Risk-Based Investing & Asset Management
Final Examination

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Deadline: March 20th 2019

Remark 1 ***The final examination is composed of 4 exercises. Please write entirely your answers. Precise the different concepts and the different statistics you use. Define the optimization program associated to each portfolio. Provide also one excel file or one program by exercise.***

- ***Concerning risk decomposition[1], present the results as follows:***

Asset	x_i	$\mathcal{MR}_i$	$\mathcal{RC}_i$	$\mathcal{RC}_i^\star$
1				
2				
$\vdots$				
n				
$\mathcal{R}(x)$			✓	

- ***The report is a zipped file, whose filename is <u>yourname.zip</u> if you do the project alone or <u>yourname1-yourname2.zip</u> if you do the project in groups of two.***
- ***The zipped file is composed of four files:***
  1. ***the pdf document that contains the answers to the four exercises and a cover sheet with your names;***
  2. ***the program of each exercise with an explicit filename, e.g. exercise1.xls (if you use excel), exercise1.m (if you use matlab), exercise1.py (if you use python), exercise1.r (if you use R), etc.***

[1] x_i is the weight (or the exposure) of the i^{th} asset in the portfolio, $\mathcal{MR}_i$ is the marginal risk, $\mathcal{RC}_i$ is the nominal risk contribution, $\mathcal{RC}_i^\star$ is the relative risk contribution and $\mathcal{R}(x)$ is the risk measure of the portfolio.

1 Mean-variance optimized portfolios

We consider an investment universe with 4 assets. We assume that their expected returns are equal to 3%, 4%, 5% and 6%, and their volatilities are equal to 8%, 10%, 12% and 14%. The correlation matrix C is given by:

$$C = \begin{pmatrix} 100\% & & & \\ 20\% & 100\% & & \\ 60\% & 50\% & 100\% & \\ 50\% & 50\% & 20\% & 100\% \end{pmatrix}$$

We assume that the risk-free rate is equal to 1%.

1. Calculate Σ the covariance matrix.

2. Perform the principal component analysis of Σ and Σ^{-1}. Comment on these results.

3. We consider long-short portfolios x without any constraints (γ-problem).

   (a) Find the optimized portfolio $x^{\star}(\gamma)$ when the coefficient γ is equal to 0.5.

   (b) Find the optimized portfolio $y^{\star}(\gamma)$ when the coefficient γ is equal to 0.5 and the correlation matrix C is the identity matrix.

   (c) For each asset i, estimate the following linear regression[2]:

   $$R_{i,t} = \alpha_i + \beta_i^{\top} R_t^{(-i)} + \varepsilon_{i,t}$$

   where $R_t^{(-i)}$ denotes the vector of asset returns excluding the i^{th} asset and $\varepsilon_{i,t} \sim \mathcal{N}\left(0, s_i^2\right)$. What is the interpretation of α_i, β_i and s_i? Give the values of α_i, β_i, s_i and the coefficient of determination $\mathfrak{R}_i^2$ for the 4 assets.

   (d) Find the optimized portfolio $z^{\star}(\gamma)$ of the four hedging portfolios that have been estimated in Question 3(c).

   (e) Verify that[3]:

   $$x_i^{\star}(\gamma) = y_i^{\star}(\gamma) + \frac{\mathfrak{R}_i^2}{1 - \mathfrak{R}_i^2}\left(y_i^{\star}(\gamma) - z_i^{\star}(\gamma)\right)$$

   Comment on these results.

4. We consider long-short portfolios x with $\sum_{i=1}^{n} x_i = 1$.

   (a) Compute the minimum variance portfolio.

   (b) Calculate the MVO portfolio if we assume that γ is respectively equal to 0.1, 0.2, 0.3, 0.4 and 0.5.

   (c) Calculate the MVO portfolio if we target an ex-ante volatility of 5%, 8%, 9% and 10%.

   (d) We would like to calculate the tangency portfolio[4].

      i. Calculate MVO portfolios $x^{\star}(\gamma)$ for $\gamma \in [0.10, 0.30]$ and select the portfolio which has the largest Sharpe ratio.

      ii. Consider an augmented QP problem by including the risk-free asset in the universe, calculate MVO portfolios $x^{\star}(\gamma)$ for $\gamma \in [0.10, 0.30]$ and select the portfolio which has the lowest exposure on the risk-free asset.

[2] Hints: use the formula of the Gaussian conditional expectation.
[3] See Bourgeron *et al.* (2018).
[4] Hints: the tangency portfolio is located in the region $\gamma \in [0.10, 0.30]$.

(e) We note $\tilde{x}$ the tangency portfolio. Give the composition of the tangency portfolio, its expected return $\mu(\tilde{x})$, its volatility $\sigma(\tilde{x})$ and its Sharpe ratio $\text{SR}(\tilde{x} \mid r)$.

(f) Calculate the beta $\beta_i(\tilde{x})$ of each asset with respect to the tangency portfolio. Deduce the risk premium $\tilde{\pi}_i$ of each asset. Verify that:

$$\tilde{\pi}_i = \mu_i - r$$

5. We new consider long-only portfolios x with $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 15\%$.

(a) Compare the efficient frontier with this obtained in Question 4.

(b) Calculate the MVO portfolio if we target an ex-ante volatility of 5%, 8%, 9% and 10%. Comment on these results.

(c) Find the tangency portfolio $\tilde{x}$. Calculate the beta $\beta_i(\tilde{x})$ of each asset with respect to the tangency portfolio. Deduce the risk premium $\tilde{\pi}_i$ of each asset. Comment on these results.

2 Risk-based portfolios

We consider an investment universe with 4 assets. We assume that their volatilities are equal to 18%, 20%, 22% and 25%. The correlation matrix C is given by:

$$C = \begin{pmatrix} 100\% & & & \\ 50\% & 100\% & & \\ 0\% & 0\% & 100\% & \\ -30\% & 20\% & 10\% & 100\% \end{pmatrix}$$

We assume that the risk measure is the portfolio volatility $\sigma(x)$.

1. Calculate the covariance matrix.
2. Give the risk decomposition of the equally-weighted (EW) portfolio.
3. Give the risk decomposition of the long-only minimum-variance (MV) portfolio.
4. Give the risk decomposition of the equal risk contribution (ERC) portfolio.
5. Give the risk decomposition of the risk budgeting (RB) portfolio, when the risk budgets are equal to $(10\%, 20\%, 30\%, 40\%)$.
6. Give the risk decomposition of the most diversified portfolio (MDP).
7. Calculate the diversification ratio $\mathcal{DR}(x)$ of the five previous risk-based portfolios. Comment on these results.
8. Calculate the correlation $\rho(e_i, x_{\mathrm{mdp}})$ between each asset i and the MDP portfolio x_{mdp}. Comment on these results.
9. We assume now that the correlation matrix C is equal to:

$$C = \begin{pmatrix} 100\% & & & \\ 0\% & 100\% & & \\ 0\% & 20\% & 100\% & \\ 0\% & 50\% & 95\% & 100\% \end{pmatrix}$$

   Calculate the long-only MDP portfolio and the correlations $\rho(e_i, x_{\mathrm{mdp}})$. Explain these results. Why the MDP is not invested in Asset 4?

3 Portfolio optimization with an initial allocation

We consider a current portfolio $\bar{x}$, which is composed of 5 assets. The weights are equal to 29%, 25%, 23%, 18% and 5%. We assume that the expected returns of the assets are equal to -10%, -10%, 2%, 5% and 10%, whereas the volatilities are equal to 20%, 20%, 25%, 15% and 25%. The correlation matrix is given by:

$$\rho = \begin{pmatrix} 100\% & & & & \\ 40\% & 100\% & & & \\ 70\% & 75\% & 100\% & & \\ 60\% & 55\% & 90\% & 100\% & \\ 70\% & 60\% & 70\% & 65\% & 100\% \end{pmatrix}$$

We note Σ the covariance matrix. In what follows, we consider long-only portfolios.

1. Calculate the covariance matrix.

2. We consider the tracking error approach.

   (a) Give the mean-variance objective function when there is a benchmark. Write this problem as a quadratic programming (QP) problem.

   (b) Find the optimized portfolio $x^\star$ when we target an excess return $\mu(x \mid \bar{x})$ of 5%.

   (c) Calculate the corresponding tracking error $\sigma(x^\star \mid \bar{x})$ and turnover $\tau(x^\star \mid \bar{x})$ with respect to the current allocation $\bar{x}$.

3. We consider the lasso approach.

   (a) We consider the following optimization problem ($\mathcal{P}_{\mathcal{L}asso}$):

   $$\begin{aligned} x^\star(\gamma, \lambda) &= \arg\min \frac{1}{2} x^\top \Sigma x - \gamma x^\top \mu + \lambda \sum_{i=1}^{n} |x_i - \bar{x}_i| \\ &\text{u.c.} \quad \begin{cases} \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \end{aligned}$$

   What is the underlying idea of this optimization problem?

   (b) Write this problem as an augmented QP problem by considering the parametrization:

   $$x_i = \bar{x}_i + \Delta x_i^+ - \Delta x_i^-$$

   where $\Delta x_i^+ \geq 0$ and $\Delta x_i^- \geq 0$.

   (c) Give the expression of the turnover $\tau(x \mid \bar{x})$ with respect to the vectors Δx^+ and Δx^-.

   (d) Give the expression of the tracking error $\sigma(x \mid \bar{x})$ with respect to the vectors Δx^+ and Δx^-.

   (e) Solve Program ($\mathcal{P}_{\mathcal{L}asso}$) when $\gamma = 0$ and λ is respectively equal to 0, 0.2%, 0.5%, 0.8%, 0.95% and 1%. Calculate $\mu(x \mid \bar{x})$, $\sigma(x \mid \bar{x})$ and $\tau(x \mid \bar{x})$ for each optimized portfolios $x^\star(\gamma, \lambda)$. Comment on these results.

   (f) Solve Program ($\mathcal{P}_{\mathcal{L}asso}$) when $\gamma = 5\%$ and λ is respectively equal to 0, 0.2%, 0.5%, 0.8%, 0.95% and 1%. Calculate $\mu(x \mid \bar{x})$, $\sigma(x \mid \bar{x})$ and $\tau(x \mid \bar{x})$ for each optimized portfolios. Comment on these results.

4. We consider the risk budgeting approach.

   (a) Describe the CCD algorithm for computing the risk budgeting portfolio when the vector of risk budgets is equal to b.

(b) Compute the ERC portfolio with the CCD method by considering the current portfolio $\bar{x}$ as starting values of the algorithm. How many iterations do we need to obtain the convergence? Give the intermediary solution after the first cycle of the CCD algorithm.

(c) What is the turnover of the ERC portfolio $\tau(x \mid \bar{x})$ with respect to the current allocation $\bar{x}$?

(d) We would like to reduce this turnover. For that, we compute the constrained ERC portfolio by imposing that:

$$|x_i - \bar{x}_i| \leq \delta$$

How to introduce these constraints in the CCD algorithm[5]?

(e) Find the constrained ERC portfolio when $\delta = 2\%$.

(f) Same question when $\delta = 7\%$.

(g) Comment on these results.

[5]Hints: use the box constraint method described in Richard and Roncalli (2019).

4 Factor Investing & Alternative Risk Premia

4.1 Factor Investing in Equities

1. Define the five equity risk factors: size, value, momentum, low beta and quality.
2. Give the metrics that allow to rank the stocks according to size, momentum and low beta factors.
3. Give two metrics for defining the value risk factor. Same question for the quality risk factor.
4. What is the intrinsic difference between value and quality?

4.2 Alternative Risk Premia

1. What is the difference between a skewness risk premium and a market anomaly?
2. Explain the difference between convex and concave strategies. Give an example of each strategy.
3. The carry risk premium
   (a) Explain how the carry risk premium is implemented in the case of currencies.
   (b) Explain how the carry risk premium is implemented in the case of commodities.
   (c) Define the volatility carry risk premium. Why it is related to the robustness of the Black-Scholes formula?
   (d) What is the adverse scenario of a carry strategy?
4. The momentum risk premium
   (a) What is the difference between cross-section and time-series momentum?
   (b) Explain why the loss frequency of a long/short trend-following strategy is higher than its gain frequency?
   (c) What is the adverse scenario of a trend-following strategy?

References

[1] FTSE, FTSE Global Factor Index Series, http://www.ftse.com/products/indices/factor.

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[3] Blin, O., Lee, J., and Teiletche, J. (2017), Alternative Risk Premia Investing: From Theory to Practice, *Unigestion*, https://www.unigestion.com.

[4] Bourgeron, T., Lezmi, E., and Roncalli, T. (2018), Robust Asset Allocation for Robo-Advisors, *SSRN*, www.ssrn.com/abstract=3261635.

[5] Griveau-Billion, T., Richard, J-C., and Roncalli, T. (2013), A Fast Algorithm for Computing High-dimensional Risk Parity Portfolios, *SSRN*, www.ssrn.com/abstract=2325255.

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