Risk-Based Investing & Asset Management
Final Examination

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Remark 1 ***The final examination is composed of 4 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file or one program by exercise.***

- ***Concerning risk decomposition[1], present the results as follows:***

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			✓	

- ***The report is a zipped file, whose filename is yourname.zip if you do the project alone or yourname1-yourname2.zip if you do the project in groups of two.***

- ***The zipped file is composed of four files:***

  1. ***the pdf document that contains the answers to the four exercises and a cover sheet with your names;***
  2. ***the program of each exercise with an explicit filename, e.g. exercise1.xls (if you use excel), exercise1.m (if you use matlab), exercise1.py (if you use python), exercise1.r (if you use R), etc.***

[1] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^\star$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

1 Mean-variance optimized & Risk-based portfolios

We consider an investment universe with 5 assets. We assume that their expected returns are equal to 3%, 6%, 8%, 9% and 8%, and their volatilities are equal to 5%, 8%, 15%, 15% and 15%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 50\% & 100\% & & & \\ 20\% & 20\% & 100\% & & \\ 50\% & 50\% & 70\% & 100\% & \\ 20\% & 20\% & 30\% & 30\% & 100\% \end{pmatrix}$$

The risk-free asset r is equal to 2%.

1. Compute the covariance matrix of asset returns.
2. We consider long-short portfolios x with $\sum_{i=1}^{5} x_i = 1$.
   (a) Draw the efficient frontier.
   (b) Compute the minimum variance portfolio.
   (c) Using the γ-problem and a bisection algorithm, calculate the MVO portfolio if we target an ex-ante volatility of 5%. What is the optimal value of γ?
   (d) Sam question if we target an ex-ante volatility of 10%.
   (e) Compare the 3 optimized portfolios in terms of expected return, volatility and Sharpe ratio.
3. We restrict the analysis to long-only portfolios x meaning that $\sum_{i=1}^{5} x_i = 1$ and $x_i \geq 0$. Moreover, we impose an upper bound of 75%, implying that $x_i \leq 0.75$.
   (a) Draw the efficient frontier and compare it with the long/short efficient frontier.
   (b) Compute the minimum variance portfolio.
   (c) Using the γ-problem and a bisection algorithm, calculate the MVO portfolio if we target an ex-ante volatility of 5%. What is the optimal value of γ?
   (d) Same question if we target an ex-ante volatility of 10%.
   (e) Compare the 3 optimized portfolios in terms of expected return, volatility and Sharpe ratio.
   (f) What is the impact of the constraints?
4. We consider long-only portfolios. For each portfolio, compute the volatility decomposition.
   (a) Determine the equally weighted (EW) portfolio.
   (b) Compute the minimum variance (MV) portfolio.
   (c) Calculate the most diversified portfolio (MDP).
   (d) Find the ERC portfolio.
   (e) Compare the four optimized portfolios in terms of expected return, volatility and Sharpe ratio.
   (f) Compare the diversification ratio $\mathcal{DR}(x)$, the weight concentration[2] $\mathcal{H}^\star(x)$ and the risk concentration $\mathcal{H}^\star(\mathcal{RC})$ of the previous portfolios (EW, MV, MDP and ERC).
   (g) Compute the correlation between each asset and the MDP portfolio. Comment on these results.

[2] We remind that the normalized Herfindahl index is defined as follows:

$$\mathcal{H}^\star(\pi) = \frac{n \sum_{i=1}^{n} \pi_i^2 - 1}{n - 1}$$

where $\pi = (\pi_1, \ldots, \pi_n)$ is a vector satisfying $\pi_i \geq 0$ and $\sum_{i=1}^{n} \pi_i = 1$.

2 ESG Investing

We consider a capitalization-weighted equity index, which is composed of 8 stocks. The weights are equal to 23%, 19%, 17%, 13%, 9%, 8%, 6% and 5%. We assume that their volatilities are equal to 22%, 20%, 25%, 18%, 35%, 23%, 13% and 29%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & & & & \\ 80\% & 100\% & & & & & & \\ 70\% & 75\% & 100\% & & & & & \\ 60\% & 65\% & 80\% & 100\% & & & & \\ 70\% & 50\% & 70\% & 85\% & 100\% & & & \\ 50\% & 60\% & 70\% & 80\% & 60\% & 100\% & & \\ 70\% & 50\% & 70\% & 75\% & 80\% & 50\% & 100\% & \\ 60\% & 65\% & 70\% & 75\% & 65\% & 70\% & 80\% & 100\% \end{pmatrix}$$

In what follows, we consider long-only portfolios, and the CW portfolio is the benchmark.

1. We would like to replicate the benchmark with a given number of stocks.

   (a) Describe the heuristic algorithm for performing index sampling.

   (b) Using the heuristic algorithm, find the sampling portfolio with 7 stocks. What is the tracking error volatility of this portfolio?

   (c) Same question if we want to keep only 6 stocks. How do you explain that the fourth asset is deleted?

   (d) Same question if we want to keep only 5 stocks.

   (e) Same question if we want to keep only 1 stock. How do you explain that the second asset is selected?

   (f) Give the elimination order of the stocks[3].

2. The ESG scores of the eight assets are the following:

Stock	#1	#2	#3	#4	#5	#6	#7	#8
CW weight	0.23	0.19	0.17	0.13	0.09	0.08	0.06	0.05
Volatility	0.22	0.20	0.25	0.18	0.35	0.23	0.13	0.29
ESG score	−1.20	0.80	2.75	1.60	−2.75	−1.30	0.90	−1.70

   We would like to tilt the benchmark by improving its ESG score.

   (a) Calculate the ESG score of the benchmark.

   (b) We consider the EW and ERC portfolios. Calculate the ESG score of these two portfolios. Define the ESG excess score with respect to the benchmark. Comment on these results.

   (c) Write the γ-problem of the ESG optimized portfolio when the goal is to improve the ESG score of the benchmark and control at the same time the tracking error volatility.

   (d) Draw the efficient frontier between the tracking error volatility and the ESG excess score[4].

   (e) Using the bisection algorithm, find the optimal portfolio if we would like to improve the ESG score of the benchmark by 0.5. Give the optimal value of γ.

   (f) Same question if we would like to improve the ESG score of the benchmark by 1.0.

   (g) We impose that the portfolio weights can not be greater than 30%. Find the optimal portfolio if we would like to improve the ESG score of the benchmark by 1.0.

[3] From the first deleted stock to the eighth deleted stock.

[4] We notice that $\gamma \in [0, 1.2\%]$ is sufficient for drawing the efficient frontier.

(h) Comment on these results.

3. Comparison of ESG rating methodologies (two pages maximum)

   (a) Download[5] the SUSTAINALYTICS ESG risk rating report of ABC Corp. Comment on these results. What can we say about the ESG risk of this company?

   (b) How ESG ratings are defined by MSCI[6]? Give the metrics that are combined for building the social score.

   (c) What is the place of the three pillars **E**, **S** and **G** in the corporate policy of Danone[7]?

   (d) Carbon intensity

[5] It is available at this adress: `https://www.sustainalytics.com/sustainable-finance/wp-content/uploads/2019/02/ESG-Risk-Rating-PDF-Sample-Company-Report.pdf`

[6] The methodology can be found at `https://www.msci.com/esg-ratings`.

[7] See for instance `https://www.danone.com/fr/about-danone/at-a-glance/external-recognition.html`.

3 Optimization Algorithms & Portfolio Allocation

1. We consider the following optimization program:

$$x^\star = \arg\min \frac{1}{2} x^\top \Sigma x - \lambda \sum_{i=1}^{n} b_i \ln x_i$$

where Σ is the covariance matrix, b is the vector of risk budgets and x is the vector of portfolio weights.

(a) Write the first-order condition with respect to the vector x and show that the solution $x^\star$ corresponds to a risk-budgeting portfolio.

(b) Write the first-order condition with respect to the coordinate x_i. Find the optimal value $x_i^\star$ when we consider the other coordinates $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ as fixed.

(c) We note $x_i^{(k)}$ the value of the i^{th} coordinate at the k^{th} iteration. Deduce the corresponding CCD algorithm.

(d) We consider a universe of three assets, whose volatilities are equal to 20%, 25% and 30%. The correlation matrix is equal to:

$$\rho = \begin{pmatrix} 100\% & & \\ 50\% & 100\% & \\ 60\% & 70\% & 100\% \end{pmatrix}$$

We would like to compute the ERC portfolio, meaning that $b_i = \frac{1}{3}$.

i. Starting from the initial value $x^{(0)} = (33.3\%, 33.3\%, 33.3\%)$, find the optimal coordinate $x_1^{(1)}$ for the first asset.

ii. Compute then the optimal coordinate $x_2^{(1)}$ for the second asset.

iii. Compute then the optimal coordinate $x_3^{(1)}$ for the third asset.

iv. Give the CCD solution $x^{(2)}$.

v. Give the CCD solution $x^{(10)}$. Deduce the ERC portfolio.

2. We recall that the ADMM algorithm considers the following optimization problem:

$$\begin{aligned} \{x^\star, y^\star\} &= \arg\min f_x(x) + f_y(y) \\ &\text{s.t. } Ax + By = c \end{aligned}$$

(a) Describe the ADMM algorithm.

(b) We consider the following optimization problem:

$$\begin{aligned} w^\star(\gamma) &= \arg\min \frac{1}{2} (w - b)^\top \Sigma (w - b) - \gamma (w - b)^\top \mu \\ &\text{s.t. } \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \sum_{i=1}^{n} |w_i - b_i| \leq \tau^+ \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

i. Give the meaning of the symbols w, b, Σ, and μ. What is the goal of this optimization program? What is the meaning of the constraint $\sum_{i=1}^{n} |w_i - b_i| \leq \tau^+$?

ii. What is the best way to specify $f_x(x)$ and $f_y(y)$ in order to find numerically the solution. Justify your choice.

(c) We consider the following optimization problem:

$$\begin{aligned} w^\star &= \arg\min \left\| w - \tilde{w} \right\|_1 \\ &\text{s.t.} \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \sqrt{(w-b)^\top \Sigma (w-b)} \leq \sigma^+ \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

i. What is the meaning of the objective function $\left\| w - \tilde{w} \right\|_1$? What is the meaning of the constraint $\sqrt{(w-b)^\top \Sigma (w-b)} \leq \sigma^+$?

ii. Propose an equivalent optimization problem such that $f_x(x)$ is a QP problem. How to solve the y-update?

(d) We consider the following optimization problem:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2} w^\top (Q + A) w - w^\top R \\ &\text{s.t.} \left\{ \begin{array}{l} \mathbf{1}_n^\top w + w^\top A w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

where Q is an Hermitian matrix and A is a diagonal matrix.

i. What is the best way to specify $f_x(x)$ and $f_y(y)$? Justify your choice.

ii. We assume that the number of assets n is equal to 5, Q is the constant correlation matrix $\mathcal{C}_5(50\%)$, and $R = (5\%, 6\%, 7\%, 8\%, 9\%)$. Using a QP algorithm, find the optimal value $w^\star_{\mathrm{QP}}$ by assuming that $A = \mathbf{0}_{5\times 5}$.

iii. We assume that $A = \mathrm{diag}(1\%, 2\%, 3\%, 4\%, 5\%)$. Program the ADMM algorithm by considering a fixed value $\varphi = 1$ for the penalization. Starting from the initial vector $w^\star_{\mathrm{QP}}$, find the optimal solution $w^\star_{\mathrm{ADMM}}$.

References

[1] Boyd, S., Parikh, N., Chu, E., Peleato, B., and Eckstein, J. (2011), Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers, *Foundations and Trends® in Machine learning*, 3(1), pp. 1-122 (available at `https://web.stanford.edu/~boyd/papers/admm_distr_stats.html`).

[2] Chen, P., Lezmi, E., Roncalli, T., and Xu, J. (2020), A Note on Portfolio Optimization with Quadratic Transaction Costs, *arXiv*, `https://arxiv.org/abs/2001.01612`.

[3] Griveau-Billion, T., Richard, J-C., and Roncalli, T. (2013), A Fast Algorithm for Computing High-dimensional Risk Parity Portfolios, *SSRN*, `www.ssrn.com/abstract=2325255`.

[4] Parikh, N., and Boyd, S. (2014), Proximal Algorithms, *Foundations and Trends® in Optimization*, 1(3), pp. 127-239 (available at `https://web.stanford.edu/~boyd/papers/prox_algs.html`).

[5] Perrin, S., and Roncalli, T. (2019), Machine Learning Optimization Algorithms & Portfolio Allocation, *arXiv*, `https://arxiv.org/abs/1909.10233`.

4 Factor Investing & Alternative Risk Premia

4.1 Factor Investing in Equities

1. Define the five equity risk factors: size, value, momentum, low beta and quality.
2. Give the metrics that allow to rank the stocks according to size, momentum and low beta factors.
3. Give two metrics for defining the value risk factor. Same question for the quality risk factor.
4. What is the intrinsic difference between value and quality?

4.2 Alternative Risk Premia

1. What is the difference between a skewness risk premium and a market anomaly?
2. Explain the difference between convex and concave strategies. Give an example of each strategy.
3. The carry risk premium
   (a) Explain how the carry risk premium is implemented in the case of currencies.
   (b) Explain how the carry risk premium is implemented in the case of commodities.
   (c) Define the volatility carry risk premium. Why it is related to the robustness of the Black-Scholes formula?
   (d) What is the adverse scenario of a carry strategy?
4. The momentum risk premium
   (a) What is the difference between cross-section and time-series momentum?
   (b) Explain why the loss frequency of a long/short trend-following strategy is higher than its gain frequency?
   (c) What is the adverse scenario of a trend-following strategy?

References

[1] FTSE, FTSE Global Factor Index Series, `http://www.ftse.com/products/indices/factor`.

[2] MSCI, MSCI factor tilt indexes, `https://www.msci.com/factor-indexes`.

[3] Blin, O., Lee, J., and Teiletche, J. (2017), Alternative Risk Premia Investing: From Theory to Practice, *Unigestion*, `https://www.unigestion.com`.

[4] Hamdan, R., Pavlowsky F., Roncalli, T., and Zheng, B. (2016), A Primer on Alternative Risk Premia, *SSRN*, `www.ssrn.com/abstract=2766850`.

[5] Koijen, R.S., Moskowitz, T., Pedersen, L.H. and Vrugt, E.B. (2018), Carry, *Journal of Financial Economics*, 127(2), pp. 197-225 (available at SSRN: `www.ssrn.com/abstract=2298565`)

[6] Roncalli, T. (2017), Alternative Risk Premia: What Do We Know?, in Jurczenko, E. (Ed.), *Factor Investing and Alternative Risk Premia*, ISTE Press – Elsevier (available at SSRN: `www.ssrn.com/abstract=2868425`).

[7] Roncalli, T. (2018), Keep Up the Momentum, *Journal of Asset Management*, 19(5), pp 351-361 (available at SSRN: `www.ssrn.com/abstract=3083921`.