Risk-Based Investing & Asset Management
Final Examination

Thierry Roncalli

March 2th 2017

Remark 1 *The final examination is composed of 4 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file by exercise.*

1 Portfolio optimization

We consider a universe of 4 assets. We assume that their expected returns are 10%, 10%, 7.5% and 7.5% and their volatilities are 20%, 20%, 15% and 15%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & \\ 80\% & 100\% & & \\ 50\% & 50\% & 100\% & \\ 30\% & 50\% & 80\% & 100\% \end{pmatrix}$$

1. We consider long-short portfolios. Solve the Markowitz γ-problem when γ is equal to -1.0, 0.0, 0.5, 1.0 and 2.0. Draw the efficient frontier.

2. We consider long-only portfolios. Solve the Markowitz γ-problem when γ is equal to -1.0, 0.0, 0.5, 1.0 and 2.0. Draw the efficient frontier. Comment on these results.

3. We consider long-short portfolios. Compute the MVO portfolio when we target a volatility of 14%, 15% and 16%.

4. We consider long-only portfolios. Compute the MVO portfolio when we target a volatility of 14%, 15% and 16%.

5. Compare long-short and long-only solutions of Questions 3 and 4. How do you explain these results?

6. We consider a fund manager, whose benchmark is the equally-weighted portfolio.

   (a) Compute the implied expected returns of this benchmak if we assume that the risk-free rate is 1.5% and the Sharpe ratio of the EW portfolio is 0.50.

   (b) The fund manager thinks that the expected return of the first asset is underestimated by 1%. Calculate the optimal portfolio if we target the same volatility than the EW portfolio. How do yo explain the new allocation? Comment on these results.

   (c) The fund manager thinks that the expected return of the third asset is overestimated by 50 bps and the expected return of the fourth asset is underestimated by 50 bps. Calculate the optimal portfolio if we target the same volatility than the EW portfolio. How do yo explain the new allocation? Comment on these results.

   (d) The fund manager thinks that the expected return of the third asset is overestimated by 2% and the expected return of the fourth asset is underestimated by 2%. Calculate the optimal portfolio if we target the same volatility than the EW portfolio. How do yo explain the new allocation? Comment on these results.

2   Smart beta portfolios

We consider a capitalization-weighted equity index, which is composed of 5 stocks. The weights are equal to 29%, 25%, 23%, 18% and 5%. We assume that the stock volatilities are 21%, 20%, 25%, 18% and 35%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 30\% & 100\% & & & \\ 70\% & 75\% & 100\% & & \\ 60\% & 55\% & 90\% & 100\% & \\ 70\% & 50\% & 70\% & 55\% & 100\% \end{pmatrix}$$

We note Σ the covariance matrix. In what follows, we consider long-only portfolios.

1. Calculate the covariance matrix.

2. For each portfolio, compute the volatility decomposition and present the results as follows[1]:

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			$\checkmark$	

   (a) Give the volatility decomposition of the capitalization-weighted (CW) portfolio.

   (b) Determine the equally weighted (EW) portfolio.

   (c) Compute the minimum variance (MV) portfolio.

   (d) Compute the most diversified portfolio (MDP).

   (e) Find the ERC portfolio.

3. Compare the diversification ratio $\mathcal{DR}(x)$, the volatility $\sigma(x)$, the weight concentration[2] $\mathcal{H}^\star(x)$ and the risk concentration $\mathcal{H}^\star(\mathcal{RC})$ of the previous portfolios (CW, EW, MV, MDP and ERC).

4. We consider the CW portfolio as the benchmark.

   (a) Describe the heuristic algorithm for performing index sampling.

   (b) Using the heuristic algorithm, find the sampling portfolio with 4 stocks.

   (c) Same question if we want to keep only 3, 2 and 1 stocks.

   (d) Calculate the volatility of the five portfolios (CW + the four sampled portfolios).

   (e) Calculate the tracking error volatility, the beta and the correlation of the five portfolios with respect to the CW portfolio.

   (f) Show that the correlation $\rho(x, b)$ can be determined using the volatility of the benchmark $\sigma(b)$, the volatility of the portfolio $\sigma(x)$ and the tracking error volatility $\sigma(x \mid b)$.

---

[1] x_i is the weight (or the exposure) of the ith asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^\star$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

[2] We remind that the normalized Herfindahl index is defined as follows:

$$\mathcal{H}^\star(\pi) = \frac{n \sum_{i=1}^{n} \pi_i^2 - 1}{n - 1}$$

where $\pi = (\pi_1, \dots, \pi_n)$ is a vector satisfying $\pi_i \geq 0$ and $\sum_{i=1}^{n} \pi_i = 1$.

3   Factor investing

1. Define the five equity risk factors: size, value, momentum, low beta and quality.

2. For each risk factor, give a metric or a ratio that allows to rank the stocks according to this factor.

3. Explain the methodology used by FTSE Russell to build the FTSE Global Factor Index Series. Compare it with the methodology used by MSCI to build the MSCI factor tilt indexes. Comment on these results.

4. In the attached Excel file, we provide the cumulative performance of the five risk factors[3]. We also provide the cumulative performance of the MSCI Europe index, the EW multi-factor strategy[4] and the ERC multi-factor strategy[5].

    (a) For each single-factor index, calculate the annualized performance, the volatility, the beta and the maximum drawdown. Comment on these results.

    (b) Compare the multi-factor strategies with respect to the MSCI Europe index.

References

[1] FTSE, FTSE Global Factor Index Series, `http://www.ftse.com/products/indices/factor`.

[2] MSCI, MSCI factor tilt indexes, `https://www.msci.com/factor-indexes`.

[3] CAZALET, Z., and RONCALLI, T. (2014), Facts and Fantasies About Factor Investing, *SSRN*, `www.ssrn.com/abstract=2524547`.

4   Alternative Risk Premia

1. What is the difference between a skewness risk premium and a market anomaly?

2. Define the option profile of mean-reverting and trend-following strategies.

3. The carry risk premium

    (a) Explain how the carry risk premium is implemented in the case of currencies.
    (b) Explain how the carry risk premium is implemented in the case of commodities.
    (c) What is the adverse scenario of a carry strategy?

4. The momentum risk premium

    (a) What is the difference between cross-section and time-series momentum?
    (b) Explain why the loss frequency of a long/short trend-following strategy is higher than its gain frequency?
    (c) What is the adverse scenario of a trend-following strategy?

5. The short volatility strategy

    (a) Define the VIX index.

---

[3]They correspond to the JP Morgan factor indices.
[4]We assume that the strategy is rebalanced at the end of each month.
[5]It uses the same rebalancing scheme than the EW multi-factor strategy. The weights are calculated using the one-year rolling covariance matrix of daily returns.

(b) What is the difference between the implied volatility and the realized volatility?

(c) How do you explain the existence of the volatility risk premium in the universe of equities?

6. We provide the cumulative performance of 5 indices (US equities, US bonds, US short volatility, Carry FX G10 & SG CTA index) in the attached Excel file.

(a) For each index, calculate the annualized performance, the volatility and the skewness coefficient.

(b) Calculate the empirical correlation matrix between these 5 indices.

(c) Comment on these results.

References

[1] BLIN, O., LEE, J., and TEILETCHE, J. (2017), Alternative Risk Premia Investing: From Theory to Practice, *Unigestion*.

[2] HAMDAN, R., PAVLOWSKY F., RONCALLI, T., and ZHENG, B. (2016), A Primer on Alternative Risk Premia, *SSRN*, www.ssrn.com/abstract=2766850.

[3] KOIJEN, R.S.J., MOSKOWITZ, T.J., PEDERSEN, L.H., and VRUGT, E.B. (2015), Carry, *SSRN*, www.ssrn.com/abstract=2298565.

[4] POTTERS, M., and BOUCHAUD, J-P. (2006), Trend Followers Lose More Often Than They Gain, *Wilmott Magazine*, 26, pp. 58-63.

[5] RONCALLI, T. (2017), Alternative Risk Premia: What Do We Know?, *SSRN*, www.ssrn.com/abstract=2868425.