Risk-Based Investing & Asset Management
Final Examination

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Remark 1 ***The final examination is composed of 4 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file or one program by exercise.***

- ***Concerning risk decomposition[1], present the results as follows:***

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			✓	

- ***The report is a zipped file, whose filename is yourname.zip if you do the project alone or yourname1-yourname2.zip if you do the project in groups of two.***

- ***The zipped file is composed of four files:***

  1. ***the pdf document that contains the answers to the four exercises and a cover sheet with your names;***
  2. ***the program of each exercise with an explicit filename, e.g. exercise1.xls (if you use excel), exercise1.m (if you use matlab), exercise1.py (if you use python), exercise1.r (if you use R), etc.***

[1] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^\star$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

1 Mean-variance optimized and risk-based portfolios

We consider an investment universe with 5 assets. We assume that their expected returns are equal to 3%, 4%, 5%, 5% and 6%, and that their volatilities are equal to 5%, 10%, 15%, 15% and 20%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 60\% & 100\% & & & \\ 70\% & 50\% & 100\% & & \\ 50\% & 50\% & 30\% & 100\% & \\ 20\% & 0\% & 30\% & -30\% & 100\% \end{pmatrix}$$

We assume that the risk-free rate is equal to 1%.

1. Calculate Σ the covariance matrix.
2. We consider long-short portfolios x with $\sum_{i=1}^{n} x_i = 1$.
   (a) Compute the minimum variance portfolio.
   (b) Calculate the MVO portfolio if we target an ex-ante volatility of 10%.
   (c) Calculate the MDP portfolio.
   (d) Compare the three optimized portfolios in terms of expected return, volatility and Sharpe ratio.
   (e) Calculate the beta $\beta_i(x)$ and the correlation $\rho_i(x)$ for each asset i when Portfolio x is respectively the previous long/short MV, MVO and MDP portfolios. Explain why we obtain the following properties:
   $$\beta_i(x_{\mathrm{MV}}) = 1 \tag{1}$$
   and:
   $$\rho_i(x_{\mathrm{MDP}}) = \rho_j(x_{\mathrm{MDP}}) \tag{2}$$
3. We restrict the analysis to long-only portfolios meaning that $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0$.
   (a) Compute the minimum variance portfolio.
   (b) Calculate the MVO portfolio if we target an ex-ante volatility of 10%.
   (c) Calculate the MDP portfolio.
   (d) Compare the three optimized portfolios in terms of expected return, volatility and Sharpe ratio. What is the impact of the long-only constraint?
   (e) Calculate the beta $\beta_i(x)$ and the correlation $\rho_i(x)$ for each asset i when Portfolio x is respectively the previous long-only MV, MVO and MDP portfolios. What do Properties (1) and (2) become in the long-only case?
4. We consider long-only portfolios. For each portfolio, compute the volatility decomposition.
   (a) Determine the equally weighted (EW) portfolio.
   (b) Compute the minimum variance (MV) portfolio.
   (c) Calculate the most diversified portfolio (MDP).
   (d) Find the ERC portfolio.
5. Draw the 4 previous risk-based portfolios on the efficient frontier. Comment on these results.

6. The previous results have been obtained by considering ex-ante expected returns, volatilities and correlations. One year later, we calculate these statistics and obtained the following ex-post values: the asset returns were equal to 2%, 7%, 1%, 6% and −2%, the realized volatilities were equal to 7%, 12%, 19%, 11% and 30%, and the realized correlation matrix was equal to:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 80\% & 100\% & & & \\ 80\% & 80\% & 100\% & & \\ 80\% & 70\% & 70\% & 100\% & \\ -20\% & -20\% & -20\% & -20\% & 100\% \end{pmatrix}$$

Draw the ex-post efficient frontier. Put the 4 risk-based portfolios and compare these ex-post figures with those obtained in an ex-ante framework. Comment on these results.

Remark 2 *For Question 6, we obtain the following result:*

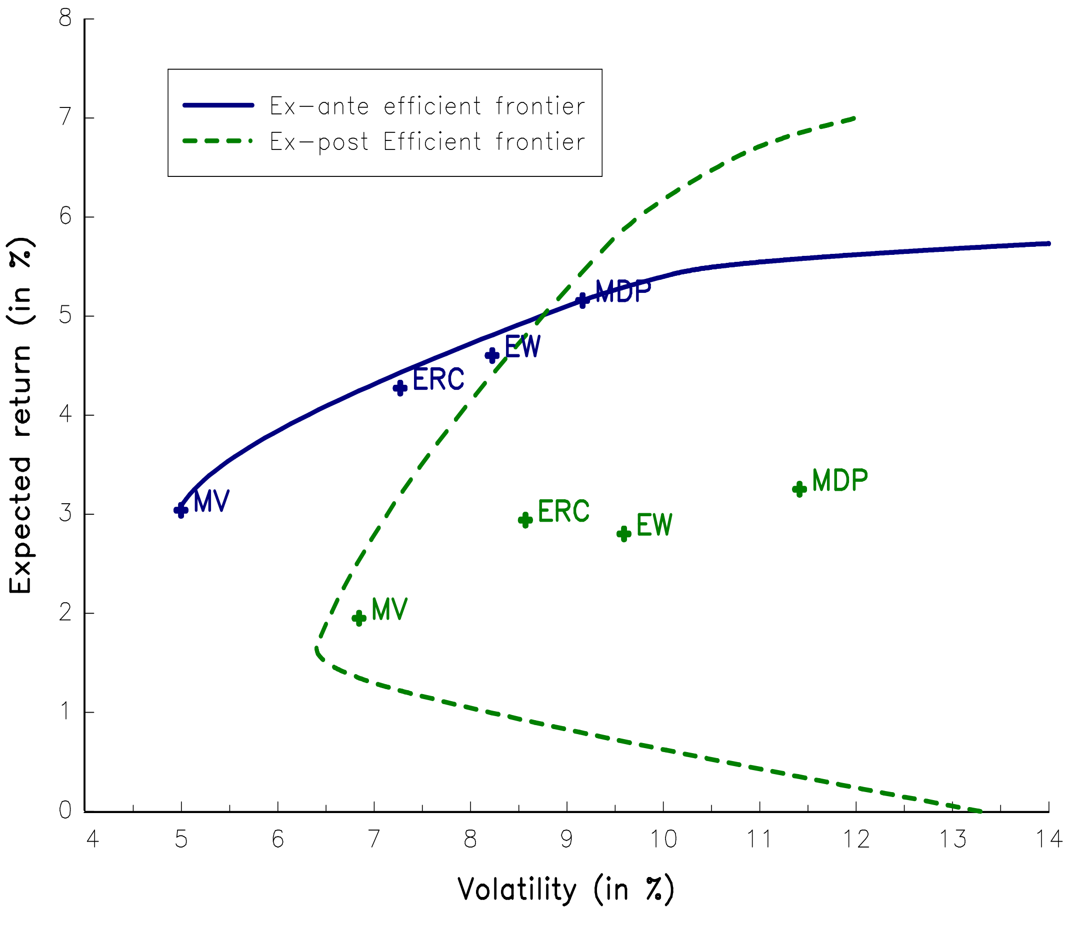

Figure 1: Comparison of ex-ante and ex-post cases

2 Index sampling and tilted portfolios

We consider a capitalization-weighted equity index, which is composed of 5 stocks. The weights are equal to 29%, 25%, 23%, 18% and 5%. We assume that the stock volatilities are 20%, 20%, 25%, 15% and 25%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 40\% & 100\% & & & \\ 70\% & 75\% & 100\% & & \\ 60\% & 55\% & 90\% & 100\% & \\ 70\% & 60\% & 70\% & 65\% & 100\% \end{pmatrix}$$

We note Σ the covariance matrix. In what follows, we consider long-only portfolios.

1. Calculate the covariance matrix.
2. We consider the CW portfolio as the benchmark.
   (a) Describe the heuristic algorithm for performing index sampling.
   (b) Using the heuristic algorithm, find the optimal sampling portfolio with 4 stocks.
   (c) Same question if we want to keep only 3, 2 and 1 stocks.
   (d) Calculate the volatility of the five portfolios (CW + the four sampled portfolios).
   (e) Calculate the tracking error volatility, the beta and the correlation of the five portfolios with respect to the CW portfolio.
3. We consider a fund manager who would like to tilt the CW benchmark. His stock picking model forecasts the following expected returns: 5% for Asset 1, 6% for Asset 2, 7% for Asset 3, 8% for Asset 4 and 9% for Asset 5.
   (a) Calculate the optimal portfolio if the fund manager targets a 1% tracking error volatility.
   (b) Same question if the targeted tracking error volatility is equal to 2%.
   (c) Same question if the targeted tracking error volatility is equal to 4%.
   (d) For the three previous tilted portfolios, calculate the expected excess return, the tracking error volatility and the information ratio. Comment on these results.
   (e) The fund manager imposes then that the allocation in one asset cannot exceed 27.5%. Find the optimal portfolio if the targeted tracking error volatility is 4%. Calculate the information ratio of this tilted portfolio. Why is this result different than the information ratio calculated in Question 3(d)?

3 Regularizing portfolio optimization

We consider an investment universe with 6 assets. We assume that their expected returns are 4%, 6%, 7%, 8%, 10% and 10%„ and their volatilities are 6%, 10%, 11%, 15%, 15% and 20%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & & \\ 50\% & 100\% & & & & \\ 20\% & 20\% & 100\% & & & \\ 50\% & 50\% & 80\% & 100\% & & \\ 0\% & -20\% & -50\% & -30\% & 100\% & \\ 0\% & 20\% & 30\% & 0\% & 0\% & 100\% \end{pmatrix}$$

1. We restrict the analysis to long-only portfolios meaning that $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0$.

   (a) Define the ridge regression.

   (b) We consider the Herfindahl index $\mathcal{H}(x) = \sum_{i=1}^{n} x_i^2$. What are the two limit cases of $\mathcal{H}(x)$? What is the interpretation of the statistic $\mathcal{N}(x) = \mathcal{H}^{-1}(x)$?

   (c) We consider the following optimization problem ($\mathcal{P}_1$):

   $$\begin{aligned} x^{\star}(\lambda) &= \arg\min \frac{1}{2} x^{\top} \Sigma x + \lambda x^{\top} x \\ &\text{u.c.} \quad \begin{cases} \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \end{aligned}$$

   What is the link between this constrained optimization program, the ridge regression and the weight diversification based on the Herfindahl index?

   (d) Solve Program ($\mathcal{P}_1$) when λ is equal to respectively 0, 0.001, 0.01, 0.05, 0.10 and 10. Compute the statistic $\mathcal{N}(x)$. Comment on these results.

   (e) Using the bisection algorithm, find the optimal value of $\lambda^{\star}$ that satisfies:

   $$\mathcal{N}(x^{\star}(\lambda^{\star})) = 4$$

   Give the composition of $x^{\star}(\lambda^{\star})$. What is the interpretation of $x^{\star}(\lambda^{\star})$?

2. We consider long/short portfolios.

   (a) Define the lasso regression.

   (b) We consider the following optimization problem ($\mathcal{P}_2$):

   $$\begin{aligned} x^{\star}(\lambda) &= \arg\min \frac{1}{2} x^{\top} \Sigma x + \lambda \sum_{i=1}^{n} |x_i| \\ &\text{u.c.} \quad \sum_{i=1}^{n} x_i = 1 \end{aligned}$$

   What is the link between this constrained optimization program and the lasso regression?

   (c) Solve Program ($\mathcal{P}_2$) when λ is equal to respectively 0, 0.0001, 0.001, 0.01, 0.05, 0.10 and 10. Comment on these results.

   (d) For each optimized portfolio, calculate the following statistic:

   $$\mathcal{L}(x) = \sum_{i=1}^{n} |x_i|$$

   What is the interpretation of $\mathcal{L}(x)$? What is the impact of Lasso regularization?

(e) We assume that the investor holds an initial portfolio $x^{(0)}$ defined as follows:

$$x^{(0)} = \begin{pmatrix} 10\% \\ 15\% \\ 20\% \\ 25\% \\ 30\% \\ 0\% \end{pmatrix}$$

We consider the optimization problem ($\mathcal{P}_3$):

$$\begin{aligned} x^{\star}(\lambda) &= \arg\min \frac{1}{2} x^{\top} \Sigma x + \lambda \sum_{i=1}^{n} \left| x_i - x_i^{(0)} \right| \\ \text{u.c.} & \sum_{i=1}^{n} x_i = 1 \end{aligned}$$

i. Solve Program ($\mathcal{P}_3$) when λ is equal respectively to 0, 0.0001, 0.001, 0.0015 and 0.01. Compute the turnover of each optimized portfolio. Comment on these results.

ii. Using the bisection algorithm, find the optimal value of $\lambda^{\star}$ such that the two-way turnover is equal to 60%. Give the composition of $x^{\star}(\lambda^{\star})$.

iii. Same question when the two-way turnover is equal to 50%.

iv. What becomes the portfolio $x^{\star}(\lambda)$ when $\lambda \to \infty$? How do you explain this result?

4 Factor Investing & Alternative Risk Premia

4.1 Factor Investing in Equities

1. Define the five equity risk factors: size, value, momentum, low beta and quality.
2. Give the metrics that allow to rank the stocks according to size, momentum and low beta factors.
3. Give two metrics for defining the value risk factor. Same question for the quality risk factor.

4.2 Alternative Risk Premia

1. What is the difference between a skewness risk premium and a market anomaly?
2. Explain the difference between convex and concave strategies. Give an example of each strategy.
3. The carry risk premium
   (a) Explain how the carry risk premium is implemented in the case of currencies.
   (b) Explain how the carry risk premium is implemented in the case of commodities.
   (c) Define the volatility carry risk premium.
   (d) What is the adverse scenario of a carry strategy?
4. The momentum risk premium
   (a) What is the difference between cross-section and time-series momentum?
   (b) Explain why the loss frequency of a long/short trend-following strategy is higher than its gain frequency?
   (c) What is the adverse scenario of a trend-following strategy?

References

[1] FTSE, FTSE Global Factor Index Series, `http://www.ftse.com/products/indices/factor`.

[2] MSCI, MSCI factor tilt indexes, `https://www.msci.com/factor-indexes`.

[3] Blin, O., Lee, J., and Teiletche, J. (2017), Alternative Risk Premia Investing: From Theory to Practice, *Unigestion*, `https://www.unigestion.com`.

[4] Hamdan, R., Pavlowsky F., Roncalli, T., and Zheng, B. (2016), A Primer on Alternative Risk Premia, *SSRN*, `www.ssrn.com/abstract=2766850`.

[5] Koijen, R.S., Moskowitz, T., Pedersen, L.H. and Vrugt, E.B. (2018), Carry, *Journal of Financial Economics*, forthcoming.
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[6] Roncalli, T. (2017), Alternative Risk Premia: What Do We Know?, in Jurczenko, E. (Ed.), *Factor Investing and Alternative Risk Premia*, ISTE Press – Elsevier.
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[7] Roncalli, T. (2017), Keep Up the Momentum, *SSRN*, `www.ssrn.com/abstract=3083921`.