Risk Management & Financial Regulation
Final Examination

Thierry Roncalli

December 17^{th} 2024

Please write entirely your answers.

1 The BCBS regulation

1. What are the main differences between the first Basel Accord and the second Basel Accord?
2. Explain how the Basel III Accord strengthens the banking regulation?

2 Market risk

1. What is the difference between the banking book and the trading book? Define the perimeter of assets for capital requirements.
2. How is calculated the capital requirement with the internal model-based approach in Basel II?
3. How is calculated the capital requirement with the internal model-based approach in Basel III?

3 Credit risk

1. What is the definition of the default in Basel II?
2. Describe the standard approach (SA) to compute the capital requirement in Basel III.
3. Define the different parameters of the internal ratings-based (IRB) approach. What are the differences between FIRB and AIRB?
4. Explain the concept of CCF? Why is it difficult to estimate a CCF?

4 Counterparty credit risk (CCR) and credit value adjustment (CVA)

1. Define the concept of counterparty credit risk.
2. What differences do you make between the CCR capital charge in Basel II and the CVA capital charge in Basel III?
3. How is calculated the CCR capital requirement? Explain why the exposure-at-default (EAD) has to be estimated.
4. How is calculated the CVA capital requirement?

5 Operational risk

1. What is the definition of operational risk? Give two examples.
2. Describe the standardized approach (TSA) to calculate the capital in Basel II charge.
3. Describe the loss distribution approach (LDA) to calculate the capital charge.
4. How is the capital requirement calculated in Basel III?

6 Market risk of a long-short portfolio

Consider a portfolio with three assets and assume that the daily PnL of the portfolio (expressed in euros) is equal to:

$$\text{PnL} = (2R_1 - 5R_2 + \beta R_3) \times 10^6$$

where R_1 is the daily return of asset 1, R_2 is the daily return of asset 2, and R_3 is the daily return of asset 3. The annual volatility of asset 1 is 26% while the annual volatility of asset 2 is 39%. The correlation between assets 1 and 2 is set to 50%.

1. Assume that β is equal to 0 and that the asset returns are Gaussian.
   (a) Calculate the annual volatility of the PnL in euros.
   (b) Find the daily value-at-risk at the confidence interval $\alpha = 99\%$.
   (c) Find the daily expected shortfall at the confidence interval $\alpha = 97.5\%$.
2. During the last 300 trading days, the ten worst daily PnLs are $-317\,000$, $-215\,000$, $-191\,000$, $-185\,000$, $-175\,000$, $-164\,000$, $-163\,000$, $-150\,000$, $-145\,000$, and $-139\,000$ euros.
   (a) Calculate the daily historical value-at-risk at the confidence interval $\alpha = 99\%$.
   (b) Calculate the daily historical expected shortfall at the confidence interval $\alpha = 97.5\%$.
3. We assume that the third asset is an at-the-money (ATM) call option on the first asset.
   (a) Give an approximation of the daily PnL of the portfolio.
   (b) What is the value of β such that the daily PnL does not depend on R_1? Deduce the daily Gaussian VaR at the confidence interval $\alpha = 99\%$. Comment on these results and explain the sign of β.
   (c) Using the Cholesky decomposition of the covariance matrix of (R_1, R_2), write R_2 as a function of R_1 and another Gaussian random variable X. Deduce the value of β such that the Gaussian value-at-risk is minimum and compute the daily Gaussian VaR at the confidence interval $\alpha = 99\%$.
   (d) Comment on these results and explain the sign of β.

7 Credit default swaps

We consider a CDS 6M with five-year maturity and \$1 mn notional principal. The recovery rate $\mathcal{R}$ is equal to 50% whereas the spread $\mathcal{s}$ is equal to 500 bps at the inception date. We assume that the protection leg is paid at the default time.

1. Give the cash flow chart.
2. What is the P&L of the protection seller A if the reference entity does not default? What is the P&L of the protection seller A if the reference entity defaults in one year and one month?
3. What is the P&L of the protection buyer B if the reference entity defaults in two years and three months? Same question if the reference entity defaults in three years and one month?
4. What is the relationship between $\mathcal{s}$, $\mathcal{R}$ and λ? What is the implied one-year default probability at the inception date?
5. Two years later, the CDS spread has decreased and is equal to 200 bps. Estimate the new default probability. The protection seller A decides to realize his P&L. For that, he reassigns the CDS contract to the counterparty C. Explain the offsetting mechanism if the risky PV01 is equal to 3.524.

8 Calculation of the effective expected positive exposure

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N_0, μ and σ be the notional, the expected return and the volatility of the underlying contract. We assume that $e(t) = N_0 X$ where X follows a log-normal distribution $\mathcal{LN}(\mu t, \sigma^2 t)$:

$$\Pr\{X \leq x\} = \Phi\left(\frac{\ln x - \mu t}{\sigma\sqrt{t}}\right)$$

1. Calculate the peak exposure $\mathrm{PE}_\alpha(t)$ and the expected exposure $\mathrm{EE}(t)$, and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.

2. We assume that $\mu \ll 1$ and $\sigma \ll 1$. Find an approximation of $\mathrm{EEPE}(0;t)$ and show that $\mathrm{EEPE}(0;t)$ is a linear function of μ, σ^2 and t. How can you justify this approximation?

3. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N_0 is equal to \$1 mn, the maturity T is one year, the volatility σ is set to 50% and μ is equal to 0.

   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.

   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[1]. What is the implied risk weight?

   (c) What does this result become if the bank manages the credit risk with the SA approach and the counterparty credit risk with an internal model in the case where the credit rating of the counterparty is **A+**.

9 Estimation of the loss severity distribution

We consider a sample of n individual losses $\{x_1, \ldots, x_n\}$. We assume that they can be described by the Pareto distribution $\mathcal{P}(\alpha, x^-)$ defined by:

$$\Pr\{X \leq x\} = 1 - \left(\frac{x}{x_-}\right)^{-\alpha}$$

where $x \geq x_-$ and $\alpha > 0$.

1. Find the maximum likelihood estimator $\hat{\alpha}_{\mathrm{ML}}$.

2. Calculate the first two moments of X.

3. Deduce the GMM conditions for estimating the parameter α.

4. Find the estimator $\hat{\alpha}_{\mathrm{MM}}$ of the method of moments using the first moment.

5. We now assume that the losses $\{x_1, \ldots, x_n\}$ have been collected beyond a threshold H meaning that $X \geq H$.

   (a) Compute the conditional distribution $\Pr\{X \leq x \mid X \geq H\}$.

   (b) Find the maximum likelihood estimator $\hat{\alpha}_{\mathrm{ML}}$.

   (c) Find the estimator $\hat{\alpha}_{\mathrm{MM}}$ of the method of moments using the first moment.

6. Application: $x_1 = 1\,274$, $x_2 = 854\,646$, $x_3 = 48\,180$, $x_4 = 686\,806$, $x_5 = 100\,539$, $x_6 = 34\,831\,239$, $x_7 = 39\,442$, $x_8 = 94\,818$, $x_9 = 1\,469$,and $x_{10} = 31\,528$. We have $\sum_{i=1}^{10} x_i = 36\,689\,941$ and $\sum_{i=1}^{10} \ln x_i = 113.608$.

   (a) We assume that $x_- = 100$. Compute $\hat{\alpha}_{\mathrm{ML}}$ and $\hat{\alpha}_{\mathrm{MM}}$.

   (b) We assume that $H = 1\,000$. Compute $\hat{\alpha}_{\mathrm{ML}}$ and $\hat{\alpha}_{\mathrm{MM}}$.

   (c) Which estimator (ML or MM) is the most relevant? Explain why one estimator is largely better than the other?

10 Extreme-value copula

1. Give the definition of a copula function $\mathbf{C}(u_1, u_2)$. Define the lower bound copula $\mathbf{C}^-$, the product copula $\mathbf{C}^\perp$ and the upper bound copula $\mathbf{C}^+$.

2. We consider the Gumbel-Barnett copula function:

$$\mathbf{C}(u_1, u_2) = u_1 u_2 e^{-\theta \ln u_1 \ln u_2}$$

   where $\theta \geq 0$.

[1] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

(a) What is the copula function when $\theta = 0$? What is the copula function when $\theta = 1$?

(b) Let (τ_1, τ_2) be two correlated default times. Assume that $\tau_1 \sim \mathcal{E}(\lambda_1)$ and $\tau_2 \sim \mathcal{E}(\lambda_2)$. Find the cumulative probability density function $\mathbf{F}(t_1, t_2)$ of the random vector (τ_1, τ_2) when the dependence function is the Gumbel-Barnett copula.

(c) Is the Gumbel-Barnett copula an extreme-value (EV) copula function? What does this imply?

3. We consider the G2 function:

$$\mathbf{C}(u_1, u_2) = \exp\left(-\sqrt{\ln^2 u_1 + \ln^2 u_2}\right)$$

where $\ln^2 u = (\ln u)^2$.

(a) Calculate the cross-derivative $\dfrac{\partial^2 \mathbf{C}(u_1, u_2)}{\partial u_1 \partial u_2}$.

(b) Deduce that $\mathbf{C}(u_1, u_2)$ is a copula function.

(c) Is this copula an EV copula function? What does this imply?

(d) Show[2] that the tail dependence λ^+ is equal to $2 - \sqrt{2}$.

(e) We consider the previous default times (τ_1, τ_2), but we assume that the dependence is the G2 copula. Without any additional calculations, do you think that the correlation between τ_1 and τ_2 is close to 0%, 50% or 100%. Justify your choice.

[2] Hints: use the L'Hôpital's rule.