Financial Risk Management

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Tutorial exercices #2

1 Risk contributions in Basle II model

Let us consider a portfolio of I credits (debts) of maturity M_i. We denote L the loss of portfolio:

$$L = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathrm{LGD}_i \times 1\left\{\tau_i \leq M_i\right\}$$

We show that, under certain hypothesis, the expectation of loss conditionally in factors $X_1, \ldots, X_m$ is:

$$\mathbb{E}\left[L \mid X_1, \ldots, X_m\right] = \sum_{i=1}^{I} \mathrm{EAD}_i \times \mathbb{E}\left[\mathrm{LGD}_i\right] \times \mathrm{PD}_i\left(X_1, \ldots, X_m\right)$$

1. How do we obtain this expression? Which are the necessary (H) hypothesis? What do we call an infinitely granular portfolio?
2. What is the credit risk contribution?
3. Define the notions of *expected loss* (EL) and *unexpected loss* (UL). Show that the VaR measure is equal to the EL measure under the previous hypothesis (H) if the default times are independent of the factors.
4. Write the expression of quantile α of loss $\mathbf{F}^{-1}(\alpha)$ in the case of one factor $X \sim H$. Why this expression isn't it available if at least one of the exposures EAD_i is negative? What can you conclude for the management of the credit portfolio?
5. In the Basle II model, we assume that default before maturity M_i if a latent variable Z_i falls below some barrier B_i:
$$\tau_i \leq M_i \Leftrightarrow Z_i \leq B_i$$
We model $Z_i = \sqrt{\rho} X + \sqrt{1-\rho}\varepsilon_i$ with Z_i, X et ε_i three standard normal random variables and independent. X is the factor (or the systemic risk) and ε_i is the individual risk. Calculate the conditionally default probability. Verify that the expectation of the conditionally default probability is equal to $\Phi^{-1}(B_i)$.
6. Calculate the quantile $\mathbf{F}^{-1}(\alpha)$. Deduce an expression of the loss density function L.
7. Interpret the correlation ρ. How could we estimate this parameter?
8. The previous risk contribution was obtained considering the hypothesis (H) and under the framework of the default model of question 5. Deduce the implications in terms of Pillar II.

2 Counterparty credit risk on market transactions

1. The following table gives the actual values of mark-to-market of 7 OTC contracts between the banks A and B:

	Equities			Fixed income		Currencies	
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
A	10	-5	6	17	-5	-5	1
B	-11	6	-3	-12	9	5	1

One can read this table in the following way: bank A has declared a mark-to-market of 10 for contract $\mathcal{C}_1$ while bank B thinks of having a mark-to-market of -11 for the same contract.

(a) Explain why there are incoherences for certain mark-to-market.

(b) Calculate the default exposition of bank A (resp. of bank B).

(c) Same question if there exists a global compensation contract.

(d) Same question if there exists a compensation contract covering only equities.

2. Let us denote $e(t)$ the default exposition of a OTC contract of maturity T. The actual date is fixed at $t = 0$.

(a) Define the concepts of potential future exposure $\mathrm{PFE}_\alpha(0;t)$, maximal exposure $\mathrm{PE}_\alpha(0)$, expected exposure $\mathrm{EE}(0;t)$, positive expected exposure $\mathrm{EPE}(0;h)$, effective expected exposure $\mathrm{EEE}(0;h)$ and positive expected effective exposure effective $\mathrm{EEPE}(0;h)$.

(b) Calculate these different quantities while the default exposure is respectively:

- $e(t) = \sigma\sqrt{t}\varepsilon$ with $\varepsilon \sim \mathcal{U}_{[0,1]}$;
- $e(t) = \exp\left(\sigma\sqrt{t}\varepsilon\right)$ with $\varepsilon \sim \mathcal{N}(0,1)$;
- $e(t) = \sigma\left(t^3 - \frac{7}{3}Tt^2 + \frac{4}{3}T^2 t\right)\varepsilon$ with $\varepsilon \sim \mathcal{U}_{[0,1]}$.

(c) Comment these results.

3. In this exercise, we are looking for measuring the impact of a compensation contract in the default exposure of a bank.

(a) We consider a first OTC contract $\mathcal{C}_1$ between bank A and bank B. The mark-to-market $\mathrm{MtM}_1(t)$ of bank A for contract $\mathcal{C}_1$ progresses in the following way:

$$\mathrm{MtM}_1(t) = x_1 + \sigma_1 W_1(t)$$

where $W_1(t)$ is a brownian motion. Calculate the average default exposure of bank A.

(b) The counter party risk of bank A relative to bank B now focuses on two contracts, the previous contract $\mathcal{C}_1$ and contract $\mathcal{C}_2$ whose the mark-to-market is:

$$\mathrm{MtM}_2(t) = x_2 + \sigma_2 W_2(t)$$

where $W_2(t)$ is a brownian motion such that $\mathbb{E}[W_1(t)W_2(t)] = \rho t$. Calculate the average exposure at default of bank A in the case where it exists a global compensation contract.

(c) Comment these results.

3 Calibrating operational risk models

1. Let us consider a sample of unitary losses $\{L_1, \ldots, L_n\}$. We assume that these losses can be modeled in two ways: *(i)* $L_i \sim \mathcal{LN}(\mu, \sigma)$ and *(ii)* L_i follows a Pareto distribution $\mathcal{P}(\theta; x^-)$ defined by $\Pr\{L \leq x\} = 1 - \left(\frac{x}{x_-}\right)^{-\theta}$ with $x \geq x_-$ et $\theta > 1$.

   (a) Let us consider the case *(i)*.Calculate the maximum-likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$. Calculate the first two moments of L_i and deduce a generalized method of moments for estimating parameters μ and σ.

   (b) Let us consider case *(ii)*. Calculate the maximum-likelihood estimator $\hat{\theta}$. Calculate the first two moments of L_i and deduce a generalized method of moments to estimate parameter θ.

   (c) We assume that the losses $\{L_1, \ldots, L_n\}$ have been collected beyond a threshold H. Calculate the maximum-likelihood estimator (MLE) $\hat{\theta}$ in the case of distribution *(ii)*. Calculate the first two conditional moments of L_i in the case of distribution as in *(i)*.

   (d) Suggest two adequacy tail distribution tests.

2. Let us consider a sample of T numbers of losses $\{N_1, \ldots, N_T\}$. We assume that the number of losses follows a Poisson distribution $\mathcal{P}(\lambda)$.

   (a) Calculate MLE $\hat{\lambda}$ if the frequency of loss measure is annual.

   (b) Same questions if the losses $N_1, \ldots, N_T$ are quarterly measures.

3. We assume that the annual number of losses follows a Poisson distribution $\mathcal{P}(\lambda)$. We assume as well that the losses are independent and follow the distribution as in *(ii)*.

   (a) Show that the duration between the two consecutive superior losses ℓ is an exponential distribution of parameter $\lambda x_-^{\theta} \ell^{-\theta}$.

   (b) How can we use this result to calibrate experts' scenarios?