose (*Alopochen aegyptiaca*), western mosquitofish (*Gambusia affinis*), fox squirrel (*Sciurus niger*), tropical fire ant (*Solenopsis geminata*), African clawed frog (*Xenopus laevis*), Senegal tea plant (*Gymnocoronis spilanthoides*), floating pennywort (*Hydrocotyle ranunculoides*), water primrose (*Ludwigia grandiflora*), kudzu vine (*Pueraria montana*)

Invasive species

"One of the worst such disasters was the introduction of the ***rosy wolf snail*** *(Euglandina rosea), native to Central America and Florida, to many Pacific islands to* ***control the previously introduced giant African snail*** *(Achatina fulica). The predator not only failed to control the targeted prey (which grows to be too large for the rosy wolf snail to attack it) but caused the extinction of over 50 species of native land snails [...] The small* ***Indian mongoose****, implicated as the sole cause or a contributing cause in the extinction of several island species of birds, mammals, and frogs, was deliberately introduced to all these islands as a* ***biological control agent for introduced rats*** *[...] The* ***mosquitofish*** *(Gambusia affinis) from Mexico and Central America has been introduced to Europe, Asia, Africa, Australia, and many islands for* ***mosquito control****. Its record on this score is mixed [...] However, it preys on native invertebrates and small fishes and in Australia is implicated in extinction of several fish species." (Simberloff, 2010, page 137).*

Invasive species

4 main reasons for the introduction of alien species:

1. Biological control
2. European colonization (birds, mammals, and fish for food)
3. Agriculture (including horticulture and aquaculture)
4. Accidental transport (especially rats, snakes, and insects)

Invasive species

New Zealand

- **Non-native predators kill over 25 million native birds annually**
- Many native land species have already been lost, including 60 bird species, 3 frog species, 7 vascular plants, and numerous invertebrates
- Currently, more than 3 000 native land species are either threatened or endangered
- In July 2016, the New Zealand government launched the **Predator Free 2050** initiative
- The initiative targets three primary groups of invasive species:
  1. Mustelids: stoats (*Mustela erminea*), ferrets (*Mustela furo*), and weasels (*Mustela nivalis*).
  2. Rats: ship rats (*Rattus rattus*), Norway rats (*Rattus norvegicus*), and kiore (*Rattus exulans*).
  3. Possums: brushtail possums (*Trichosurus vulpecula*).

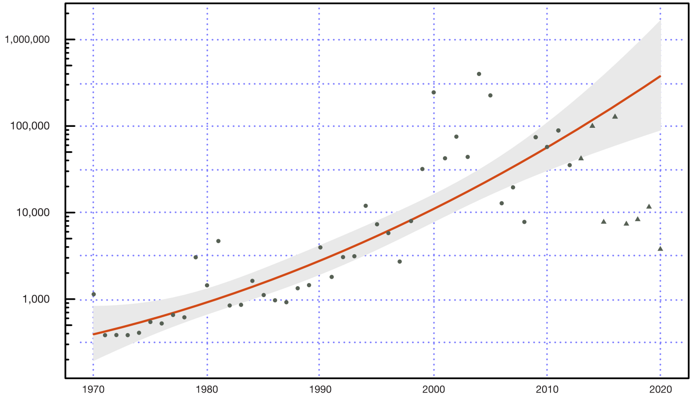
Invasive species

The vulnerability of endemic species helps explain why invasive alien species have contributed to 60% of recent species extinctions, of which 90% occurred on islands

> *"Because they evolved in the absence of selective pressures from mammalian grazers and predators, many endemic island plants and animals have evolutionarily lost or never developed defenses against these enemies and often lack a fear of them. Many island plants do not produce the bad-tasting, tough vegetative tissue that discourages herbivores, nor do they have the ability to resprout rapidly following damage. Some birds have lost the power of flight and simply build their nests on the ground." (Primack, 2014, pages 228-229).*

Invasive species

Figure 47: Average annual cost of invasive species (in 2017 $ mn)



Source: IPBES (2023, Figure 4.25, page 455).

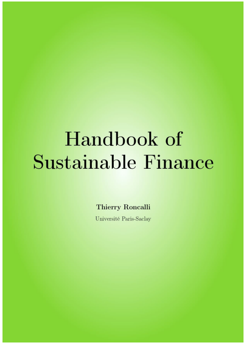
Invasive species

IPBES estimates

- Global annual economic cost of biological invasions: **$423 billion in 2019**
- 92% are due to the negative impacts of invasive alien species on nature's contribution to people or quality of life
- 8% are related to the management of biological invasions
- 66% of these costs are attributed to reductions in food supply

Climate change

See Chapter 12 (Physical Risk Modeling) in the Handbook of Sustainable Finance



A Course on Biodiversity
Lecture 3b. Biodiversity Threats and Risks

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March 2025

[6]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Pollution

Definition

- IPBES
  "*Pollution is the introduction of contaminants into the natural environment that cause adverse change*".
- UN Data glossary
  Pollution is the "*presence of substances and heat in environmental media (air, water, land) whose nature, location, or quantity produces undesirable environmental effects*" and is the "*activity that generates pollutants*"

$\Rightarrow$ A pollutant or contaminant is a substance present in concentrations that can harm organisms (humans, plants, and animals) or exceed an environmental quality standard

Pollution

- **Point source** pollution comes from a single, identifiable source, such as a pipe, drain, or specific location. It is easier to monitor and control because the source is clearly identified
- **Non-point source** pollution comes from multiple diffuse sources rather than a single point of origin. It is often carried into water bodies by rainfall, snowmelt, or runoff

Pollution

Characterization of pollutants

- By **source**:
  - Natural pollutants occur without human intervention, such as volcanic eruptions, forest fires, and natural decomposition processes;
  - Anthropogenic pollutants result from human activities, including industrial emissions, vehicle exhaust, and agricultural practices;
- By **chemical composition**:
  - Organic pollutants contain carbon-based compounds, such as pesticides, petroleum products, and plastic waste;
  - Inorganic pollutants lack carbon in their structure, including heavy metals (such as mercury and lead), mineral acids, and inorganic salts;
- Through **environmental persistence**:
  - Persistent pollutants remain in the environment for long periods of time without breaking down, such as certain pesticides, heavy metals, and some industrial chemicals;
  - Non-persistent pollutants break down relatively quickly through natural processes, such as many biological wastes and some air pollutants.

Types of biodiversity pollution

1. Air pollution
2. Biological pollution
3. Chemical pollution
4. Light pollution
5. Noise pollution
6. Plastic pollution
7. Soil pollution
8. Water pollution

Air pollution

- Indoor air pollution $\neq$ outdoor air pollution
- **Particulate matter** ($PM_{2.5}$, PM_{10}, ultrafine particles, and dust)
- Primary pollutants (carbon monoxide CO, nitrogen oxides NO_x, and sulfur dioxide SO_2)
- Secondary pollutants (acid rain, aerosols and ozone O_3)

Biological pollution

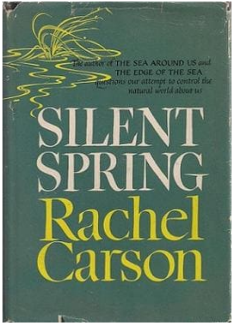
Biological pollution refers to the introduction of harmful or invasive living organisms into an ecosystem where they do not occur naturally

- Invasive species
- Pathogens
- Biologically active agents (GMOs, antibiotic-resistant microbes)

Cholera is an example of biological pollution, because it involves the contamination of water with a pathogenic microorganism, the bacterium *Vibrio cholerae*

Chemical pollution

Figure 48: Silent Spring by Rachel Carson



Silent Spring is a landmark environmental book published in 1962 by **Rachel Carson**. The book documents the harmful effects of pesticides on the environment and wildlife. Carson focused particularly on **DDT**, which was widely used after World War II

Chemical pollution

- Wang *et al.* (2020) estimated that more than 350 000 chemicals and chemical mixtures have been produced and synthesized by humans
- The global use of chemicals is expected to **increase by 70% between 2020 and 2030**, with the largest growth anticipated in China, which is projected to account for nearly 50% of the global chemical market by 2030
- The European Environment Agency (EEA) estimates that about 60% of the total volume consumed in Europe is hazardous to health, and that 8% of deaths can be attributed to hazardous chemicals

Chemical pollution

Table 39: Globally harmonized system of classification and labelling of chemicals (GHS) pictogram for hazardous substances

GHS01: Explosive

GHS02: Flammable

GHS03: Oxidizing

GHS04: Compressed gas

GHS05: Corrosive

GHS06: Fatal or toxic

GHS07: Low level toxicity

GHS08: Serious health hazard

GHS09: Hazardous to the environment

Source: `https://unece.org/transport/dangerous-goods/ghs-pictograms`.

Chemical pollution

Origin of the chemicals

- Industrial emissions
- Agricultural runoff
- Household and municipal waste
- Petroleum products
- Transportation emissions

Hazardous chemicals

- Conventional pollutants: CO, NO_x, SO_2, PM
- Heavy metals: arsenic (As), cadmium (Cd), chromium (Cr), lead (Pb), mercury (Hg)
- Persistent organic pollutants: 32 POPs including chlordane, DDT, HCB, PCB
- Emerging contaminants: pharmaceuticals and personal care products (PPCP), endocrine-disrupting chemicals (EDC), micro and nanoplastics, antibiotic-resistant genes (ARG)

Light pollution

- Astronomical light pollution, which obscures the view of the night sky
- Ecological light pollution, which alters the natural light regime in terrestrial and aquatic ecosystems.

$\Rightarrow$ Impacts on wildlife migration, reproduction, and feeding

Examples include disorientation of sea turtles, bird migration, insect disruption, human sleep disturbance, metabolic disorders, and increased risk of some cancers

Light pollution

"For most of history, the only lights made by humans were naked flames. Daily life was governed by the times of sunrise and sunset, outdoor nighttime activities depended on the phase of the Moon, and viewing the stars was a common and culturally important activity. Today, the widespread deployment of outdoor electric lighting means that the night is no longer dark for most people — few can see the Milky Way from their homes. Outdoor lighting has many legitimate uses that have benefited society. However, it often leads to illumination at times and locations that are unnecessary, excessive, intrusive, or harmful: light pollution." (Smith et al., 2023).

Noise pollution

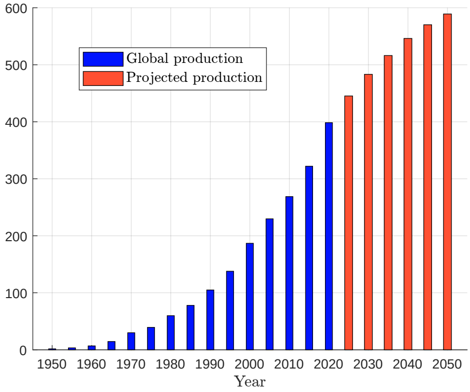
- Human health: hearing loss, sleep disturbance, mental health issues, and cognitive impairments
- Wildlife: communication disruption, habitat abandonment, interferences with pollination processes
- Marine noise pollution (because fish rely on sound for essential activities such as mating calls, territorial defense, predator alerts, and navigation during migration)

Plastic pollution

- Plastic pollution is the accumulation of plastic materials and particles in the environment
- Plastic pollutants are categorized by size as nano-, micro-, or macro-debris
- Plastics pose a critical environmental threat to wildlife through multiple mechanisms:
  1. Direct physical harm through suffocation and entanglement
  2. Chemical toxicity through leaching

Plastic pollution

Figure 49: Global production of thermoplastics with projections, 1950–2050 (in Mt)

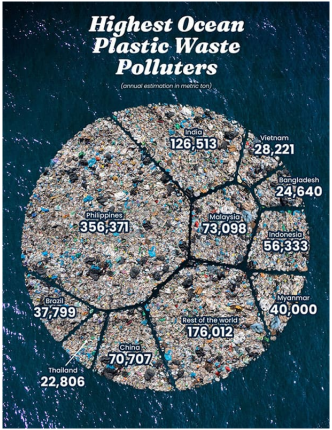


Source: `www.iea.org/data-and-statistics/charts/production-of-key-thermoplastics-1980-2050`.

Plastic pollution — Formation of garbage patches

- Great Pacific Garbage Patch (GPGP), located in the subtropical waters between California and Hawaii
- This patch covers an area of approximately 1.6 million square kilometers — comparable to the size of Mongolia or Iran (**2.5 times the size of France**)
- At least 79 000 tonnes of marine plastics are currently floating within the GPGP
- Over 75% of the mass of the GPGP consists of debris larger than 5 centimeters

Plastic pollution — Marine plastic pollution



Graphic by Louis Lugas, Visual Capitalist

- Marine plastic pollution is estimated to be between 0.8 and 2.7 million tonnes of plastic waste discharged into the oceans annually (Meijer *et al.*, 2021)
- About 1 600 rivers are responsible for 80% of marine plastic pollution
- The top 10 plastic-emitting rivers are Pasig (Philippines), Tullahan (Philippines), Ulhas (India), Klang (Malaysia), Meycauayan (Philippines), Pampanga (Philippines), Libmanan (Philippines), Ganges (India), Rio Grande de Mindanao (Philippines), and Agno (Philippines)
- Asia accounts for 81% of marine plastic pollution, followed by Africa (8%), South America (5.5%) and North America (4.5%)

Soil pollution

Definition

Soil pollution (or soil contamination) is the presence or accumulation of toxic substances, harmful chemicals, salts, pathogens, or other contaminants in soil that adversely affect soil quality, reduce soil fertility, and pose risks to human health and ecosystems.

The most common sources of soil pollution are:

- Industrial activities (chemical pollutants, heavy metals, radioactive contaminants)
- Agricultural practices (soil degradation and the use of fertilizers, pesticides and herbicides)
- Waste disposal
- Mining (petroleum hydrocarbons, solvents)

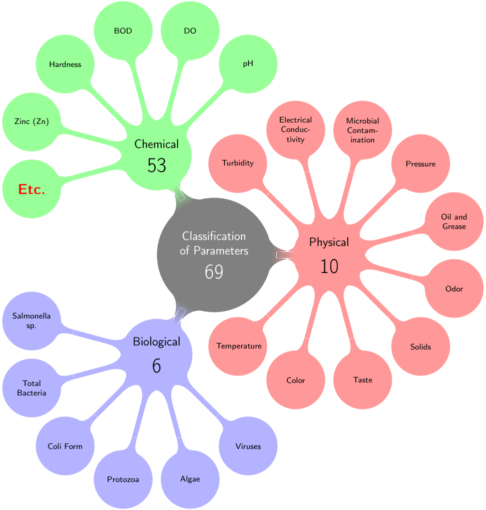
$\Rightarrow$ Impacts on health risks, food security, ecosystem degradation and habitat loss

Water pollution

Definition

Water pollution is the contamination of water bodies (*e.g.*, rivers, lakes, oceans, groundwater, and streams) by harmful substances that degrade water quality and make water unsafe for drinking, swimming, agriculture, and other uses.

Water pollution



How to measure water quality?

Source: Syeed *et al.* (2023, Figure 5, page 7).

Dose-response relationship

Definition

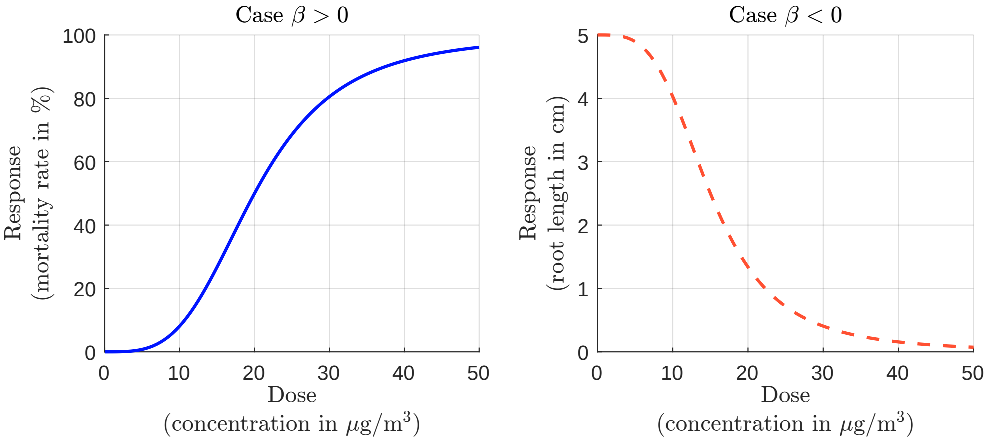
The dose-response model describes how the amount of a contaminant (dose) affects health or environmental outcomes (response). Toxicologists use this model to understand how different doses of pollutants cause different levels of harm.

Different names for the same concept:

- Dose-response relationship or curve
- Concentration-response relationship
- Exposure-response relationship
- Concentration-response function (CRF)
- Risk-response function
- Toxicity curve

Dose-response relationship

Figure 50: Sigmoidal dose-response curve



Dose-response relationship

Two distinct forms depending on the nature of the response:

- **Increasing response**
  In this form, the response increases with the concentration of the substance. For example, mortality rates may increase with increasing levels of air, soil, or water pollution.
- **Decreasing response**
  In this form, the response decreases with increasing concentration. For example, physical characteristics such as production levels, weight, or height may decrease in response to increasing levels of air, soil, or water pollution.

Dose-response relationship

- Generalized log-logistic function

$$y = y_{\min} + \frac{y_{\max} - y_{\min}}{1 + \exp\left(-\beta\left(\ln\left(x\right) - \ln\left(\alpha\right)\right)\right)} = y_{\min} + \frac{y_{\max} - y_{\min}}{1 + \left(\dfrac{x}{\alpha}\right)^{-\beta}}$$

- Log-normal model

$$y = y_{\min} + \left(y_{\max} - y_{\min}\right)\Phi\left(\beta \ln\left(\frac{x}{\alpha}\right)\right)$$

- Weibull model

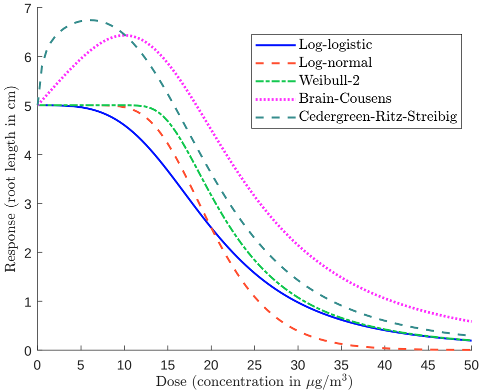
$$y = y_{\min} + \left(y_{\max} - y_{\min}\right)\exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)$$

Dose-response relationship

Hormesis

- The hormesis phenomenon occurs when a substance or environmental factor produces opposite effects at low and high doses
- The substance may have a stimulatory effect at low doses, while the same substance becomes toxic at high doses
- Hydrocarbons may have statistically significant hormetic effect on alfalfa (*Medicago sativa*)
- Carbon monoxide and oxygen are examples of hormesis

Figure 51: Hormesis biological phenomenon



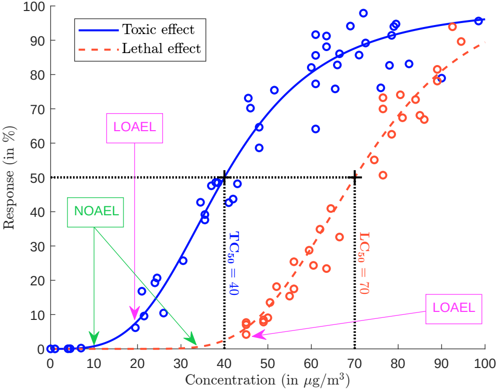
Dose-response relationship

- When the response is the percentage of individuals who respond to a given dose of a drug, we can calculate the statistic ED_p, which is the dose required to achieve the desired therapeutic effect in $p\%$ of the population
- When $p = 50\%$, we obtain the **median effective dose**: ED_{50}
- The **median toxic dose** TD_{50} and the **median lethal dose** LD_{50} represent the dose at which 50% of the population will experience a specific toxic effect or die, respectively
- Concentration of a pollutant $\Rightarrow$ **half-maximal effective concentration** EC_{50} and **half-lethal concentration** LC_{50}
- In the log-logistic and log-normal models, the median effective concentration corresponds to the parameter α
- The Hill equation is:

$$E = E_0 + (E_{\max} - E_0) \frac{[C]^n}{\mathrm{EC}_{50}^n + [C]^n}$$

Dose-response relationship

Figure 52: Threshold concentration



Critical values:

- TC_{50} and LC_{50}
- **NOAEL** (no observed adverse effect level) represents the highest dose or concentration at which no adverse effects are observed
- **LOAEL** (lowest observed adverse effect level) is the lowest dose or concentration at which adverse effects are first observed

Application to air quality standards

Table 40: Air quality standards (limit values for the protection of human health)

Pollutant	Symbol	Unit	Period	WHO 2021	EU 2008	EU 2030	US 1990	China 2012	India 2009	Brazil 1990	Switzerland 2024
Particulate matter	$PM_{2.5}$	$\mu g/m^3$	Annual	5	25	10	9/15	15/35	40		10
			24-hour	15		25	35	35/75	60		
	PM_{10}		Annual	15	40	20		40/70	60		20
			24-hour	45	50	45	150	50/150	100		50
Ozone	O_3	$\mu g/m^3$	Peak-season	60							
			8-hour	100	120	120	137	100/160	100		
Nitrogen dioxide	NO_2	$\mu g/m^3$	Annual	10	40	20	100	40	30/40	100	30
			24-hour	25		50		80	80		80
			1-hour	200	200	200	188	200		190/320	
Sulfur dioxide	SO_2	$\mu g/m^3$	Annual			20	26	20/60	20/50	40/80	30
			24-hour	40	125	50		50/150	80	100/365	100
			1 hour		350	350	196	150/500			
			10-minute	500							
Carbon monoxide	CO	mg/m^3	24-hour	4		4		4			8
			8-hour	10	10	10	10		2	10	
			1-hour	35			40	10	4	40	
			15-minute	100							

Application to air quality standards

Table 41: Air quality standards (limit values for the protection of human health)

Pollutant	Symbol	Unit	Period	WHO	EU		US	China	India	Brazil	Switzerland
				2021	2008	2030	1990	2012	2009	1990	2024
Lead	Pb	$\mu g/m^3$	Annual	0.5	0.5	0.5		0.5	0.5		
			3-month				0.15				
Amonia	NH_3	$\mu g/m^3$	Annual						100		
Arsenic	As	ng/m^3	Annual	6.6	6	6			6		
Benzene	C_6H_6	$\mu g/m^3$	Annual	1.7	5	3.4			5		
Benzo[a]pyrene	BaP	ng/m^3	Annual	0.12		1		1	1		
Cadmium	Cd	ng/m^3	Annual	5	5	5					
Nickel	Ni	ng/m^3	Annual	25	20	20			20		
Total suspended particles	TSP	$\mu g/m^3$	Annual					80/200		60/80	

Source: World Health Organization (2021, Tables 0.1 & 0.2, pages xvii-xviii), Directive 2008/50/EC (https://eur-lex.europa.eu/eli/dir/2008/50/oj), EU Directive 2024/2881 (https://eur-lex.europa.eu/eli/dir/2024/2881/oj), NAAQS Table (www.epa.gov/criteria-air-pollutants/naaqs-table), www.transportpolicy.net/topic/air-quality-standards (Brazil, China and India), Switzerland OAPC (www.fedlex.admin.ch/eli/cc/1986/208_208_208) & Author's calculations.

Air quality index (AQI)

- No globally harmonized AQI system
- The AQI developed by the US EPA has served as a model for many other countries' AQI frameworks

Air quality index (AQI)

- The US AQI is calculated based on five key pollutants:
  1. Carbon monoxide (CO)
  2. Nitrogen dioxide (NO_2)
  3. Ground-level ozone (O_3)
  4. Particulate matter (PM_{10} and $PM_{2.5}$)
  5. Sulfur dioxide (SO_2)
- For each pollutant, a sub-index is calculated based on its concentration:

$$I_j = I_{j,\mathrm{low}} + \left(\frac{[C_j] - [C_j]_{\mathrm{low}}}{[C_j]_{\mathrm{high}} - [C_j]_{\mathrm{low}}} \right) \left(I_{j,\mathrm{high}} - I_{j,\mathrm{low}} \right)$$

  where j represents the pollutant, $[C_j]$ is the concentration of the pollutant in the air, $[C_j]_{\mathrm{low}}$ and $[C_j]_{\mathrm{high}}$ are the breakpoints for the concentration range, and $I_{j,\mathrm{low}}$ and $I_{j,\mathrm{high}}$ are the sub-index values corresponding to the low and high breakpoints, respectively
- The overall Air Quality Index (AQI) is:

$$\mathrm{AQI} = \max_j I_j$$

Air quality index (AQI)

Table 42: US AQI categories

Category	AQI band	Levels of concern	Daily AQI color
1	0 to 50	Good	Green
2	51 to 100	Moderate	Yellow
3	101 to 150	Unhealthy for sensitive groups	Orange
4	151 to 200	Unhealthy	Red
5	201 to 300	Very unhealthy	Purple
6	301 and higher	Hazardous	Maroon

Source: US EPA (2024, Tables 1 and 5, paged 3–12).

Air quality index (AQI)

AQI category	1	2	3	4	5	6
$[C_j]_{\mathrm{low}}$ (in $\mu\mathrm{g}/\mathrm{m}^3$)	0.0	9.1	35.5	55.5	125.5	225.5
$[C_j]_{\mathrm{high}}$ (in $\mu\mathrm{g}/\mathrm{m}^3$)	9.0	35.4	55.4	125.4	225.4	500.4
$I_{j,\mathrm{low}}$	0	51	101	151	201	301
$I_{j,\mathrm{high}}$	50	100	150	200	300	500

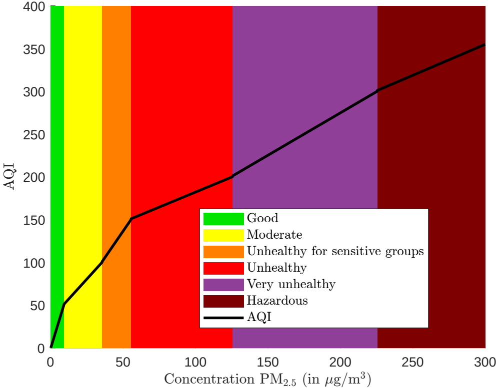
Example #2

- Suppose we have a 24-hour $\mathrm{PM}_{2.5}$ value of $27.4\,\mu\mathrm{g}/\mathrm{m}^3$
- This value falls into the second AQI category, those concentration values that are above 9.1 and below 35.4
- We have:

$$I_{\mathrm{PM}_{2.5}} = 51 + \left(\frac{27.4 - 9.1}{35.4 - 9.1}\right)(100 - 51) = 85.0951$$

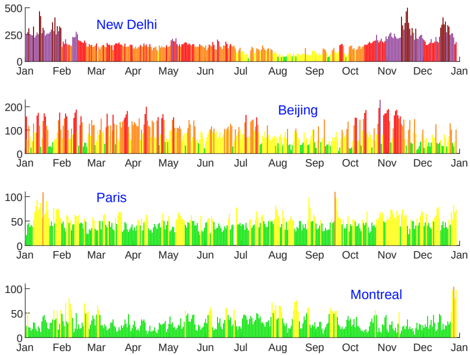
Air quality index (AQI)

Figure 53: US AQI for $PM_{2.5}$



Source: US EPA (2024) & Author's calculations.

Figure 54: 2024 AQI $PM_{2.5}$ values



Source: World Air Quality Index (WAQI) & Author's calculations.

Air quality index (AQI)

Figure 55: European air quality index (12/03/2025 12:00 pm)

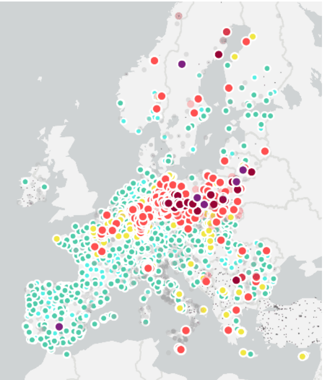
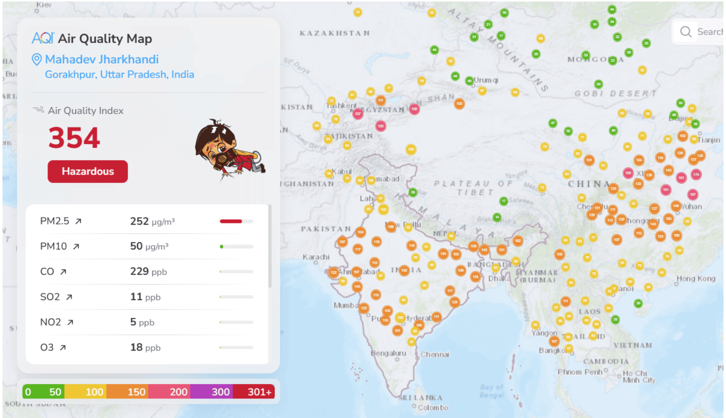


Figure 56: Legend

Source: `https://airindex.eea.europa.eu/AQI/index.html`.

Air quality index (AQI)

Figure 57: India & Western China air quality index (12/03/2025 12:00 pm)



Source: https://www.aqi.in.

The cost of pollution

Table 43: Human health effects of pollution

Health impact	Air	Biological	Chemical	Light	Noise	Plastic	Soil	Water
Cancer	✓		✓			✓	✓	✓
Cardiovascular problems	✓		✓		✓			
Cognitive development	✓	✓	✓					
Endocrine disruption			✓			✓	✓	✓
Food contamination		✓				✓	✓	✓
Hearing loss					✓			
Infectious diseases		✓					✓	✓
Mental health	✓		✓	✓	✓			
Neurological effects	✓		✓					
Physical development	✓		✓					✓
Poisoning	✓	✓	✓			✓	✓	✓
Respiratory problems	✓	✓	✓			✓		
Skin problems	✓	✓						✓
Sleep disruption	✓			✓	✓			

The cost of pollution

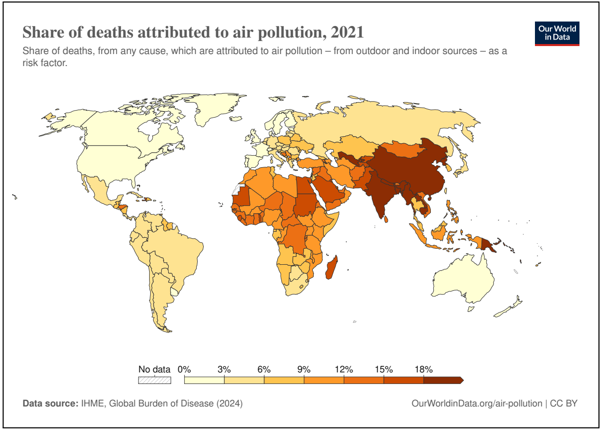
Table 44: Global estimated pollution-attributable deaths (in millions) in 2019

Pollution type	Female	Male	Total	in %
Total air pollution	2.92	3.75	6.67	74.0
Household air	1.13	1.18	**2.31**	25.6
Ambient particulate	1.70	2.44	**4.14**	45.9
Ambient ozone	0.16	0.21	0.37	4.1
Total water pollution	0.73	0.63	1.36	15.1
Unsafe sanitation	0.40	0.36	0.76	8.4
Unsafe source	0.66	0.57	**1.23**	13.7
Total occupational pollution	0.22	0.65	0.87	9.7
Carcinogens	0.07	0.28	0.35	3.9
Particulates	0.15	0.37	0.52	5.8
Lead pollution	0.35	0.56	0.90	10.0
Total pollution	3.92	5.09	**9.01**	100.0

Source: Fuller *et al.* (2022).

The cost of pollution

Figure 58: Share of deaths attributed to air pollution (2021)



Source: Our World in Data, `https://ourworldindata.org/air-pollution`.

The cost of pollution

We have to distinguish:

1. Death and premature mortality
2. Morbidity due to illness and disability

Non-monetary value of health impacts

- Years of life lost (YLL)
- Years lost/lived with disability (YLD)
- Disability-adjusted life years (DALY)
- Days lived with illness (DLI)

$$\mathrm{DLI} = \frac{365 \times \mathrm{YLD}}{\mathrm{DW}}$$

where DW is the disability weight

The cost of pollution

- For premature deaths, World Bank (2022) reported that **6.45 million premature deaths** in 2019 were attributed to exposure to $PM_{2.5}$ pollution
- $PM_{2.5}$ was responsible for about 8.1% of global mortality
- Globally, 64.2% of all $PM_{2.5}$ deaths were due to ambient air pollution (outdoor pollution), while 35.8% were due to household air pollution from the use of solid fuels (indoor pollution)
- About 95% of these deaths occurred in low- and middle-income countries, with 27.7% in China, 24.6% in India, 3.6% in Pakistan, 3.1% in Nigeria, and 2.9% in Indonesia
- In terms of morbidity, World Bank (2022) estimated that air pollution has caused **21 million years lived with disability** (YLD) and **93 billion days lived with illness** (DLI) in 2019

The cost of pollution

Table 45: Global burden of morbidity from $PM_{2.5}$ exposure in 2019

Disease	YLD (in mn)	YLD (in %)	DLI (in bn)	DLI (in %)	DW (in %)
Type 2 diabetes	6.653	31.34	30.252	32.52	8.03
COPD	5.831	27.47	22.780	24.49	9.34
Stroke	5.028	23.68	10.497	11.29	17.48
Cataracts	2.143	10.10	11.370	12.22	6.88
IHD	1.248	5.88	16.654	17.90	2.74
LRI	0.198	0.93	1.214	1.31	5.95
Lung cancer	0.099	0.47	0.214	0.23	16.92
Neonatal disorders	0.019	0.09	0.035	0.04	20.32
Other	0.008	0.04	0.000	0.00	
Total	21.229	100.00	93.016	100.00	8.33

Source: World Bank (2022, Table 3.6, page 22) & Author's calculations.

Global economic costs

- The *Lancet* Commission estimated that welfare losses due to pollution are more than $4.6 trillion per year, or **6.2% of global economic output**
- 81.5% of these economic losses are due to ambient air pollution and household air pollution
- The remainder is explained by lead exposure (9.8%) and water pollution (8.7%)

Global economic costs

> *"The economic costs of air pollution from fossil fuels are estimated at $2.9 trillion in 2018, or 3.3% of global GDP [...] An estimated 4.5 million people died in 2018 due to exposure to air pollution from fossil fuels. On average, each death was associated with a loss of 19 years of life. [...] Fossil fuel $PM_{2.5}$ pollution was responsible for 1.8 billion days of work absence, 4 million new cases of child asthma and 2 million preterm births, among other health impacts that affect healthcare costs, economic productivity and welfare." (Myllyvirta, 2020, page 2).*

The distribution of this total cost is as follows:

- 84% is attributed to adult deaths
- The other factors are disability due to chronic diseases (7%), sick leave (3.5%), preterm births (3.15%), child deaths (1.75%), and asthma (0.6%)

Global economic costs

Table 46: Economic costs of air pollution from fossil fuels (% of GDP, 2018)

Country	Cost	Country	Cost	Country	Cost	Country	Cost
China	6.6%	Bulgaria	6.0%	Hungary	6.0%	Ukraine	5.8%
Serbia	5.8%	Belarus	5.4%	India	5.4%	Romania	5.3%
Bangladesh	5.1%	Moldova	5.0%	Poland	4.9%	Russia	4.1%
Germany	3.5%	South Korea	3.4%	USA	3.0%	Japan	2.5%
UK	2.3%	France	2.0%	Spain	1.7%	Brazil	0.8%

Source: Myllyvirta (2020, page 6) & Centre for Research on Energy and Clean Air (CREA).

Global economic costs

Monetary economic value of health impacts

- Quality-adjusted life year (QALY)

$$\text{QALY} = \text{Years of Life Gained} \times \text{Average Quality of Life Weight}$$

- Quality-adjusted life expectancy (QALE)

$$\text{QALE} = \frac{1}{S(t)} \int_t^T e^{-\varrho(u-t)} \mathbf{S}(u) \, \mathbb{E}[Q(u)] \, \mathrm{d}u$$

where $\mathbf{S}(t)$ is the survival function at time t, ϱ is the discount rate and $Q(t) \in [0,1]$ is the quality life of weight at time t

Global economic costs

Monetary economic value of health impacts

- Value of a statistical life (VSL)

$$\mathrm{VSL} = \frac{v}{\Delta L}$$

where v is the monetary value of the risk reduction and ΔL is the expected number of lives saved

- Value per statistical life year (VSLY) or value of a life year (VOLY)

$$\mathrm{VSLY} = \frac{\mathrm{VSL}}{\dfrac{1}{\mathbf{S}(t)} \displaystyle\int_t^T e^{-\varrho(u-t)} \mathbf{S}(u) \, \mathrm{d}u}$$

- Willingness-to-pay (WTP)

Global economic costs

Table 47: Annual cost of health damages from $PM_{2.5}$ by region (% of GDP, 2019)

Region	Outdoor	Indoor	Mortality	Morbidity	Total
East Asia and Pacific (EAP)	7.3%	2.0%	8.1%	1.2%	9.3%
Europe and Central Asia (ECA)	4.4%	0.2%	4.0%	0.6%	4.6%
Latin America and Carribean (LAC)	2.7%	0.7%	2.9%	0.5%	3.4%
Middle Easth and North Africa (MNA)	5.5%	0.0%	4.7%	0.8%	5.5%
North America (NA)	1.7%	0.0%	1.3%	0.4%	1.7%
South Asia (SA)	5.9%	4.3%	8.3%	2.0%	10.3%
Sub-Saharan Africa (SSA)	3.6%	2.4%	5.2%	0.8%	6.0%
Low-income countries	1.3%	4.6%	5.0%	0.9%	5.9%
Lower-middle-income countries	5.4%	3.6%	7.5%	1.5%	9.0%
Upper-middle-income countries	7.1%	1.8%	7.8%	1.1%	8.9%
High-income non OECD countries	4.3%	0.2%	4.0%	0.5%	4.5%
High-income OECD	2.8%	0.0%	2.3%	0.5%	2.8%

Source: World Bank (2022, Figures 3.13 & 3.14, pages 20 & 21) & Author's calculations.

Global economic costs

Table 48: Annual cost of health damages from $PM_{2.5}$ by country (% of GDP, 2019)

Region	Top 1 country		Top 2 country		Top 3 country	
EAP	China	12.9%	Papua New Guinea	12.0%	Myanmar	11.4%
ECA	Serbia	18.9%	Bulgaria	16.3%	North Macedonia	15.9%
LAC	Barbados	8.8%	Haiti	8.1%	Trinidad/Tobago	7.8%
MNA	Egypt	8.6%	Morocco	7.3%	Tunisia	6.5%
NA	USA	1.7%	Canada	1.2%		
SA	India	10.6%	Nepal	10.2%	Pakistan	8.9%
SSA	Burkina Faso	9.1%	Mali	9.1%	Central African Republic	8.7%

Source: World Bank (2022, Table 3.5, page 21).

Global economic costs

Table 49: Annual cost of health damages from $PM_{2.5}$ by country in 2019 (Top 15)

Country	Total (in \$ bn)	Total (in % of GDP)	Outdoor (in %)	Indoor (in %)	Mortality (in %)	Morbidity (in %)
China	3 029	12.9	79	21	88	12
India	1 022	10.6	60	40	81	19
United States	373	1.7	100	0	78	22
Russia	241	5.7	97	3	91	9
Indonesia	220	6.6	56	44	85	15
Japan	210	3.8	100	0	82	18
Germany	178	3.8	100	0	81	19
Turkey	134	5.8	99	1	85	15
Italy	132	5.0	99	1	83	17
Poland	127	9.8	91	9	86	14
South Korea	114	5.1	100	0	83	17
Egypt	105	8.6	100	0	87	13
Mexico	104	4.0	78	22	84	16
Saudi Arabia	96	5.7	100	0	89	11
Pakistan	94	8.9	48	52	82	18

Source: World Bank (2022, Table A.3, page 40) & Author's calculations.

Global economic costs

A basic economic model of pollution costs

- $Y = F(K, L, P)$, where Y is economic output, K is capital, L is labor, and P is pollution
- The labor input L is expressed as $L = N \cdot A_L \cdot T_L$ where N is the workforce size (or the population), A_L is labor productivity, $T_L = \tau - \varsigma$ is the time individuals spend working, which is the difference between the total endowment of labor time τ and sick time ς
- The output function can be rewritten as:

$$Y = F\left(K, N(P) A_L(P) (\tau - \varsigma(P)), P\right)$$

- Let $\varepsilon_L \geq 0$ be the elasticity of output with respect to labor and $\theta = \varsigma / (\tau - \varsigma)$ be the ratio of sick time to effective labor time. We get:

$$\frac{\mathrm{d} \ln Y}{\mathrm{d} P} = \underbrace{\epsilon_L \left(\frac{\partial \ln N}{\partial P} + \frac{\partial \ln A_L}{\partial P} - \theta \frac{\partial \ln \varsigma(P)}{\partial P} \right)}_{\text{Pollution-related labor impact}} + \frac{\partial \ln Y}{\partial P}$$

Global economic costs

The previous equation can be expressed in the following compact form:

$$\beta_P = \epsilon_L \beta_{L,P} + \beta_{L,P}$$

Pollution has an indirect impact on output through the labor factor, which operates through three dimensions:

1. Pollution increases mortality, reducing the size of labor force: $\dfrac{\partial \ln N}{\partial P} < 0$.
2. Pollution increases morbidity, decreasing labor productivity: $\dfrac{\partial \ln A_L}{\partial P} < 0$.
3. Pollution increases morbidity, leading to more work absences: $\frac{\partial \ln \varsigma(P)}{\partial P} > 0$.

$\Rightarrow$ The labor-related impact of pollution is **always negative** ($\epsilon_L \beta_{L,P} \leq 0$), while the direct impact of pollution on output can be **either positive or negative** ($\beta_{L,P} = \dfrac{\partial \ln Y}{\partial P} \lessgtr 0$)

Economic costs in Europe

- Dechezleprêtre *et al.* (2019) estimated that a $1\,\mu\mathrm{g}/\mathrm{m}^3$ increase in $\mathrm{PM}_{2.5}$ concentration leads to a 0.8% reduction in real GDP within the same year
- 95% of this impact is attributed to a decline in output per worker (higher absenteeism or reduced labor productivity)

Economic costs in Europe

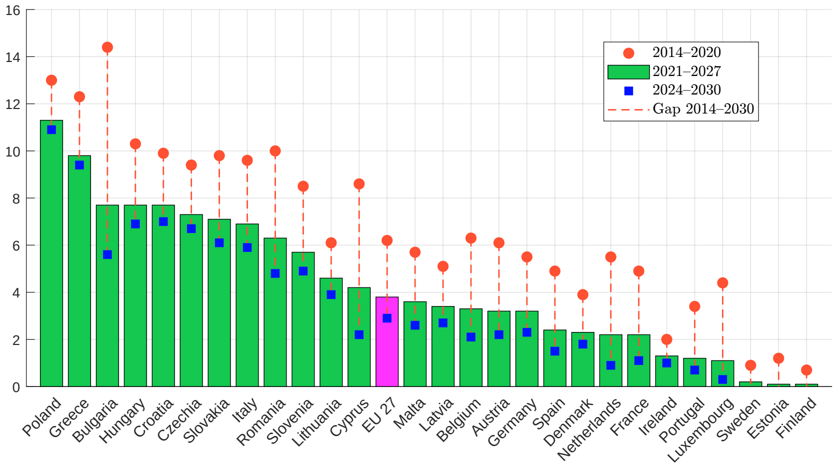
Mejino-López and Oliu-Barton (2024) estimated the costs of particulate matter $PM_{2.5}$ as follows:

$$\mathcal{C} = \beta_P \cdot \left([C] - \mathrm{AQG} \right)^+ \cdot \mathrm{GDP} = 0.8\% \cdot \max \left([C]_{\mathrm{PM}_{2.5}} - 5\,\mu\mathrm{g}/\mathrm{m}^3, 0 \right) \cdot \mathrm{GDP}$$

"Despite significant progress, air pollution still causes €600 billion in losses each year in the European Union — equal to 4% of its annual GDP. These costs stem from productivity losses such as increased absenteeism, the reduction of in-job productivity and harm to ecosystems. Air pollution costs are disproportionately high in eastern Europe and Italy, where losses are projected to remain above 6% of GDP until 2030. The EU's 10% most-polluted regions suffer 25% of the burden of mortality attributable to air pollution." (Mejino-López and Oliu-Barton, 2024, page 1).

Economic costs in Europe

Figure 59: Estimated cost of air pollution by country (in % of GDP)



Economic costs in Europe

EEA (2024) considers the following groups of pollutants:

- Main air pollutants: particulate matter ($PM_{2.5}$, PM_{10}), sulfur dioxide (SO_2), ammonia (NH_3), nitrogen oxides (NO_x), and non-methane volatile organic compounds (NM VOC);
- Greenhouse gases (GHG): carbon dioxide (CO_2), methane (CH_4) and nitrous oxide(N_2O);
- Heavy metals: arsenic (As), cadmium (Cd), chromium VI (Cr), lead (Pb), mercury (Hg), and nickel (Ni);
- Organic pollutants: 1,3 butadiene, benzene, formaldehyde, polycyclic aromatic hydrocarbons, dioxins and furans.

The study examines only air pollution and estimates the external costs of these facilities, taking into account both health effects (mortality and morbidity and non-health effects (damage to buildings, crops, and forests).

Economic costs in Europe

Table 50: External costs of air pollution (2021)

Country	Relative cost		Breakdown of external costs	
	VOLY/GDP	VSL/GDP	VOLY	VSL
Austria	0.71%	1.07%	1.43%	1.34%
Belgium	1.59%	2.68%	3.96%	4.16%
Bulgaria	7.87%	16.11%	2.77%	3.53%
Croatia	1.72%	2.89%	0.50%	0.52%
Cyprus	2.89%	3.02%	0.34%	0.22%
Denmark	0.47%	0.64%	0.78%	0.66%

Economic costs in Europe

Table 51: External costs of air pollution (2021)

Country	Relative cost		Breakdown of external costs	
	VOLY/GDP	VSL/GDP	VOLY	VSL
Estonia	3.35%	3.75%	0.52%	0.36%
Finland	2.12%	2.36%	2.63%	1.82%
France	0.68%	1.07%	8.39%	8.25%
Germany	1.54%	2.63%	27.41%	29.19%
Greece	2.93%	4.76%	2.63%	2.67%
Hungary	2.21%	4.27%	1.69%	2.03%

Economic costs in Europe

Table 52: External costs of air pollution (2021)

Country	Relative cost		Breakdown of external costs	
	VOLY/GDP	VSL/GDP	VOLY	VSL
Ireland	0.50%	0.62%	1.06%	0.82%
Italy	1.06%	1.79%	9.35%	9.87%
Latvia	0.69%	1.05%	0.11%	0.11%
Luxembourg	0.35%	0.65%	0.13%	0.15%
Netherlands	1.37%	1.91%	5.81%	5.03%
Poland	5.63%	8.68%	16.06%	15.43%

Economic costs in Europe

Table 53: External costs of air pollution (2021)

Country	Relative cost		Breakdown of external costs	
	VOLY/GDP	VSL/GDP	VOLY	VSL
Portugal	1.67%	2.18%	1.77%	1.45%
Romania	2.47%	4.55%	2.96%	3.38%
Slovenia	1.76%	2.99%	0.46%	0.48%
Spain	1.07%	1.77%	6.40%	6.58%
Sweden	1.07%	1.18%	2.86%	1.95%
EU-27	1.39%	2.23%		

Source: EEA (2024, Figure 4.2, Tables 4.2 & 4.3, pages 31–34), Author's calculations & icons taken from `https://icons8.com/icons`.

Economic costs in Europe

Table 54: External costs of air pollution in € mn (2021)

Pollutants	2012	2021	Cumulative	2021 Breakdown
Main air (VOLY)	119 042	59 728	834 066	SO_2: 45.3%, NO_x: 41.1%, NH_3: 7.5%, PM_{10}: 5.0%, NM VOC: 1.2%
Main air (VSL)	329 152	193 056	2 426 585	SO_2: 45.7%, NO_x: 40.5%, NH_3: 7.7%, PM_{10}: 5.0%, NM VOC: 1.0%
GHG	193 641	150 657	1 728 224	CO_2: 94.5%, CH_4: 4.9%, N_2O: 0.6%
Heavy metals	13 803	8 924	120 622	Pb: 79.6%, Cd: 15.9%, Hg: 3.2%, As: 1.3%, Cr(VI): 0.1%, Ni: 0.0%
Organic	66	69	1 071	Dioxins: 82.6%, B(a)P: 15.9%, Benzene: 1.4%
Total (VOLY)	326 553	219 378	2 683 984	GHG: 68.7%, Main air: 27.2%, Heavy metals: 4.1%, Organic: 0.0%
Total (VSL)	536 663	352 707	4 276 503	Main air: 54.7%, GHG: 42.7%, Heavy metals: 2.5%, Organic: 0.0%

Source: EEA (2024, Table 4.1, Figures 4.4–4.7, pages 28 & 35–37) & Author's calculations.

Economic costs in Europe

For the main air pollutants, the average marginal costs for 2019, expressed in 2021 thousand euros per kg of pollutant emitted into the air, are as follows:

Pollutant	NO_x	SO_2	PM_{10}	$PM_{2.5}$	NM VOC	NH_3
MDC (VOLY)	15.4	16.2	51.5	86.5	1.8	19.0
MDC (VSL)	43.0	38.3	141.1	237.1	4.5	52.3

while for heavy metals and organic compounds they find these values:

Pollutant	Arsenic	Cadmium	Chromium VI	Lead	Mercury
MDC	10.3	253.1	0.7	45.2	16.8
Pollutant	Nickel	1,3 Butadiene	Benzene	B(a)P	Dioxins
MDC	0.02	0.00	0.00	1.4	132 600

Overexploitation and resource extraction

"Although there is considerable variation in detail, there is remarkable consistency in the history of resource exploitation: ***resources are inevitably overexploited****, often to the point of collapse or extinction." (Ludwig et al., 1993, page 17).*

The Tragedy of the Commons

- Published in Science in 1968, **The Tragedy of the Commons** by Garrett Hardin is a seminal essay that explores the conflict between individual interests and the common good in the context of shared resources
- The article focuses on how individuals, acting in their own self-interest, can deplete or degrade shared resources (referred to as '*commons*'), even when it is not in the collective best interest to do so
- Hardin (1968) illustrates this dilemma with a hypothetical example of a shared pasture (the commons) where each herder individually benefits by adding more cattle to the pasture. Each herder reasons that adding one more animal will bring him personal gain, while the negative consequences (overgrazing) are shared by all users of the commons
- From an individual perspective, it's rational to add more animals. However, if every herder follows this logic, the collective overuse of the pasture leads to its destruction, as the resources become insufficient to sustain the community
- Dilemma of **individual gain versus collective ruin**

Overexploitation and resource extraction

"Extraction of living biomass and nonliving materials is increasing as both populations and per capita consumption increased sixfold from 1970 to 2010, while the demand for materials used in construction and industry quadrupled during that time. [...] Materials for construction and industry increased 4-fold, with the most dramatic increases for lower-middle (7-fold) and upper-middle income countries (11-fold) and the Asia and the Pacific region (10-fold for whole region) and, generally, the growing economies. The use of biomass, fossil fuels, metal ores and non-metallic minerals doubled from 2005 (26.3 billion tons) to 2015 (46.4 billion tons), growing an annual rate of 6.1%." (IPBES, 2019, page 121).

The example of megafauna

Svenning *et al.* (2021) found that among terrestrial mammals, only 11 of 57 species of megaherbivores (mean adult body mass greater than 1 000 kg) survived to 1 000 AD:

Status	Cetartiodactyla	Cingulata	Diprotodontia	Litopterna	Notoungulata	Rhinocerotidae	Pilosa	Elephantidae	Proboscidea	Total
Survivor	4					4		3		11
Extinct	4	4	1	2	3	4	16	7	5	46

Survivors include:

- 3 species of elephant
- 4 species of rhinoceros
- The common hippopotamus
- The giraffe
- 2 species of cattle

The example of megafauna

Table 55: Current threats to megafauna

Megafauna	n	Animal husbandry	Climate	Cropping	Development	Harvesting	Invasives	Pollution	Transportation
Cartilaginous fish	38	3%	11%		13%	100%		13%	8%
Mammals	73	53%		61%	56%	98%	54%		51%
Ray-finned fish	29				14%	100%	31%	40%	28%
Reptiles	20		40%	25%	40%	90%	65%		25%

Source: Ripple *et al.* (2019, Figure 2, page 5).

The example of megafauna

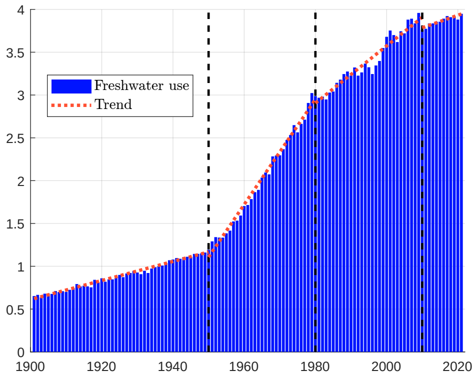
An example with the North American buffalo (*Bison bison*)

> *"Prior to European exploration and settlement of North America, the buffalo or American bison inhabited vast stretches of the continent. [...] At its greatest moment, the total numbers for the continent may have been as high as 25 to 30 million before white settlement. On the Great Plains, where the bison were most suited and most plentiful, its population is estimated to have been* ***20 million as late as 1800****. Even by 1850, substantially more than 10 million bison roamed the plains. Yet,* ***by 1890, these plains held just 1 000 bison****." (Lueck, 2002, page 609).*

- After 1890, the story of the American bison took a more hopeful turn, thanks to conservation efforts. Over time, the bison population began to recover
- Today, bison are no longer endangered. There are approximately 400 000 bison in North America, but most are managed as livestock on private ranches. In fact, only about 30 000 are wild bison

The example of freshwater

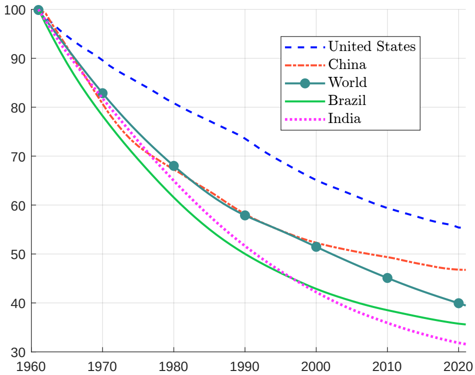
Figure 60: Global freshwater withdrawals (in trillion m^3 per year)



Source: `https://databank.worldbank.org/source/world-development-indicators` & Author's calculations.

The example of freshwater

Figure 61: Renewable internal freshwater resources per capita (base 100 in 1961)



Source: `https://databank.worldbank.org/source/world-development-indicators` & Author's calculations.

The example of freshwater

Table 56: Freshwater withdrawals by country

Country	Total		Agriculture	Industry	Domestic	Renewable resources	Renewable per capita	Water stress
	2000	2021	2021			2021		
	(in bn m^3)		(in %)			(in bn m^3)	(in m^3)	(in %)
Algeria	5.6	9.8	63.8	1.8	34.4	11	251	137.9
Brazil	56.1	67.3	61.3	14.5	24.2	5 661	27 015	1.5
Canada	41.9	36.3	11.4	74.2	14.4	2 850	74 530	3.7
China	550.9	568.5	62.1	17.7	20.1	2 813	1 992	41.5
Congo (DRC)	0.6	0.7	10.5	21.5	68.0	900	9 077	0.2
Egypt	57.0	77.5	79.2	7.0	13.9	1	9	141.2

The example of freshwater

Table 57: Freshwater withdrawals by country

France	32.7	24.7	13.9	64.3	21.7	200	2 948	21.6
Iceland	0.2	0.3	0.1	71.1	28.7	170	456 351	0.4
India	610.4	647.5	90.4	2.2	7.4	1 446	1 022	66.5
Iran	88.5	93.0	92.2	1.2	6.6	128	1 453	81.3
Madagascar	13.4	13.5	95.9	1.2	2.9	337	11 350	11.3
Pakistan	172.6	264.2	94.0	0.8	5.3	55	230	162.1
Qatar	0.2	0.2	33.3	4.3	62.4	0	22	431.0
Russia	75.9	64.8	28.8	44.8	26.5	4 312	29 790	4.1
Saudi Arabia	19.7	23.4	81.6	5.4	13.1	2	78	974.2
Spain	36.1	29.0	65.3	19.0	15.7	111	2 345	43.3
Sudan		26.9	96.2	0.3	3.5	4	83	118.7
United Kingdom	12.5	8.4	14.0	12.0	74.0	145	2 163	14.4
United States	473.5	444.4	39.7	47.2	13.1	2 818	8 487	28.2
World	3 679.8	3 948.7	71.6	15.1	13.1	42 809	5 429	

Source: `https://databank.worldbank.org/source/world-development-indicators`.

The example of freshwater

- The **water stress** is defined as the ratio of total freshwater withdrawals to total renewable freshwater resources, referring specifically to internal freshwater resources
- Water stress is considered **high when the ratio exceeds 75%** and **critical when it exceeds 100%**
- Some countries are experiencing critical water stress, including Algeria, Egypt, Libya, Pakistan, Qatar, Saudi Arabia, and Sudan
- As of 2021, there are 25 countries with water stress levels exceeding 75%.

Mathematical models of population and resource ecology with harvesting

- **Malthus model** (1798)

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \delta N(t)$$

  where $N(t)$ is the number of individuals, $N(t_0) = N_0$ is the initial population size and $\delta = \lambda - \mu$ is the difference between the birth rate and the death/mortality rate

- **Verlust model** (1838)

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \delta N(t) - \varphi(N(t))$$

- By considering the specific case where $\varphi(x) = \eta x^2$, we obtain the **logistic population model**:

$$N(t) = \frac{\delta N_0 e^{\delta(t-t_0)}}{\delta + \eta N_0 \left(e^{\delta(t-t_0)} - 1\right)}$$

Mathematical models of population and resource ecology with harvesting

- The **Lotka-Volterra model** (1928)

$$\left\{\begin{array}{l}\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)y(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = cx(t)y(t) - dy(t)\end{array}\right.$$

  where $x(t)$ is the prey population, $y(t)$ is the predator population, a is the intrinsic growth rate of the prey (in the absence of predators), b is the predation rate coefficient, c is the reproduction rate of predators per prey consumed, and d is the natural mortality rate of predators

Logistic growth model with harvesting

- $x(t)$ represents the size of a population, the amount of a resource or the biomass stock of a species in a finite environment
- When the stock $x(t)$ is small, the size of the environment has no impact, and $x(t)$ can grow at an exponential rate δ, also known as the intrinsic growth rate
- Beyond a certain threshold, the density of the stock or population becomes too high to sustain the growth rate, and the regenerative rate of the stock decreases
- This suggests the existence of a threshold κ at which $x(t)$ remains constant
- 3 factors that drive the population dynamics:
  1. The **constant intrinsic growth rate** δ (or biotic potential)
  2. The **regenerative rate** $\xi(t) = \dfrac{\kappa - x(t)}{\kappa}$ (or biotic resistance), which depends on the distance between the current stock $x(t)$ and the threshold κ which is called the **carrying capacity**
  3. The **current stock** $x(t)$

Logistic growth model with harvesting

- We obtain following differential equation:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \delta\xi(t)\,x(t) = \delta\left(\frac{\kappa - x(t)}{\kappa}\right)x(t)$$

- This model corresponds to the logistic model, where $\eta = \delta/\kappa$:

$$x(t) = \frac{\kappa N_0 e^{\delta(t-t_0)}}{\kappa + N_0\left(e^{\delta(t-t_0)} - 1\right)}$$

- We verify that $x(t)$ tends asymptotically to κ as $t \to \infty$
- The model can be easily modified by considering exploitation:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \delta x(t)\left(1 - \frac{x(t)}{\kappa}\right) - h(x(t))$$

where $h(x(t))$ is the harvest, catch or removal rate

Logistic growth model with harvesting

- The **maximum sustainable yield** (MSY) is the largest harvest rate of a renewable resource that can be sustained indefinitely without causing the population to decline
- The sustainable harvest rate is given by:

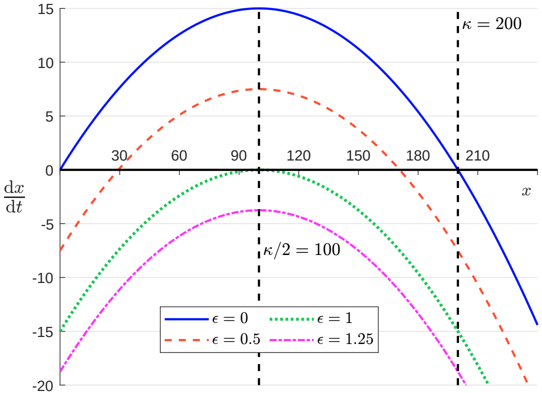
$$h(x(t)) = \delta x(t) \left(1 - \frac{x(t)}{\kappa}\right)$$

- The maximum sustainable yield is achieved when the stock is at half the carrying capacity: $x(t) = \frac{\kappa}{2}$
- We deduce that:

$$\text{MSY} := \max h(x(t)) = h\left(\frac{\kappa}{2}\right) = \frac{\delta}{4}\kappa$$

Logistic growth model with harvesting

Figure 62: Stability analysis of the logistic model with harvesting



Logistic growth model with harvesting

Figure 63: Simulation of the logistic model with harvesting

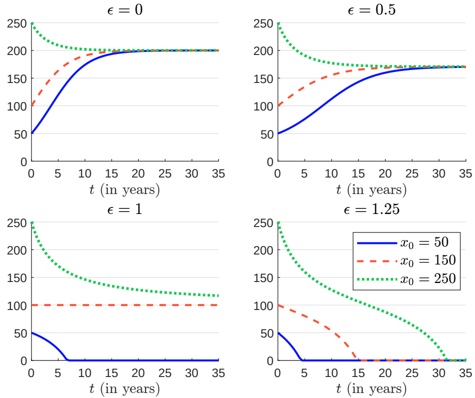
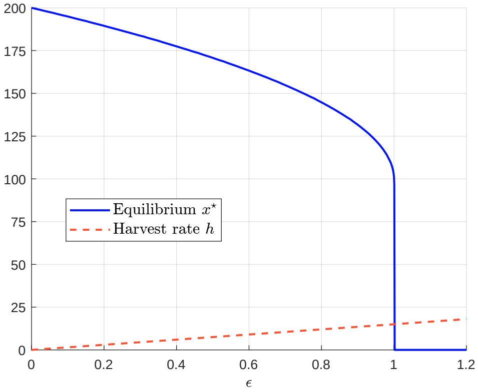


Figure 64: Equilibrium $x^\star$ and harvest rate h



Mathematical models of population with harvesting

- The relative harvest rate, defined as the ratio of the absolute harvest rate to the carrying capacity, must be less than one quarter of the intrinsic growth rate:

$$\eta = \frac{h}{\kappa} < \eta^\star = \frac{\delta}{4}$$

- For example, if $\delta = 30\%$ per year, then $\eta^\star = 7.5\%$, meaning that the relative harvest rate must be less than 7.5% per year
- Conversely, if $\delta = 2\%$, the relative harvest rate must be much lower, less than 0.5% per year ($\eta^\star = 0.5\%$

Mathematical models of population with harvesting

Here are some typical values of intrinsic growth rate:

- Fish species
  In general, small and fast growing fish species such as sardines and anchovies can have rates of 100% or higher, while large and slow growing fish species such as sharks and tuna often have rates below 10%. Many commercial fish species, such as cod and haddock, have intrinsic growth rates between 10% and 50% per year.
- Mammals
  Large mammals such as whales and elephants typically have low rates (5%–20% per year) because they reproduce slowly and have long gestation periods, while small mammals such as rodents and rabbits can have higher rates (50%–200% per year), reflecting their ability to reproduce rapidly.
- Bacteria
  Bacteria can have very high intrinsic growth rates, sometimes doubling several times a day. For example, Escherichia coli (E. coli) can have rates of 20+ per day under optimal conditions.

Mathematical models of population with harvesting

Example #3

Consider the collapse of the American buffalo. Suppose the intrinsic growth rate is 10% per year, and the initial buffalo population is at its carrying capacity ($x_0 = \kappa$).

Table 58: Proportion in % of remaining buffalo under different harvest rate assumptions

	η (in %)	1.00	2.00	3.00	4.00	4.10	4.20	4.30	4.40
t	10 years	93.55	86.81	79.77	72.40	71.65	70.89	70.12	69.36
	20 years	90.88	80.64	68.95	55.35	53.85	52.34	50.79	49.22
	30 years	89.71	77.32	61.63	40.10	37.45	34.68	31.78	28.74
	40 years	89.18	75.41	55.85	21.32	16.04	10.18	3.62	0.00

Mathematical models of population with harvesting

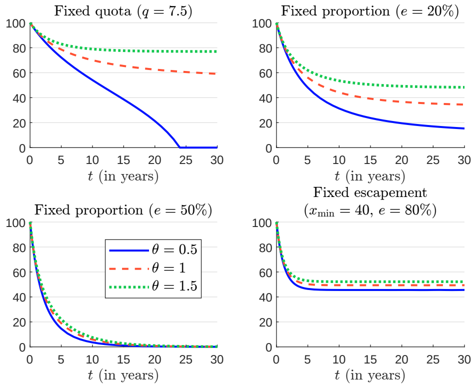
The three most popular functions $h(x)$ are:

1. **Fixed quota** (or constant catch) management
   This specifies a predetermined, fixed number of animals that can be harvested. In this case, $h(x) = q$ is a constant, corresponding to the case we have already studied.
2. **Fixed proportion** harvesting
   This specifies a proportion e of animals that can be harvested, rather than a specific number:
   $$h(x) = ex$$
   where e is the exploitation rate expressed as a percentage.
3. **Fixed escapement** (or constant escapement rule)
   This specifies not the number of animals to be harvested, but rather the number of animals to remain unharvested. In this approach, harvest occurs only when the population exceeds a threshold $x_{\min}$, ensuring a minimum escapement:
   $$h(x) = e(x - x_{\min})^{+}$$

Mathematical models of population with harvesting

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \delta x(t)\left(1-\left(\frac{x(t)}{\kappa}\right)^{\theta}\right) - h(x(t))$$

Figure 65: Impact of the harvest function and the inflection point



Lotka-Volterra model with harvesting

Overexploitation can be studied using a second class of multi-species models based on the Lotka-Volterra formulation:

$$\left\{\begin{array}{l}\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = ax(t) - bx(t)\,y(t) - h_x(x(t), y(t)) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = cx(t)\,y(t) - dy(t) - h_y(x(t), y(t))\end{array}\right.$$

where $h_x(x, y)$ and $h_{x,y}(y)$ are the harvest functions of the prey and predator species, respectively

Lotka-Volterra model with harvesting

Figure 66: Simulation ($a = 2$, $b = 3$, $c = 2$, $d = 4$)

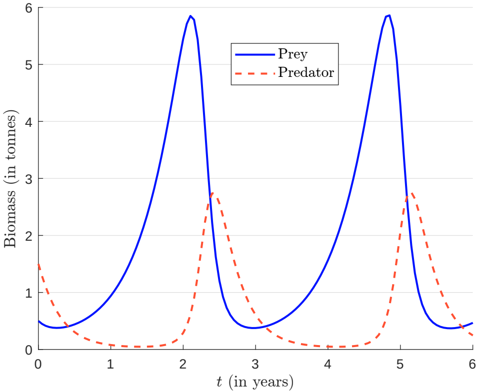
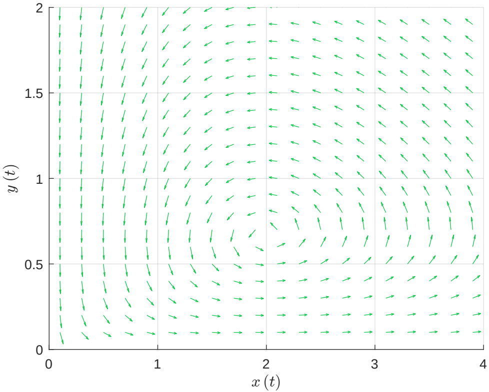


Figure 67: Vector field representation



Lotka-Volterra model with harvesting

Figure 68: Phase portrait ($a = 2$, $b = 3$, $c = 2$, $d = 4$)

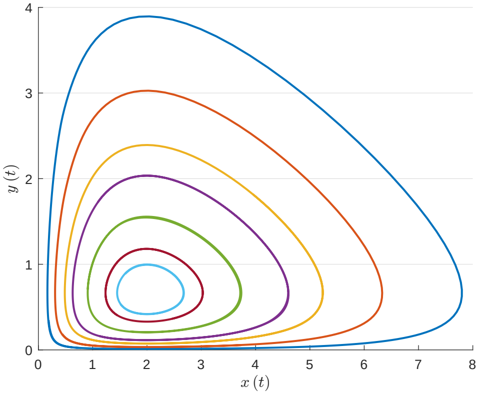
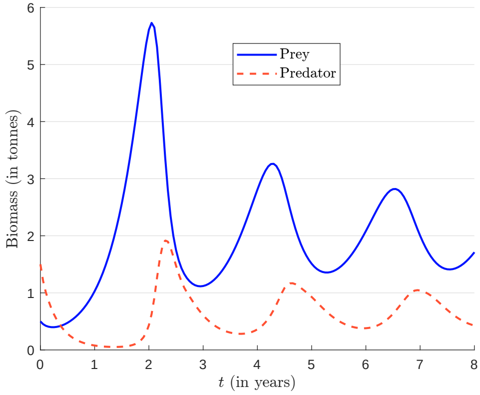


Figure 69: Simulation with harvesting ($e_y = 5$, $y_{\min} = 1$)



Predation models

1. The **functional response** describes how the predation rate (*i.e.*, the number of prey consumed per predator) varies with prey density. It quantifies the efficiency of individual predators in capturing and consuming prey.
2. The **numerical response** describes how predator population density changes in response to prey density. It reflects the total population-level effect of prey availability on predator numbers, including factors such as predator consumption, reproduction and migration, but excluding natural mortality.

From a mathematical point of view, we can write:

$$\left\{ \begin{array}{l} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = \delta(t)\, x(t) - f(x(t), y(t))\, y(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = g(x(t), y(t))\, y(t) - \mu(t)\, y(t) \end{array} \right.$$

where $\delta(t)$ is the growth rate of the prey species, $f(x, y)$ is the functional response, $g(x, y)$ is the numerical response, and $\mu(t)$ is the mortality rate of the predator species

Predation models

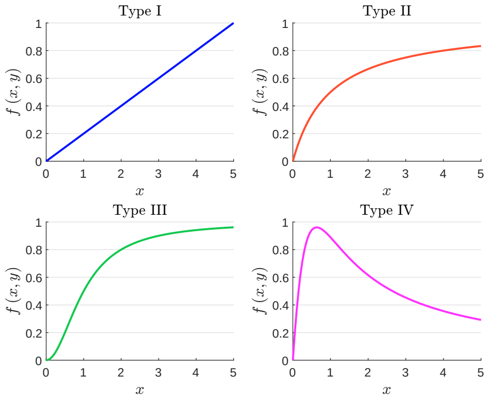
Holling (1959) introduced a classification of predation models based on the form of their functional responses:

- **Type I (linear)**
  Predation rate increases linearly with prey density until a maximum is reached. This simple, though somewhat unrealistic, model assumes that predators can process prey immediately upon encounter. It's often used in simplified models.
- **Type II (decelerating)**
  Predation rate still increases with prey density, but the rate of increase slows as prey becomes more abundant. This reflects factors such as predator handling time (the time it takes to consume each prey item) or satiation (when the predator becomes full).
- **Type III (sigmoidal)**
  Predation rate follows a sigmoidal curve, *i.e.*, a slow initial increase at low prey densities, followed by an accelerated increase at moderate densities, and finally saturation at high densities. This pattern often results from more complex predator behavior, such as learning, improved search efficiency, or the presence of prey refugia. It suggests that predators initially struggle to find or handle prey, but become more efficient over time.

Predation models

Type	I	II	III	IV
$f(x,y)$	cx	$\dfrac{\alpha x}{\beta + x}$	$\dfrac{\alpha x^2}{\beta + x^2}$	$\dfrac{\alpha x}{\beta + x + \gamma x^2}$

Figure 70: Holling functional responses

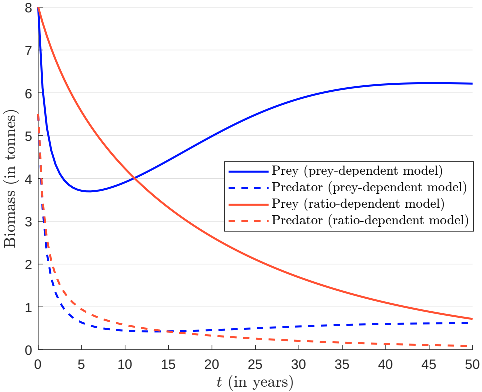


Holling-Tanner predator-prey model

$$\left\{\begin{array}{l}\dfrac{\mathrm{d}x(t)}{\mathrm{d}t}=\delta x(t)\left(1-\dfrac{x(t)}{\kappa}\right)-\dfrac{\alpha x(t)}{\beta+x(t)}y(t)\\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t}=\sigma y(t)\left(1-\dfrac{y(t)}{\gamma x(t)}\right)\end{array}\right.$$

where $x(t)$ is the prey population and $y(t)$ is the predator population

Figure 71: Simulation of the Holling-Tanner predator-prey model



Symbiosis and interspecific interactions

7 broad types of symbiosis:

1. Amensalism $(-/0)$
2. Competition $(-/-)$
3. Mutualism $(+/+)$
4. Neutralism $(0/0)$
5. Parasitism $(+/-)$
6. Predation $(+/-)$

Multi-species competition model

- Lotka-Volterra multi-species competition model

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \delta_i x_i(t)\left(1 - \frac{\sum_{j=1}^{n}\omega_{i,j}x_j(t)}{\kappa_i}\right)$$

- Interference competition model

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \delta_i x_i(t)\left(1 - \frac{x_i(t)}{\kappa_i}\right) - \sum_{j\neq i}\alpha_{i,j}x_i(t)\,x_j(t)$$

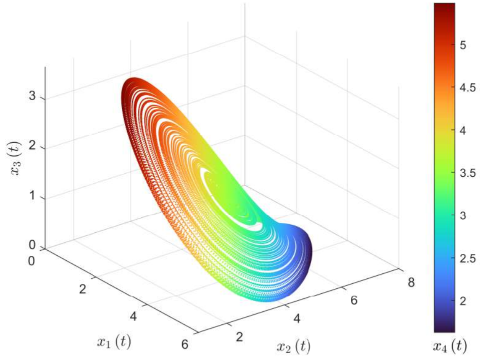
- Interference competition model II

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \delta_i x_i(t)\left(1 - \frac{x_i(t)}{\kappa_i}\right) - \sum_{j\neq i} f_{i,j}\left(x_1(t),\ldots,x_n(t)\right)x_j(t)$$

with Holling type I linear response — $f_{i,j}(x_1,\ldots,x_n) = \alpha_{i,j}x_i$ — or Holling type II responses — $f_{i,j}(x_1,\ldots,x_n) = \dfrac{\alpha_{i,j}x_i}{\beta_{i,j}+x_i}$

Multi-species competition model

Figure 72: Phase portrait of the Lotka-Volterra four-species competition model



Source: Vano *et al.* (2006, Figure 3, page 2386).

MacArthur consumer-resource model

$$
\begin{cases}
\dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} = \beta_i x_i(t) \left(\displaystyle\sum_{j=1}^{m} \alpha_{i,j} \omega_j y_j(t) \right) - \mu_i x_i(t) \\
\dfrac{\mathrm{d}y_j(t)}{\mathrm{d}t} = \delta_j y_j(t) \left(1 - \dfrac{y_j(t)}{\kappa_j} \right) - \displaystyle\sum_{i=1}^{n} \alpha_{i,j} x_i(t)
\end{cases}
$$

where $x_i(t)$ represents the population density of species i, $y_j(t)$ represents the population density of resource j (*e.g.*, food resources), β_i is a conversion factor that translates resource consumption into per capita growth rate, $\alpha_{i,j}$ is the rate at which species i captures resource j, ω_j is the value of a unit of resource j to the consumer (*e.g.*, caloric energy), μ_i is the mortality rate of species i, and δ_j is the intrinsic growth rate of resource j

Figure 73: Robert H. MacArthur

Competitive exclusion principle

Gause's law

- The competitive exclusion principle states that two species competing for exactly the same resource cannot stably coexist indefinitely
- One will outcompete the other
- For coexistence to occur, the species must occupy slightly different niches or use resources differently (niche theory)
- In fact, the relationship between biodiversity and competition is more complex
- There is a gap between previous theoretical and empirical results

Fishing down marine food webs

"With the development of industrial fishing, and the resulting invasion of the refuges previously provided by distance and depth, our interactions with fisheries resources have come to resemble the wars of extermination that newly arrived hunters conducted 40 000–50 000 years ago in Australia, and 12 000–13 000 years ago against large terrestrial mammals in North America." (Pauly et al., 2005, page 5).

The oceans will be empty by 2048

Myers and Worm (2003) estimated the following biomass time-trend model:

$$N_i(t) = N_i(0)\left((1-\varphi_i)\, e^{-r_i(t-t_0)} + \varphi_i\right)$$

where $N_i(t)$ is the biomass at time t, $N_i(0)$ is the initial biomass before industrialized exploitation, r_i is the rate of decline and φ_i is the fraction of the community that remains at equilibrium as $t \to \infty$

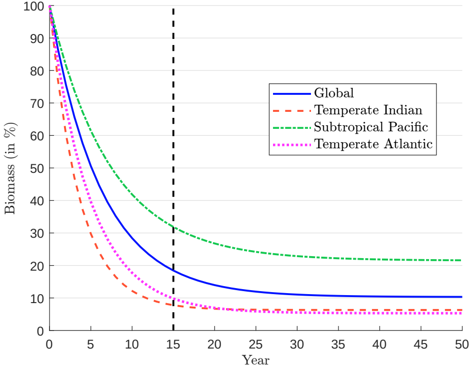
- Estimation by the method of maximum likelihood
- Nonlinear mixed-effects models assuming that $r_i \sim \mathcal{N}(\mu_r, \sigma_r^2)$ and $\varphi_i \sim \mathcal{N}(\mu_\varphi, \sigma_\varphi^2)$

The oceans will be empty by 2048

Region	$\hat{r}_i$	$\hat{\mu}_r$	$\hat{\varphi}_i$	$\hat{\mu}_\varphi$
Tropical Atlantic	16.6	16.7	12.1	11.9
Subtropical Atlantic	12.9	13.0	8.1	8.3
Temperate Atlantic	21.4	20.3	4.7	5.3
Tropical Indian	9.2	9.5	17.6	16.8
Subtropical Indian	6.5	6.8	8.2	9.2
Temperate Indian	30.7	27.7	5.5	6.3
Tropical Pacific	12.1	12.4	15.5	14.9
Subtropical Pacific	12.8	13.5	23.5	21.5
Temperate Pacific	20.8	20.4	8.2	8.5
Gulf of Thailand	25.6	22.2	9.3	9.8
South Georgia	166.6	30.8	20.9	16.0
Southern Grand Banks	4.0	5.7	0.0	10.0
Saint Pierre Banks	5.1	6.3	2.7	7.9
Global		16.0		10.3

Source: Myers and Worm (2003, Table 1, page 281).

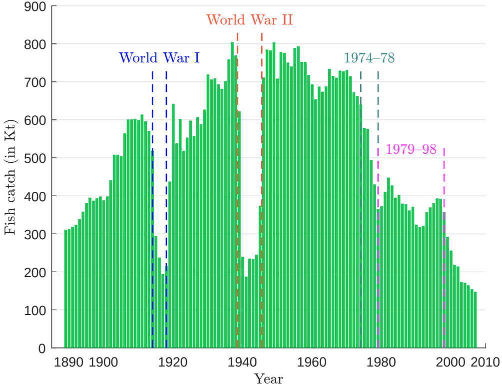
Figure 74: Time trends of community biomass in oceanic and shelf ecosystems



Source: Myers and Worm (2003, Figure 1, page 280).

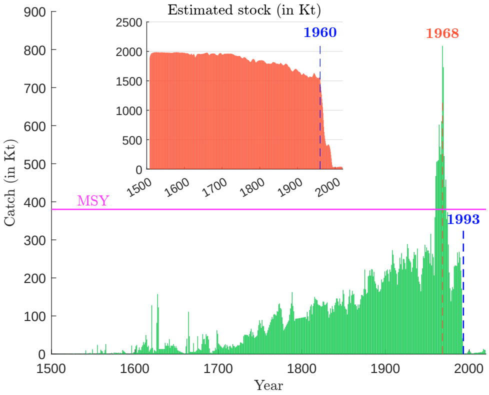
Overexploitation in aquatic systems

Figure 75: Fish catch in the United Kingdom



Source: Thurstan *et al.* (2010, Figure 1a, page 2).

Figure 76: Northern cod catch in eastern Canada



Source: Schijns *et al.* (2021, Figure 1, page 2679).

Overexploitation in aquatic systems

Figure 77: Global seafood production in Mt

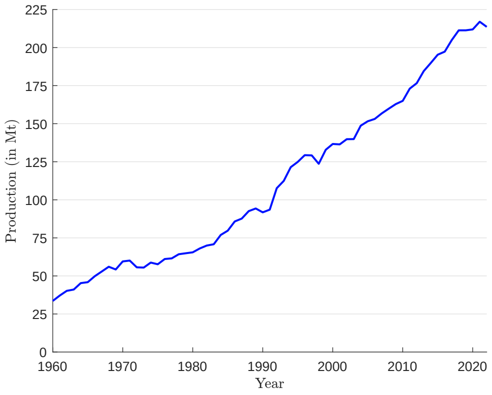
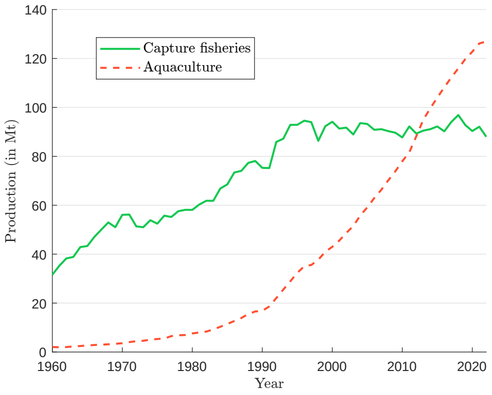


Figure 78: Global fisheries and aquaculture in Mt



Overexploitation in aquatic systems

- Annual growth rate of 3%: multiplication factor of 19 over one century and 369 over two centuries!
- Two distinct phases:
  - From 1960 to 1990, production growth was driven primarily by capture fisheries
  - Since 1990, capture fisheries have remained constant, while aquaculture has experienced impressive growth.
- In 1960, capture fisheries accounted for 95% of seafood production
- Today aquaculture contributes about 60% of total seafood production

Overexploitation in aquatic systems

Table 59: Capture fisheries production

	Country	1960	1980	2000	2010	2020	2022
	World (in Mt)	31.6	58.1	94.1	87.7	90.3	88.0
Top 10 (in %)	China	7.0	5.4	15.8	17.2	14.9	15.0
	Indonesia	2.2	2.8	4.4	6.1	7.7	8.4
	India	3.5	3.6	4.0	5.4	5.2	6.3
	Peru	11.1	4.7	11.3	4.9	6.3	6.1
	Russia	0.0	0.0	4.3	4.6	5.6	5.7
	United States	8.6	6.4	5.1	5.0	4.7	4.8
	Vietnam	1.4	0.8	1.7	2.6	3.9	4.1
	Japan	18.7	17.3	5.5	4.8	3.6	3.4
	Chile	1.1	5.0	4.8	3.5	2.4	3.1
	Norway	4.4	4.4	3.1	3.2	2.9	3.0

Table 60: Aquaculture production

	Country	1960	1980	2000	2010	2020	2022
	World (in Mt)	2.0	7.6	43.0	78.0	122.8	126.9
Top 10 (in %)	China	46.8	40.8	69.2	61.3	57.4	59.4
	Indonesia	3.9	3.0	2.3	8.0	12.1	11.5
	India	2.2	4.8	4.5	4.9	7.0	8.1
	Vietnam	1.8	1.3	1.2	3.5	3.8	4.1
	Bangladesh	2.4	1.2	1.5	1.7	2.1	2.2
	Philippines	3.0	4.4	2.6	3.3	1.9	1.9
	Norway	0.1	0.1	1.1	1.3	1.2	1.3
	Egypt	0.2	0.2	0.8	1.2	1.3	1.2
	Chile	0.0	0.0	1.0	0.9	1.2	1.2
	Myanmar	0.0	0.0	0.2	1.1	0.9	0.9

Source: `https://databank.worldbank.org/source/world-development-indicators` & Author's calculations.

Overexploitation in aquatic systems

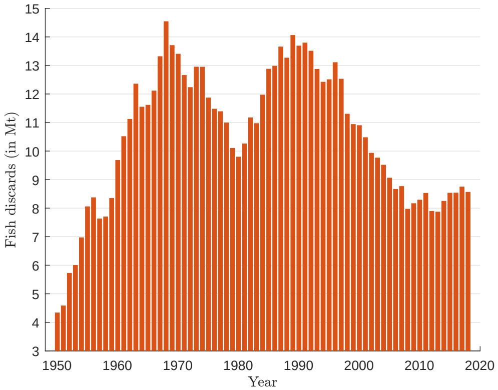
- FAO (2024) assesses the sustainability of fisheries by comparing the biomass stock $x(t)$ with the maximum sustainable yield (MSY) and defines three categories:

Category	Overfished	Maximally sustainably fished	Underfished
$\dfrac{x(t)}{\mathrm{MSY}}$	$[0, 0.8[$	$[0.8, 1.2]$	$]1.2, +\infty)$
Unsustainable	✓		
Sustainable		✓	✓

- The proportion of fishery stocks within biologically sustainable levels has declined from 94% in 1974 to 62% in 2021
- Large discrepancy between the 15 major FAO fishing regions
- 4 areas have more than 50% of their fish stocks at unsustainable levels: the eastern central Atlantic (51.3%), the northwest Pacific (56%), the Mediterranean and Black Sea (62.5%), and the southeast Pacific (66.7%)
- 4 areas have more than 75% of fisheries stocks that are sustainable: the eastern central Pacific (84.2%), the northeast Atlantic (79.4%), the northeast Pacific (76.5%), and the southwest Pacific (75.9%)

Overexploitation in aquatic systems

Figure 79: Estimated fish discards in Mt (1950–2018)



Source: FishStat & `www.ourworldindata.org/grapher/fish-discards`.

Overexploitation in tropical forests

By definition, overexploitation in tropical forest ecosystems is related to deforestation and habitat degradation. However, we need to distinguish between the two concepts: deforestation and overexploitation

According to Peres (2010), overexploitation of tropical forests involves three main issues:

- Timber extraction refers to the process of harvesting trees for commercial purposes, such as logging for wood, paper, and construction materials;
- Tropical forest vertebrates have been hunting in tropical forests for over 100 000 years, but their consumption increased during the 20th century;
- Non-timber forest products are biological resources such as plants and raw materials.

Opportunity cost

- Let Δp and Δv be the annual changes in timber prices and tree volume relative to a reference age t_0
- The market value $W(t)$ of the trees is given by:

$$W(t) = p(t)\, v(t) = p_0 v_0 (1 + \Delta p)^{t - t_0} (1 + \Delta v)^{t - t_0}$$

  where p_0 and v_0 are the price level and the volume of trees at age t_0
- Assume that trees are harvested at age $\tau \leq t$ and the proceeds are invested at the risk-free rate r. The resulting wealth conditional on this harvest is equal to:

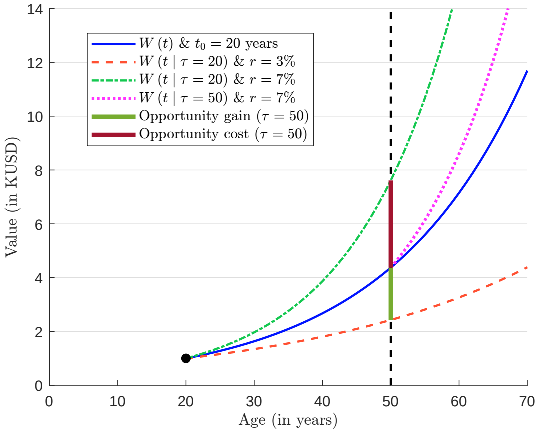
$$W(t \mid \tau \leq t) = p(\tau)\, v(\tau)\, (1 + r)^{t - \tau}$$

- The economic objective is to maximize the conditional wealth $W(t \mid \tau \leq t)$ or to find the optimal harvest age τ that maximizes the return

$\Rightarrow$ **opportunity cost** when $r \geq \Delta p + \Delta v$

Opportunity cost

Figure 80: Opportunity cost of logging in tropical forests



The bushmeat crisis

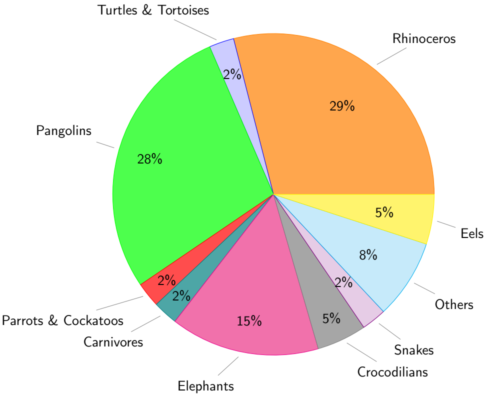
- **Bushmeat** is the meat of wild animals that are hunted for food
- The meat is either eaten by the hunter or sold to make money
- In the media, the term bushmeat is generally used to refer to the illegal hunting of protected animals in Africa
- This includes various species such as antelopes, monkeys, rodents and other wild animals
- Bushmeat has historically been a vital protein source in parts of Africa, Latin America and South Asia
- While more than 1 000 animal species are affected by bushmeat hunting, Ripple *et al.* (2016) estimated that approximately 300 of these terrestrial mammal species are threatened with extinction
- The term '*bushmeat crisis*' refers to this paradox: the need for local people to hunt wild animals to improve their food security and well-being, while at the same time this practice has a significant negative impact on biodiversity

Trafficking in protected species of wild fauna and flora

- According to Traffic (`www.traffic.org`), "*the illegal trade in wild species is one of the most profitable criminal activities in the world, estimated to be worth up to $23 billion each year.*"
- The UNODC reports that from 2015 to 2021, more than 140 000 seizures of wildlife products were made in 162 countries, involving 4 000 different species
- Analysis of seizure records shows that coral pieces were the most common item found in the illegal wildlife trade, accounting for 16% of the total number of seizures.
- Other seizures included crocodiles (9%), elephants (6%), bivalves and carnivores (5% each), parrots and cockatoos (4%), orchids (4%), and many other species

Trafficking in protected species of wild fauna and flora

Figure 81: Percentage share by species group aggregated by standardized seizure index (2015–2021)



Source: UNDOC (2021, Figure 2.3, page 63).

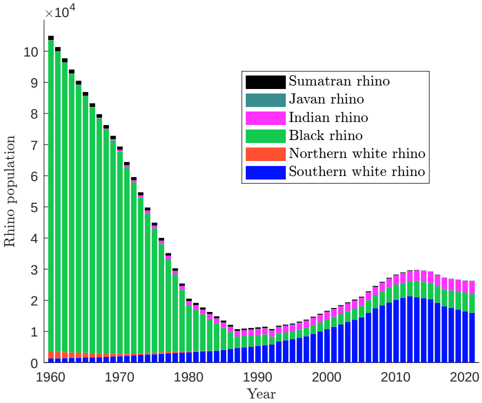
Operation Thunder 2024

- On 4 February 2025, INTERPOL announced that nearly 20 000 live animals, all endangered or protected species, were seized in a global operation against wildlife and forestry trafficking networks
- OT-2024 took place in 138 countries between 11 November and 6 December
- Significant seizures included 4 472 kg of pangolin scales (Nigeria); 6 500 live songbirds discovered during a vehicle inspection at the Syrian border (Turkey); 5 193 live red-eared slider turtles concealed in passenger suitcases arriving from Malaysia at Chennai Airport (India); 3 700 protected plants intercepted en route from Ecuador (Peru); 8 rhino horns found in a suspect's luggage while transiting from Mozambique to Thailand (Qatar); One tonne of sea cucumbers, considered a seafood delicacy, smuggled from Nicaragua (United States); 973 kg of dried shark fins originating from Morocco seized at the airport (Hong Kong); 8 tigers, aged between two months and two years, discovered in a suspected illegal breeding facility (Czech Republic); 846 pieces of reticulated python skin, from the world's longest snake species, concealed onboard a ship (Indonesia).

The case of the rhinoceros

- The number of black rhinos declined by over 95% from around 100 000 in 1960 to fewer than 2 500 in the 1990s due to hunting and poaching
- The northern white rhino has virtually disappeared, with only two females remaining in 2021
- Indian rhinos were reduced to less than 100 individuals in 1900 due to hunting, but have since recovered thanks to conservation efforts and now number around 4 000

Figure 82: Number of rhinoceros in the world



Zoonoses and pandemics

- Two decades ago, Bengis *et al.* (2024 and Kruse *et al.* (2004) raised concerns about the growing risk of zoonotic pandemics and the role of biodiversity loss, particularly deforestation and habitat alteration
- **Zoonoses are diseases transmissible from animals to humans caused by pathogens** — viruses, bacteria, fungi, or parasites — that circulate naturally in animal populations
- 61% of human pathogenic species are zoonotic
- Zoonotic origin for diseases such as bubonic plague, chikungunya, dengue fever, HIV/AIDS, Lyme disease, the 1918 Spanish flu, salmonellosis, and rabies
- Pandemics: avian influenza (H5N1), COVID-19, Ebola, H1N1 influenza, MERS, SARS, and Zika

A Course on Biodiversity
Lecture 4. Biodiversity Measurement

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March 2025

[7] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Essential biodiversity variables

"Despite progress in digital mobilization of biodiversity records and data standards, there is insufficient consistent national or regional biodiversity monitoring and sharing of such information. Along with inadequate human and financial resources, a key obstacle is the ***lack of consensus about what to monitor****. Many initiatives collect data that could be integrated into an EBV global observation network, though important gaps remain. Different organizations and projects adopt diverse measurements, with some important biodiversity dimensions, such as genetic diversity, often missing." (Pereira et al., 2013, page 277).*

Essential biodiversity variables

Table 61: EBV classes and names

#	EBV class	EBV theme
1	Genetic composition	Genetic diversity (richness and heterozygosity), genetic differentiation (number of genetic units and genetic distance), effective population size, inbreeding
2	**Species populations**	Species distributions, species abundances
3	Species traits	Morphology, physiology, phenology, movement, reproduction
4	Community composition	Community abundance, taxonomic/phylogenetic diversity, trait diversity, interaction diversity
5	**Ecosystem functioning**	Primary productivity, ecosystem phenology, ecosystem disturbances
6	Ecosystem structure	Live cover fraction, ecosystem distribution, ecosystem vertical profile

Source: `https://geobon.org/ebvs/what-are-ebvs`.

Essential biodiversity variables

Table 62: Eight essential biodiversity variables (Schmeller *et al.*, 2018)

Indicator	Definition
Abundance	Abundance is the number of individuals of a species within a local population.
Allelic diversity	Allelic diversity is the average number of alleles per locus in a population of a given species.
Body mass	Body mass scaled by body size, or the body mass index (BMI), indicates the condition and energy reserves of animals.
Ecosystem heterogeneity	Ecosystem heterogeneity describes the amount of variability in space and time of ecosystems.
Phenology	Phenology is defined as annually recurring life-cycle events, such as the timing of migration or flowering.
Range dynamics	Range dynamics are changes in species distributions through time, space and shape. This EBV is derived from the species distribution EBV for detecting critical ecological change early.
Size at first reproduction	Size at first reproduction is the individual body size (length) reached by an organism at the time when its first reproduction occurs.
Survival rates	Survival rate is the average probability that an organism will stay alive between two time points.

Biodiversity metrics

1. Mean species abundance (MSA)
2. Potentially disappeared fraction (PDF)
3. Biodiversity intactness index (BII)
4. Species threat abatement and reforestation (STAR)
5. Others

These metrics focus on species abundance or species richness

Can we reduce biodiversity to the counting of species?

Mean species abundance (MSA)

The MSA of area A is calculated as follows:

$$\mathrm{MSA} = \frac{1}{S_A} \sum_{i=1}^{S_A} \min \left(\frac{n_i}{n_i^\star}, 1 \right)$$

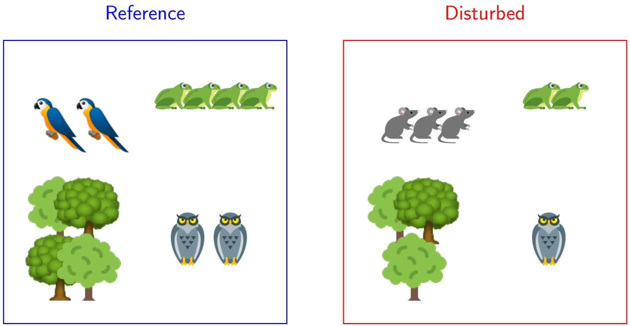
where:

- n_i is the abundance of species i in an area A
- $n_i^\star$ be the abundance of species i in a reference state
- S_A be the number of native species in the area

By construction we have $\mathrm{MSA} \in [0, 1]$

Mean species abundance (MSA)

Figure 83: Computation of the MSA

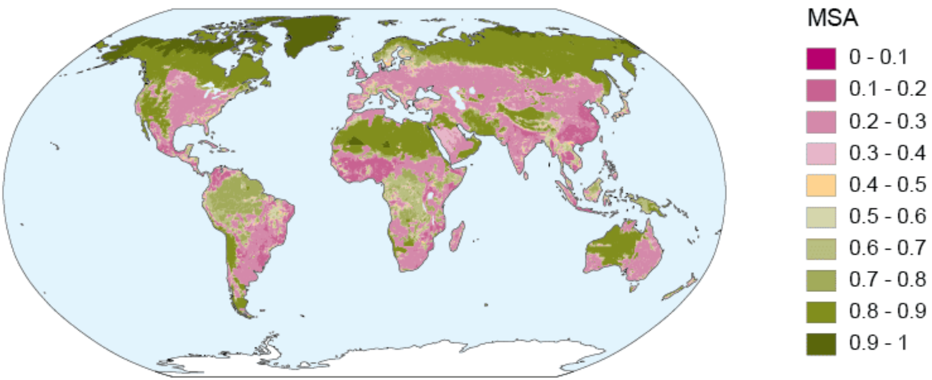


- There are four native species in the reference area
- In the disturbed area, the parrots have disappeared and have been replaced by rodents
- The MSA is equal to:

$$
\begin{aligned}
\mathrm{MSA} &= \frac{1}{4}\left(\frac{0}{2}+\frac{2}{4}+\frac{3}{4}+\frac{1}{2}\right) \\
&= 43.75\%
\end{aligned}
$$

Mean species abundance (MSA)

Figure 84: Mean species abundance values for 2015 (GLOBIO 4)



Source: Schipper *et al.* (2020, Figure 4(a), page 766).

Potentially disappeared fraction (PDF)

The PDF of area A is:

$$\mathrm{PDF} = \frac{S_0 - S_A}{S_0} = \frac{1}{S_0} \sum_{j=1}^{S_0} \mathbb{1}\left\{j \notin A\right\}$$

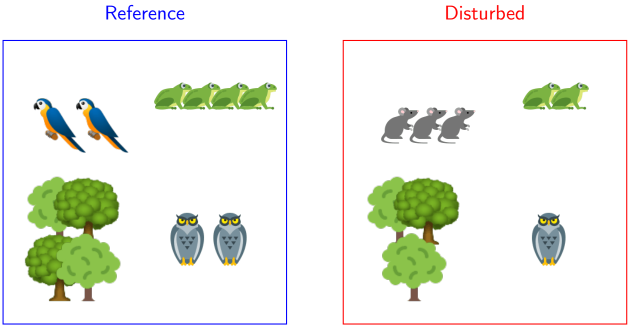
where:

- S_0 is the undisturbed species richness of A
- S_A is the current species richness of A
- j is the species index

$\Rightarrow$ PDF equals zero if no species disappear (**no biodiversity loss**), while PDF equals one if all species in the considered ecosystem disappear (**total biodiversity loss**)

Potentially disappeared fraction (PDF)

Figure 85: Computation of the MSA



- There are four native species in the reference area
- In the disturbed area, the parrots have disappeared and have been replaced by rodents
- The PDF is equal to 25% because 25% of the species in this area have disappeared

Biodiversity intactness index (BII)

- A biodiversity intactness index is a metric that aims to measure how much of a region's natural biodiversity remains, despite human impacts
- If we measure intactness at the species level, we have $\mathrm{BII} = 1 - \mathrm{PDF}$

Biodiversity intactness index (BII)

The biodiversity intactness index is equal to:

$$\mathrm{BII} = \frac{\sum_i \sum_j \sum_k S_{i,j} A_{j,k} I_{i,j,k}}{\sum_i \sum_j \sum_k R_{i,j} A_{j,k}}$$

where:

- i is the taxon index
- j is the ecosystem index
- k is the land use index
- $S_{i,j}$ is the species richness of taxon i in ecosystem j
- $A_{j,k}$ is the area of land use k in ecosystem j
- $I_{i,j,k}$ is the population impact or relative population of taxon i (compared to the reference state) under land use k in ecosystem j

Biodiversity intactness index (BII)

Example with one ecosystem and one land use

There are three taxonomic groups (birds, mammals, and plants), the species richness is 100 bird species, 50 mammal species, 200 plant species, and the population impacts are 50%, 80%, and 90%, respectively

We get:

$$\mathrm{BII} = \frac{100 \times 50\% + 50 \times 80\% + 200 \times 90\%}{100 + 50 + 200} = \frac{270}{350} = 77.14\%$$

Biodiversity intactness index (BII)

Table 63: Biodiversity intactness index in % of tropical forests (2001–2012)

Region	Metric	ISO code							
South America		ARG	BOL	BRA	CHL	COL	CUB	PER	PRY
	2001 BII	48.3	72.8	82.7	14.6	81.2	18.0	88.2	58.2
	2012 BII	46.4	71.2	80.7	16.1	80.0	19.3	87.5	53.0
	Change (in %)	−3.8	−2.2	−2.4	10.2	−1.4	7.8	−0.8	−8.9
Africa		CIV	CMR	COD	COG	GAB	LBR	SLE	TGO
	2001 BII	57.0	79.7	84.2	89.7	87.9	72.6	60.2	58.7
	2012 BII	41.9	79.3	83.2	88.8	87.3	72.8	57.2	57.7
	Change (in %)	−26.4	−0.5	−1.2	−1.1	−0.7	0.3	−4.9	−1.8
South Asia		CHN	KHM	IDN	IND	LAO	MYS	THA	VNM
	2001 BII	38.6	41.2	75.8	13.3	64.5	79.6	31.0	39.5
	2012 BII	36.4	34.5	70.4	13.0	61.3	70.5	29.7	38.2
	Change (in %)	−5.5	−16.2	−7.1	−1.5	−4.9	−11.4	−4.2	−3.4

Source: De Palma *et al.* (2021), `https://doi.org/10.5519/5wriutqz` & Author's calculations.

Species threat abatement and restoration (STAR)

STAR threat-abatement score

For a given location i and a given threat j, the STAR **threat-abatement score** is calculated as a weighted average of the species IUCN Red List status:

$$T_{i,j} = \sum_{s=1}^{S} T_{i,j}^{(s)} = \sum_{s=1}^{S} \pi_{i,s} w_s c_{j,s}$$

where:

- $\pi_{i,s} \in [0, 1]$ is the current area of habitat (AOH) of species s within location i — expressed as a percentage of the current global AOH of the species
- $w_s \in \{1, 2, 3, 4\}$ is the IUCN Red List category weight of species s
- $c_{j,s}$ is the relative contribution of threat j to the extinction risk of species s
- S is the species richness at location i

Species threat abatement and restoration (STAR)

STAR restoration score

For a given location i and a given threat j, the STAR **restoration score** is calculated as:

$$R_{i,j} = \sum_{s=1}^{S} R_{i,j}^{(s)} = \sum_{s=1}^{S} \varphi_{i,s} w_s c_{j,s} m_{i,s}$$

where:

- $\varphi_{i,s} \in [0,1]$ is the amount of restorable AOH for species s at location i — expressed as a percentage of the current global AOH of the species
- $w_s \in \{1,2,3,4\}$ is the IUCN Red List category weight of species s
- $c_{j,s}$ is the relative contribution of threat j to the extinction risk of species s
- $m_{i,s} > 0$ is a multiplier appropriate for the habitat at location i to discount restoration results — the default value is 29%
- S is the species richness at location i

Species threat abatement and restoration (STAR)

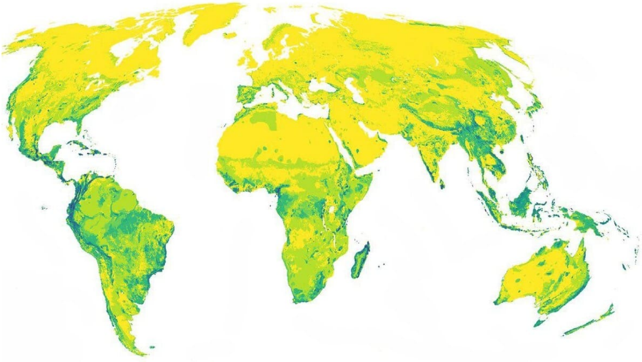
Example

Species	Red List	w_s	$\pi_{i,s}$	$c_{j,s}$	$\varphi_{i,s}$	$m_{i,s}$	$T_{i,j}^{(s)}$	$R_{i,j}^{(s)}$
Salmo salar (salmon)	NT	1	25%	90%	10%	29%	0.225	0.026
Phengaris teleius (butterfly)	VU	2	50%	90%	25%	29%	0.900	0.131
Conraua goliath (frog)	EN	3	80%	80%	25%	29%	1.920	0.174
Pericallis malvifolia (magnolia)	CR	4	75%	80%	20%	29%	2.400	0.186
Total							5.445	0.516

- The site contains four species, each belonging to a different IUCN Red List category
- Salmon occupy 25% of the habitat, indicating this is either a marine site or one with rivers. The threat contribution is high at 90%, meaning this threat is responsible for 90% of the extinction risk to salmon. Potential habitat restoration is 10%, suggesting we can reduce extinction risk by that amount.

Species threat abatement and restoration (STAR)

Figure 86: Global STAR threat-abatement score for amphibians, birds and mammals (at a 50-km grid cell resolution)



Source: Mair *et al.* (2021, Figure 2).

Species threat abatement and restoration (STAR)

"While every nation can contribute towards halting biodiversity loss, ***Indonesia, Colombia, Mexico, Madagascar and Brazil*** *combined have stewardship over* ***31% of total STAR values*** *for terrestrial amphibians, birds and mammals. Among actions, sustainable crop production and forestry dominate, contributing 41% of total STAR values for these taxonomic groups. Key Biodiversity Areas cover 9% of the terrestrial surface but capture 47% of STAR values." (Mair et al., 2021, page 836).*

Commercial solutions

1. **Scope 1**: Impacts generated within the entity's control and other impacts directly caused by the entity during the assessment period;
2. **Scope 2**: Impacts resulting from the generation of non-fuel energy (electricity, steam, heat and cooling) for use at the site level, including impacts from land use change, fragmentation and related factors;
3. **Scope 3**
    1. **Scope 3 Upstream**: Impacts that are a consequence of the company's activities, but are caused by sources not owned or controlled by the company within its upstream supply chain;
    2. **Scope 3 Downstream**: Impacts that are a consequence of the company's activities, but are caused by sources not owned or controlled by the company within its downstream consumption and waste processes.

Commercial solutions

Table 64: List of commercial solutions for measuring biodiversity

Sigle	Name	Sponsors	OS‡
ABD Index	AgroBioDiversity Index	Alliance of Bioversity International, CIAT	
B-INTACT	Biodiversity Integrated Assessment Computation Tool	FAO	✓
BFFI	Biodiversity Footprint for Financial Institutions	ASN Bank, CREM, PRé Sustainability	✓
BFM	Biodiversity Footprint Methodology	Plansup	
BIA-GBS	Biodiversity Impact Analytics – Global Biodiversity Score	CDC Biodiversité, Carbon4 Finance	
BIAT	Biodiversity Impact Assessment Tool	ISS ESG	
BII	Biodiversity Intactness Index	Natural History Museum (UK)	
BIM	Biodiversity Impact Metric	Cambridge Institute for Sustainable Leadership	
BRF	Biodiversity Risk Filter	WWF	✓
CBF	Corporate Biodiversity Footprint	Iceberg Datalab	
ENCORE	Exploring Natural Capital Opportunities, Risks and Exposure	Global Canopy, UNEP FI, UNEP-WCMC	✓
GBS	Global Biodiversity Score	CDC Biodiversité	
GBSFI	Global Biodiversity Score for Financial Institutions	CDC Biodiversité	
GID	Global Impact Database	Impact Institute	
GIST NBS	Nature & Biodiversity Suite (BIGER/SLAM/DIRO 360)	GIST Impact	
IBAT	Integrated Biodiversity Assessment Tool	BirdLife Int., Conservation Int., IUCN, UNEP-WCMC	
InVEST	Integrated Valuation of Ecosystem Services and Tradeoffs	Natural Capital Project	✓
MBFM	MSCI Biodiversity Footprint Metrics	MSCI	
NRP	Nature Risk Profile	S&P Global	
NVE	Nature Value Explorer	Flemish Institute for Technological Research	✓
PBF	Product Biodiversity Footprint	I Care	

Source: Bailon *et al.* (2024), De Ryck *et al.* (2024), & Author's research.

A Course on Biodiversity
Lecture 5. Biodiversity Governance and Regulation

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March 2025

[8]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Convention on Biodiversity (CBD)

Convention on Biological Diversity

- Adopted at the 1992 Earth Summit in Rio de Janeiro
- Entered into force in 1993
- The purpose of this international treaty is to address the global loss of biological diversity, with the following objectives:
  1. The conservation of biological diversity
  2. The sustainable use of its components
  3. The fair and equitable sharing of the benefits arising out of the utilization of genetic resources
- The Conference of the Parties (COP) serves as the governing body of the CBD (meeting every two years)

Conference of the Parties (COP)

- The International Coral Reef Initiative (1994, COP1, Bahamas), a partnership of nations and organizations to protect coral reefs
- The Cartagena Protocol on Biosafety (2000), which regulates the international movement of living modified organisms to ensure biosafety
- The **Nagoya Protocol** on Access and Benefit-Sharing (2010, COP10, Japan)
  - Legal framework for the equitable sharing of benefits arising from the use of genetic resources
  - **Aichi Biodiversity Targets** (2011–2020)
- The Pyeongchang Roadmap (2014, COP12, South Korea) $\Rightarrow$ accelerate global efforts to achieve the Aichi Biodiversity Targets
- The **Kunming-Montreal Global Biodiversity Framework**, known as the GBF (2022, COP15) $\Rightarrow$ Protecting 30% of the Earth's land and marine areas by 2030 (the 30×30 target)

Aichi Biodiversity Targets

Table 65: Aichi Biodiversity Targets (2011–2020)

Strategic Goal A	Address the underlying causes of biodiversity loss by mainstreaming biodiversity across government and society (Target 1–4)
Strategic Goal B	Reduce the direct pressures on biodiversity and promote sustainable use (Target 5–10)
Strategic Goal C	To improve the status of biodiversity by safeguarding ecosystems, species and genetic diversity (Target 11–13)
Strategic Goal D	Enhance the benefits to all from biodiversity and ecosystem services (Target 14–16)
Strategic Goal E	Enhance implementation through participatory planning, knowledge management and capacity building (Target 17–20)

Source: `https://www.cbd.int/sp/targets`.

Aichi Biodiversity Targets

Figure 87: 20 global biodiversity goals

Aichi Biodiversity Targets

Table 66: Aichi Biodiversity Targets (2011–2020)

Target 5	By 2020, the rate of loss of all natural habitats, including forests, is at least halved and, where feasible, brought close to zero, and degradation and fragmentation are significantly reduced
Target 7	By 2020 areas under agriculture, aquaculture and forestry are managed sustainably, ensuring conservation of biodiversity
Target 8	By 2020, pollution, including from excess nutrients, has been brought to levels that are not detrimental to ecosystem function and biodiversity
Target 9	By 2020, invasive alien species are identified, priority species are controlled or eradicated, and measures are in place to manage pathways to prevent their introduction and establishment
Target 11	By 2020, at least 17% of terrestrial and inland water areas and 10% of coastal and marine areas are protected by well-connected systems of protected areas
Target 12	By 2020, the extinction of known threatened species has been prevented, and their conservation status has been improved and maintained, especially for those species most in decline
Target 15	By 2020, the resilience of ecosystems has been enhanced, including restoration of at least 15% of degraded ecosystems
Target 20	By 2020, the goal is to **significantly increase financial resources** for effective implementation of the Strategic Plan for Biodiversity 2011–2020

Aichi Biodiversity Targets

Several issues:

- Many of the targets are vague and lack clear commitments (*e.g.*, Target 7)
- **Inadequate funding**
  - Target 12
    Reducing the risk of extinction of all globally threatened bird species by at least one IUCN Red List category would cost between \$875 million and \$1.23 billion, representing only 12% of current funding (McCarthy *et al.*, 2012)
  - Waldron *et al.* (2013) identified 40 severely underfunded countries, with the top five being Iraq, Djibouti, Angola, Kyrgyzstan, and Guyana
- Lack of government commitment, inadequate monitoring and reporting by the CBD, and low public awareness

Aichi Biodiversity Targets

"This report determines that, in 2019, the total global annual flow of funds toward biodiversity protection amounted to approximately $124–143 billion per year against an estimated annual need of $722–967 billion to halt the decline in global biodiversity between now and 2030. Taken together, these figures reveal a biodiversity financing gap of $598–824 billion per year. [...] this report shows that annual governmental expenditures on activities harmful to biodiversity in the form of agricultural, forestry, and fisheries subsidies — $274–542 billion per year in 2019 — are two to four times higher than annual capital flows toward biodiversity conservation." (Paulson Institute Report, 2020).

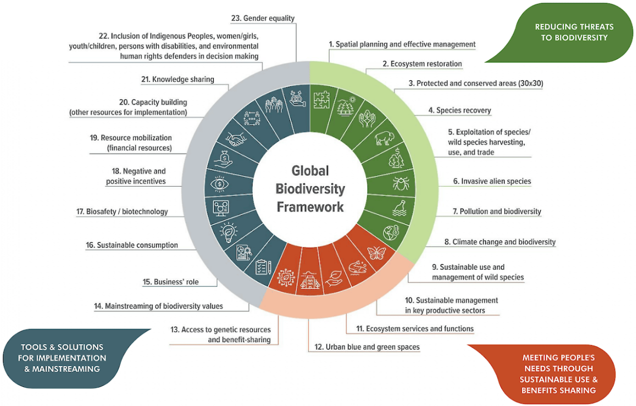
Kunming-Montreal Global Biodiversity Framework (CBF)

Kunming-Montreal Global Biodiversity Framework (GBF)

- Failure of the Strategic Plan for Biodiversity 2011–2020 and its Aichi Biodiversity Targets
- Adoption of the GBF at the CBD COP15 in December 2022
- Re-launch global biodiversity conservation efforts by addressing the gaps in national ambition and commitment, as well as previous initiatives
- Section F: 2050 vision and 2030 mission
- Section G: Global goals for 2050
- Section H: Global targets for 2030

Kunming-Montreal Global Biodiversity Framework (CBF)

Figure 88: 20 global biodiversity goals



Source: Environment and Climate Change Canada (2024).

The 23 targets of the Kunming-Montreal Global Biodiversity Framework

Reducing threats to biodiversity

Target	Purpose
1	Plan and manage all areas to reduce biodiversity loss
2	Restore 30% of all degraded ecosystems — terrestrial, inland water, and marine and coastal ecosystems
3	Conserve 30% of land, waters and seas — protected areas and other effective area-based conservation measures
4	Halt species extinction, protect genetic diversity, and manage human-wildlife conflicts
5	Ensure sustainable, safe and legal harvesting and trade of wild species (reducing the risk of pathogen spillover)
6	Reduce the introduction of invasive alien species by 50% and minimize their impact — eradicating or controlling invasive alien species, especially in priority sites, such as islands
7	Reduce pollution to levels that are not harmful to biodiversity — (a) reducing excess nutrients lost to the environment by at least half, (b) reducing the overall risk from pesticides and highly hazardous chemicals by at least half, (c) preventing, reducing, and working towards eliminating plastic pollution
8	Minimize the impacts of climate change on biodiversity and build resilience

The 23 targets of the Kunming-Montreal Global Biodiversity Framework

Meeting people's needs through sustainable use and benefit-sharing

Target	Purpose
9	Manage wild species sustainably to benefit people
10	Enhance biodiversity and sustainability in agriculture, aquaculture, fisheries, and forestry
11	Restore, maintain and enhance nature's contributions to people
12	Enhance green spaces and urban planning for human well-being and biodiversity
13	Increase the sharing of benefits from genetic resources, digital sequence information and traditional knowledge

The 23 targets of the Kunming-Montreal Global Biodiversity Framework

Tools and solutions for implementation and mainstreaming

Target	Purpose
14	Integrate biodiversity in decision-making at every level
15	Businesses assess, disclose and reduce biodiversity-related risks and negative impacts
16	Enable sustainable consumption choices to reduce waste and overconsumption
17	Strengthen biosafety and distribute the benefits of biotechnology
18	Reduce harmful incentives by at least $500 billion per year, and scale up positive incentives for biodiversity
19	Mobilize $200 billion per year for biodiversity from all sources, including $30 billion through international finance
20	Strengthen capacity-building, technology transfer, and scientific and technical cooperation for biodiversity
21	Ensure that knowledge is available and accessible to guide biodiversity action
22	Ensure participation in decision-making and access to justice and information related to biodiversity for all
23	Ensure gender equality and a gender-responsive approach for biodiversity action

Kunming-Montreal Global Biodiversity Framework (CBF)

Indicators

1. Headline indicators
2. Component indicators
3. Complementary indicators

Kunming-Montreal Global Biodiversity Framework (CBF)

Headline indicators (Reducing threats to biodiversity)

- 1.A.1 Red List of ecosystems
- 1.A.2 Extent of natural ecosystems
- 1.1 Per cent of land and seas covered by biodiversity-inclusive spatial plans
- 2.2 Area under restoration
- 3.1 Coverage of protected areas and OECMs
- 4.A.3 Red list Index
- 4.A.4 The proportion of populations within species with an effective population size > 500
- 5.1 Proportion of fish stocks within biologically sustainable levels
- 6.1 Rate of invasive alien species establishment
- 7.1 Index of coastal eutrophication potential
- 7.2 Pesticide environment concentration

Kunming-Montreal Global Biodiversity Framework (CBF)

Headline indicators (Meeting people's needs through sustainable use and benefit-sharing)

- 9.1 Benefits from the sustainable use of wild species
- 9.2 Percentage of the population in traditional occupations
- 10.1 Proportion of agricultural area under productive and sustainable agriculture
- 10.2 Progress towards sustainable forest management
- 11.B.1 Services provided by ecosystems
- 12.1 Average share of the built-up area of cities that is green/blue space for public use for all

Kunming-Montreal Global Biodiversity Framework (CBF)

Headline indicators (Tools and solutions for implementation and mainstreaming)

- 13.C.1 Indicator on monetary benefits received
- 13.C.2 Indicator on non-monetary benefits
- 15.1 Number of companies reporting on disclosures of risks, dependencies and impacts on biodiversity
- 18.1 Positive incentives in place to promote biodiversity conservation and sustainable use
- 19.D.1 International public funding, including official development assistance for conservation and sustainable use of biodiversity and ecosystems
- 19.D.2 Domestic public funding of conservation and sustainable use of biodiversity and ecosystems
- 19.D.3 Private funding (domestic and international) of conservation and sustainable use of biodiversity and ecosystems
- 21.1 Indicator on biodiversity information for monitoring the global biodiversity framework

Funding requirements

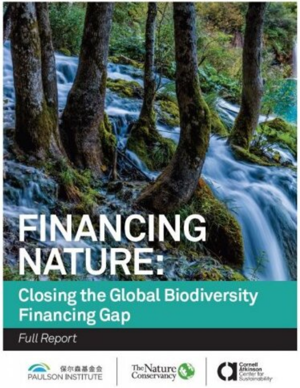
BIODIVERSITY $700 bn

CLIMATE $3 500 bn

(per year)

Funding requirements

Figure 89: Paulson Institute Report (2020)



- Paulson Institute
- Nature Conservancy
- Cornell Atkinson Center for Sustainability

Funding requirements

Table 67: Global biodiversity conservation financing in 2019 (in $ bn)

Financial flows	Lower limit	Upper limit	Midpoint	Percentage
Domestic budgets and tax policy	74.6	77.7	76.2	57.1%
Natural infrastructure	26.9	26.9	26.9	20.2%
Biodiversity offsets	6.3	9.2	7.8	5.8%
Sustainable supply chains	5.5	8.2	6.8	5.1%
Official development assistance (ODA)	4.0	9.7	6.8	5.1%
Green financial products	3.8	6.3	5.0	3.8%
Philanthropy & conservation NGOs	1.7	3.5	2.6	2.0%
Nature-based solutions & carbon markets	0.8	1.4	1.1	0.8%
Total	123.6	142.9	133.3	100.0%

Source: Deutz *et al.* (2020, Table 3.1, page 48).

Funding requirements

Table 68: Global biodiversity conservation funding needs (in $ bn)

Funding needs	Lower limit	Upper limit	Midpoint	Percentage
Croplands	315	420	367.5	43.5%
Protected areas	149	192	170.5	20.2%
Rangelands	81	81	81.0	9.6%
Urban environments	73	73	73.0	8.6%
Invasive alien species	36	84	60.0	7.1%
Coastal	27	37	32.0	3.8%
Fisheries	23	47	35.0	4.1%
Forests	19	32	25.5	3.0%
Total	**722**	**967**	844.5	100.0%

Source: Deutz *et al.* (2020,Figure 4.1, page 55).

Funding requirements

Table 69: Estimated positive and negative flows to biodiversity conservation (in $ bn)

Financial flows	2019		2030	
	Lower limit	Upper limit	Lower limit	Upper limit
Harmful subsidy reform	−542.0	−273.9	−268.1	0.0
Domestic budgets and tax policy	74.6	77.7	102.9	155.4
Natural infrastructure	26.9	26.9	104.7	138.6
Biodiversity offsets	6.3	9.2	162.0	168.0
Sustainable supply chains	5.5	8.2	12.3	18.7
Official development assistance (ODA)	4.0	9.7	8.0	19.4
Green financial products	3.8	6.3	30.9	92.5
Philanthropy & conservation NGOs	1.7	3.5		
Nature-based solutions & carbon markets	0.8	1.4	24.9	39.9
Total	123.6	142.9	445.7	632.5

Source: Deutz *et al.* (2020, Figure 5.1, page 64).

Convention on Biodiversity (CBD)

An illustration with the EU Biodiversity Strategy for 2030

- Part of the European Green Deal
- Examples:
  - Planting 3 billion trees by 2030
  - Restoring 25 000 km of free-flowing rivers by removing barriers
  - Increasing organic farming to 25% of agricultural land
  - Reducing pesticide use by 50%
- Nature Restoration Law (June 2022) $\Rightarrow$ Restore at least 30% of habitats in poor condition by 2030, 60% by 2040, and 90% by 2050

IPBES

- Intergovernmental Science-Policy Platform on Biodiversity and Ecosystem Services
- $\approx$ IPCC
- Secretariat in Bonn, Germany
- 2 900 experts

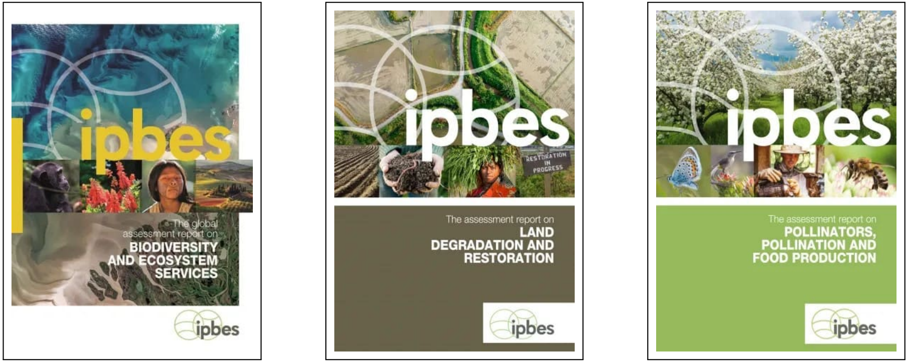
IPBES

Publications

- Global reports
  - Global assessment report on biodiversity and ecosystem services (2019)
- Thematic reports
  - Pollinators, pollination and food production (2016)
  - Land degradation and restoration (2018)
  - Sustainable use of wild species (2022)
  - Invasive alien species and their control (2023)
  - Interlinkages among biodiversity, water, food and health (2024)
- Methodological reports
  - Scenarios and models of biodiversity and ecosystem services (2016)
  - The diverse values and valuation of nature (2022)
  - Underlying causes of biodiversity loss and the determinants of transformative change and options for achieving the 2050 Vision for Biodiversity (2024)
- Regional reports
  - Biodiversity and ecosystem services for Africa (2018)
  - Biodiversity and ecosystem services for the Americas (2018)
  - Biodiversity and ecosystem services for Asia and the Pacific (2018)
  - Biodiversity and ecosystem services for Europe and Central Asia (2018)

IPBES

Figure 90: Some IPBES assessment reports



Source: IPBES & `www.ipbes.net/assessing-knowledge`.

Taskforce on Nature-related Financial Disclosures (TNFD)

- Global initiative to develop a framework
- Launched in 2021 by four founding organizations: Global Canopy, UNDP, UNEP FI and WWF
- $\neq$ TCFD (not supported by the Financial Stability Board)
- 40 members of the Taskforce: financial institutions (17), corporates (17) and market service providers (6)
- As of October 2024, there are 502 TNFD adopters (318 corporations, 129 financial institutions)
- The majority are located in Asia (236) and Europe (183)
- These adopters represent $17.7 trillion in AUM and $6.5 trillion in MCAP

Figure 91: TNFD Final Report (2023)

Taskforce on Nature-related Financial Disclosures (TNFD)

Table 70: The 14 recommended disclosures

Pillar	#	Recommended Disclosure
Governance	1	Board oversight
	2	Management's role
	3	**Human rights policies (IPLC)**
Strategy	4	Risks and opportunities
	5	Impact on organization
	6	Resilience of strategy
	7	**Locations of assets/activites/value chain**
Risk management	8	Risk identification and assessment processes
	9	**Dependencies in the value chain**
	10	Risk management processes
	11	Integration into overall risk management
Metrics and targets	12	Nature-related metrics
	13	Metrics used to manage impacts and risks
	14	Nature-related targets

Source: TNFD (2023) & `https://tnfd.global`.

Taskforce on Nature-related Financial Disclosures (TNFD)

Table 71: TNFD core global disclosure indicators for nature-related dependencies and impacts

#	Indicator	Unit	GBF Targets
C1.0	Total spatial footprint	km^2	1, 2, 5, 11
C1.1	Extent of land/freshwater/ocean-use change	km^2	1, 2, 5, 11
C2.0	Pollutants released to soil split by type	tonne	7, 11
C2.1	Wastewater discharged	m^3	7, 11
C2.2	Waste generation and disposal	m^3	7, 11
C2.3	Plastic pollution	tonne	7, 11
C2.4	Non-GHG air pollutants	$PM_{2.5}$, etc.	7, 11
C3.0	Water withdrawal and consumption from areas of water scarcity	m^3	11
C3.1	Quantity of high-risk natural commodities sourced from land/ocean/freshwater	tonne	5, 9, 11
C4.0	Measures against unintentional introduction of invasive alien species		6, 11
C5.0	Ecosystem condition		1, 2, 3, 4, 11
C5.0	Species extinction risk		1, 2, 3, 4, 11

Source: (TNFD, 2023, Table 6, pages 83–86).

Taskforce on Nature-related Financial Disclosures (TNFD)

Table 72: TNFD core global disclosure indicators for nature-related risks and opportunities

#	Indicator
C7.0	Value of assets, liabilities, revenue and expenses that are assessed as vulnerable to nature-related transition risks (total and proportion of total)
C7.1	Value of assets, liabilities, revenue and expenses that are assessed as vulnerable to nature-related physical risks (total and proportion of total)
C7.2	Description and value of significant fines/penalties received/litigation action in the year due to negative nature-related impacts
C7.3	Amount of capital expenditure, financing or investment deployed towards nature-related opportunities, by type of opportunity, with reference to a government or regulator green investment taxonomy or third-party industry or NGO taxonomy, where relevant
C7.4	Increase and proportion of revenue from products and services producing demonstrable positive impacts on nature with a description of impacts

Source: TNFD (2023, Table 7, page 87).

Taskforce on Nature-related Financial Disclosures (TNFD)

Table 73: Examples of TNFD additional global disclosure indicators

#	Indicator
A2.3	Light and noise pollution
A3.4	Area used for the production of natural commodities
A3.5	Use of wild species
A4.0	Number/extent of unintentionally introduced species, varieties or strains
A7.0	Value of write-offs and early retirements of assets due to nature-related risks
A8.4	Capital expenditure on adaption due to nature-related physical risks
A14.0	Expenditure on R&D for new and alternative technologies related to mitigation and adaptation of nature-related risks
A20.0	Proportion of sites that have active engagement with local stakeholders on nature-related issues
A21.1	Investment in nature-related solutions as defined in relevant government or regulator green investment taxonomy
A22.4	Proportion of suppliers committed to and effectively implementing sustainable production

Source: TNFD (2023, Tables 8–10, pages 89–99).

A Course on Biodiversity
Lecture 6. Investment approaches

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March 2025

[9]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Biodiversity finance

Table 74: Biodiversity finance initiatives

Acronym	Name	Website	Year
BCA	Biodiversity Credit Alliance	www.biodiversitycreditalliance.org	2022
BIOFIN	Biodiversity Finance Initiative	www.biofin.org	2012
BfN	Business for Nature	www.businessfornature.org	2019
FfB	Finance for Biodiversity Foundation	www.financeforbiodiversity.org	2021
NCIA	Natural Capital Investment Alliance	www.sustainable-markets.org	2021
NA 100	Nature Action 100	www.natureaction100.org	2023

Financial instruments

- Fixed-income instruments
  - **Blue bonds**
    - Definition: Debt securities designed to raise capital for marine and ocean conservation projects (*e.g.*, protecting marine biodiversity, restoring coastal ecosystems, financing sustainable fisheries)
    - Example: Seychelles Blue Bond (2018), which raised $15 million to support sustainable marine areas and fisheries
  - **Debt-for-nature swaps**
    - Definition: Financial transactions in which a portion of a country's debt is forgiven in exchange for environmental commitments
    - Example: Gabon debt-for-nature swap (2023), which restructured $500 million of debt to protect 30% of marine and forest ecosystems
  - **Green and sustainable bonds**
    - Definition: Debt instruments that target environmental projects and sustainable land use
    - Example: Colombia Biodiversity Bond (BBVA/IFC), which raised $50 million to finance projects focused on reforestation and wildlife habitat restoration

Financial instruments

- Fixed-income instruments
  - **Natural capital bonds**, nature performance bonds and conservation performance bonds

    Definition: Bonds that directly finance the protection and restoration of natural capital, with returns linked to specific ecological performance metrics and biodiversity outcomes, or that monetize the value of ecosystem services and biodiversity

    Example: Voluntary carbon credit-linked bonds, such as the IFC Forest Bond
  - **Sustainability-linked bonds** and pay-for-success financial instruments

    Definition: Bonds whose financial characteristics can change based on the achievement of sustainability targets

    Example: Rhino Bond (2022), issued by the World Bank ($150 million), where returns are linked to the growth of the black rhino population in Africa

Financial instruments

- Market-based instruments
  - **Biodiversity credits/offsets**
    - Definition: Tradable units representing positive biodiversity outcomes (market mechanisms to promote biodiversity conservation)
    - Example: UK Biodiversity Net Gain (BNG) policy (developers must ensure a 10% net gain in biodiversity by funding conservation projects or purchasing biodiversity credits)
  - **Nature-based insurance products**
    - Definition: Insurance mechanisms to protect natural capital and ecosystem services
    - Example: Parametric insurance for coral reef protection in the Caribbean and Central America (provides insurance coverage for coastal infrastructure and triggers payouts for reef restoration after hurricanes)
  - **Payments for ecosystem services** (PES)
    - Definition: Schemes that provide financial incentives to landowners or communities to manage their land in a way that maintains or enhances ecosystem services
    - Example: Vittel (Nestlé Waters) offers PES to farmers in the Vosges mountains in France to maintain water quality

Financial instruments

- Investment funds
  - **Biodiversity impact funds**

    Definition: Specialized investment vehicles focused on biodiversity conservation

    Example: The African Forestry Impact Platform (AFIP) managed by Norfund, which invests in sustainable forestry and conservation projects
  - **Blended finance**

    Definition: Combines public and private capital to attract more capital to biodiversity projects

    Example: The Land Degradation Neutrality (LDN) Fund initiated by the United Nations Convention to Combat Desertification (UNCCD) and Mirova, which finances the rehabilitation of degraded land

Financial instruments

- Investment funds
  - **Conservation trust funds**

    Definition: Long-term financing mechanisms for conservation and sustainable development

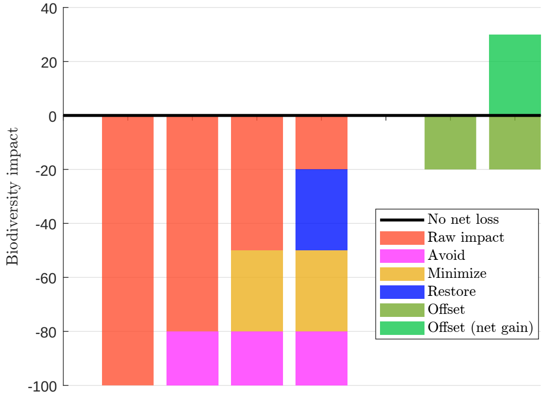
    Example: Bhutan Trust Fund for Environmental Conservation (BTFEC)
  - **Private equity and venture capital funds**

    Definition: Investment funds focused on companies and technologies that support biodiversity

    Example: Regeneration VC Fund

The avoid-minimize-restore-offset approach

Figure 92: Mitigation hierarchy for nature conservation



The avoid-minimize-restore-offset approach

Double materiality principle

Dependency	$\Longleftarrow$	How does biodiversity affect companies and investments?
Impact	$\Longrightarrow$	How do companies and investments affect biodiversity?

The avoid-minimize-restore-offset approach

Figure 93: Linking ESG investment strategies and biodiversity mitigation dimensions

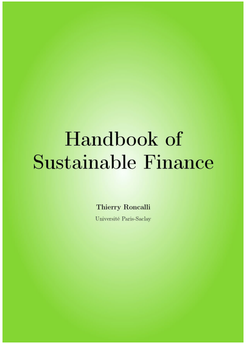
1. Exclusion	**Avoid**
2. Values	**Avoid**
3. Selection	**Minimize**
4. Thematic	**Restore/Offset**
5. Integration	**Minimize**
6. Engagement	**Minimize/Restore**
7. Impact	**Restore/Offset**

The impact investing approach

- 1 080 biodiversity-related funds
- Market size of €129 billion in open-end investment funds
- 69% of these funds primarily address food-related needs rather than direct biodiversity conservation
- 162 funds targeting SDG 14 (life below water)
- 278 funds targeting SDG 15 (life on land
- Only 11 real asset impact funds focus on timber and forests in Asia, 12 in Africa, and 26 in South America

Investment approaches

For more information on green bonds, blue bonds, blended finance, debt-for-nature swaps, impact funds & SDG funds, see the Handbook of Sustainable Finance (Chapter 5)



A Course on Biodiversity
Exercise 1. Calculating the Prevalence of Undernourishment

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March 2025

[10]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Let X and R be the random variables representing energy intake and energy requirement, respectively.

Question 1

We assume that the random vector (X, R) follows a bivariate log-normal distribution: $(X, R) \sim \mathcal{LN}\left(\mu_x, \sigma_x^2, \mu_r, \sigma_r^2, \rho\right)$.

Question (a)

Find the probability distribution of $D = X/R$. Then, calculate the prevalence of undernourishment, denoted by $\mathrm{PoU}^{\star} = \Pr\{X < R\}$.

We have:

$$\ln D = \ln X - \ln R \sim \mathcal{N}\left(\mu_d, \sigma_d^2\right)$$

where:

$$\mu_d = \mathbb{E}\left[\ln X - \ln R\right] = \mathbb{E}\left[\ln X\right] - \mathbb{E}\left[\ln R\right] = \mu_x - \mu_r$$

and:

$$\begin{aligned}\sigma_d^2 &= \operatorname{var}\left(\ln X - \ln R\right) \\ &= \operatorname{var}\left(\ln X\right) + \operatorname{var}\left(\ln R\right) - 2\operatorname{cov}\left(\ln X, \ln R\right) \\ &= \sigma_x^2 + \sigma_r^2 - 2\rho\sigma_x\sigma_r\end{aligned}$$

We deduce that $D = X/R$ is a log-normal random variable: $D \sim \mathcal{LN}\left(\mu_d, \sigma_d^2\right)$.

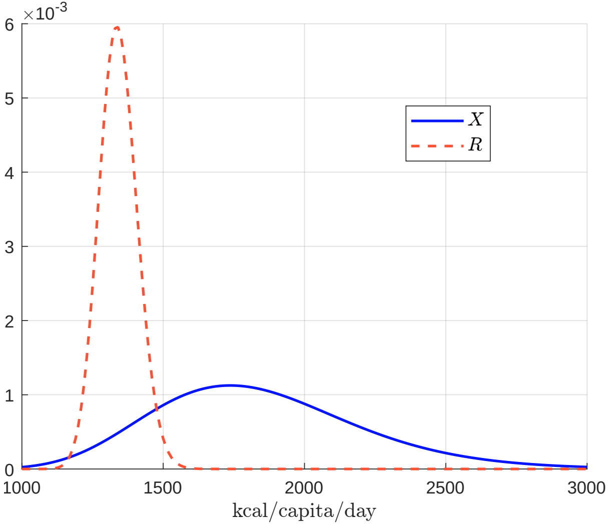
It follows that:

$$\begin{aligned}
\mathrm{PoU}^{\star} &= \Pr\{X < R\} \\
&= \Pr\{D < 1\} \\
&= \Pr\{\ln D < 0\} \\
&= \Phi\left(-\frac{\mu_d}{\sigma_d}\right) \\
&= \Phi\left(-\frac{\mu_x - \mu_r}{\sqrt{\sigma_x^2 + \sigma_r^2 - 2\rho\sigma_x\sigma_r}}\right)
\end{aligned}$$

Question (b)

Assume that $\mu_x = 7.50$, $\sigma_x = 0.20$, $\mu_r = 7.20$, and $\sigma_r = 0.05$. Plot the density functions of X and R.

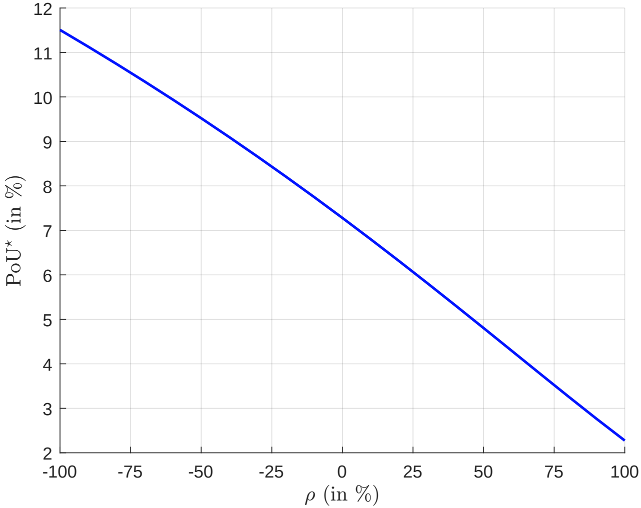
Figure 94: Probability density functions of X and R



Question (c)

Plot the function of $\mathrm{PoU}^{\star}$ as ρ varies in the interval $[-1, 1]$. Comment on the results.

Figure 95: Relationship between the correlation ρ and the prevalence of undernourishment $\mathrm{PoU}^{\star}$



Remark

We observe a decreasing function between ρ and $\mathrm{PoU}^{\star}$, because the standard deviation of D is a decreasing function of the correlation between X and R and $\mu_x > \mu_r$. Conversely, if $\mu_x < \mu_r$, the relationship between ρ and $\mathrm{PoU}^{\star}$ becomes increasing.

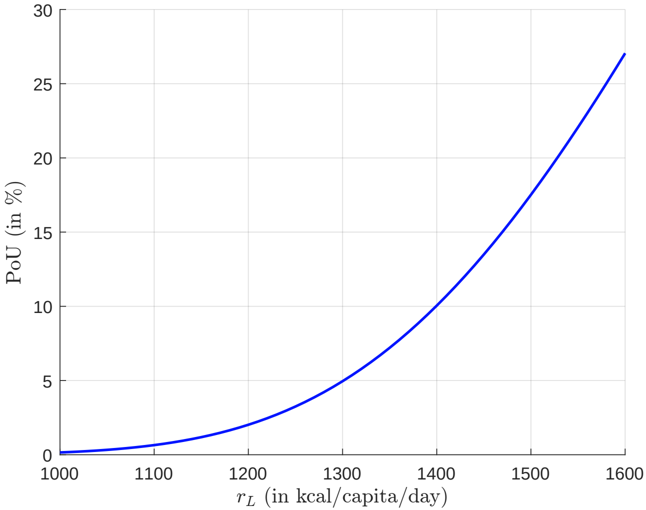
Question (d)

Compute the prevalence of undernourishment defined by $\mathrm{PoU} = \Pr\{X < r_L\}$. Plot the relationship between r_L and PoU when $r_L \in [1000, 1600]$.

We have:

$$\mathrm{PoU} = \Pr\left\{X < r_L\right\} = \Pr\left\{\ln X < \ln r_L\right\} = \Phi\left(\frac{\ln r_L - \mu_x}{\sigma_x}\right)$$

Figure 96: Prevalence of undernourishment PoU



Question (e)

Find the value of $r_L^\star$ such as $\mathrm{PoU} = \mathrm{PoU}^\star$. Calibrate the parameter $r_L^\star$ for the prevalence of undernourishment calculated in Question 1.(c).

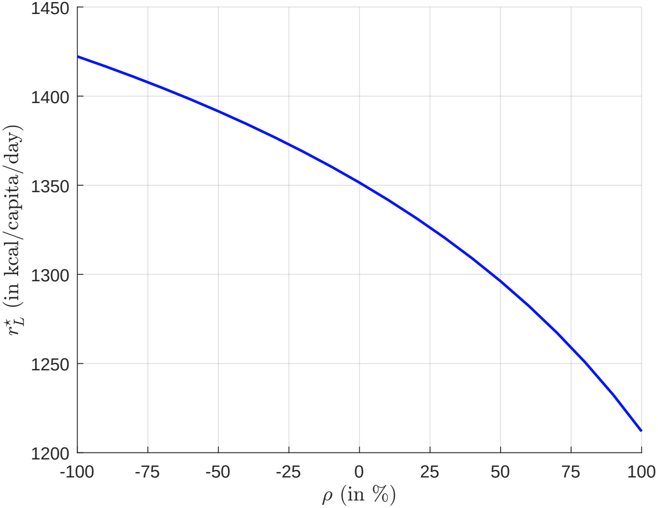
We deduce that:

$$
\begin{aligned}
\mathrm{PoU} = \mathrm{PoU}^{\star} \quad & \Leftrightarrow \quad \Phi\left(\frac{\ln r_L^{\star} - \mu_x}{\sigma_x}\right) = \Phi\left(-\frac{\mu_x - \mu_r}{\sqrt{\sigma_x^2 + \sigma_r^2 - 2\rho_{x,r}\sigma_x\sigma_r}}\right) \\
& \Leftrightarrow \quad \frac{\ln r_L^{\star} - \mu_x}{\sigma_x} = -\frac{\mu_x - \mu_r}{\sqrt{\sigma_x^2 + \sigma_r^2 - 2\rho_{x,r}\sigma_x\sigma_r}}
\end{aligned}
$$

which implies:

$$
r_L^{\star} = \exp\left(\mu_x - \frac{(\mu_x - \mu_r)\,\sigma_x}{\sqrt{\sigma_x^2 + \sigma_r^2 - 2\rho_{x,r}\sigma_x\sigma_r}}\right)
$$

Figure 97: Calibration of $r_L^\star$



Question 2

We want to calibrate the probability distribution function of X. We assume that $X \sim \mathcal{LN}\left(\mu_x, \sigma_x^2\right)$.

Question (a)

Give the first two moments of X. We will denote them by $\mu(X)$ and $\sigma^2(X)$.

We have:

$$\mu\left(X\right) = e^{\mu_x + \frac{1}{2}\sigma_x^2}$$

and:

$$\sigma^2\left(X\right) = e^{2\mu_x + \sigma_x^2}\left(e^{\sigma_x^2} - 1\right) = \mu^2\left(X\right)\left(e^{\sigma_x^2} - 1\right)$$

Question (b)

Deduce the coefficient of variation $\mathrm{CV}(X)$.

We deduce that:

$$\mathrm{CV}\left(X\right) = \frac{\sigma\left(X\right)}{\mu\left(X\right)} = \sqrt{e^{\sigma_x^2} - 1}$$

Question (c)

Find the moment estimators of μ_x and σ_x from $\mu(X)$ and $\mathrm{CV}(X)$.

We have:

$$
\begin{aligned}
\mathrm{CV}(X) = \sqrt{e^{\sigma_x^2} - 1} \quad &\Leftrightarrow \quad e^{\sigma_x^2} = \mathrm{CV}^2(X) + 1 \\
&\Leftrightarrow \quad \sigma_x = \sqrt{\ln\left(\mathrm{CV}^2(X) + 1\right)}
\end{aligned}
$$

and:

$$
\begin{aligned}
\mu(X) = e^{\mu_x + \frac{1}{2}\sigma_x^2} \quad &\Leftrightarrow \quad \mu_x = \ln \mu(X) - \frac{1}{2}\sigma_x^2 \\
&\Leftrightarrow \quad \mu_x = \ln \mu(X) - \frac{1}{2} \ln\left(\mathrm{CV}^2(X) + 1\right) \\
&\Leftrightarrow \quad \mu_x = \ln \mu(X) - \ln \sqrt{\mathrm{CV}^2(X) + 1} \\
&\Leftrightarrow \quad \mu_x = \ln \frac{\mu(X)}{\sqrt{\mathrm{CV}^2(X) + 1}}
\end{aligned}
$$

Question (d)

We consider an hypothetical country with a population of 1 million and two food components (cereals and fruits/vegetables), whose the food balance sheet is as follows:

	Production	Imports	Exports	ΔStocks	Feed	Waste
#1	349 000	1 025	40 000	−1 000	45 000	9 000
#2	50 000	9 010	1 000	6 000	500	500

All the figures are expressed in tonnes. Calculate the food available for human consumption. Assuming that the average energy density is 3 500 and 500 Calories/kg for cereals and fruits/vegetables respectively, find the average dietary energy consumption $\mu(X)$ expressed in kcal/capita/day.

The food available for human consumption is equal to:

$$
\begin{aligned}
Q_1 &= 349\,000 + 1\,025 - 40\,000 - (-1\,000) - 45\,000 - 9\,000 \\
&= 257\,025 \text{ tonnes}
\end{aligned}
$$

and

$$
\begin{aligned}
Q_2 &= 50\,000 + 9\,010 - 1\,000 - 6\,000 - 500 - 500 \\
&= 51\,010 \text{ tonnes}
\end{aligned}
$$

We deduce that the ADEC value is:

$$
\begin{aligned}
\mu(X) &= \frac{257\,025 \times 10^3 \times 3\,500 + 51\,010 \times 10^3 \times 500}{365 \times 10^6} \\
&= 2\,534.50 \text{ kcal/capita/day}
\end{aligned}
$$

Question (e)

The average dietary energy consumption by household expenditure deciles are 1 650 (first decile), 1 985, 2 150, 2 350, 2 530, 2 550, 2 650, 2 750, 3 100 and 3 630 (last decile). Calculate the coefficient of variation $\mathrm{CV}(X \mid Y)$. Check that $\mu(X \mid Y) = \mu(X)$. Assuming that $\mathrm{CV}(X \mid R) = 0.20$, calculate $\mathrm{CV}(X)$.

Let x_j^y be the average dietary energy consumption for the j^{th} household expenditure decile. We have:

$$\sigma(X \mid Y) = \sqrt{\sum_{j=1}^{10} f_j \left(x_i^y - \bar{x}^y\right)^2} = 533.4906$$

where $f_j = 10\%$ is the frequency of each decile group and $\bar{x}^y = \sum_{j=1}^{10} f_j x_i^y$ is the mean. We verify that $\bar{x}^y = 2\,534.50 = \mu(X)$. It follows that:

$$\mathrm{CV}(X \mid Y) = \frac{\sigma(X \mid Y)}{\mu(X \mid Y)} = \frac{533.4906}{2\,534.50} = 0.2105$$

We deduce that:

$$\mathrm{CV}(X) = \sqrt{\mathrm{CV}^2(X \mid Y) + \mathrm{CV}^2(X \mid R)} = \sqrt{0.2105^2 + 0.20^2} = 0.2904$$

Question (f)

Calibrate the parameters μ_x and σ_x using the previous figures. What is the prevalence of undernourishment if we assume that $r_L = 1\,850$? Give an estimate of the number of undernourished people.

We have:

$$\sigma_x = \sqrt{\ln\left(0.2904^2 + 1\right)} = 0.2845$$

and:

$$\mu_x = \ln \frac{2\,534.50}{\sqrt{0.2904^2 + 1}} = 7.7973$$

The prevalence of undernourishment is equal to:

$$\mathrm{PoU} = \Phi\left(\frac{1\,850 - 7.7973}{0.2845}\right) = 16.75\%$$

Finally, the number of undernourished people is around 167 500:

$$\mathrm{NoU} = N \cdot \mathrm{PoU} = 10^6 \times 16.75\% = 167\,451$$

Question 3

We seek to estimate the minimum dietary energy requirement (MDER).

Question (a)

We recall that the body mass index (BMI) is expressed in kg/m^2 and is defined as the ratio of the weight (in kg) to the square of the height (in meter):

$$\mathrm{BMI} = \frac{\mathrm{weight}}{\mathrm{height}^2}$$

The ideal value of BMI is 22. To define undernourished people, we assume that their weight is below a reference value:

$$\mathrm{weight} \leq \mathrm{weight}^\star$$

where:

$$\mathrm{weight}^\star = \mathrm{BMI}\,(p) \cdot \mathrm{height}^2$$

where p is a percentile value that depends on age.

Question (a) (Cont'd)

If the age is less than ten years, p is set to 50%, while for individuals 10 years and older, it is set to 5%. Below, we give the values of $\mathrm{BMI}(p)$ and the average height per age and sex:

	Age	−3	3–10	10–18	18–30	30–60	60+
$\mathrm{BMI}(p)$	Female	15.5	15.5	17.0	17.5	17.5	17.5
	Male	15.5	15.5	17.0	18.5	18.5	18.5
Height	Female	0.80	1.20	1.55	1.60	1.60	1.60
	Male	0.88	1.24	1.58	1.72	1.72	1.72

Calculate the reference value $\mathrm{weight}^{\star}$ per age and sex to determine the undernourishment.

We have:

$$\mathrm{weight}^{\star}_{age,sex} = \mathrm{BMI}_{age,sex}\left(p\right) \cdot \mathrm{height}^{2}_{age,sex}$$

For example, we have:

$$\begin{aligned}\mathrm{weight}^{\star}{}_{-3yr,female} &= \mathrm{BMI}_{-3yr,female}\left(5\%\right) \cdot \mathrm{height}^{2}{}_{-3yr,female} \\ &= 15.5 \times 0.80^{2} \\ &= 9.92\,\mathrm{kg}\end{aligned}$$

We obtain the following results:

Age	−3	3–10	10–18	18–30	30–60	60+
Female	9.92	22.32	40.84	44.80	44.80	44.80
Male	12.00	23.83	42.44	54.73	54.73	54.73

This means that a 40-year-old woman is considered undernourished if she weighs less than 44.8 kg.

Question (b)

We assume that the basal metabolic rate (BMR) is given by the Schofield equation:

$$\text{BMR} = \alpha + \beta \cdot \text{weight}^{\star}$$

where α_j and β_j are the estimates of the linear regression between weight and BMR:

	Age	-3	3–10	10–18	18–30	30–60	60+
α	Female	-31.1	485.9	692.6	486.6	845.6	658.5
	Male	-30.4	504.3	658.2	692.2	873.1	587.7
β	Female	58.317	20.315	13.384	14.818	8.126	9.082
	Male	59.512	22.706	17.686	15.057	11.472	11.711

Calculate the basal metabolic rate for each group.

The basal metabolic rate (BMR) is given by the Schofield equation:

$$\mathrm{BMR}_{age,sex} = \alpha_{age,sex} + \beta_{age,sex} \cdot \mathrm{weight}^{\star}_{age,sex}$$

For example, we have:

$$\begin{aligned}\mathrm{BMR}_{-3yr,female} &= \alpha_{-3yr,female} + \beta_{-3yr,female} \cdot \mathrm{weight}^{\star}_{-3yr,female} \\ &= -31.1 + 58.317 \times 9.92 \\ &= 547.40 \,\mathrm{kcal/capita/day}\end{aligned}$$

We obtain the following results:

Age	−3	3–10	10–18	18–30	30–60	60+
Female	547	939	1 239	1 150	1 210	1 065
Male	684	1 045	1 409	1 516	1 501	1 229

Question (c)

The minimum dietary energy requirement is equal to the physical activity level (PAL) times the basal metabolic rate:

$$\mathrm{MDER} = \mathrm{PAL} \cdot \mathrm{BMR}$$

We assume that the population is on average lightly active, implying that $\mathrm{PAL} = 1.55$. Calculate the minium dietary energy requirement for the different groups.

The minimum dietary energy requirement is equal to the physical activity level times the basal metabolic rate:

$$\mathrm{MDER}_{age,sex} = \mathrm{PAL}_{age,sex} \cdot \mathrm{BMR}_{age,sex}$$

We obtain the following results:

Age	-3	3–10	10–18	18–30	30–60	60+
Female	848	1 456	1 921	1 783	1 875	1 651
Male	1 060	1 620	2 184	2 350	2 326	1 904

This means that a 40-year-old woman is considered undernourished if her dietary energy consumption is less than 1 875 Calories per day.

Question (d)

We assume that the proportion of females and males is the same, while the distribution of the population by age is as follows:

Age	-3	3–10	10–18	18–30	30–60	60+
Frequency	9.9%	15%	16%	18%	29%	12.1%

Calculate the minium dietary energy requirement of the population.

The minimum dietary energy requirement of the population is the weighted average of the different MDER values:

$$\mathrm{MDER} = \sum_{sex} \sum_{age} f_{age,sex} \cdot \mathrm{MDER}_{age,sex}$$

where $f_{age,sex}$ is the frequency of the group in the population. Finally, we obtain:

$$\mathrm{MDER} = 1\,849.90\,\text{kcal/capita/day}$$

Question 4

We want to calculate the depth of the food deficit:

$$\mathrm{FD} = \int_{x<r_L} (\bar{r} - x) f_x(x) \, \mathrm{d}x$$

where r_L is the minimum dietary energy requirement (MDER), $\bar{r}$ is the average dietary energy requirement (ADER), and $f_x(x)$ is the probability density function of the dietary energy consumption X.

Question (a)

What is the interpretation of the indicator FD?

FD indicates how many calories would be needed to ensure that undernourished would be eliminated if properly distributed.

Question (b)

Find the probabilistic expression of the indicator FD.

We have:

$$\begin{aligned}
\mathrm{FD} &= \int_{x<r_L} (\bar{r} - x) f_x(x)\,\mathrm{d}x \\
&= \int_0^{r_L} \bar{r} f_x(x)\,\mathrm{d}x - \int_0^{r_L} x f_x(x)\,\mathrm{d}x \\
&= \bar{r}\int_0^{r_L} f_x(x)\,\mathrm{d}x - \frac{\int_0^{r_L} x f_x(x)\,\mathrm{d}x}{\int_0^{r_L} f_x(x)\,\mathrm{d}x}\int_0^{r_L} f_x(x)\,\mathrm{d}x \\
&= (\bar{r} - \mathbb{E}[X \mid X < r_L]) \cdot \Pr\{X < r_L\} \\
&= \mathrm{PoU} \cdot (\bar{r} - \mathbb{E}[X \mid X < r_L])
\end{aligned}$$

The depth of the food deficit is the product of the prevalence of undernourishment and the difference between the average dietary energy requirement and the average dietary energy consumption, conditional on consumption being below the minimum requirement. Another expression is:

$$\mathrm{FD} = \mathrm{PoU} \cdot \mathbb{E}\left[(\bar{r} - X)_+ \mid X < r_L\right]$$

because $\bar{r} \geq r_L$. $\mathbb{E}\left[(\bar{r} - X)_+ \mid X < r_L\right]$ is the expected shortfall of food security.

Question (c)

Find the analytical value of FD when $X \sim \mathcal{LN}\left(\mu_x, \sigma_x^2\right)$.

Following Roncalli (2021, page 319), we introduce the notation $\Phi_c(x) = \Phi((x - \mu_x)/\sigma_x)$, and we calculate the conditional moment $\mu'_m(X) = \mathbb{E}[X^m \mid X < r_L]$ for $m \geq 1$ by using the change of variable $y = \ln x$:

$$\begin{aligned}
\mu'_m(X) &= \frac{1}{\Phi_c(\ln r_L)} \int_0^{r_L} \frac{x^m}{x\sigma_x\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu_x}{\sigma_x}\right)^2\right) \mathrm{d}x \\
&= \frac{1}{\Phi_c(\ln r_L)} \int_{-\infty}^{\ln r_L} \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y - \mu_x}{\sigma_x}\right)^2 + my\right) \mathrm{d}y
\end{aligned}$$

We have:

$$\begin{aligned}
-\frac{1}{2}\left(\frac{y - \mu_x}{\sigma_x}\right)^2 + my &= -\frac{1}{2}\left(\frac{y^2 - 2y\left(\mu_x + m\sigma_x^2\right) + \mu_x^2}{\sigma_x^2}\right) \\
&= -\frac{1}{2}\left(\frac{y - \left(\mu_x + m\sigma_x^2\right)}{\sigma_x}\right)^2 + \left(m\mu_x + \frac{1}{2}m^2\sigma_x^2\right)
\end{aligned}$$

We deduce that:

$$\mu'_m(X) = \frac{\exp\left(m\mu_x + m^2\sigma_x^2/2\right)}{\Phi_c(\ln r_L)} \int_{-\infty}^{\ln r_L} \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-\left(\mu_x + m\sigma_x^2\right)}{\sigma_x}\right)^2\right) \mathrm{d}y$$

Using the change of variable $z = \dfrac{y-\left(\mu_x + m\sigma_x^2\right)}{\sigma_x}$, it follows that:

$$\mu'_m(X) = \frac{\exp\left(m\mu_x + m^2\sigma_x^2/2\right)}{\Phi_c(\ln r_L)} \int_{-\infty}^{z_L} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) \mathrm{d}z$$

where:

$$z_L = \frac{\ln r_L - \left(\mu_x + m\sigma_x^2\right)}{\sigma_x}$$

Finally, we obtain:

$$\mu'_m(X) = \frac{\Phi_c\left(\ln r_L - m\sigma_x^2\right)}{\Phi_c\left(\ln r_L\right)} \exp\left(m\mu_x + \frac{1}{2}m^2\sigma_x^2\right)$$

and:

$$\mathbb{E}\left[X \mid X < r_L\right] = \mu'_1(X) = \frac{\Phi\left(\dfrac{\ln r_L - \mu_x - \sigma_x^2}{\sigma_x}\right)}{\Phi\left(\dfrac{\ln r_L - \mu_x}{\sigma_x}\right)} \exp\left(\mu_x + \frac{1}{2}\sigma_x^2\right)$$

The analytical expression of the depth of the food deficit is:

$$\mathrm{FD} = \bar{r}\Phi\left(\frac{\ln r_L - \mu_x}{\sigma_x}\right) - e^{\mu_x + \frac{1}{2}\sigma_x^2}\Phi\left(\frac{\ln r_L - \mu_x - \sigma_x^2}{\sigma_x}\right)$$

because:

$$\mathrm{PoU} = \Pr\left\{X < r_L\right\} = \Phi\left(\frac{\ln r_L - \mu_x}{\sigma_x}\right)$$

Question (d)

Calculate the depth of the food deficit in the case of Question 2 if we assume that the average dietary energy requirement is equal to 2 500 Calories per person per day.

The depth of the food deficit is equal to:

$$\begin{aligned}
\mathrm{FD} &= 2\,500 \times \Phi\left(\frac{\ln 1\,850 - 7.7973}{0.2845}\right) - \\
&\quad \exp\left(7.7973 + \frac{1}{2}0.2845^2\right) \times \Phi\left(\frac{\ln 1\,850 - 7.7973 - 0.2845^2}{0.2845}\right) \\
&= 150.2968 \text{ kcal/capita/year}
\end{aligned}$$

A Course on Biodiversity
Exercise 2. Calculating the species-area relationship using the theory of island biogeography

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[11]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Question 1

We consider the model of island biogeography developed by Beaugrand *et al.* (2024). We denote species richness by $S(t)$. At the initial date t_0, we have $S(t_0) = S_0$ (we can assume that $S_0 = 0$). The authors assume the existence of a saturation date t_s, *i.e.*, the species richness cannot exceed a limit value:

$$0 \leq S(t) \leq S_s = S(t_s)$$

Question (a)

Let $f(x) = ae^{b(x/c)^d}$ where $a > 0$, $c > 0$ and $d > 0$. The parameter b can take two values: -1 or $+1$. We also assume that $0 \leq x \leq c$. In which case do we get a decreasing, increasing, concave and convex function?

We have:

$$
\begin{aligned}
f'(x) &= ae^{b(x/c)^d} bd \left(\frac{x}{c}\right)^{d-1} \frac{1}{c} \\
&= \frac{abd}{c} \left(\frac{x}{c}\right)^{d-1} e^{b(x/c)^d}
\end{aligned}
$$

and:

$$
\begin{aligned}
f''(x) &= \frac{abd}{c}(d-1)\left(\frac{x}{c}\right)^{d-2} \frac{1}{c} e^{b(x/c)^d} + \frac{abd}{c}\left(\frac{x}{c}\right)^{d-1} e^{b(x/c)^d} bd \left(\frac{x}{c}\right)^{d-1} \frac{1}{c} \\
&= \frac{abd(d-1)}{c^2}\left(\frac{x}{c}\right)^{d-2} e^{b(x/c)^d} + \frac{ab^2d^2}{c^2}\left(\frac{x}{c}\right)^{2d-2} e^{b(x/c)^d} \\
&= \frac{abd}{c^2}\left(\frac{x}{c}\right)^{d-2}\left(d-1+bd\left(\frac{x}{c}\right)^{d}\right) e^{b(x/c)^d}
\end{aligned}
$$

Since a, c and d are positive, we deduce that $\dfrac{x}{c} \geq 0$, $e^{b(x/c)^d} \geq 0$ and:

$$f'(x) \geq 0 \Leftrightarrow b \geq 0$$

$f(x)$ is an increasing function of x if the parameter b is positive, otherwise $f(x)$ is a decreasing function of x. We have:

$$f''(x) \geq 0 \Leftrightarrow g(x; b) = \underbrace{b(d-1)}_{-/+} + \underbrace{b^2 d \left(\frac{x}{c}\right)^d}_{+} \geq 0$$

because we have:

$$0 \leq b^2 d \left(\frac{x}{c}\right)^d \leq b^2 d$$

We consider two cases:

1. $b = -1$
   If $d \leq 1$, then $g(x; -1) = 1 - d + d\left(\frac{x}{c}\right)^d$, $f''(x) \geq 0$ and the function is convex. If $d > 1$, the function is neither convex nor concave.
2. $b = 1$
   If $d \geq 1$, then $g(x; 1) = (d - 1) + d\left(\frac{x}{c}\right)^d$, $f''(x) \geq 0$ and the function is convex. If $d \leq \dfrac{1}{2}$, then $g(0; 1) = d - 1 < 0$, $g(c; 1) = 2d - 1 \leq 0$, $f''(x) \leq 0$ and the function is concave. If $d \in \left(\dfrac{1}{2}, 1\right)$, the function is neither convex nor concave.

Question (b)

Let $\lambda(t)$ represent the immigration rate. We assume that:

$$\lambda(t) = \lambda_0 \frac{e^{-\left(\frac{S(t)}{S_s}\right)^{\beta_1 \lambda_0}} - e^{-1}}{1 - e^{-1}} \qquad \text{for } \lambda_0 \geq \lambda(t) \geq \lambda_s = \lambda(t_s)$$

where $\lambda_0 \geq 0$ is the initial immigration rate, and $\beta_1 > 0$ is a parameter. Prove that $\lambda(t)$ is a decreasing function of $S(t)$, with $\lambda(t_0) = \lambda_0$ and $\lambda(t_s) = 0$. Determine whether $\lambda(t)$ is a convex or concave function. Plot the function $\lambda(t)$ for the following parameter sets $(\lambda_0, \beta_1, S_s)$: $(1.0, 0.2, 200)$, $(1.0, 0.5, 200)$, $(1.0, 1.0, 200)$, and $(0.9, 2.0, 150)$. Analyze and comment on these results.

We set $a = \dfrac{\lambda_0}{1 - e^{-1}}$, $b = -1$, $c = S_s$ and $d = \beta_1 \lambda_0$. We have:

$$\frac{\partial \lambda (t)}{\partial S (t)} = -\frac{\beta_1 \lambda_0^2}{(1 - e^{-1}) S_s} \left(\frac{S (t)}{S_s} \right)^{\beta_1 \lambda_0 - 1} e^{-\left(\frac{S(t)}{S_s} \right)^{\beta_1 \lambda_0}} \leq 0$$

with:

$$\lambda (t_0) = \lambda_0 \frac{e^{-0} - e^{-1}}{1 - e^{-1}} = \lambda_0$$

and:

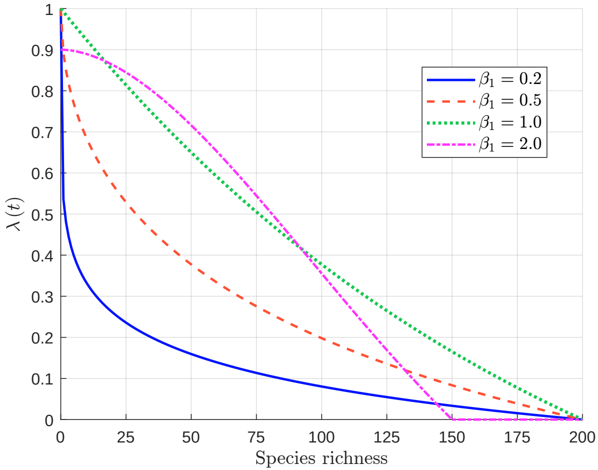
$$\lambda (t_s) = \lambda_0 \frac{e^{-1} - e^{-1}}{1 - e^{-1}} = 0$$

The second-order derivative is:

$$\frac{\partial^2 \lambda (t)}{\partial S (t)^2} = -\frac{\beta_1 \lambda_0^2}{(1 - e^{-1}) S_s^2} \left(\frac{S (t)}{S_s} \right)^{\beta_1 \lambda_0 - 2} \left(\beta_1 \lambda_0 - 1 - \beta_1 \lambda_0 \left(\frac{S (t)}{S_s} \right)^{\beta_1 \lambda_0} \right) e^{-\left(\frac{S(t)}{S_s} \right)^{\beta_1 \lambda_0}}$$

$\lambda (t)$ is convex if and only if $\beta_1 \lambda_0 \leq 1$.

Figure 98: Immigrate rate function $\lambda(t)$



We verify that the function $\lambda(t)$ is decreasing and convex for the first three sets of parameters because $\beta_1 \lambda_0 \leq 1$. For the last set of parameters, it is decreasing but not convex.

Question (c)

Let $\mu^{\mathrm{long}}(t)$ be the long-term extinction rate. We assume that:

$$\mu^{\mathrm{long}}(t) = \mu_s \frac{e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s}} - 1}{e^1 - 1} \qquad \text{for } \mu_0^{\mathrm{long}} \leq \mu^{\mathrm{long}}(t) \leq \mu_s = \mu^{\mathrm{long}}(t_s)$$

where $\mu_s \geq 0$ is the extinction rate at the saturation date, and $\beta_2 > 0$ is a parameter. Prove that $\mu^{\mathrm{long}}(t)$ is an increasing function of $S(t)$, with $\mu^{\mathrm{long}}(t_0) = 0$ and $\mu^{\mathrm{long}}(t_s) = \mu_s$. Determine whether $\mu^{\mathrm{long}}(t)$ is a convex or concave function. Plot the function $\mu^{\mathrm{long}}(t)$ for the following parameter sets (μ_s, β_2, S_s): $(1.0, 0.2, 200)$, $(1.0, 0.5, 200)$, $(1.0, 1.0, 200)$, and $(0.5, 2.0, 150)$. Analyze and comment on these results.

We set $a = \dfrac{\mu_s}{e^{-1} - 1}$, $b = 1$, $c = S_s$ and $d = \beta_2 \mu_s$. We have:

$$\frac{\partial\, \mu^{\text{long}}(t)}{\partial\, S(t)} = \frac{\beta_2 \mu_s^2}{\left(e^{-1} - 1\right) S_s} \left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s - 1} e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s}} \geq 0$$

with:

$$\mu^{\text{long}}(t_0) = \mu_s \frac{e^0 - 1}{e^1 - 1} = 0$$

and:

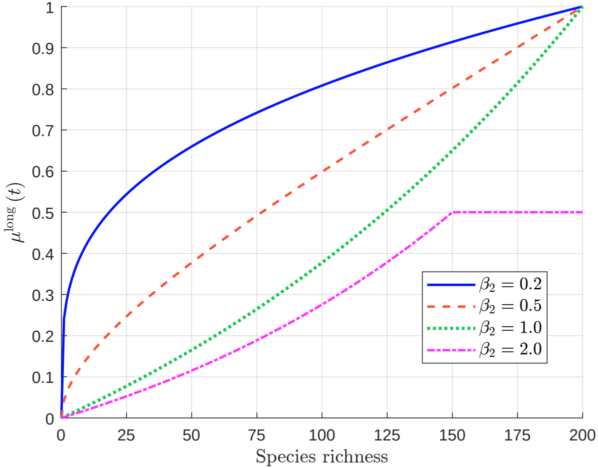
$$\mu^{\text{long}}(t_s) = \mu_s \frac{e^1 - 1}{e^1 - 1} = \mu_s$$

The second-order derivative is:

$$\frac{\partial^2\, \mu^{\text{long}}(t)}{\partial\, S(t)^2} = \frac{\beta_2 \mu_s^2}{\left(e^{-1} - 1\right) S_s^2} \left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s - 2} \left(\beta_2 \mu_s - 1 + \beta_2 \mu_s \left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s}\right) e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2 \mu_s}}$$

$\mu^{\text{long}}(t)$ is convex if and only if $\beta_2 \mu_s \geq 1$ and concave if and only if $\beta_2 \mu_s \leq \dfrac{1}{2}$.

Figure 99: Long-term extinction rate function $\mu^{\text{long}}(t)$



We verify that the function $\mu^{\text{long}}(t)$ is increasing and concave for the first two sets of parameters because $\beta_2 \mu_s \leq \frac{1}{2}$. For the last two sets of parameters, it is increasing and convex.

Question (d)

Let $\mu^{\text{short}}(t)$ be the short-term extinction rate. We assume that:

$$\mu^{\text{short}}(t) = \beta_3 \lambda(t) e^{-\beta_4 S(t)} \qquad \text{for } \mu_0^{\text{short}} \geq \mu^{\text{short}}(t) \geq \mu_s^{\text{short}} = \mu^{\text{short}}(t_s)$$

where β_3 and β_4 are two positive parameters. Prove that $\mu^{\text{short}}(t)$ is a decreasing function of $S(t)$, with $\mu^{\text{short}}(t_0) \leq \beta_3 \lambda_0$ and $\mu^{\text{short}}(t_s) = 0$. Plot the function[a] $\mu^{\text{short}}(t)$ for the following parameter sets (β_3, β_4, S_s): $(0.2, 0.02, 200)$, $(0.5, 0.02, 200)$, $(0.8, 0.02, 200)$, and $(1.0, 0.10, 150)$.

[a] We assume $\lambda_0 = 1.0$ and $\beta_1 = 0.5$ to define $\lambda(t)$.

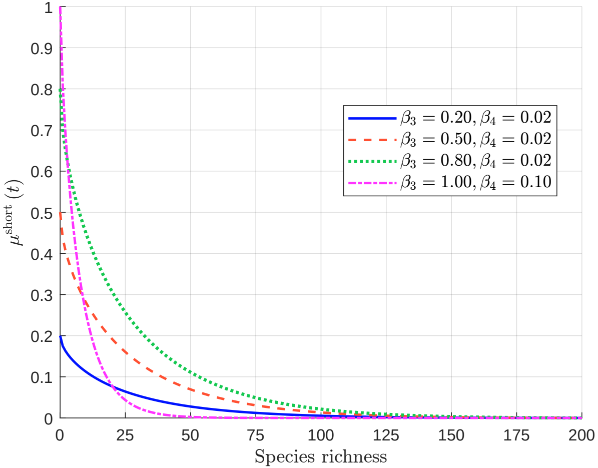
$\mu^{\text{short}}(t)$ is the product of two positive and decreasing functions: $\lambda(t)$ and $\beta_3 \lambda(t) e^{-\beta_4 S(t)}$. We deduce that it is decreasing. We also have:

$$\mu^{\text{short}}(t_0) = \beta_3 \lambda_0 e^{-\beta_4 S_0} \leq \beta_3 \lambda_0$$

and:

$$\mu^{\text{short}}(t_s) = \beta_3 \lambda_s e^{-\beta_4 S_s} = 0$$

Figure 100: Short-term extinction rate function $\mu^{\text{short}}(t)$



Question (e)

Let $\mu(t) = \mu^{\mathrm{short}}(t) + \mu^{\mathrm{long}}(t)$ be the extinction rate. Show that $\mu(t_0) = \mu^{\mathrm{short}}(t_0)$ and $\mu(t_s) = \mu_s$. Plot the function $\mu(t)$ for the following set of parameters: $\lambda_0 = 1$, $\beta_1 = 0.5$, $\mu_s = 1$, $\beta_2 = 2$, $\beta_3 = 0.70$, $\beta_4 = 0.01$, and $S_s = 200$. Comment on these results.

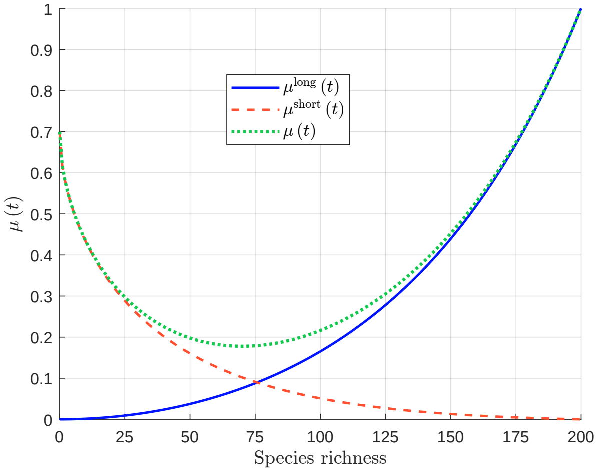
We have:

$$\begin{aligned}\mu(t_0) &= \mu^{\text{short}}(t_0) + \mu^{\text{long}}(t_0) \\ &= \mu^{\text{short}}(t_0) + 0 \\ &= \mu^{\text{short}}(t_0)\end{aligned}$$

and:

$$\begin{aligned}\mu(t_s) &= \mu^{\text{short}}(t_s) + \mu^{\text{long}}(t_s) \\ &= 0 + \mu_s \\ &= \mu_s\end{aligned}$$

Figure 101: Extinction rate function $\mu(t)$



- Figure 101 illustrates the function $\mu(t)$
- Since the extinction rate is the sum of two functions, $\mu^{\text{short}}(t)$ which is decreasing, and $\mu^{\text{long}}(t)$ which is increasing, $\mu(t)$ is not monotonic and can exhibit several shapes
- For reasonable values of the parameters, we observe that $\mu(t)$ first decreases and then increases. This behavior arises because $\mu^{\text{short}}(t_0) > \mu^{\text{long}}(t_0)$ and $\mu^{\text{short}}(t_s) < \mu^{\text{long}}(t_s)$
- Consequently, when $S(t)$ is low and close to zero, the short-term extinction rate dominates the long-term extinction rate, leading to a general decrease in $\mu(t)$. In contrast, when $S(t)$ is high and approaches the saturation state S_s, the long-term extinction rate becomes dominant, implying that $\mu(t)$ generally increases for high values of species richness
- In summary, the extinction rate $\mu(t)$ has a U-shaped relationship with $S(t)$

Question (f)

The dynamics of species richness is governed by the following differential equation:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = \delta(t) = \lambda(t) - \mu(t)$$

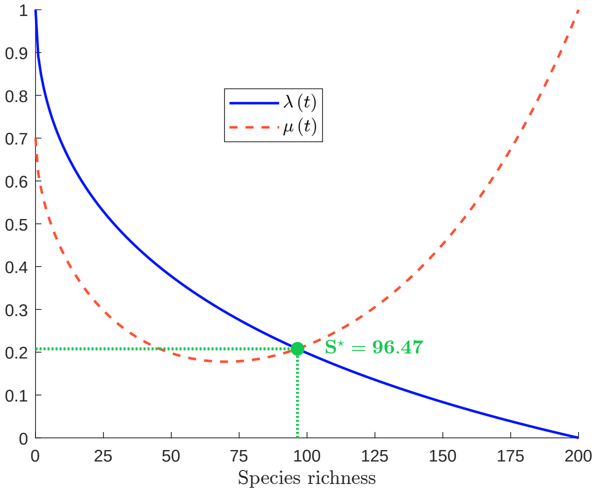
What is the condition for $S(t)$ to reach an equilibrium? Illustrate and determine the equilibrium $S^{\star}$ using the set of parameters provided in Question 1.e.

An equilibrium occurs if and only if $\delta(t) = 0$ or $\lambda(t) = \mu(t)$. We deduce that such an equilibrium $S^\star$ is achieved if the following equation has at least one solution:

$$\lambda_0 \frac{e^{-\left(\frac{S^\star}{S_s}\right)^{\beta_1 \lambda_0}} - e^{-1}}{1 - e^{-1}} \left(1 - \beta_3 e^{-\beta_4 S^\star}\right) = \mu_s \frac{e^{\left(\frac{S^\star}{S_s}\right)^{\beta_2 \mu_s}} - 1}{e^1 - 1} \tag{1}$$

Figure 102 shows the functions $\lambda(t)$ and $\mu(t)$. Solving Equation (1) gives $S^\star = 96.472$. This equilibrium is stable, because $\frac{\mathrm{d}S(t)}{\mathrm{d}t} < 0$ when $S(t) > S^\star$ and $\frac{\mathrm{d}S(t)}{\mathrm{d}t} > 0$ when $S(t) < S^\star$. This implies that any perturbation from $S^\star$ will return to the steady state $S^\star$.

Figure 102: Immigration rate $\lambda(t)$, extinction rate $\mu(t)$ and equilibrium $S^{\star}$



Question (g)

Under what conditions can two equilibria exist? Let $S_1^\star$ and $S_2^\star$ be the two equilibria, with $S_1^\star \leq S_2^\star$. Show that $S_1^\star$ is an unstable steady state, while $S_2^\star$ is a stable steady state. Deduce that there is a third equilibrium $S_0^\star$, with $S_0^\star \leq S_1^\star \leq S_2^\star$. Illustrate the three-equilibrium case with the following set of parameters: $\lambda_0 = 1$, $\beta_1 = 0.5$, $\mu_s = 1$, $\beta_2 = 2$, $\beta_3 = 1.70$, $\beta_4 = 0.02$, and $S_s = 200$.

In Table 75, we give the direction of variation of the functions $\lambda(t)$, $\mu^{\text{short}}(t)$, $\mu^{\text{long}}(t)$ and $\mu(t)$.

Table 75: Table of variation for the immigration and extinction rates

S	0	S^*	S_s
$\lambda(t)$	λ_0	$\searrow$	0
$\mu^{\text{short}}(t)$	$\beta_3\lambda_0$	$\searrow$	0
$\mu^{\text{long}}(t)$	0	$\nearrow$	μ_s
$\mu(t)$ $(\beta_3 = 0)$	0	$\nearrow$	μ_s
$\mu(t)$ $(\beta_3 > 0)$	$\beta_3\lambda_0$	μ^*	μ_s

We can then derive the table of variation of the net diversification rate $\delta(t) = \lambda(t) - \mu(t)$. When $\beta_3 = 0$, $\mu^{\text{short}}(t) = 0$ and $\delta(t)$ is a decreasing function with $\delta(t_0) = \delta_0 = \lambda_0 > 0$ and $\delta(t_s) = -\mu_s < 0$. We conclude that there is only one equilibrium. When $0 < \beta_3 \leq 1$, $\delta(t)$ increases up to a threshold S^* and then decreases. We have $\delta_0 = \lambda_0 (1 - \beta_3) > 0$ and $\delta(t_s) = -\mu_s < 0$. We obtain the table of variation of the net diversification rate given in Table 76.

Table 76: Table of variation for the net diversification rate $\delta(t)$

S	0	S^*	$S^\star$	S_s
$\lambda(t) - \mu(t)$ $(0 < \beta_3 \leq 1)$	δ_0	δ^*	0	$-\mu_s$

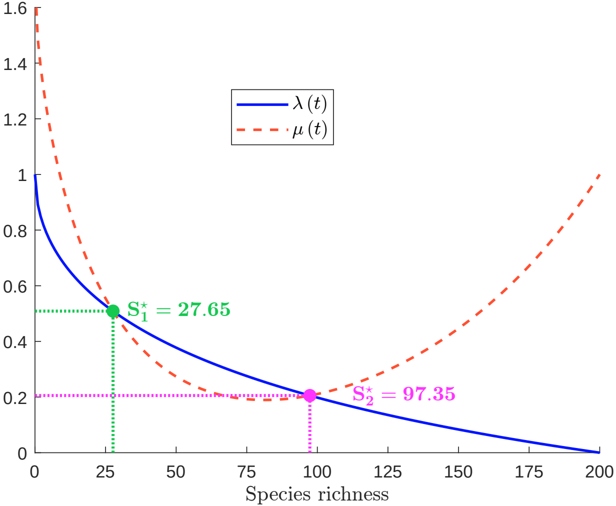
When $\beta_3 > 1$, $\delta(t)$ has the same behavior as the previous case, but $\delta_0 = \lambda_0 (1 - \beta_3) < 0$ and we obtain the table of variation of the net diversification rate given in Table 77.

Table 77: Table of variation for the net diversification rate $\delta(t)$

S	0	$S_1^\star$	S^*	$S_2^\star$	S_s
$\lambda(t) - \mu(t)$ $(\beta_3 > 1)$	δ_0	0	δ^*	0	$-\mu_s$

We deduce that there are two equilibria $S_1^\star$ and $S_2^\star$, with $S_1^\star \leq S_2^\star$. The equilibrium $S_1^\star$ is unstable because $\dfrac{\mathrm{d}S(t)}{\mathrm{d}t} < 0$ when $S(t) < S_1^\star$ and $\dfrac{\mathrm{d}S(t)}{\mathrm{d}t} > 0$ when $S(t) > S_1^\star$. Only $S_2^\star$ is a stable steady state. Furthermore, the net diversification rate is negative when $S(t) < S_1^\star$, which means that $S(t + \mathrm{d}t) < S(t)$ and $S(t)$ tends to the equilibrium $S_0^\star = 0$.

Figure 103: The three-equilibrium case



Question (h)

Using the two sets of parameters defined in Questions 1.e and 1.g, simulate the process $S(t)$ when the initial species richness S_0 is 0, 25, 30, 50 and 120, respectively. Comment on these results.

- The simulations of the species richness process are presented in Figures 104 and 105
- In the one-equilibrium case, the process $S(t)$ converges to the equilibrium $S^{\star} = 96.47$, regardless of the initial state S_0
- In the three-equilibrium case, the process $S(t)$ converges to either $S_0^{\star} = 0$ or $S_2^{\star} = 97.35$, depending on the initial state S_0. The equilibrium $S_1^{\star} = 27.65$ is reached only if $S_0 = S_1^{\star}$. Otherwise, we get the equilibrium $S_0^{\star} = 0$ if $S_0 < S_1^{\star}$, and the equilibrium $S_2^{\star} = 97.35$ if $S_0 > S_1^{\star}$

Figure 104: Simulation of the species richness process under a one-equilibrium scenario

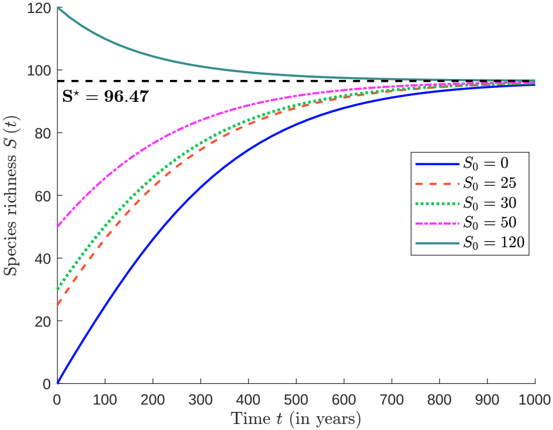
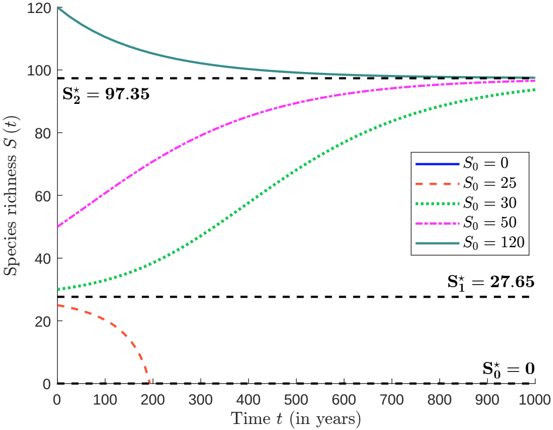


Figure 105: Simulation of the species richness process under a three-equilibrium scenario



Question 2

We aim to analyze the dynamics of the TIB steady state $S^{\star}$ under the assumption that there is no short-term extinction rate, meaning $\beta_3 = 0$ or $\mu^{\text{short}}(t) = 0$. The default parameter values are $\lambda_0 = 1$, $\beta_1 = 0.5$, $\mu_s = 1$, $\beta_2 = 1$, and $S_s = 200$.

Question (a)

What is the impact of β_2 on the equilibrium $S^\star$? Compare the solutions for $\beta_2 = 0.2$, $\beta_2 = 1$, and $\beta_2 = 5$.

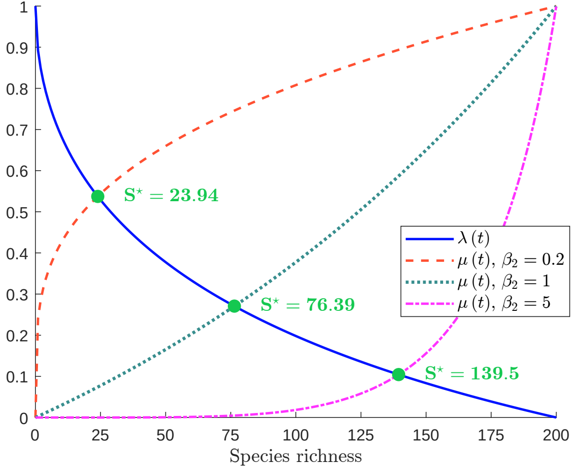
We have:

$$\begin{aligned}\frac{\partial\,\mu(t)}{\partial\,\beta_2} &= \frac{\mu_s^2}{e^1-1}e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2\mu_s}} \cdot \\ &\quad \left(\frac{S(t)}{S_s}\right)^{\beta_2\mu_s}\ln\left(\frac{S(t)}{S_s}\right) \\ &\leq 0\end{aligned}$$

This means that the relationship $\mu(t)$ shifts downward as β_2 increases

Figure 106 illustrates the equilibrium for the three values of β_2. From this, we deduce that if $\beta_2' \geq \beta_2$, then $S^\star(\beta_2') \geq S^\star(\beta_2)$

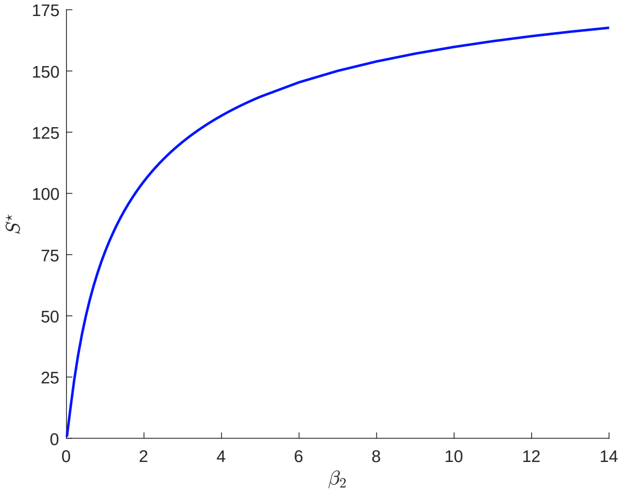
Figure 106: How the equilibrium shifts with the parameter β_2



Question (b)

Plot the relationship between β_2 and $S^{\star}$.

Figure 107: Relationship between β_2 and $S^\star$



Question (c)

What is the impact of μ_s on the equilibrium $S^\star$?

We have:

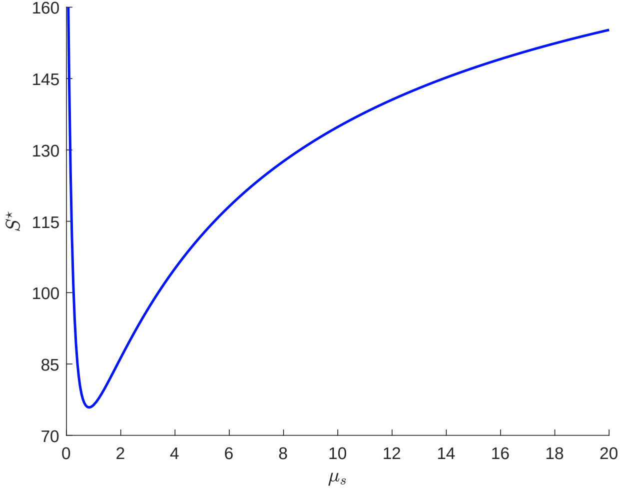
$$\frac{\partial\,\mu\left(t\right)}{\partial\,\mu_s}=\frac{e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2\mu_s}}-1}{e^1-1}+\frac{\beta_2\mu_s}{e^1-1}e^{\left(\frac{S(t)}{S_s}\right)^{\beta_2\mu_s}}\left(\frac{S\left(t\right)}{S_s}\right)^{\beta_2\mu_s}\ln\left(\frac{S\left(t\right)}{S_s}\right)$$

Using the default parameter values and $S\left(t\right)=100$, we find that $\partial_{\mu_s}\mu\left(t\right)=0.045$ when $\mu_S=1$ and $\partial_{\mu_s}\mu\left(t\right)=-0.047$ when $\mu_S=5$. Since the derivative can be either positive or negative, we cannot determine the effect of μ_s on the equilibrium $S^\star$.

Question (d)

Plot the relationship between μ_s and $S^\star$.

Figure 108: Relationship between μ_s and $S^\star$



Question (e)

What conclusion can we draw about the relationship between the area A and the equilibrium $S^{\star}$.

- We can assume that the relationship between A and β_2 is increasing. In this case, we obtain the species-area relationship
- We can also assume that the relationship between A and μ_s is increasing. In this case, we retrieve the species-area relationship only if μ_s is greater than a threshold.

Question (f)

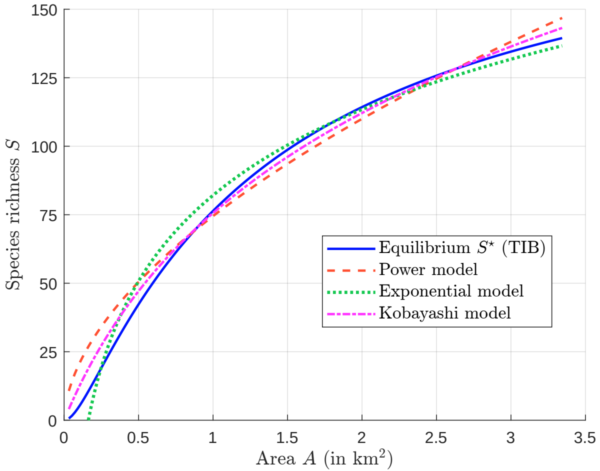
We assume that the area A (expressed in km^2) is related to the parameter β_2 as follows: $A = \beta_2^{0.75}$. A sampling of β_2 is taken between 0.01 and 5.00 with a step size of 0.01. Use nonlinear least squares to estimate the power model $S = cA^z$, the exponential model $S = c + z \ln(A)$, and the Kobayashi model $S = c \ln(1 + zA)$. Compare the TIB equilibrium $S^\star$ with the forecasts of the fitted models. Comment on these results.

We obtain the following estimated values for the parameters:

Model	c	z
Power	74.513	0.562
Exponential	82.098	45.199
Kobayashi	73.539	1.797

We observe that the fitted curves closely align with the equilibrium curve predicted by the theory of island biogeography

Figure 109: Species-area relationship (TIB, power, exponential and Kobayashi models)



A Course on Biodiversity
Exercise 3. Species abundance models

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[12]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

This exercise is inspired by the research articles of Coleman (1981), and He and Legendre (2002).

Question 1

We consider an area (or an ecosystem) A containing S species. Let n_i denote the abundance of the i^{th} species and let $n = \sum_{i=1}^{S} n_i$ represent the total abundance in the area. We focus on a subarea $a \subseteq A$ and denote by $\tilde{S}_a$ the random variable representing the number of species in this subarea. We introduce the notation $S_a = \mathbb{E}\left[\tilde{S}_a\right]$ to represent the expected number of species in the subarea.

Question (a)

Let p_i be the probability that species i is present in the subarea a, and $\tilde{S}_i \sim \mathcal{B}(p_i)$ the random variable indicating its presence ($\tilde{S}_i = 1$) or its absence ($\tilde{S}_i = 0$). What is the probability distribution of $\tilde{S}_a$? Deduce the value of S_a.

Application: Given $p = (p_1, \ldots, p_5) = (20\%, 30\%, 10\%, 65\%, 10\%)$, calculate $\tilde{S}_a$ and S_a.

We have:

$$\tilde{S}_a = \sum_{i=1}^{S} \tilde{S}_i$$

Assuming that the random variables $\tilde{S}_1, \ldots, \tilde{S}_S$ are independent, $\tilde{S}_a$ follows a Poisson binomial distribution:

$$\tilde{S}_a \sim \mathcal{PB}\left(p_1, \ldots, p_S\right)$$

The probability mass function is given by:

$$\Pr\left\{\tilde{S}_a = k\right\} = \sum_{\mathcal{E} \in \mathcal{F}_k} \prod_{i \in \mathcal{E}} p_i \prod_{j \in \mathcal{E}^c} \left(1 - p_j\right)$$

where $\mathcal{F}_k$ is the set of all subsets of k integers that can be selected from $\{1, \ldots, n\}$ and $\mathcal{E}^c$ denotes the complement of the subset $\mathcal{E}$. From this, we deduce the expected value:

$$S_a = \mathbb{E}\left[\tilde{S}_a\right] = \mathbb{E}\left[\sum_{i=1}^{S} \tilde{S}_i\right] = \sum_{i=1}^{S} p_i$$

Given $p = (20\%, 30\%, 10\%, 65\%, 10\%)$, we obtain the following probability mass function for $\tilde{S}_a$:

k	0	1	2	3	4	5
$\Pr\left\{\tilde{S}_a = k\right\}$	15.88%	43.79%	30.85%	8.48%	0.97%	0.04%

The expected number of species is $S_a = 1.35$.

Question (b)

We assume a random placement of the species within the area A. What is the probability of observing k individuals of species i in the subarea a? Show that:

$$\tilde{S}_i \sim \mathcal{PB}\left(p_i = 1 - \left(1 - \frac{a}{A}\right)^{n_i}\right)$$

Find the value of S_a.

Application: Given $(n_1, \ldots, n_5) = (10, 4, 25, 6, 8)$, $A = 10\,\mathrm{km}^2$, and $a = 2\,\mathrm{km}^2$, calculate $\tilde{S}_a$ and S_a.

Let π be the probability that an individual occupies the subarea a. We have:

$$\pi = \frac{a}{A}$$

For example, if $A = 10\,\mathrm{km}^2$, the occupancy probability is 20% when $a = 2\,\mathrm{km}^2$. Let $\tilde{N}_i$ be the random variable indicating the number of individuals of species i present in the subarea a. $\tilde{N}_i$ follows a binomial distribution with parameters n_i (the total number of individuals of species i) and π. Thus, we have:

$$\Pr\left\{\tilde{N}_i = k\right\} = C_{n_i}^{k}\pi^{k}\left(1-\pi\right)^{n_i-k}$$

The probability p_i of observing the species i on the subarea is then:

$$\begin{aligned} p_i = \Pr\left\{\tilde{S}_i = 1\right\} &= \Pr\left\{\tilde{N}_i > 0\right\} \\ &= 1 - \Pr\left\{N_i = 0\right\} \\ &= 1 - \left(1-\pi\right)^{n_i} \\ &= 1 - \left(1 - \frac{a}{A}\right)^{n_i} \end{aligned}$$

Consequently, $\tilde{S}_a$ follows then a Poisson binomial distribution:

$$\tilde{S}_a \sim \mathcal{PB}\left(p_i = 1 - \left(1 - \frac{a}{A}\right)^{n_i}\right)$$

It follows that:

$$\begin{aligned} S_a &= \sum_{i=1}^{S} p_i \\ &= \sum_{i=1}^{S} \left(1 - \left(1 - \frac{a}{A}\right)^{n_i}\right) \\ &= S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i} \end{aligned}$$

Given $(n_1, \ldots, n_5) = (10, 30, 25, 66, 8)$, $A = 10\,\mathrm{km}^2$, and $a = 2\,\mathrm{km}^2$, $\pi = 20\%$ and we obtain the following results:

k or i	0	1	2	3	4	5
p_i		89.26%	59.04%	99.62%	73.79%	83.22%
$\Pr\left\{\tilde{S}_a = k\right\}$	0.00%	0.21%	3.45%	19.78%	44.32%	32.34%

The expected number of species is $S_a = 4.05$.

Question (c)

Calculate S_a for the following models.

Question (c.i)

Most even model:

$$n_i = \frac{n}{S} \quad \text{for } i = 1, \ldots, S$$

If the distribution is homogenous across species ($n_i = \frac{n}{S}$), we have:

$$p_i = 1 - \left(1 - \frac{a}{A}\right)^{\frac{n}{S}}$$

$\tilde{S}_a$ follows then a binomial distribution:

$$\tilde{S}_a \sim \mathcal{B}\left(S, p = 1 - \left(1 - \frac{a}{A}\right)^{\frac{n}{S}}\right)$$

The expected number of species is then equal to:

$$S_a = Sp = S\left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n}{S}}\right)$$

Question (c.ii)

Most uneven model:

$$n_i = 1 \quad \text{for } i < S \quad \text{and} \quad n_S = n - S + 1$$

If the distribution is the most uneven ($n_i = 1$ for $i < S$ and $n_S = n - S + 1$), we have:

$$\begin{aligned} S_a &= S - \sum_{i=1}^{S-1} \left(1 - \frac{a}{A}\right)^1 - \left(1 - \frac{a}{A}\right)^{n-S+1} \\ &= S - (S-1)\left(1 - \frac{a}{A}\right) - \left(1 - \frac{a}{A}\right)^{n-S+1} \\ &= 1 + (S-1)\frac{a}{A} - \left(1 - \frac{a}{A}\right)^{n-S+1} \end{aligned}$$

Question (c.iii)

Mixed even-uneven model:

$$n_i = 1 \quad \text{for } i \leq s \quad \text{and} \quad n_i = \frac{n - s}{S - s} = n_{s+1} \text{ for } i > s$$

We have:

$$\begin{aligned} S_a &= S - \sum_{i=1}^{s} \left(1 - \frac{a}{A}\right)^1 - \sum_{i=s+1}^{S} \left(1 - \frac{a}{A}\right)^{n_{s+1}} \\ &= S - s\left(1 - \frac{a}{A}\right) - (S - s)\left(1 - \frac{a}{A}\right)^{n_{s+1}} \\ &= s\frac{a}{A} + (S - s)\left(1 - \left(1 - \frac{a}{A}\right)^{n_{s+1}}\right) \\ &= s\frac{a}{A} + (S - s)\left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n-s}{S-s}}\right) \end{aligned}$$

If $s = 0$, we retrieve the most even model:

$$S_a = S\left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n}{S}}\right)$$

If $s = S - 1$, we retrieve the most uneven model:

$$\begin{aligned} S_a &= (S - 1)\frac{a}{A} + \left(1 - \left(1 - \frac{a}{A}\right)^{\frac{n-S+1}{S-S+1}}\right) \\ &= 1 + (S - 1)\frac{a}{A} - \left(1 - \frac{a}{A}\right)^{n-S+1} \end{aligned}$$

Question (c.iv)

He-Gaston model:

$$p_i = 1 - \left(1 - \frac{a}{A}\right)\left(1 + \frac{n_i a}{\kappa_i A}\right)^{-\kappa_i}$$

where $\kappa_i \in (-\infty, m_i) \cup [0, \infty)$ is a parameter which describes the spatial pattern of species i and $m_i = n_i a / A$ is the mean density of species i in the subarea a.

We have:

$$
\begin{aligned}
S_a &= \sum_{i=1}^{S} \left(1 - \left(1 - \frac{a}{A}\right)\left(1 + \frac{n_i a}{\kappa_i A}\right)^{-\kappa_i}\right) \\
&= S - \left(1 - \frac{a}{A}\right) \sum_{i=1}^{S} \left(1 + \frac{n_i a}{\kappa_i A}\right)^{-\kappa_i}
\end{aligned}
$$

Question (c.v)

Broken-stick model[a]:

$$n_i = \frac{n}{S} \sum_{k=i}^{S} \frac{1}{k}$$

[a]Hint: Approximate the harmonic sum by $\ln\left(\frac{S}{i}\right)$, and replace the sum $\sum_{i=1}^{S}\left(1 - \frac{a}{A}\right)^{n_i}$ with its integral form.

We can approximate the harmonic sum by:

$$\sum_{k=i}^{S} \frac{1}{k} \approx \int_{i}^{S} \frac{1}{x} \,\mathrm{d}x = \ln \left(\frac{S}{i} \right)$$

We deduce that:

$$n_i = \frac{n}{S} \sum_{k=i}^{S} \frac{1}{k} \approx \frac{n}{S} \ln \left(\frac{S}{i} \right)$$

We have:

$$\left(1 - \frac{a}{A} \right)^{n_i} = e^{\left(n_i \ln\left(1 - \frac{a}{A}\right)\right)} = \exp \left(\frac{n}{S} \ln \left(\frac{S}{i} \right) \ln \left(1 - \frac{a}{A} \right) \right) = \left(\frac{i}{S} \right)^{-\frac{n}{S} \ln\left(1 - \frac{a}{A}\right)}$$

and:

$$\sum_{i=1}^{S} \left(1 - \frac{a}{A} \right)^{n_i} = \sum_{i=1}^{S} \left(\frac{i}{S} \right)^{-\frac{n}{S} \ln\left(1 - \frac{a}{A}\right)} \approx \int_{0}^{S} \left(\frac{x}{S} \right)^{-\frac{n}{S} \ln\left(1 - \frac{a}{A}\right)} \mathrm{d}x$$

Using the change of variable $u = \frac{x}{S}$, we obtain:

$$\begin{aligned}
\int_0^S \left(\frac{x}{S}\right)^{-\frac{n}{S}\ln\left(1-\frac{a}{A}\right)} \mathrm{d}x &= S \int_0^1 u^{-\frac{n}{S}\ln\left(1-\frac{a}{A}\right)} \mathrm{d}u \\
&= S \left[\frac{u^{-\frac{n}{S}\ln\left(1-\frac{a}{A}\right)+1}}{-\frac{n}{S}\ln\left(1-\frac{a}{A}\right)+1}\right]_0^1 \\
&= \frac{S}{-\frac{n}{S}\ln\left(1-\frac{a}{A}\right)+1} \\
&= \frac{S^2}{S - n\ln\left(1-\frac{a}{A}\right)}
\end{aligned}$$

Since we have:

$$S_a = S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i} \approx S - \frac{S^2}{S - n \ln\left(1 - \frac{a}{A}\right)} = \frac{-Sn \ln\left(1 - \frac{a}{A}\right)}{S - n \ln\left(1 - \frac{a}{A}\right)}$$

we conclude that:

$$S_a = \frac{S \ln\left(1 - \dfrac{a}{A}\right)}{\ln\left(1 - \dfrac{a}{A}\right) - \dfrac{S}{n}}$$

Question (d)

The species abundance distribution is defined as the series $\{s(1), s(2), \ldots\}$ where $s(j)$ is the number of species with j individuals. Show that:

$$S_a = S - \sum_j s(j) \left(1 - \frac{a}{A}\right)^j$$

Application: Calculate S_a for the log-series distribution:

$$s(j) = \alpha \frac{x^j}{j}$$

where α is a parameter related to the total diversity and $x \in (0, 1)$ is a parameter determining the relative abundance.

We have:

$$S_a = \sum_{i=1}^{S} p_i = \sum_j \sum_{i \in j} p_i = \sum_j s(j)\, p_j$$

where p_j is the probability that a species with abundance j is present in the subarea a. Since $p_j = 1 - \left(1 - \dfrac{a}{A}\right)^j$, we get:

$$S_a = \sum_j s(j) \left(1 - \left(1 - \frac{a}{A}\right)^j\right) = S - \sum_j s(j) \left(1 - \frac{a}{A}\right)^j$$

where $S = \sum_j s(j)$ is the total number of species.

To calculate S_a for the log-series distribution, we use a preliminary result: $\sum_{j=1}^{\infty} x^j/j$ is the series expansion for $-\ln(1-x)$ when $|x| < 1$. We have:

$$S = \sum_{j=1}^{\infty} s(j) = \alpha \sum_{j=1}^{\infty} \frac{x^j}{j} = -\alpha \ln(1-x)$$

and:

$$\sum_{j=1}^{\infty} s(j) \left(1 - \frac{a}{A}\right)^j = \alpha \sum_{j=1}^{\infty} \frac{x^j}{j} \left(1 - \frac{a}{A}\right)^j = -\alpha \ln\left(1 - x\left(1 - \frac{a}{A}\right)\right)$$

We deduce that:

$$\begin{aligned} S_a &= -\alpha \ln(1-x) + \alpha \ln\left(1 - x\left(1 - \frac{a}{A}\right)\right) \\ &= \alpha \ln\left(\frac{1 - x\left(1 - \frac{a}{A}\right)}{1-x}\right) \\ &= \alpha \ln\left(1 + \frac{x}{1-x}\frac{a}{A}\right) \end{aligned}$$

Question 2

We say that the species is locally endemic to subarea $a \subseteq A$ if it is found exclusively in subarea a and not in the complementary area $A - a$.

Question (a)

Calculate the probability $\breve{p}_i$ of endemism of species i.

Species i is locally endemic to subarea $a \subseteq A$ if all the individuals of this species are found in a. Let $E_i \sim \mathcal{B}(\breve{p}_i)$ be the Bernoulli random variable that takes the value 1 if species i is locally endemic to a. We have:

$$\breve{p}_i = \Pr\{E_i = 1\} = \prod_{k=1}^{n_i} \left(\frac{a}{A}\right) = \left(\frac{a}{A}\right)^{n_i}$$

Question (b)

What is the expected number E_a of locally endemic species?

The expected number E_a of locally endemic species is equal to:

$$E_a = \mathbb{E}\left[E_1 + E_2 + \ldots + E_S\right] = \sum_{i=1}^{S} \mathbb{E}\left[E_i\right] = \sum_{i=1}^{S} \breve{p}_i = \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$$

Question (c)

Calculate S_{A-a} and E_{A-a} for the complementary area $A-a$. Deduce that:

$$S_{A-a} + E_a = S$$

Since we have $S_a = S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i}$ and $E_a = \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$, we deduce that:

$$S_{A-a} = S - \sum_{i=1}^{S} \left(1 - \frac{A-a}{A}\right)^{n_i} = S - \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$$

and:

$$E_{A-a} = \sum_{i=1}^{S} \left(\frac{A-a}{A}\right)^{n_i} = \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i}$$

It follows that:

$$S_{A-a} + E_a = S - \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i} + \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i} = S$$

Question (d)

Show that:

$$0 \leq S_a + E_a \leq 2S$$

We have:

$$T_a = S_a + E_a = S - \sum_{i=1}^{S} \left(1 - \frac{a}{A}\right)^{n_i} + \sum_{i=1}^{S} \left(\frac{a}{A}\right)^{n_i}$$

It follows that:

$$\begin{aligned}
\frac{\partial\, T_a}{\partial\, a} &= -\sum_{i=1}^{S} n_i \left(1 - \frac{a}{A}\right)^{n_{i-1}} \left(-\frac{1}{A}\right) + \sum_{i=1}^{S} n_i \left(\frac{a}{A}\right)^{n_{i-1}} \left(\frac{1}{A}\right) \\
&= \frac{1}{A} \left(\sum_{i=1}^{S} n_i \left(1 - \frac{a}{A}\right)^{n_{i-1}} + \sum_{i=1}^{S} n_i \left(\frac{a}{A}\right)^{n_{i-1}}\right) \\
&\geq 0
\end{aligned}$$

This implies that T_a is an increasing function of a. Moreover, we have:

$$T_\emptyset = S_\emptyset + E_\emptyset = S - \sum_{i=1}^{S} \left(1 - \frac{0}{A}\right)^{n_i} + \sum_{i=1}^{S} \left(\frac{0}{A}\right)^{n_i} = S - S + 0 = 0$$

and:

$$T_A = S_A + E_A = S - \sum_{i=1}^{S} \left(1 - \frac{A}{A}\right)^{n_i} + \sum_{i=1}^{S} \left(\frac{A}{A}\right)^{n_i} = S - 0 + S = 2S$$

We deduce that:

$$0 \leq S_a + E_a \leq 2S$$

A Course on Biodiversity
Exercise 4. Valuation of life and health

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[13]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

This exercise is inspired by the research article of Hammitt (2023).

Let τ be a survival time[14], whose survival function is $\mathbf{S}(t) = \Pr\{\tau > t\}$ and density function is $f(t) = -\partial\,\mathbf{S}(t)$. We have $\mathbf{S}(0) = 1$ and $\mathbf{S}(\infty) = 0$.

[14] In mortality analysis, this is referred to as time-to-event or time-to-death.

Question 1

Assume that the individual is alive at time t. What is the conditional survival function $\mathbf{S}(u \mid t) = \Pr\{\tau > u \mid \tau > t\}$?

Using Bayes theorem, we have:

$$\mathbf{S}(u \mid t) = \Pr\{\tau > u \mid \tau > t\} = \frac{\Pr\{\tau > u, \tau > t\}}{\Pr\{\tau > t\}} = \frac{\Pr\{\tau > u, t\}}{\Pr\{\tau > t\}} = \frac{\mathbf{S}(u)}{\mathbf{S}(t)}$$

Question 2

Show that the average length of life (or life expectancy) for an individual who is alive at time t is given by:

$$\mathrm{LE}(t) = \mathbb{E}[\tau \mid \tau > t] = \frac{1}{\mathbf{S}(t)} \int_t^\infty \mathbf{S}(u)\,\mathrm{d}u$$

Since $\mathrm{d}\mathbf{S}(t) = \partial\,\mathbf{S}(t)\,\mathrm{d}t = -f(t)\,\mathrm{d}t$, we can express the expected survival time as:

$$\mathbb{E}[\tau] = \int_0^\infty t f(t)\,\mathrm{d}t = -\int_0^\infty t\,\mathrm{d}\mathbf{S}(t)$$

Using integration by parts, we get:

$$\mathbb{E}[\tau] = -\left[t\mathbf{S}(t)\right]_0^\infty + \int_0^\infty \mathbf{S}(t)\,\mathrm{d}t = 0 + \int_0^\infty \mathbf{S}(t)\,\mathrm{d}t = \int_0^\infty \mathbf{S}(t)\,\mathrm{d}t$$

We deduce the conditional life expectancy for an individual alive at time t:

$$\mathrm{LE}(t) = \mathbb{E}[\tau \mid \tau > t] = \frac{\mathbb{E}[\mathbb{1}\{\tau > t\}\cdot\tau]}{\Pr\{\tau > t\}} = \frac{\int_t^\infty \mathbf{S}(u)\,\mathrm{d}u}{\mathbf{S}(t)} = \int_t^\infty \mathbf{S}(u \mid t)\,\mathrm{d}u$$

Question 3

Let ϱ be the discount rate and $X(t)$ a payoff function. Following the seminal paper of Yaari (1965), we define the expected present value of the payoff, taking into account the future lifetime, as:

$$\mathbb{E}[X; t, \varrho] = \int_t^\infty e^{-\varrho(u-t)} \mathbf{S}(u \mid t) X(u) \, \mathrm{d}u$$

What is the rationale behind this formula?

The expected present value of a payoff, taking into account the future lifetime, is given by:

$$\mathbb{E}\left[X; t, \varrho\right] = \int_t^\infty e^{-\varrho(u-t)} \mathbf{S}\left(u \mid t\right) X\left(u\right) \, \mathrm{d}u = \mathbb{E}\left[\delta\left(u\right) X\left(u\right)\right] \tag{2}$$

where $\delta\left(u\right) = B\left(t, u\right) \mathbf{S}\left(u \mid t\right)$ is the discount factor under uncertain lifetime and $B\left(t, u\right) = e^{-\varrho(u-t)}$ is the standard discount factor at time u. Therefore, we have:

$$\delta\left(u\right) = e^{-\varrho(u-t)} \Pr\left\{\tau > u \mid \tau > t\right\}$$

Equation (2) is the classical formula for the present value when the discount rate accounts for uncertainty about the individual's lifetime.

Question 4

The discounted remaining life expectancy is defined as the expected present value of the future lifetime:

$$\mathrm{LE}\left(t;\varrho\right) = \mathbb{E}\left[1;t,\varrho\right] = \frac{1}{\mathbf{S}\left(t\right)}\int_t^{\infty} e^{-\varrho(u-t)}\mathbf{S}\left(u\right)\,\mathrm{d}u$$

Shows that:

$$\mathrm{LE}\left(t;\varrho\right) \geq \mathrm{DLE}\left(t;\varrho\right) = \int_t^{\infty} e^{-\varrho(u-t)}\left(u-t\right)f\left(u \mid t\right)\,\mathrm{d}u$$

What is the interpretation of $\mathrm{DLE}\left(t;\varrho\right)$? Under which condition do we have equality?

$\mathrm{DLE}\,(t;\varrho)$ is the mathematical expectation of the discounted survival time, given that the survival time exceeds t:

$$\mathrm{DLE}\,(t;\varrho) = \mathbb{E}\left[e^{-\varrho t}\tau \mid \tau > t\right]$$

Using integration by parts with $u = e^{-\varrho t}t$ and $v' = f\,(t)$, we can express the discounted life expectancy at $t = 0$ as:

$$\begin{aligned}
\mathrm{DLE}\,(0;\varrho) &= \int_0^\infty e^{-\varrho t} t f\,(t)\,\mathrm{d}t \\
&= \left[-e^{-\varrho t} t\mathbf{S}\,(t)\right]_0^\infty + \int_0^\infty \left(-\varrho e^{-\varrho t} t + e^{-\varrho t}\right)\mathbf{S}\,(t)\,\mathrm{d}t \\
&= \int_0^\infty e^{-\varrho t}\mathbf{S}\,(t)\,\mathrm{d}t - \varrho \int_0^\infty e^{-\varrho t} t\mathbf{S}\,(t)\,\mathrm{d}t
\end{aligned}$$

because $u' = -\varrho e^{-\varrho t} t + e^{-\varrho t}$ and $v = -\mathbf{S}\,(t)$.

We deduce that:

$$\mathrm{DLE}(0;\varrho) = \mathrm{LE}(0;\varrho) - \underbrace{\varrho \int_0^\infty e^{-\varrho t} t \mathbf{S}(t) \,\mathrm{d}t}_{\geq 0}$$

and:

$$\mathrm{LE}(0;\varrho) \geq \mathrm{DLE}(0;\varrho) = \int_0^\infty e^{-\varrho t} t f(t) \,\mathrm{d}t$$

The generalization to the case $t \neq 0$ is straightforward. Using the change of variable $x = u - t$, we obtain $\mathrm{DLE}(t;\varrho) = \mathrm{LE}(t;\varrho) - \varrho \int_0^\infty e^{-\varrho x} x \mathbf{S}(t + x) \,\mathrm{d}x$. Equality is achieved if and only if $\varrho = 0$.

Question 5

Assume that the survival time follows an exponential distribution: $\tau \sim \mathcal{E}(\lambda)$. Derive the formulas for $\mathrm{LE}(t; \varrho)$ and $\mathrm{DLE}(t; \varrho)$? Comment on these results.

Since $\mathbf{S}(t) = e^{-\lambda t}$, we have:

$$\begin{aligned}
\mathrm{LE}(t;\varrho) &= \frac{1}{e^{-\lambda t}} \int_t^{\infty} e^{-\varrho(u-t)} e^{-\lambda u} \, \mathrm{d}u \\
&= \int_t^{\infty} e^{-(\varrho+\lambda)(u-t)} \, \mathrm{d}u \\
&= \left[\frac{e^{-(\varrho+\lambda)(u-t)}}{(\varrho+\lambda)} \right]_t^{\infty} \\
&= \frac{1}{\varrho+\lambda}
\end{aligned}$$

and:

$$\begin{aligned}
\mathrm{DLE}\left(t;\varrho\right) &= \int_t^\infty e^{-\varrho(u-t)}\left(u-t\right) f\left(u \mid t\right)\,\mathrm{d}u \\
&= \int_t^\infty e^{-\varrho(u-t)}\left(u-t\right)\lambda e^{-\lambda(u-t)}\,\mathrm{d}u \\
&= \int_0^\infty \lambda e^{-(\varrho+\lambda)s} s\,\mathrm{d}s \\
&= \left[-\lambda s\frac{e^{-(\varrho+\lambda)s}}{(\varrho+\lambda)}\right]_0^\infty + \lambda\int_0^\infty \frac{e^{-(\varrho+\lambda)s}}{(\varrho+\lambda)}\,\mathrm{d}s \\
&= \lambda\left[-\frac{e^{-(\varrho+\lambda)s}}{(\varrho+\lambda)^2}\right]_0^\infty \\
&= \frac{\lambda}{(\varrho+\lambda)^2}
\end{aligned}$$

We verify that:

$$\operatorname{LE}(t;\varrho) = \left(1 + \frac{\varrho}{\lambda}\right) \operatorname{DLE}(t;\varrho) \geq \operatorname{DLE}(t;\varrho)$$

Moreover, we note that $\operatorname{LE}(t;\varrho)$ does not depend on time t because exponential survival times satisfy the property of lack of memory.

Question 6

We assume that life expectancy is 70 years ($\mathbb{E}[\tau] = 70$). We consider two survival functions:

1. $\tau \sim \mathcal{E}(\lambda)$
2. $\tau \sim \mathcal{W}\mathrm{eibull}(a, b)$, where the survival function is defined as $\mathbf{S}(t) = \exp\left(-\left(\frac{x}{a}\right)^{b}\right)$

Question (a)

Calibrate the parameters λ and a so that $b = 5$. Plot the survival functions $\mathbf{S}(t)$ and $\mathbf{S}(u \mid t = 50)$. Comment on these results.

In the case of exponential survival time, we have:

$$\mathbb{E}[\tau] = \frac{1}{\lambda}$$

It follows that:

$$\lambda = \frac{1}{\mathbb{E}[\tau]} = \frac{1}{70} = 0.01429$$

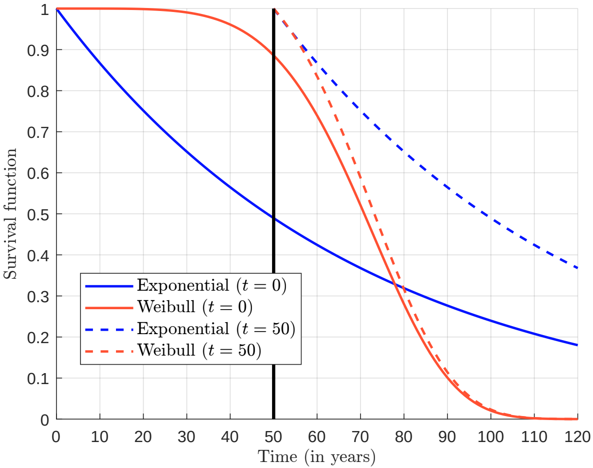
For the Weibull distribution, we have:

$$\mathbb{E}[\tau] = a\Gamma\left(1 + \frac{1}{b}\right)$$

We deduce that:

$$a = \frac{\mathbb{E}[\tau]}{\Gamma\left(1 + \frac{1}{b}\right)} = \frac{70}{\Gamma\left(1 + \frac{1}{5}\right)} = 76.23871$$

Figure 110: Survival function $\mathbf{S}(u \mid t)$



It is evident that the exponential distribution is not suitable for modeling human lifetimes, as the probability of dying before reaching 50 years of age is approximately 50%. In contrast, the Weibull distribution provides a more realistic model for human lifetime.

Question (b)

We assume that $\varrho = 3\%$. Calculate $\mathrm{LE}\,(0;\varrho)$ and $\mathrm{DLE}\,(0;\varrho)$ if the survival time is exponential.

We have:

$$\mathrm{LE}\left(0; 3\%\right) = \frac{1}{\varrho + \lambda} = \frac{1}{3\% + 1.43\%} = 22.58 \text{ years}$$

and:

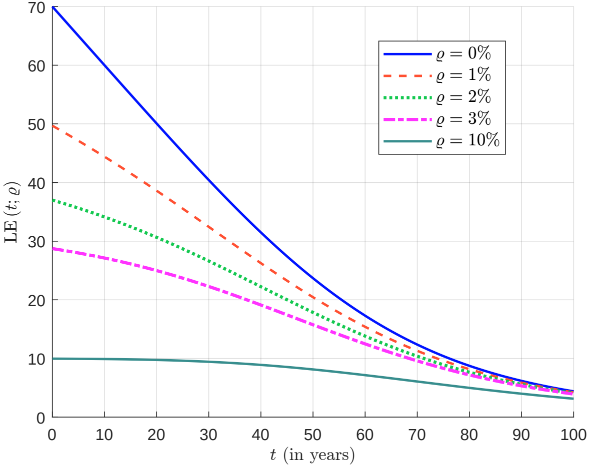
$$\mathrm{DLE}\left(0; 3\%\right) = \frac{\lambda}{\left(\varrho + \lambda\right)^2} = \frac{1.43\%}{\left(3\% + 1.43\%\right)^2} = 7.28 \text{ years}$$

The function $\mathrm{DLE}\left(t; \varrho\right)$ discounts lifetime too rapidly and is not a suitable approach for calculating present values while accounting for future lifetime.

Question (c)

Assume that the survival time is Weibull distributed. Using numerical integration, plot the discounted life expectancy $\mathrm{LE}(t; \varrho)$ for $t \in [0, 100]$ and for different discount rates ϱ (0%, 1%, 2%, 3% and 10%). Comment on these results.

Figure 111: Discounted lifetime expectancy $\mathrm{LE}(t; \varrho)$



We observe that $\mathrm{LE}(t; \varrho)$ is a decreasing function with respect to both time t and the discount rate ϱ. Using a standard discount rate of 3%, a life expectancy of 70 years corresponds to a discounted life expectancy of approximately 30 years at birth and 20 years at age 40.

Question 7

We assume that the quality life weight $\hat{Q}(t)$ is a random variable between 0 and 1, and we note $Q(t) = \mathbb{E}\left[\hat{Q}(t)\right]$ its expected value. The quality-adjusted life expectancy (QALE) is defined as $\mathrm{QALE}(t;\varrho) = \mathbb{E}[Q(t); t, \varphi]$.

Question (a)

Give the formula of $\mathrm{QALE}(t;\varrho)$ when the function $Q(t)$ is constant.

We have:

$$
\begin{aligned}
\mathrm{QALE}(t;\varrho) &= \frac{1}{\mathbf{S}(t)}\int_t^\infty e^{-\varrho(u-t)}\mathbf{S}(u)\,Q(t)\,\mathrm{d}u \\
&= \left(\frac{1}{\mathbf{S}(t)}\int_t^\infty e^{-\varrho(u-t)}\mathbf{S}(u)\,\mathrm{d}u\right)\cdot Q(t) \\
&= \mathrm{LE}(t;\varrho)\cdot Q(t)
\end{aligned}
$$

The quality-adjusted life expectancy is the product of the discounted life expectancy and the average quality of life weight.

Question (b)

Show that:

$$\mathrm{QALE}\left(t;\varrho\right) \leq \mathrm{LE}\left(t;\varrho\right)$$

Since $Q(t) \leq 1$, we deduce that:

$$0 \leq e^{-\varrho(u-t)} \frac{\mathbf{S}(u)}{\mathbf{S}(t)} Q(u) \leq e^{-\varrho(u-t)} \frac{\mathbf{S}(u)}{\mathbf{S}(t)}$$

and:

$$\int_t^\infty e^{-\varrho(u-t)} \frac{\mathbf{S}(u)}{\mathbf{S}(t)} Q(u)\,\mathrm{d}u \leq \int_t^\infty e^{-\varrho(u-t)} \frac{\mathbf{S}(u)}{\mathbf{S}(t)}\,\mathrm{d}u$$

We conclude that $\mathrm{QALE}(t;\varrho) \leq \mathrm{LE}(t;\varrho)$.

Question (c)

Consider the previously calibrated Weibull distribution. We assume that:

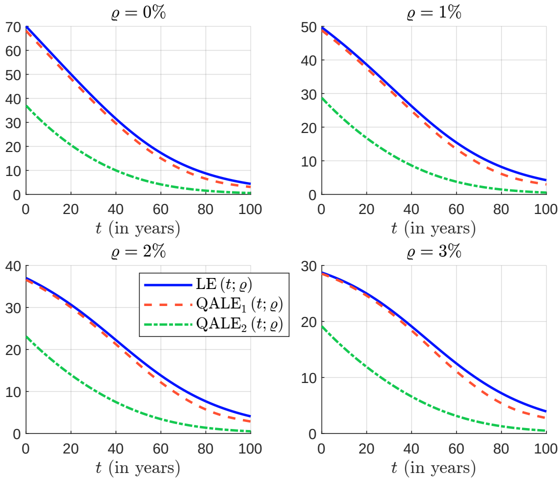
$$Q_1(t) = \begin{cases} 1 & \text{if } t \leq t_1 \\ 1 - \kappa_1 \dfrac{(t - t_1)}{t_2 - t_1} & \text{if } t_1 \leq t \leq t_2 \\ 1 - \kappa_1 - \kappa_2 \dfrac{(t - t_2)}{t_3 - t_2} & \text{if } t_2 \leq t \leq t_3 \\ 1 - \kappa_1 - \kappa_2 & \text{if } t \geq t_3 \end{cases}$$

What is the rationale for this specification? We consider a second HRQL function:

$$Q_2(t) = \exp(-\kappa t)$$

Compare $\mathrm{QALE}(t; \varrho)$ and $\mathrm{LE}(t; \varrho)$ for the following set of parameters: $t_1 = 50$, $\kappa_1 = 10\%$, $t_2 = 70$, $\kappa_2 = 20\%$, $t_3 = 90$, and $\kappa = 2\%$.

Figure 112: Discounted lifetime expectancy $\mathrm{LE}(t;\varrho)$



- $Q_1(t)$ is a piecewise linear function representing three phases of life. Before t_1, the quality of life is equal to 1. Between t_1 and t_2, the quality of life decreases linearly, reaching $1-\kappa_1$ at $t=t_2$. The rate of decrease per year in this phase is $\kappa_1/(t_2-t_1)$. In the third phase, between t_2 and t_3, the quality of life continues to decline linearly at a rate of $\kappa_2/(t_3-t_2)$.
- Figure 112 shows the functions $\mathrm{LE}(t;\varrho)$, $\mathrm{QALE}_1(t;\varrho)$ calculated with $Q_1(t)$, and $\mathrm{QALE}_2(t;\varrho)$ calculated with $Q_2(t)$.
- The difference between $\mathrm{LE}(t;\varrho)$ and $\mathrm{QALE}_1(t;\varrho)$ is small, especially when $t\leq t_1$. It is more pronounced for large values of t because quality of life is strongly affected when people are old.
- In contrast, $\mathrm{QALE}_2(t;\varrho)$ is much lower than $\mathrm{LE}(t;\varrho)$, because the quality of life decreases exponentially 2% per year.

Question 8

Let $\mathcal{R}(t)$ be the impact on mortality risk. We use the convention that $\mathcal{R}(t)$ is positive if the impact is a risk reduction and negative if the impact is a risk increase. We define $\Delta L(t) = \mathcal{R}(t) \cdot \Delta t$ as the expected number of lives saved (or the decrease in the number of deaths) during the short time interval Δt. Hammitt (2023) also defines the increase in life expectancy as $\Delta \mathrm{LE}(t) = \Delta L(t) \cdot \mathrm{LE}(t; \varrho)$ and the increase in quality-adjusted life expectancy as $\Delta \mathrm{QALE}(t) = \Delta L(t) \cdot \mathrm{QALE}(t; \varrho)$. The monetary value v of the risk reduction $\mathcal{R}(t)$ is the product of the value of a statistical life (VSL) and the expected number of lives saved. Similarly, v can be expressed as the product of the value per statistical life year (VSLY) and the increase in life expectancy or as the product of the value per quality-adjusted life year (VQALY) and the increase in quality-adjusted life expectancy. Following Hammitt (2023), we have:

$$v = \mathrm{VSL}(t) \cdot \Delta L(t) = \mathrm{VSLY}(t) \cdot \Delta \mathrm{LE}(t) = \mathrm{VQALY}(t) \cdot \Delta \mathrm{QALE}(t)$$

Question (a)

Find $\mathrm{VSL}(t)$ and express $\mathrm{VSLY}(t)$ and $\mathrm{VQALY}(t)$ as functions of $\mathrm{VSL}(t)$.

We have:

$$\mathrm{VSL}\left(t\right)=\frac{v}{\Delta L\left(t\right)}=\frac{v}{\mathcal{R}\left(t\right)\cdot\Delta t}$$

We deduce that:

$$\mathrm{VSLY}\left(t\right)=\frac{\mathrm{VSL}\left(t\right)\cdot\Delta L}{\Delta\mathrm{LE}\left(t\right)}=\frac{\mathrm{VSL}\left(t\right)}{\mathrm{LE}\left(t;\varrho\right)}$$

and:

$$\mathrm{VQALY}\left(t\right)=\frac{\mathrm{VSL}\left(t\right)\cdot\Delta L}{\Delta\mathrm{QALE}\left(t\right)}=\frac{\mathrm{VSL}\left(t\right)}{\mathrm{QALE}\left(t;\varrho\right)}$$

Question (b)

Hammitt (2023) considers persistent risk impact $\mathcal{I}(t)$ and defines the economic gain from risk reduction as:

$$\mathcal{G}(t;\varrho) = \frac{1}{\mathbf{S}(t)} \int_t^\infty e^{-\varrho(u-t)} \mathbf{S}(u) \, \mathrm{VSL}(u) \, \mathcal{I}(u) \, \mathrm{d}u$$

What is the interpretation of $\mathcal{G}(t;\varrho)$? Assume that $\mathcal{I}(t) = \mathcal{R}(t)$. What is the associated payoff function. Give the expression for the payoff function using $\mathrm{VSLY}(t)$ or $\mathrm{VQALY}(t)$.

Since $\Delta L(t) = \mathcal{R}(t) = \mathcal{I}(t)$ because $\Delta t = 1$ year, the associated payoff is:

$$X(t) = \mathrm{VSL}(t) \cdot \mathcal{R}(t) = \mathrm{VSL}(t) \cdot \Delta L(t)$$

It follows that:

$$X(t) = \mathrm{VSLY}(t) \cdot \Delta\mathrm{LE}(t) = \mathrm{VQALY}(t) \cdot \Delta\mathrm{QALE}(t)$$

Question (c)

What is the value of $\mathcal{G}(t;\varrho)$ if $\mathrm{VSL}(t)$ and $\mathcal{I}(t)$ are assumed to be constant?

If $\mathrm{VSL}(t)$ and $\mathcal{I}(t)$ are constant, we get:

$$\begin{aligned}\mathcal{G}(t;\varrho) &= \mathrm{VSL}\cdot\left(\frac{1}{\mathbf{S}(t)}\int_t^\infty e^{-\varrho(u-t)}\mathbf{S}(u)\,\mathrm{d}u\right)\cdot\mathcal{I}(t) \\ &= \mathrm{VSL}\cdot\mathrm{LE}(t;\varrho)\cdot\Delta L \qquad (3)\end{aligned}$$

The economic gain is the value of a statistical life multiplied by the discounted life expectancy and the expected number of lives saved.

Question (d)

In the case of air pollution, the economic cost is sometimes estimated using the following formula:

$$\mathcal{C} = \mathrm{VSLY} \cdot \mathrm{YLL}$$

What is the rationale for this formula?

In the case of air pollution, $\mathcal{I}(t)$ is negative and the economic cost becomes:

$$\mathcal{C}(t;\varrho) = -\mathcal{G}(t;\varrho) = \mathrm{VSL}\cdot\mathrm{LE}(t;\varrho)\cdot\Delta L$$

where ΔL is the expected number of deaths due to air pollution. If we compare with the formula:

$$\mathcal{C} = \mathrm{VSLY}\cdot\mathrm{YLL}$$

we can deduce that the years of life lost (YLL) is equal to:

$$\mathrm{YLL} = \Delta L\cdot\mathrm{LE}(t;\varrho)$$

In this model, years of life lost is the product of the expected number of deaths and the discounted life expectancy.

The traditional formula is:

$$\mathrm{YLL} = \Delta L \cdot \Delta \tau$$

where $\Delta \tau$ is the difference between life expectancy without air pollution and life expectancy with air pollution. Therefore, we assume that:

$$\Delta \tau = \mathrm{LE}\left(t; \varrho\right) = \frac{1}{\mathbf{S}\left(t\right)} \int_t^\infty e^{-\varrho(u-t)} \mathbf{S}\left(u\right) \,\mathrm{d}u$$

This means that we take the discount rate into account when calculating years of life lost.

Question (e)

Consider a numerical application using the previously calibrated Weibull distribution and the HRQL function $\mathcal{Q}_1(t)$. Compute the values of $\mathrm{LE}(t;\varrho)$ and $\mathrm{QALE}(t;\varrho)$ for $t \in \{0, 40, 80\}$ and $\varrho \in \{0, 3\%\}$. Workers in an industry are paid an additional \$1 000 per year to face a 1 in 10 000 increased risk of death. Compute the value of a statistical life. Deduce the values of $\mathrm{VSLY}(t)$ and $\mathrm{VQALY}(t)$. What is the drawback of assuming a constant VSL regardless the worker's age. Therefore, we prefer to assume a constant $\mathrm{VSLY}(t)$, equal to \$150 000. Deduce the values of $\mathrm{VSL}(t)$ and $\mathrm{VQALY}(t)$. Calculate the economic gain $\mathcal{G}(t;\varrho)$ of a policy that avoids 5 deaths among 100 000 people. Comment on these results.

Table 78: Calculation of the economic gain $\mathcal{G}(t;\varrho)$

Discount rate	$\varrho = 0\%$			$\varrho = 3\%$			Unit
Age	Birth	40 years	80 years	0	40 years	80 years	
$\mathrm{LE}(t;\varrho)$	70.00	31.49	8.74	28.73	19.11	7.19	years
$\mathrm{QALE}(t;\varrho)$	68.13	29.54	6.50	28.52	18.39	5.38	years
$\mathrm{VSL}(t)$	10 000						\$1 000
$\mathrm{VSLY}(t)$	143	318	1 144	348	523	1 390	\$1 000
$\mathrm{VQALY}(t)$	147	339	1 538	351	544	1 858	\$1 000
$\mathrm{VSL}(t)$	10 500	4 723	1 312	4 310	2 867	1 079	\$ 1000
$\mathrm{VSLY}(t)$	150						\$ 1000
$\mathrm{VQALY}(t)$	154	160	202	151	156	200	\$ 1000
$\mathcal{G}(t;\varrho)$	36 750	7 437	573	6 191	2 740	388	\$

- By construction, the life expectancy of an individual at age 0 is 70 years. Using a discount rate of 3%, the discounted life expectancy is 28.73 years.
- At age 40, the remaining life expectancy is 31.49 years, so the average lifetime of an individual alive at age 40 is 71.49 years. This corresponds to a discounted life expectancy of 19.11 years.
- At age 80, the remaining life expectancy is 8.74 years, so the average lifetime of an individual alive at age 80 is 88.74 years.
- The impact of the quality-of-life weight is shown in the second row.

- The estimated value of a statistical life is calculated as:

$$\mathrm{VSL}\left(t\right)=\frac{\$1\,000}{1/10\,000}=\$10\,\mathrm{mn}$$

- We can then compute the value of a statistical life year. Assuming an age of 40 years and a discount rate of 3%, we obtain:

$$\mathrm{VSLY}\left(40\right)=\frac{\mathrm{VSL}\left(40\right)}{\mathrm{LE}\left(40;3\%\right)}=\frac{\$10\,\mathrm{mn}}{19.11}=\$523\,206$$

and:

$$\mathrm{VQALY}\left(40\right)=\frac{\mathrm{VSL}\left(40\right)}{\mathrm{QALE}\left(40;3\%\right)}=\frac{\$10\,\mathrm{mn}}{18.39}=\$543\,817$$

- The assumption that $\mathrm{VSL}(t)$ is constant is not realistic. For instance, the value per statistical life year for a newborn is \$142 857, while for an 80-year-old individual it is \$1 390 448.
- On the other hand, the assumption that $\mathrm{VSLY}(t)$ is constant is more realistic, even if it does not fully match reality. In fact, we can expect $\mathrm{VSLY}(40) \geq \mathrm{VSLY}(0)$ and $\mathrm{VSLY}(40) \geq \mathrm{VSLY}(80)$.
- Finally, we calculate the economic gain using Equation (3). For example, assuming an age of 40 years and a discount rate of 3%, the economic gain is:

$$\mathcal{G}(40; 3\%) = \$2\,866\,938 \times 19.11 \times \frac{5}{100\,000} = \$2\,740$$

We observe that the economic gain is greater for newborns than for older individuals.