Course 2024–2025 in Sustainable Finance
Lecture 11. Economic Models & Climate Change

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[1]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Agenda

Sustainable growth and climate change

"*There is no Plan B, because there is no Planet B*"

Ban Ki-moon, UN Secretary-General, September 2014

Is it a question of climate-related issues?

In fact, it is more an economic growth issue

"*The Golden Rule of Accumulation: A Fable for Growthmen*"

Edmund Phelps, *American Economic Review*, 1961
Nobel Prize in Economics, 2006

Sustainable growth and climate change

Environmental	Economy
Social	Sustainability
Gouvernance	Growth

Adam Smith (1776)

An Inquiry into the Nature and Causes of The Wealth of Nations

Limits of economic models

The Solow growth model

The model

- Production function:

$$Y(t) = F(K(t), A(t) L(t))$$

  where $K(t)$ is the capital, $L(t)$ is the labor and $A(t)$ is the knowledge factor

- Law of motion for the capital per unit of effective labor $k(t) = K(t) / (A(t) L(t))$:

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = s\, f(k(t)) - (g_L + g_A + \delta_K)\, k(t)$$

  where s is the saving rate, δ_K is the depreciation rate of capital and g_A and g_L are the productivity and labor growth rates

Limits of economic models
The golden rule

Golden rule with the Cobb-Douglas production and Hicks neutrality

The equilibrium to respect the '*fairness*' between generations is:

$$k^{\star} = \left(\frac{s}{g_L + g_A + \delta_K} \right)^{\dfrac{1}{1-\alpha}}$$

"*Each generation in a boundless golden age of natural growth will prefer the same investment ratio, which is to say the same natural growth path*" (Phelps, 1961, page 640).

"*By a golden age I shall mean a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital-output ratio is stationary over time*" (Phelps, 1961, page 639).

Limits of economic models

Golden rule and climate risk

What is economic growth and what is the balanced growth path?

- There is a saving rate that maximizes consumption over time and between generations ("**the fair rate to preserve future generations**")
- Economic growth corresponds to the exponential growth of capital and output to answer the needs of the growing population
- Introducing human and natural capitals add constraints and therefore **reduce growth**!

$$\text{Economic growth} \Rightarrow \begin{cases} \text{productivity} \nearrow \text{and labor} \nearrow \\ \text{maximization of } \textbf{consumption-based utility} \text{ function} \end{cases}$$

Limits of economic models

Extension to natural capital

What are the effects of environmental constraints on growth?

Introducing a decreasing natural capital (Romer, 2006)

The balanced growth path $g_Y^\star$ is equal to:

$$g_Y^\star = g_L + g_A - \frac{g_L + g_A + \delta_{N_c}}{1 - \alpha} \vartheta$$

where δ_{N_c} is the depreciation rate of natural capital and ϑ is the elasticity of output with respect to (normalized) natural capital $N_c(t)$

"*The static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline*" (Hotteling, 1931, page 138-139)

Accounting for environment... changes the definition of economic growth

Limits of economic models

Inter-temporal utility functions

Preferences modeling (Ramsey model)

- ρ is the discount rate (time preference)
- $c(t)$ is the consumption per capita and u is the CRRA utility function:

$$u(c(t)) = \begin{cases} \dfrac{1}{1-\theta} c(t)^{1-\theta} & \text{if } \theta > 0, \quad \theta \neq 1 \\ \ln c(t) & \text{if } \theta = 1 \end{cases}$$

  where θ is the risk aversion parameter

- Maximization of the welfare function:

$$\int_t^{\infty} e^{-\rho t} u(c(t)) \, \mathrm{d}t$$

Limits of economic models
The discounting issue

Does the golden rule of saving rates hold in a Keynesian approach with discounted maximization of consumption?

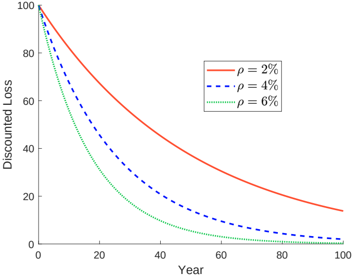


Figure 1: Discounted value of \$100 loss

- "*There is still time to avoid the worst impacts of climate change, if we take strong action now*" (Stern, 2007)
- "*I got it wrong on climate change – it's far, far worse*" (Stern, 2013)

The value of a loss in 100 years almost disappears... while it is only the next generation!

Limits of economic models

Does consumption maximization make sense?

How many planets do we need?

To achieve the current levels of consumption for the world population, we need:

Qatar: 9.0 planets	Canada: 5.1 planets	US: 5.1 planets
Sweden: 4.0 planets	Germany: 3.0 planets	France: 2.8 planets
China: 2.4 planets	**World: 1.75 planets**	Brazil: 1.6 planets
India: 0.8 planets	Afghanistan: 0.4 planets	Yemen: 0.3 planets

https://overshoot.footprintnetwork.org/how-many-earths-or-countries-do-we-need

Source: Global Footprint Network, http://www.footprintcalculator.org

Limits of economic models
Fairness between generations

Keynes

"*In the long run, we are all dead*"

John Maynard Keynes[a], *A Tract on Monetary Reform*, 1923.

[a] "*Men will not always die quietly*", *The Economic Consequences of the Peace*, 1919.

Carney

"*The Tragedy of the Horizon*"

Mark Carney, Chairman of the Financial Stability Board, 2015

$\Rightarrow$ Back to the Golden Rule and the Fable for Growthmen...

Integrated assessment models (IAMs)

Main categories

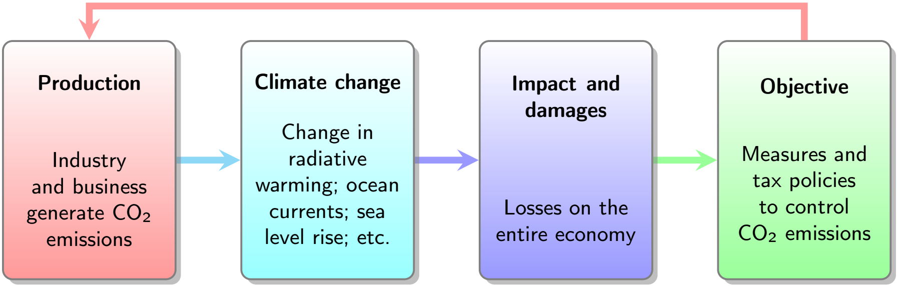
- **Optimization models**
  The inputs of these models are parameters and assumptions about the structure of the relationships between variables. The outputs provided by optimization process are scenarios depending on a set of constraints
- **Evaluation models**
  Based on exogenous scenarios, the outputs provide results from partial equilibriums between variables

Three main components of IAMs

1. Economic growth relationships
2. Dynamics of climate emissions
3. Objective function

Modeling framework

Figure 2: Economic models of climate risk



Modeling framework

1. Economic module
   1. Production function $\Longrightarrow$ GDP
   2. Impact of the climate risk on GDP (damage losses, mitigation and adaptation costs)
   3. The climate loss function depends on the temperature
2. Climate module
   1. Dynamics of GHG emissions
   2. Modeling of Atmospheric and lower ocean temperatures
3. Optimal control problem
   1. Maximization of the utility function
   2. We can test many variants

Modeling framework

The most famous IAM is the **Dynamic Integrated model of Climate and the Economy** (or DICE) developed by William Nordhaus[2]

[2] 2018 Nobel Laureate

Economic module

Production and consumption functions

- The **gross production** $Y(t)$ is given by a Cobb-Douglas function:

$$Y(t) = A(t) K(t)^{\gamma} L(t)^{1-\gamma}$$

  where:
  - $A(t)$ is the total productivity factor
  - $K(t)$ is the capital input
  - $L(t)$ is the labor input
  - $\gamma \in]0,1[$ measures the elasticity of the capital factor:
- Climate change impacts the **net output**:

$$Q(t) = \Omega_{\text{climate}}(t) Y(t) \leq Y(t)$$

- Classical identities $Q(t) = C(t) + I(t)$ and $I(t) = s(t) Q(t)$

Economic module
Production and consumption functions

- The dynamics of the state variables are:

$$\left\{ \begin{array}{l} A(t) = (1 + g_A(t))\, A(t-1) \\ K(t) = (1 - \delta_K) K(t-1) + I(t) \\ L(t) = (1 + g_L(t))\, L(t-1) \end{array} \right.$$

- We have:

$$\left\{ \begin{array}{l} g_A(t) = \dfrac{1}{1 + \delta_A} g_A(t-1) \\ g_L(t) = \dfrac{1}{1 + \delta_L} g_L(t-1) \end{array} \right.$$

Economic module

Labor input

Example #1

The world population was equal to 7.725 billion in 2019 and 7.805 billion in 2020. At the beginning of the 1970s, we estimate that the annual growth rate was equal to 2.045%. According to the United Nations, the global population could surpass 10 billion by 2100.

Economic module
Labor input

- In 2020, the annual growth rate was equal to:

$$g_L(2020) = \frac{L(2020)}{L(2019)} - 1 = \frac{7.805}{7.725} - 1 = 1.036\%$$

- Since we have $g_L(t) = \left(\frac{1}{1+\delta_L}\right)^{t-t_0} g_L(t_0)$, we deduce that:

$$\delta_L = \left(\frac{g_L(t_0)}{g_L(t)}\right)^{1/(t-t_0)} - 1$$

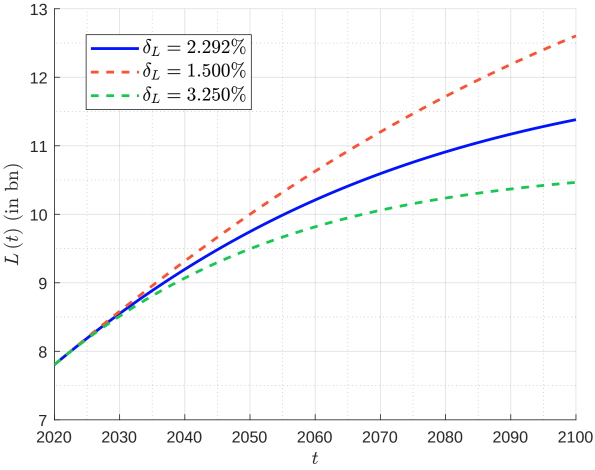
- An estimate of δ_L is then:

$$\delta_L = \left(\frac{g_L(1970)}{g_L(2020)}\right)^{1/30} - 1 = 2.292\%$$

Economic module

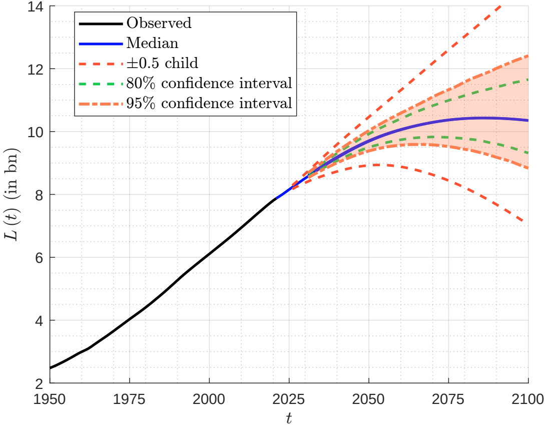
Labor input

Figure 3: Evolution of the labor input $L(t)$



Economic module
Labor input

Figure 4: Projection of the world population



Source: United Nations (2022), `https://population.un.org/wpp`.

Economic module
Labor input

- AR(1) model:

$$g_L(t) = \phi g_L(t-1) + \varepsilon(t)$$

We have

$$\hat{\delta}_L = \frac{\left(1 - \hat{\phi}\right)}{\hat{\phi}}$$

- Log-linear model:

$$\ln g_L(t) = \beta_0 + \beta_1 (t - t_0) + \varepsilon(t)$$

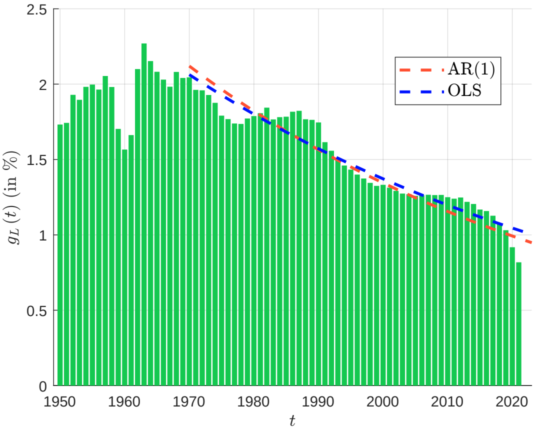
We have:

$$\hat{\delta}_L = e^{-\hat{\beta}_1} - 1$$

Economic module

Labor input

Figure 5: Population growth rate



Source: United Nations (2022), `https://population.un.org/wpp` & Author's calculations

Economic module

Total factor productivity

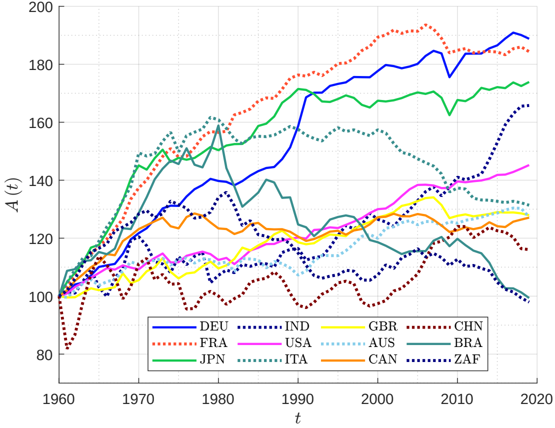
Table 1: Average productivity growth rate (in %)

Country	1960-1970	1970-1980	1980-1990	1990-2000	2000-2010	2010-2020
AUS	1.02	0.07	−0.23	1.02	0.36	0.13
BRA	2.39	2.05	−1.04	−1.12	−0.17	−1.63
CAN	2.18	0.38	−0.25	0.21	−0.21	0.40
CHN	−0.03	−0.06	−0.04	−0.41	2.24	−0.35
FRA	3.59	1.63	1.12	0.61	−0.11	0.02
DEU	2.33	1.63	0.75	1.52	0.01	0.74
IND	2.37	−1.22	1.06	1.04	0.70	1.89
ITA	3.71	1.66	−0.19	−0.20	−1.32	−0.34
JPN	4.05	0.77	1.09	−0.22	−0.15	0.69
ZAF	2.37	0.30	−0.84	−1.11	0.50	−1.20
GBR	0.50	0.72	0.75	0.42	0.12	0.08
USA	1.00	0.42	0.46	0.73	0.65	0.56

Source: Penn World Table 10.01 (Feenstra *et al.*, 2015) & Author's calculations.

Economic module
Total factor productivity

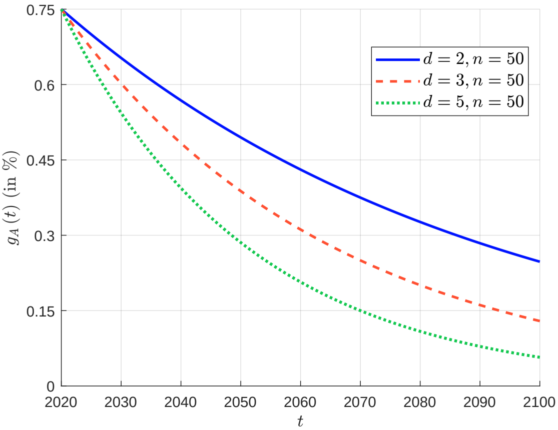
Figure 6: Total factor productivity index (base 100 = 1960)



Source: Penn World Table 10.01 (Feenstra *et al.*, 2015) & Author's calculations.

Economic module
Total factor productivity

Figure 7: Dynamics of the TFP growth rate*



*We use the following calibration rule: $\delta_A = \sqrt[n]{d} - 1$

Economic module

Investment, capital stock and gross output

- Penn World Table/IMF's ICSD
- In 2019, we obtain $I(2019) = \$30.625$ tn, $K(2019) = \$318.773$ tn and $Y(2019) = \$124.418$ tn
- We also have:

$$\delta_K(t) = \frac{K(t-1) - K(t) + I(t)}{K(t-1)}$$

  and we obtain $\delta_K(2019) = 6.25\%$
- To calibrate the initial value of $A(t)$, we inverse the Coob-Douglas function:

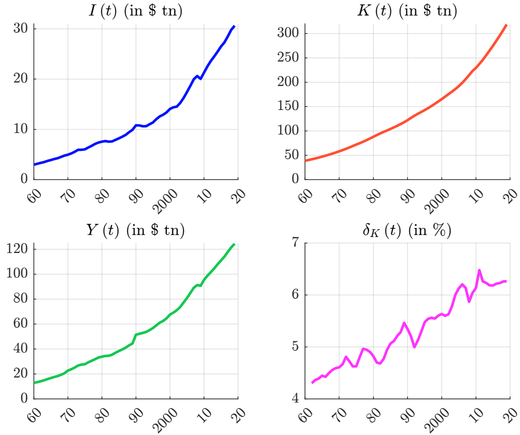
$$A(2019) = \frac{Y(t)}{K(t)^{\gamma} L(t)^{1-\gamma}} = \frac{124.418}{318.773^{0.30} \times 7.725^{0.70}} = 5.276$$

- The saving rate $s(t)$ is exogenous

Economic module

Investment, capital stock and gross output

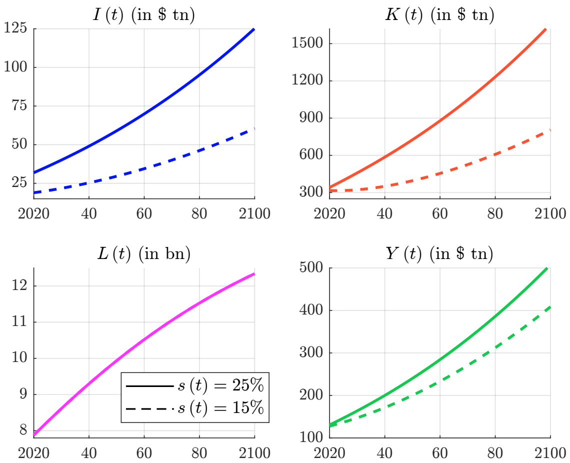
Figure 8: Historical estimates of $I(t)$, $K(t)$, $Y(t)$ and $\delta_K(t)$



Source: IMF Investment and Capital Stock Dataset (2021) & Author's calculations.

Economic module

Figure 9: Simulation of the DICE macroeconomic module



Economic module

Cost function of climate change

- The survival function is given by:

$$\Omega_{\text{climate}}(t) = \Omega_D(t)\,\Omega_\Lambda(t) = \frac{1}{1 + D(t)}\left(1 - \Lambda(t)\right)$$

where:

- $D(t) \geq 0$ is the **climate damage function** (**physical risk**)
- $\Lambda(t) \geq 0$ is **the mitigation or abatement cost** (**transition risk**)

Economic module

Cost function of climate change

- The cost $D(t)$ resulting from natural disasters depends on the atmospheric temperature $\mathcal{T}_{\mathrm{AT}}(t)$:

$$D(t) = \psi_1 \mathcal{T}_{\mathrm{AT}}(t) + \psi_2 \mathcal{T}_{\mathrm{AT}}(t)^2$$

- The abatement cost function depends on the control variable $\mu(t)$:

$$\Lambda(t) = \theta_1(t)\,\mu(t)^{\theta_2}$$

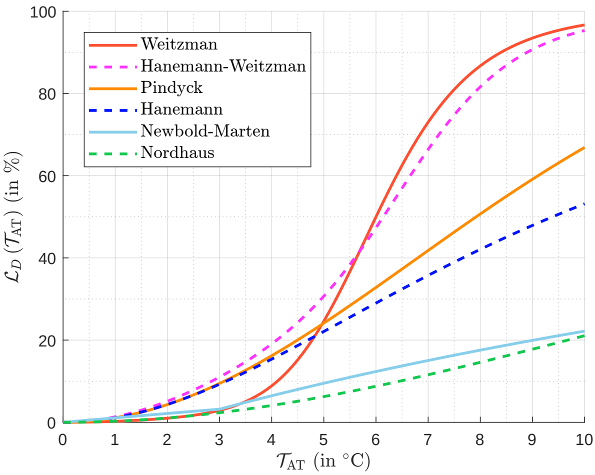
- The global impact of climate change is equal to:

$$\Omega_{\mathrm{climate}}(t) = \frac{1 - \theta_1(t)\,\mu(t)^{\theta_2}}{1 + \psi_1 \mathcal{T}_{\mathrm{AT}}(t) + \psi_2 \mathcal{T}_{\mathrm{AT}}(t)^2}$$

Economic module

Cost function of climate change

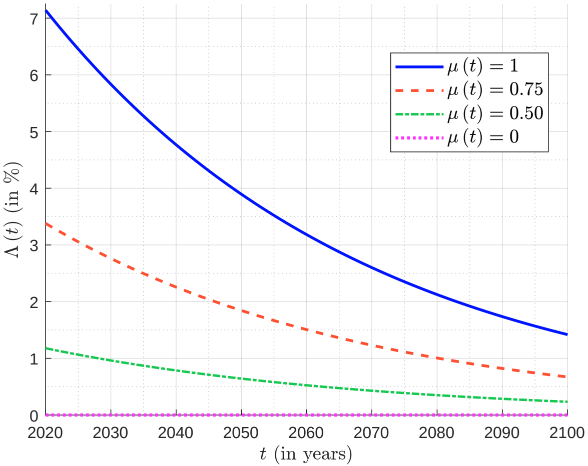
Figure 10: Loss function due to climate damage costs



Economic module

Cost function of climate change

Figure 11: Abatement cost function



Climate module

GHG emissions

- The **total GHG emissions** depends on the production $Y(t)$ and the land use emissions $\mathcal{CE}_{\mathrm{Land}}(t)$:

$$\begin{aligned}\mathcal{CE}(t) &= \mathcal{CE}_{\mathrm{Industry}}(t) + \mathcal{CE}_{\mathrm{Land}}(t) \\ &= (1-\mu(t))\sigma(t)Y(t) + \mathcal{CE}_{\mathrm{Land}}(t)\end{aligned}$$

- $\sigma(t)$ is the anthropogenic carbon intensity of the economy:

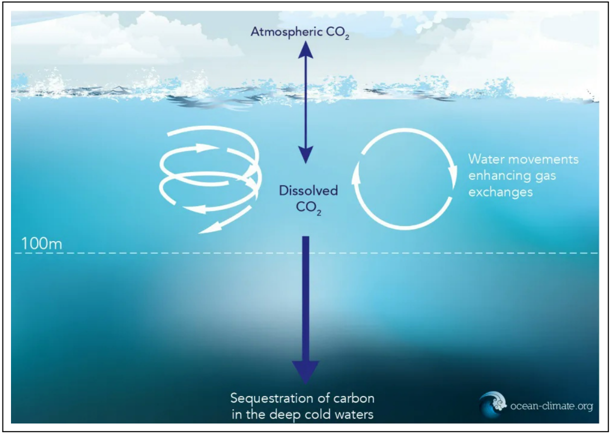
$$\sigma(t) = (1 + g_\sigma(t))\sigma(t-1)$$

where:

$$g_\sigma(t) = \frac{1}{1+\delta_\sigma} g_\sigma(t-1)$$

Climate module
Temperature modeling

Figure 12: Physical carbon pump



Source: `ocean-climate.org`.

Climate module
Concentration modeling

- We have:

$$\left\{\begin{array}{l}\mathcal{CC}_{\mathrm{AT}}\left(t\right)=\phi_{1,1}\mathcal{CC}_{\mathrm{AT}}\left(t-1\right)+\phi_{1,2}\mathcal{CC}_{\mathrm{UP}}\left(t-1\right)+\phi_{1}\boldsymbol{\mathcal{CE}}\left(t\right)\\ \mathcal{CC}_{\mathrm{UP}}\left(t\right)=\phi_{2,1}\mathcal{CC}_{\mathrm{AT}}\left(t-1\right)+\phi_{2,2}\mathcal{CC}_{\mathrm{UP}}\left(t-1\right)+\phi_{2,3}\mathcal{CC}_{\mathrm{LO}}\left(t-1\right)\\ \mathcal{CC}_{\mathrm{LO}}\left(t\right)=\phi_{3,2}\mathcal{CC}_{\mathrm{UP}}\left(t-1\right)+\phi_{3,3}\mathcal{CC}_{\mathrm{LO}}\left(t-1\right)\end{array}\right.$$

- The dynamics of $\mathcal{CC}=\left(\mathcal{CC}_{\mathrm{AT}},\mathcal{CC}_{\mathrm{UP}},\mathcal{CC}_{\mathrm{LO}}\right)$ is a VAR(1) process:

$$\mathcal{CC}\left(t\right)=\Phi_{\mathcal{CC}}\mathcal{CC}\left(t-1\right)+B_{\mathcal{CC}}\boldsymbol{\mathcal{CE}}\left(t\right)$$

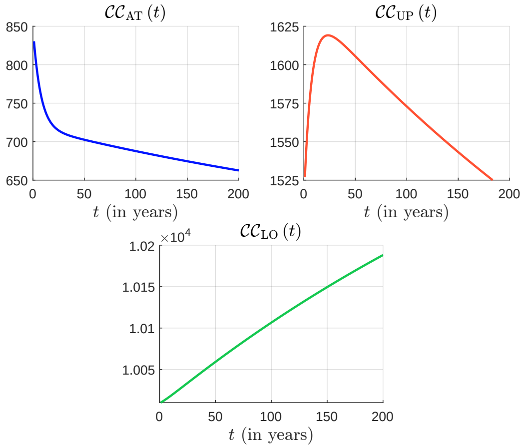
Carbon cycle diffusion matrix

We have:

$$\Phi_{\mathcal{CC}}=\left(\begin{array}{ccc}91.20\% & 3.83\% & 0\\ 8.80\% & 95.92\% & 0.03\%\\ 0 & 0.25\% & 99.97\%\end{array}\right)$$

Climate module
Concentration modeling

Figure 13: Impulse response analysis ($\Delta\mathcal{CE} = -1$ GtCO$_2$e)



Climate module
Radiative forcing

We have:

$$\mathcal{F}_{\mathrm{RAD}}(t) = \frac{\eta}{\ln 2} \ln \left(\frac{\mathcal{CC}_{\mathrm{AT}}(t)}{\mathcal{CC}_{\mathrm{AT}}(1750)} \right) + \mathcal{F}_{\mathrm{EX}}(t)$$

where:

- $\mathcal{F}_{\mathrm{RAD}}(t)$ is the change in **total radiative forcing** of GHG emissions since 1750 (expressed in W/m^2)
- η is the temperature forcing parameter
- $\mathcal{F}_{\mathrm{EX}}(t)$ is the exogenous forcing (other GHG emissions)

Climate module

Temperature modeling

- The climate system for temperatures is characterized by a **two-layer system**:

$$\left\{\begin{array}{lcl} \mathcal{T}_{\mathrm{AT}}(t) & = & \mathcal{T}_{\mathrm{AT}}(t-1)+\xi_1\left(\mathcal{F}_{\mathrm{RAD}}(t)-\xi_2\mathcal{T}_{\mathrm{AT}}(t-1)-\right. \\ & & \left.\xi_3\left(\mathcal{T}_{\mathrm{AT}}(t-1)-\mathcal{T}_{\mathrm{LO}}(t-1)\right)\right) \\ \mathcal{T}_{\mathrm{LO}}(t) & = & \mathcal{T}_{\mathrm{LO}}(t-1)+\xi_4\left(\mathcal{T}_{\mathrm{AT}}(t-1)-\mathcal{T}_{\mathrm{LO}}(t-1)\right) \end{array}\right.$$

- Let $\mathcal{T}=(\mathcal{T}_{\mathrm{AT}},\mathcal{T}_{\mathrm{LO}})$ be the temperature vector. We have:

$$\mathcal{T}(t)=\Xi_{\mathcal{T}}\mathcal{T}(t-1)+B_{\mathcal{T}}\mathcal{F}_{\mathrm{RAD}}(t)$$

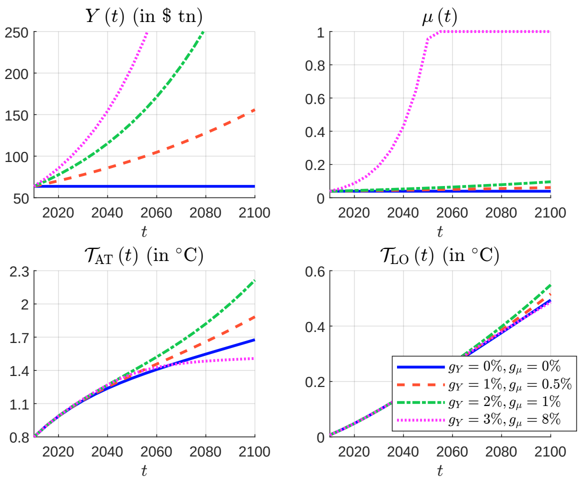
Climate module

Table 2: Output of the DICE climate module ($Y(t) = Y(t_0)$, $\mu(t) = \mu(t_0)$)

t	$\mathcal{CE}(t)$	$\sigma(t)$	$\mathcal{CC}_{\mathrm{AT}}(t)$	$\mathcal{F}_{\mathrm{RAD}}(t)$	$\mathcal{T}_{\mathrm{AT}}(t)$	$\mathcal{T}_{\mathrm{LO}}(t)$
2010	36.91	0.55	830.4	2.14	0.800	0.007
2015	36.25	0.55	825.7	2.14	0.900	0.027
2020	36.06	0.56	821.9	2.14	0.986	0.048
2025	35.97	0.57	818.9	2.14	1.061	0.072
2030	35.98	0.57	816.6	2.15	1.127	0.097
2035	36.05	0.58	814.9	2.16	1.186	0.122
2040	36.18	0.58	813.9	2.18	1.238	0.149
2045	36.36	0.59	813.3	2.20	1.286	0.176
2050	36.58	0.59	813.3	2.23	1.329	0.204
2055	36.82	0.60	813.6	2.26	1.370	0.232
2060	37.09	0.61	814.4	2.29	1.408	0.261
2065	37.39	0.61	815.4	2.32	1.445	0.289
2070	37.70	0.62	816.8	2.35	1.480	0.318
2075	38.02	0.62	818.4	2.39	1.514	0.347
2080	38.36	0.63	820.3	2.43	1.547	0.376
2085	38.71	0.64	822.4	2.46	1.580	0.406
2090	39.06	0.64	824.7	2.50	1.612	0.435
2095	39.43	0.65	827.1	2.55	1.645	0.464
2100	39.80	0.66	829.7	2.59	1.677	0.494

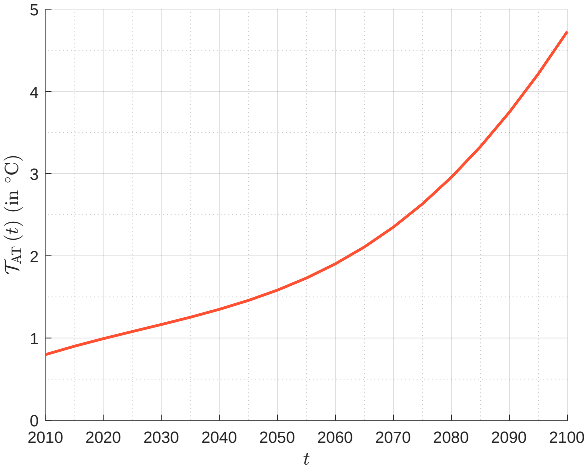
Climate module

Figure 14: Simulation of the DICE climate module



Climate module

Figure 15: The nightmare climate-economic scenario ($g_Y = 0\%$, $\mu(t) = 0$)



The optimal control problem

Optimization problem

- The social welfare function W is equal to:

$$W(s(t), \mu(t)) = \sum_{t=t_0+1}^{T} \frac{L(t)\,\mathcal{U}(c(t))}{(1+\rho)^{t-t_0}}$$

where ρ is the (generational) discount rate and $c(t) = C(t)/L(t)$ is the consumption per capita

- $\mathcal{U}(c) = \left(c^{1-\alpha} - 1\right)/(1-\alpha)$ is the CRRA utility function
- The optimal control problem is then given by:

$$\begin{aligned}
(s^\star(t), \mu^\star(t)) &= \arg\max W(s(t), \mu(t)) \\
&\text{s.t.} \quad \left\{ \begin{array}{l} \text{DICE Equations} \\ \mu(t) \in [0,1] \\ s(t) \in [0,1] \end{array} \right.
\end{aligned}$$

The optimal control problem

The important variables are:

- $\mathcal{T}_{\mathrm{AT}}(t)$ — **Atmospheric temperature**
- $\mu(t)$ — **Control rate (mitigation policies)**
- $\mathcal{CE}(t)$ — **Total emissions of GHG**
- $\mathrm{SCC}(t)$ — **Social cost of carbon**

Social cost of carbon (SCC)

"The most important single economic concept in the economics of climate change is the ***social cost of carbon*** *(SCC). This term designates the economic cost caused by an additional tonne of carbon dioxide emissions or its equivalent. In a more precise definition, it is the change in the discounted value of economic welfare from an additional unit of* CO_2*-equivalent emissions. The SCC has become a central tool used in climate change policy, particularly in the determination of regulatory policies that involve greenhouse gas emissions." (Nordhaus, 2017).*

Social cost of carbon (SCC)

Mathematical definition

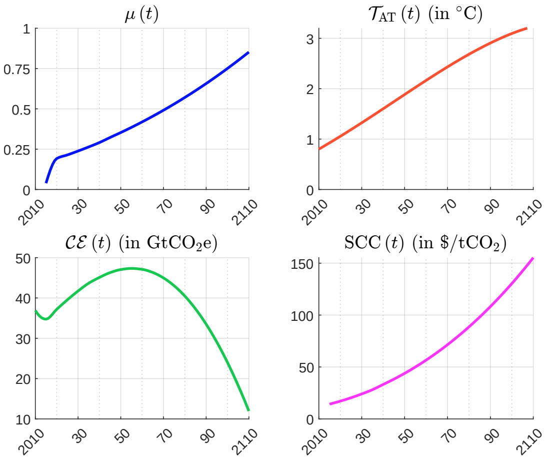
- The social cost of carbon is then defined as:

$$\mathrm{SCC}(t) = \frac{\dfrac{\partial\, W(t)}{\partial\, \boldsymbol{\mathcal{CE}}(t)}}{\dfrac{\partial\, W(t)}{\partial\, C(t)}} = \frac{\partial\, C(t)}{\partial\, \boldsymbol{\mathcal{CE}}(t)}$$

- It is expressed in $\$/\mathrm{tCO}_2$

Social cost of carbon (SCC)

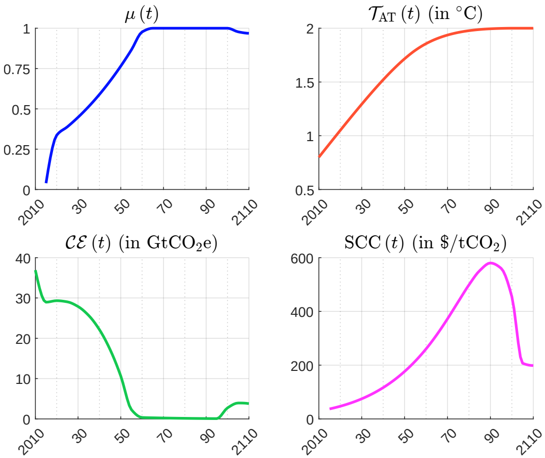
Figure 16: Optimal welfare scenario (DICE 2013R)



Source: Le Guenedal (2019).

Social cost of carbon (SCC)

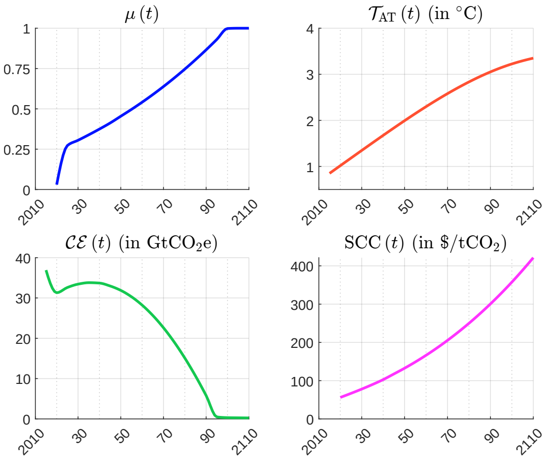
Figure 17: $2^\circ\mathrm{C}$ scenario (DICE 2013R)



Source: Le Guenedal (2019).

Social cost of carbon (SCC)

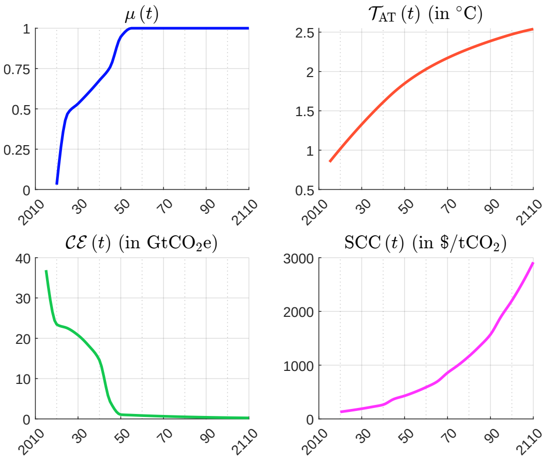
Figure 18: Optimal welfare scenario (DICE 2016R)



Source: Le Guenedal (2019).

Social cost of carbon (SCC)

Figure 19: $2^\circ\mathrm{C}$ scenario (DICE 2016R)



Source: Le Guenedal (2019).

The tragedy of the horizon

The tragedy of the horizon

Achieving the 2°C scenario

- In 2013, the DICE model suggested to reduce drastically CO_2 emissions...
- Since 2016, **the 2°C trajectory is no longer feasible!** (minimum $\approx$ 2.6°C)
- For many models, we now have:

$$\mathbb{P}\left(\Delta T > 2^{\circ}C\right) > 95\%$$

Social cost of carbon (SCC)

Table 3: Global SCC under different scenario assumptions (in $\$/\mathrm{tCO}_2$)

Scenario	2015	2020	2025	2030	2050	CAGR
Baseline	31.2	37.3	44.0	51.6	102.5	3.46%
Optimal	30.7	36.7	43.5	51.2	103.6	3.54%
2.5°C-max	184.4	229.1	284.1	351.0	1 006.2	4.97%
2.5°C-mean	106.7	133.1	165.1	203.7	543.3	4.76%

Source: Nordhaus (2017).

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

- In 2007, Nicholas Stern published a report called ***The Economics of Climate Change: The Stern Review***
- The Stern Review called for sharp and immediate action to stabilize greenhouse gases because:

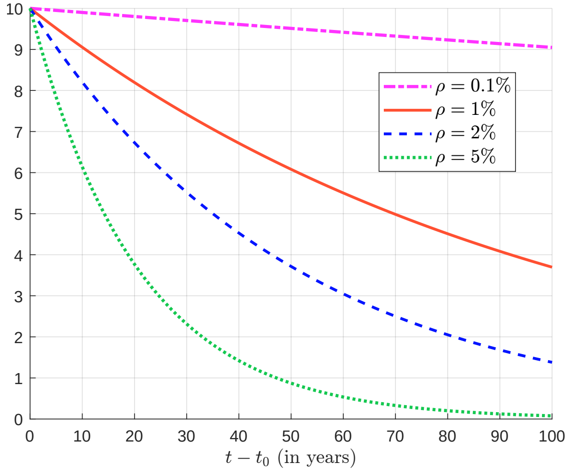
  "***the benefits of strong, early action on climate change outweighs the costs***"

- The Stern Review proposes to use $\boldsymbol{\rho = 0.10\%}$

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

Figure 20: Discounted value of $10



Social cost of carbon (SCC)

The Stern-Nordhaus controversy

- The time (or generational) discount rate ρ is also called the pure rate of time preference
- It is related to the Ramsey rule:

$$r = \rho + \alpha g$$

  where:
  - r is the real interest rate
  - $g = \partial\, c(t) / c(t)$ is the growth rate of per capita consumption
  - α is the consumption elasticity of the utility function

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

We report the computations done by Dasgupta (2008):

Model	ρ	α	g_c	r
Cline (1992)	0.0%	1.5	1.3%	2.05%
Nordhaus (2007)	3.0%	1.0	1.3%	4.30%
Stern (2007)	0.1%	1.0	1.3%	1.40%

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

Table 4: Global SCC under different discount rate assumptions

Discount rate	2015	2020	2025	2030	2050	CAGR
Stern	197.4	266.5	324.6	376.2	629.2	3.37%
Nordhaus	30.7	36.7	43.5	51.2	103.6	3.54%
2.5%	128.5	140.0	152.0	164.6	235.7	1.75%
3%	79.1	87.3	95.9	104.9	156.6	1.97%
4%	36.3	40.9	45.8	51.1	81.7	2.34%
5%	19.7	22.6	25.7	29.1	49.2	2.65%

Source: Nordhaus (2017).

Some models

- AIM ———————————— RCP 6.0
- DICE/RICE
- FUND
- GCAM
- IMACLIM (CIRED)
- IMAGE ———————————— RCP 2.6
- MESSAGE ———————————— RCP 8.5
- MiniCAM ———————————— RCP 4.5
- PAGE
- REMIND
- RESPONSE (CIRED)
- WITCH

Some models

Table 5: Main integrated assessment models

Model	Reference	Name
Stylized simple models		
DICE	Nordhaus and Sztorc (2013)	Dynamic Integrated Climate-Economy
FUND	Anthoff and Tol (2014)	Climate Framework for Uncertainty, Negotiation and Distribution
PAGE	Hope (2011)	Policy Analysis of the Greenhouse Effect
Complex models		
AIM/CGE	Fujimori *et al.* (2017)	Asia-Pacific Integrated Model/Computable General Equilibrium
GCAM	Calvin *et al.* (2019)	Global Change Assessment Model
GLOBIOM	Havlik *et al.* (2018)	Global Biosphere Management Model
IMACLIM-R	Sassi *et al.* (2010)	Integrated Model to Assess Climate Change
IMAGE	Stehfest *et al.* (2014)	Integrated Model to Assess the Greenhouse Effect
MAGICC	Meinshausen *et al.* (2011)	Model for the Assessment of Greenhouse Gas Induced Climate Change
MAgPIE	Dietrich *et al.* (2019)	Model of Agricultural Production and its Impact on the Environment
MESSAGEix	Huppmann *et al.* (2019)	Model for Energy Supply Strategy Alternatives and their General Environmental Impact
REMIND	Aboumahboub *et al.* (2020)	REgional Model of INvestments and Development
WITCH	Bosetti *et al.* (2006)	World Induced Technical Change Hybrid

Source: Grubb *et al.* (2021) & Author's research.

Stylized IAMs

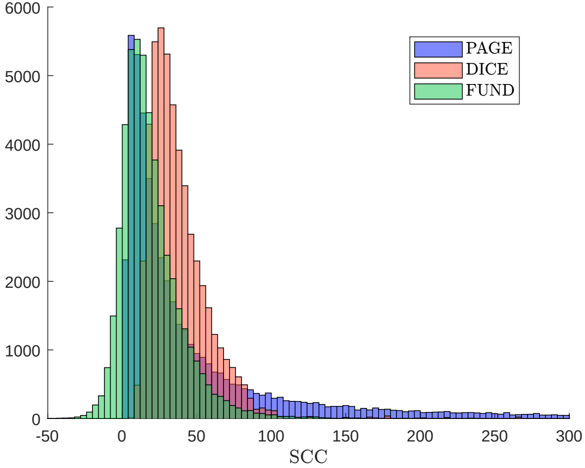
The Leaders

- DICE
- FUND
- PAGE

$\Rightarrow$ SCC: PAGE $\succ$ DICE $\succ$ FUND

Stylized IAMs

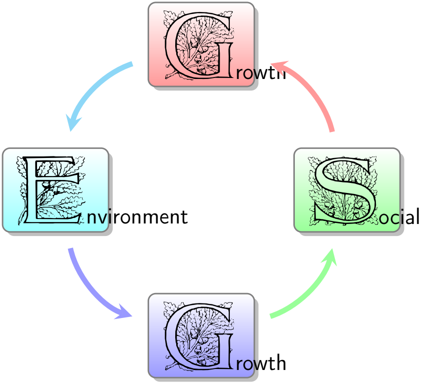
Figure 21: Histogram of the 150 000 US Government SCC estimates for 2020 with a 3% discount rate



Stylized IAMs

The liability/fairness question

Liability



Aristotle (384 BC – 322 BC)
ΗΘΙΚΩΝ ΝΙΚΟΜΑΧΕΙΩΝ

Karl Marx and Friedrich Engels (1848)
The Communist Manifesto

Stylized IAMs
The liability/fairness question

Fairness

Du Contrat Social

Stylized IAMs

Climate risk and inequalities

Three types of inequalities

- Spatial (or regional) inequalities
- Social (or intra-generation) inequalities
- Time (or inter-generation) inequalities

⇒ These issues are highly related to liability risks:

> *"[...] liability risks stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible" (Mark Carney, 2018)*

- Regional inequalities ⇒ lack of cooperation between countries (e.g., Glasgow COP 26)
- Social inequalities ⇒ climate action postponing (e.g., carbon tax in France)

Stylized IAMs
Regional inequalities

The **R**egional **I**ntegrated model of **C**limate and the **E**conomy (RICE) model is a sub-regional neoclassical climate economy model (Nordhaus and Yang, 1996)

$\Rightarrow$ Sub-regional problem of welfare:

- Each region of the world has a different utility functions
- The big issue is how the most developed regions can finance the transition to a low-carbon economy of the less developed regions

Both spacial and time (inter-generation) inequalities

Stylized IAMs
Social inequalities

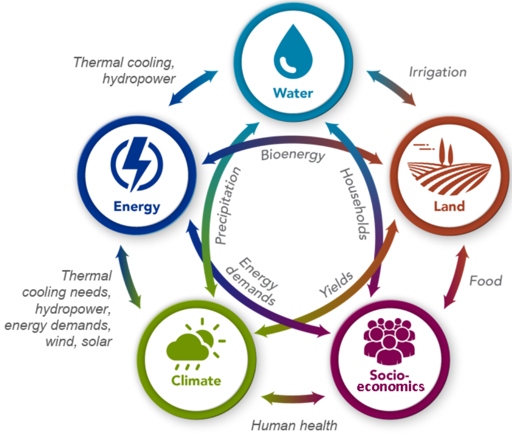
The **N**ested **I**nequalities **C**limate-**E**conomy (NICE) model integrates distributional differences of income (Dennig *et al.*, 2015)

> *"[...] If the distribution of damage is less skewed to high income than the distribution of consumption, then weak or no climate policy will result in sufficiently large damages on the lower economic strata to eventually stop their welfare levels from improving, and instead cause them to decline" (Dennig et al., 2015)*

Both social (intra-generation) and time (inter-generation) inequalities

Complex IAMs

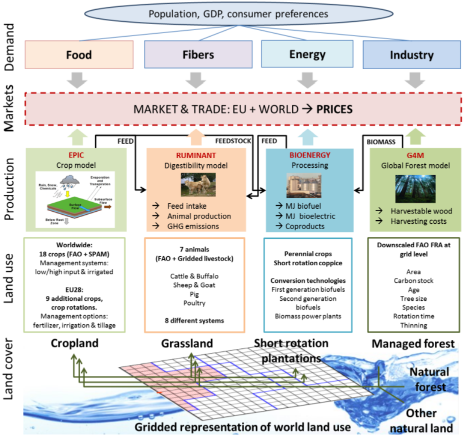
Figure 22: Linkages between the major systems in GCAM



Source: Calvin *et al.* (2019).

Complex IAMs

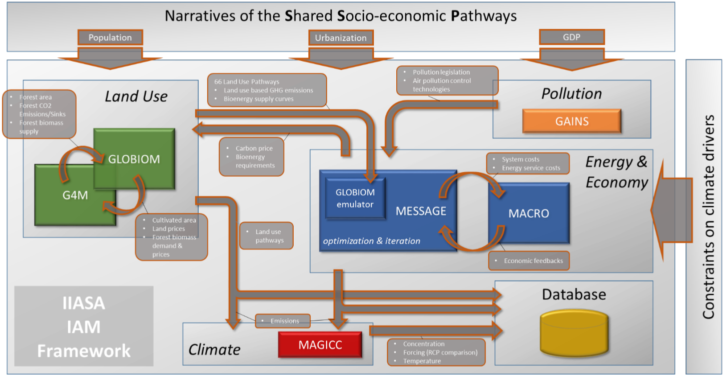
Figure 23: The main land use sectors of GLOBIOM



Source: `https://iiasa.github.io/GLOBIOM`.

Complex IAMs

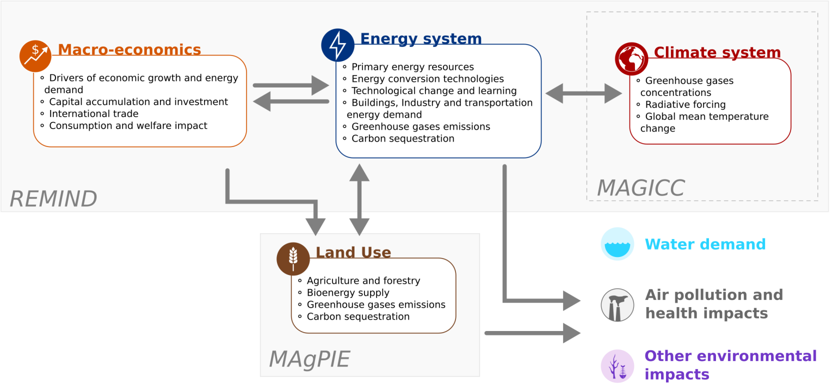
Figure 24: Overview of the IIASA IAM framework



Source: https://docs.messageix.org/projects/global/en/latest/overview/index.html.

Complex IAMs

Figure 25: The Remind-MAgPIE framework



Source: www.pik-potsdam.de/en/institute/departments/transformation-pathways/models/remind.

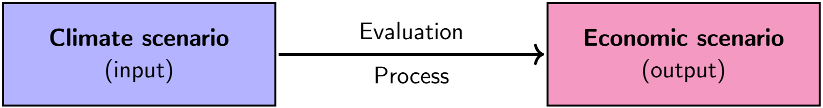
Criticisms of integrated assessment models

> *"IAM-based analyses of climate policy create a perception of knowledge and precision that is illusory and can fool policymakers into thinking that the forecasts the models generate have some kind of scientific legitimacy" (Pindyck, 2017)*

- Certain inputs, such as the discount rate, are arbitrary
- There is a lot of uncertainty about climate sensitivity and the temperature trajectory
- Modeling damage functions is arbitrary
- IAMs are unable to consider tail risk

Scenarios

Figure 26: Scenario evaluation



Climate scenarios

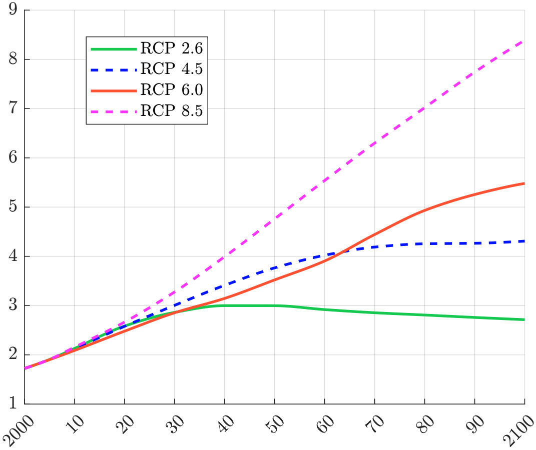
- The representative concentration pathways (RCPs) — IPCC AR5
- The IEA scenarios
- The $1.5°\mathrm{C}$ scenarios — SR15
- The scenarios for the future published — IPCC AR6

Climate scenarios
The RCP scenarios

1. **RCP 2.6**: GHG emissions start declining by 2020 and go to zero by 2100 (IMAGE)
2. **RCP 4.5**: GHG emissions peak around 2040, and then decline (MiniCAM)
3. **RCP 6.0**: GHG emissions peak around 2080, and then decline (AIM)
4. **RCP 8.5**: GHG emissions continue to rise throughout the 21st century (MESSAGE)

Climate scenarios
The RCP scenarios

Figure 27: Total radiative forcing (in $\mathrm{W/m^2}$)

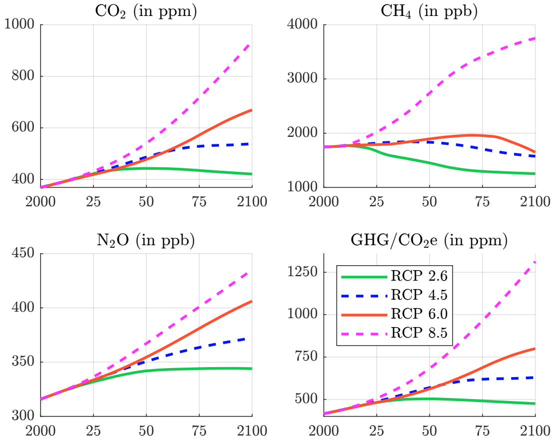


Source: https://tntcat.iiasa.ac.at/RcpDb.

Climate scenarios

The RCP scenarios

Figure 28: Greenhouse gas concentration trajectory

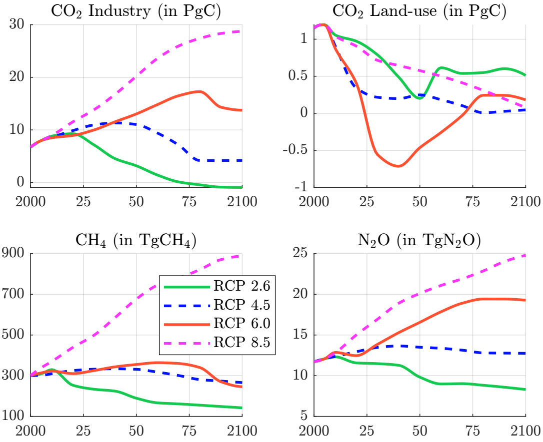


Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios

The RCP scenarios

Figure 29: Greenhouse gas emissions trajectory

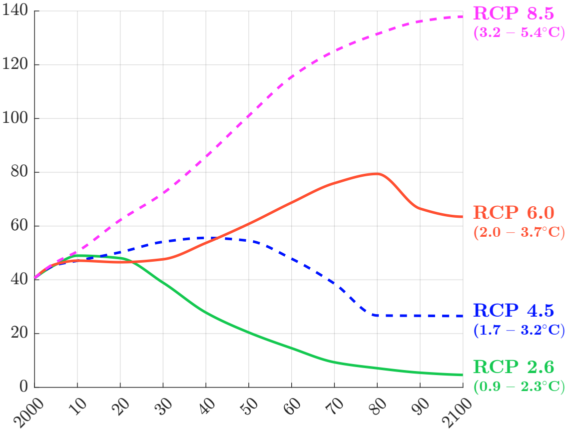


Source: https://tntcat.iiasa.ac.at/RcpDb.

Climate scenarios

The RCP scenarios

Figure 30: Total GHG emissions trajectory (in $\mathrm{GtCO_2e}$)

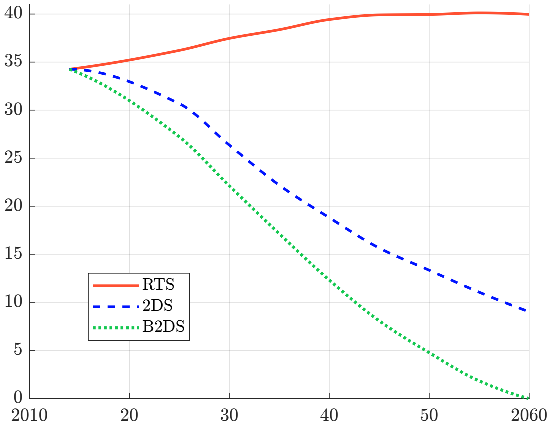


Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios

The IEA scenarios

Figure 31: Direct CO_2 emissions (in Gt)

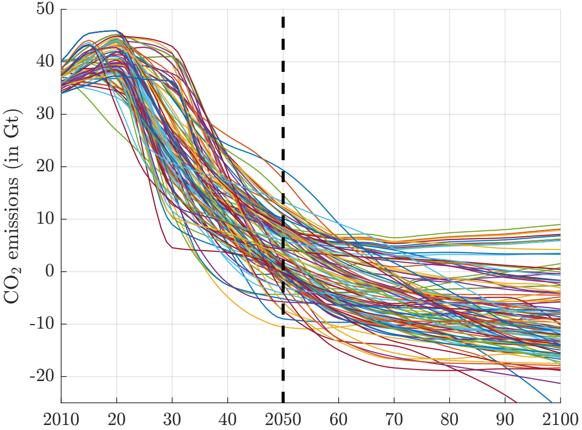


Source: IEA (2017).

Climate scenarios

The 1.5°C scenarios

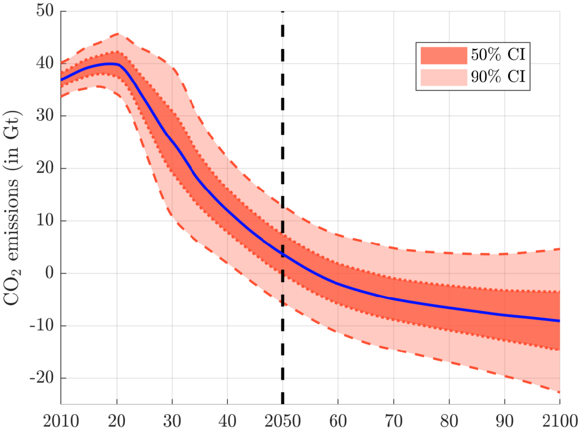
Figure 32: IPCC 1.5°C scenarios of CO_2 emissions



Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios
The 1.5°C scenarios

Figure 33: Confidence interval of the average IPCC 1.5°C scenario

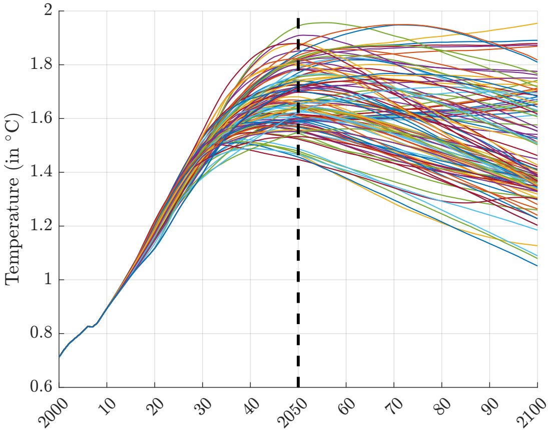


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The 1.5°C scenarios

Figure 34: IPCC 1.5°C scenarios of the global mean temperature

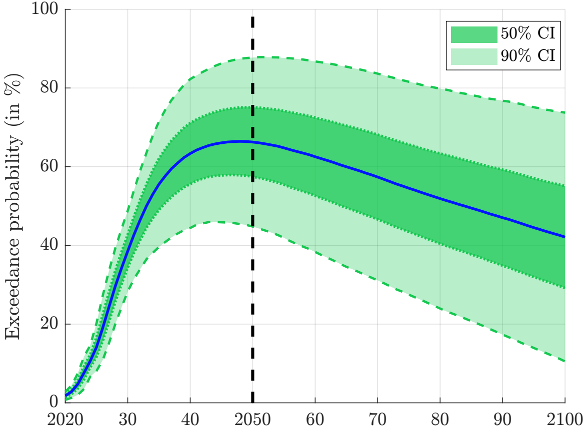


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The $1.5^{\circ}\mathrm{C}$ scenarios

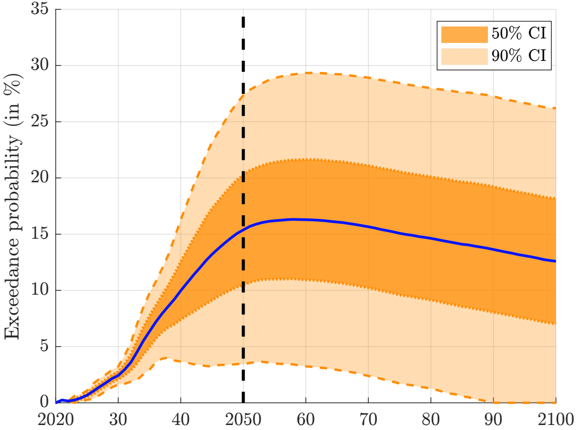
Figure 35: Confidence interval of the exceedance probability $\Pr\{\mathcal{T} > 1.5^{\circ}\mathrm{C}\}$



Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios
The 1.5°C scenarios

Figure 36: Confidence interval of the exceedance probability $\Pr\{\mathcal{T} > 2^{\circ}\mathrm{C}\}$



Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios
The AR6 scenarios

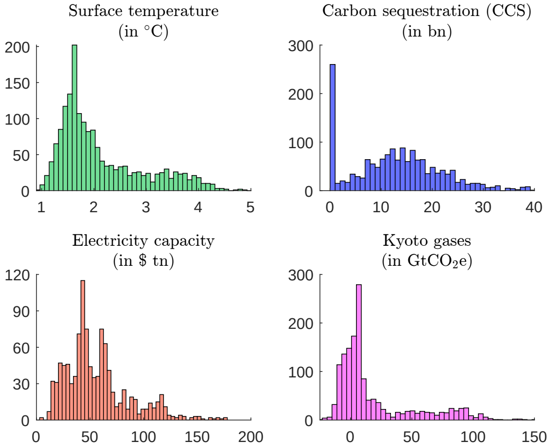
The new dataset contains 188 models, 1 389 scenarios, 244 countries and regions, and 1 791 variables, which can be split into six main categories:

- Agriculture: agricultural demand, crop, food, livestock, production, etc.
- Capital cost: coal, electricity, gas, hydro, hydrogen, nuclear, etc.
- Energy: capacity, efficiency, final energy, lifetime, OM cost, primary/secondary energy, etc.
- GHG impact: carbon sequestration, concentration, emissions, forcing, temperature, etc.
- Natural resources: biodiversity, land cover, water consumption, etc.
- Socio-economic variables: capital formation, capital stock, consumption, discount rate, employment, expenditure, export, food demand, GDP, Gini coefficient, import, inequality, interest rate, investment, labour supply, policy cost, population, prices, production, public debt, government revenue, taxes, trade, unemployment, value added, welfare, etc.

Climate scenarios

The AR6 scenarios

Figure 37: Histogram of some AR6 output variables by 2100

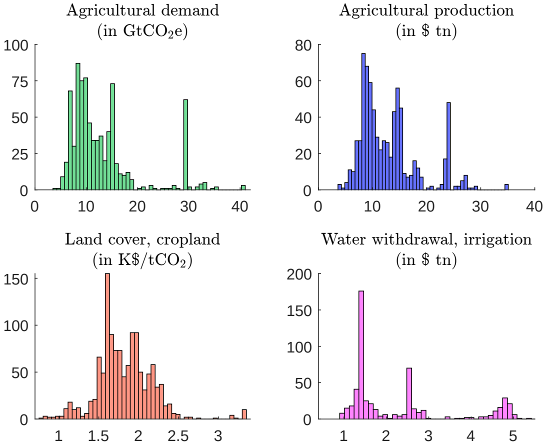


Source: `https://data.ene.iiasa.ac.at/ar6`.

Climate scenarios

The AR6 scenarios

Figure 38: Histogram of some AR6 output variables by 2100



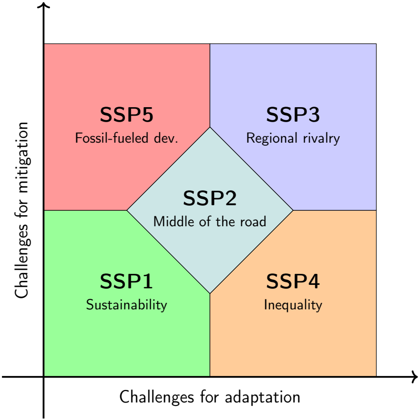
Source: `https://data.ene.iiasa.ac.at/ar6`.

Shared socioeconomic pathways

*"The **SSP narratives** [are] a set of five qualitative descriptions of future changes in demographics, human development, economy and lifestyle, policies and institutions, technology, and environment and natural resources. [...] Development of the narratives drew on expert opinion to (1) identify key determinants of the challenges [to mitigation and adaptation] that were essential to incorporate in the narratives and (2) combine these elements in the narratives in a manner consistent with scholarship on their inter-relationships. The narratives are intended as a **description of plausible future conditions at the level of large world regions** that can serve as a basis for integrated scenarios of emissions and land use, as well as climate impact, adaptation and vulnerability analyses." (O'Neill et al., 2017)*

Shared socioeconomic pathways

Figure 39: The shared socioeconomic pathways



Source: O'Neill *et al.* (2017).

Shared socioeconomic pathways

Figure 40: The shared socioeconomic pathways

- SSP1 — **Sustainability** (Taking the Green Road)
  Low challenges for both mitigation and adaptation, rapid development
- SSP2 — **Middle of the Road**
  Moderate challenges for mitigation and adaptation
- SSP3 — **Regional Rivalry** (A Rocky Road)
  High challenges for both mitigation and adaptation — Concerns about competitiveness/security and regional conflicts pushing countries to focus on regional issues
- SSP4 — **Inequality** (A Road Divided)
  Low challenges for mitigation, high for adaptation — Unequal investment in human capital, concentration of power in a small business elite
- SSP5 — **Fossil-fueled Development** (Taking the Highway)
  High challenges for mitigation, low for adaptation

Source: O'Neill *et al.* (2017).

Shared socioeconomic pathways

Relationship with the ESG dimensions

E The mitigation/adaptation trade-off is obviously an environmental issue, but the SSPs encompass other environmental narratives, e.g. land use, energy efficiency and green economy

S The social dimension is the central theme of SSPs, and concerns demography, wealth, inequality & poverty, health, education, employment, and more generally the evolution of society. This explains that SSPs and SDGs are highly interconnected

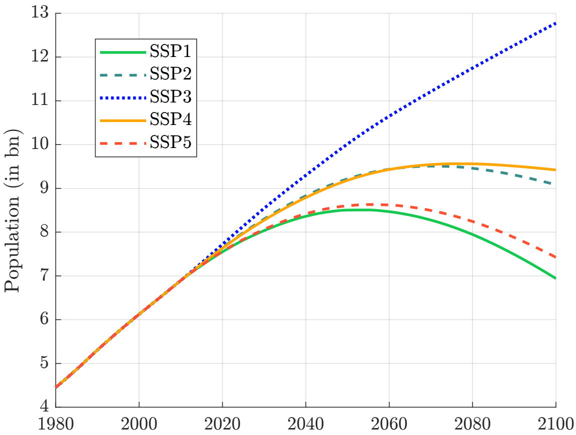
G Finally, the governance dimension is present though two major themes: international fragmentation or cooperation, and the political/economic system, including corruption, stability, rule of law, etc.

Shared socioeconomic pathways

- **SSP1**: IMAGE (PBL)
- **SSP2**: MESSAGE-GLOBIOM (IIASA)
- **SSP3**: AIM/CGE (NIES)
- **SSP4**: GCAM (PNNL)
- **SSP5**: REMIND-MAGPIE (PIK) and WITCH-GLOBIOM (FEEM)

Shared socioeconomic pathways

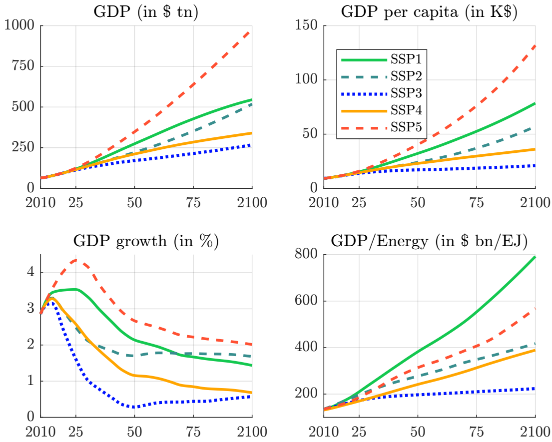
Figure 41: SSP demography projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

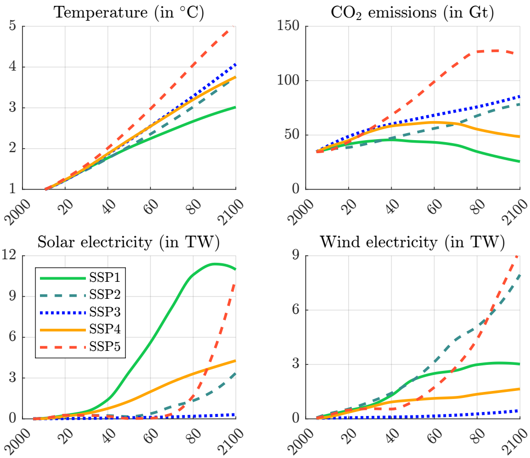
Figure 42: SSP economic projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

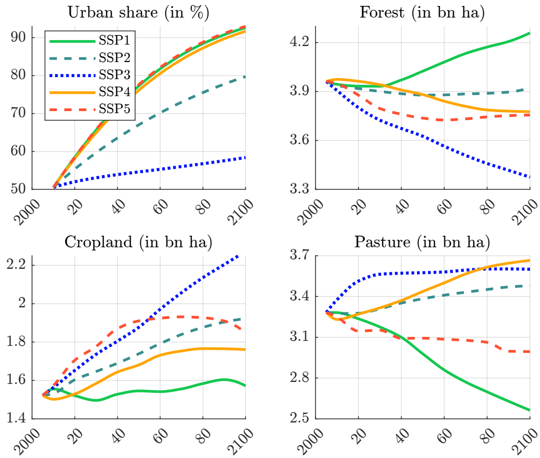
Figure 43: SSP environmental projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

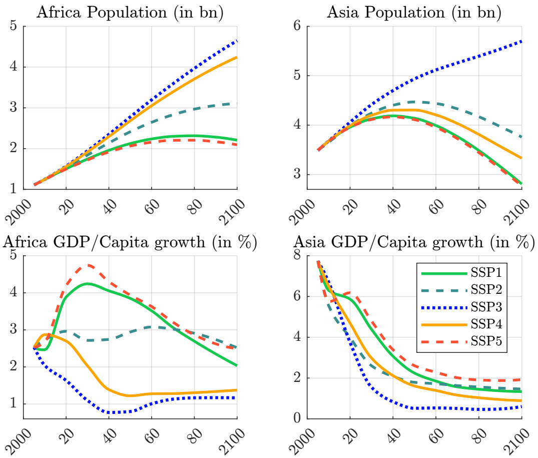
Figure 44: SSP land use projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

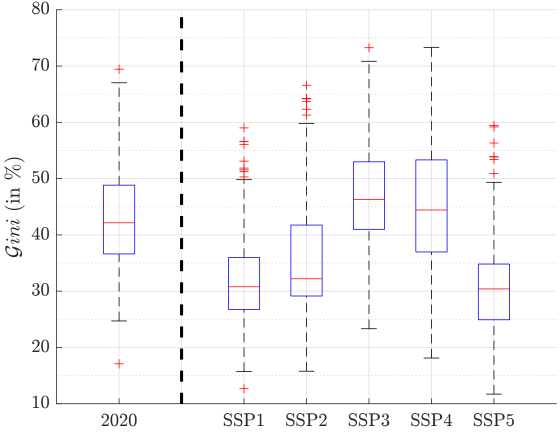
Figure 45: Example of SSP regional differences



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

Figure 46: Gini coefficient projections by 2100



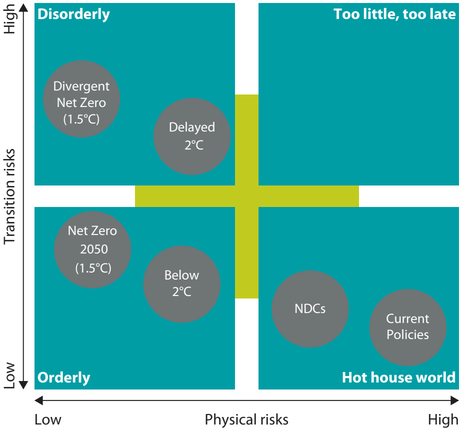
Source: `https://tntcat.iiasa.ac.at/SspDb`.

NGFS scenarios

Figure 47: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)

NGFS scenarios

Figure 48: NGFS scenarios framework (2022)



NGFS scenarios

- Orderly scenarios
  - #1 Net zero 2050 (NZ)
  - #2 Below $2^{\circ}\mathrm{C}$ (B2D)
- Disorderly scenarios
  - #3 Divergent net zero (DNZ)
  - #4 Delayed transition (DT)
- Hot house world scenarios
  - #5 Nationally determined contributions (NDC)
  - #6 Current policies (CP)

NGFS scenarios

Figure 49: Physical and transition risk level of NGFS scenarios

		Physical risk	Transition risk			
Category	**Scenario**	**Policy ambition**	**Policy reaction**	**Technology change**	**Carbon dioxide removal** ⁻	**Regional policy variation** ⁺
Orderly	Net Zero 2050	1.4°C	Immediate and smooth	Fast change	Medium-high use	Medium variation
	Below 2°C	1.6°C	Immediate and smooth	Moderate change	Medium-high use	Low variation
Disorderly	Divergent Net Zero	1.4°C	Immediate but divergent across sectors	Fast change	Low-medium use	Medium variation
	Delayed Transition	1.6 °C	Delayed	Slow / Fast change	Low-medium use	High variation
Hot house world	Nationally Determined Contributions (NDCs)	2.6°C	NDCs	Slow change	Low-medium use	Medium variation
	Current Policies	3°C +	Non-currente policies	Slow change	Low use	Low variation

NGFS scenarios

Variables (economic)

- Central bank intervention rate
- Domestic demand
- Effective exchange rate
- Exchange rate
- Exports (goods and services)
- Gross Domestic Product (GDP)
- Gross domestic income
- Imports (goods and services)
- Inflation rate
- Long term & real interest rates
- Trend output for capacity utilisation
- Unemployment

Variables (energy)

- Coal price
- Gas price
- Oil price
- Quarterly consumption of coal
- Quarterly consumption of gas
- Quarterly consumption of oil
- Quarterly consumption of renewables
- Total energy consumption

Models (IPCC)

- Meta-model: NiGEM 1.21
- Sub-models:
  1. GCAM 5.3
  2. MESSAGE-GLOBIOM 1.1
  3. REMIND-MAgPIE 2.1-4.2

6 scenarios

1. Net Zero 2050 (NZ)
2. Below 2°C (B2D)
3. Divergent Net Zero (DNZ)
4. Delayed Transition (DT)
5. Notionally Determined Contribution (NDC)
6. Current Policies (CP)

NGFS scenarios

Table 6: Impact of climate change on the GDP loss by 2050 (GCAM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−3.09	−5.64	−2.35	−3.28	−5.15	−2.56
Transition risk	−0.75		−3.66	−1.78	−0.89	−0.88
Combined risk	−3.84	−5.64	−6.00	−5.05	−6.03	−3.44
Combined + business confidence			−6.03	−5.09		

NGFS scenarios

Table 7: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−2.05	−5.26	−1.55	−2.64	−4.78	−1.59
Transition risk	−1.46		−10.00	−10.77	−1.39	−3.26
Combined risk	−3.51	−5.26	−11.53	−13.37	−6.16	−4.84
Combined + business confidence			−11.57	−13.40		

NGFS scenarios

Table 8: Impact of climate change on the GDP loss by 2050 (REMIND-MAgPIE)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−2.24	−6.05	−1.67	−2.65	−5.41	−1.76
Transition risk	−0.78		−3.01	−1.95	−0.33	−1.46
Combined risk	−3.02	−6.05	−4.68	−4.59	−5.73	−3.21
Combined + business confidence			−4.70	−4.63		

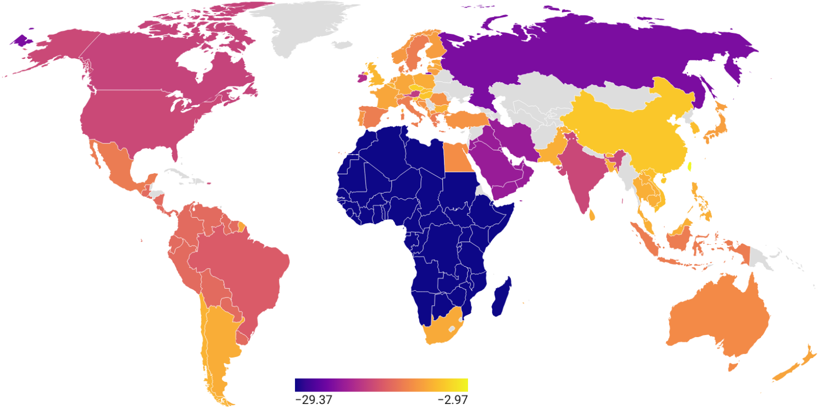
NGFS scenarios

Table 9: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Africa	−13.58	−7.50	−27.35	−29.37	−11.78	−18.36
Asia	−1.50	−7.29	−5.44	−8.76	−6.78	−1.38
Australia	−4.11	−3.90	−11.03	−11.74	−5.77	−5.19
Brazil	−4.43	−5.92	−13.15	−15.90	−6.67	−6.65
Canada	−1.02	−2.37	−15.07	−18.12	−4.33	−4.87
China	−2.33	−4.97	−5.13	−6.73	−4.67	−2.76
Developing Europe	−0.28	−3.11	−0.56	−7.38	−2.73	0.39
Europe	−1.02	−2.84	−9.64	−11.02	−4.01	−1.62
France	−1.15	−2.80	−8.35	−9.48	−3.68	−1.56
Germany	−0.77	−2.38	−8.58	−9.38	−3.63	−1.21
India	−3.45	−8.61	−16.43	−17.74	−8.71	−3.86
Italy	−0.15	−3.69	−9.23	−12.88	−4.85	−0.89
Japan	−1.26	−4.14	−7.16	−10.05	−4.61	−1.40
Latam	−4.35	−6.10	−12.70	−14.58	−6.97	−5.74
Middle East	−9.97	−7.98	−22.03	−21.96	−10.28	−15.24
Russia	−12.18	−2.26	−23.46	−23.80	−7.54	−17.11
South Africa	−2.02	−5.06	−7.24	−9.16	−5.38	−3.04
South Korea	0.11	−3.49	−3.23	−7.57	−3.33	0.12
Spain	−2.41	−3.81	−12.49	−12.89	−5.41	−3.30
Switzerland	2.32	−2.25	−9.47	−10.35	−2.18	2.30
United Kingdom	−0.86	−1.90	−6.50	−8.05	−2.56	−1.33
United States	−2.67	−4.38	−15.37	−17.66	−6.31	−4.36
World	−3.51	−5.26	−11.53	−13.37	−6.16	−4.84

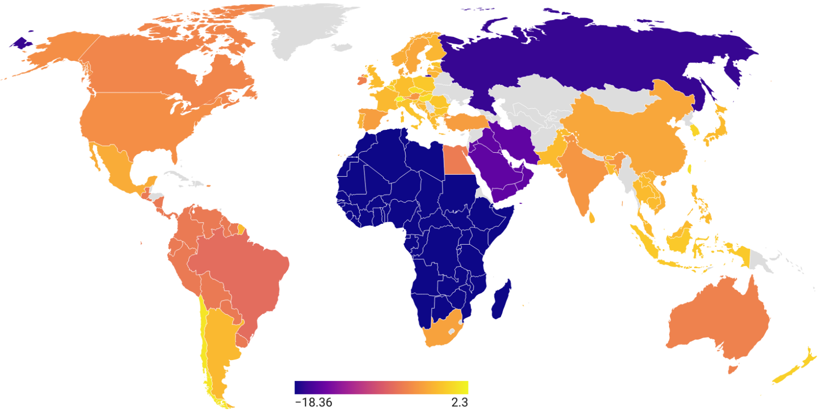
NGFS scenarios

Figure 50: GDP impact by 2050 (% change from baseline) — Delayed transition scenario



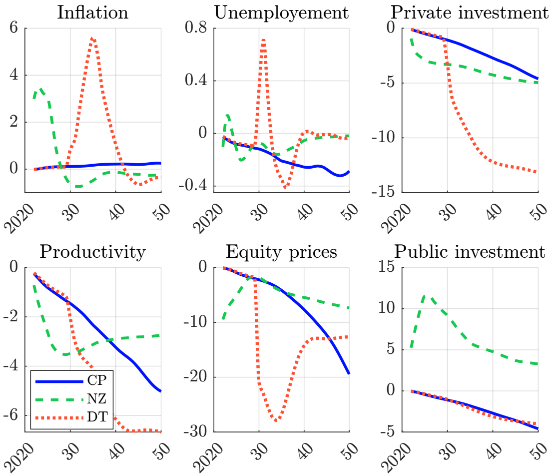
NGFS scenarios

Figure 51: GDP impact by 2050 (% change from baseline) — Net zero 2050 scenario



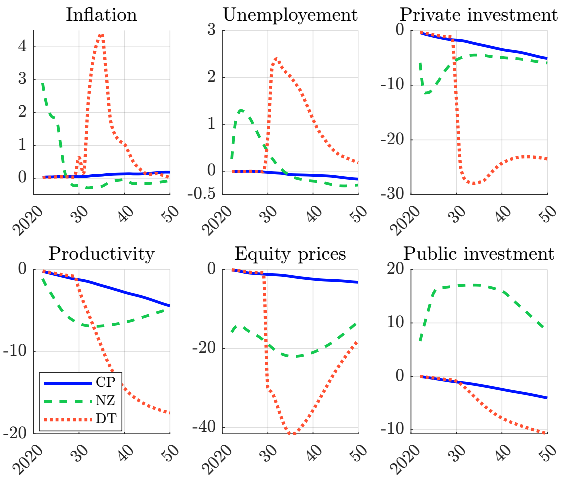
NGFS scenarios

Figure 52: Impact of climate scenarios on economics (% change from baseline) — China



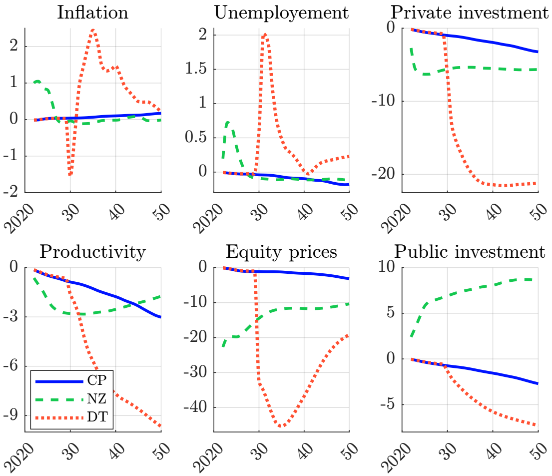
NGFS scenarios

Figure 53: Impact of climate scenarios on economics (% change from baseline) — United States



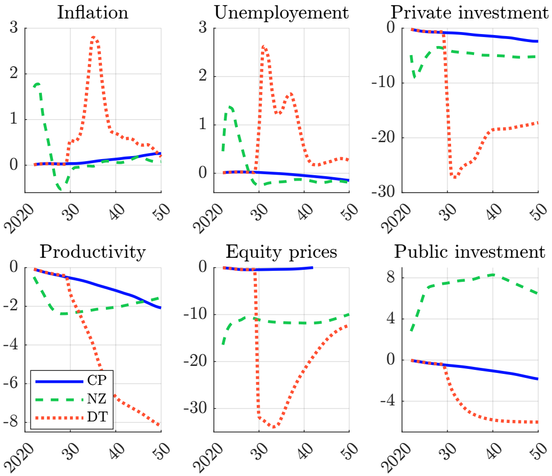
NGFS scenarios

Figure 54: Impact of climate scenarios on economics (% change from baseline) — France



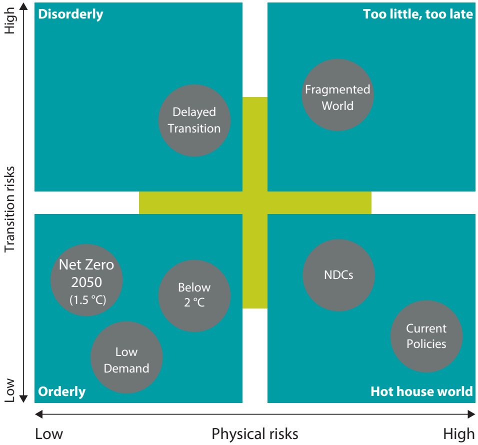
NGFS scenarios

Figure 55: Impact of climate scenarios on economics (% change from baseline) — United Kingdom



NGFS scenarios (2023)

Figure 56: NGFS scenarios framework (2023)



NGFS scenarios (2023)

Figure 57: Physical and transition risk level of NGFS scenarios (2023)

Quadrant	Scenario	Physical risk: End of century (peak) warming – model average	Transition risk: Policy reaction	Technology change	Carbon dioxide removal [-]	Regional policy variation [+]
Orderly	Low Demand NEW	1.4 °C (1.6 °C)	Immediate	Fast change	Medium use	Medium variation
	Net Zero 2050	1.4 °C (1.6 °C)	Immediate	Fast change	Medium-high use	Medium variation
	Below 2 °C	1.7 °C (1.8 °C)	Immediate and smooth	Moderate change	Medium use	Low variation
Disorderly	Delayed Transition	1.7 °C (1.8 °C)	Delayed	Slow/Fast change	Medium use	High variation
Hot house world	Nationally Determined Contributions (NDCs)	2.4 °C (2.4 °C)	NDCs	Slow change	Low use	Medium variation
	Current Policies	2.9 °C (2.9 °C)	None – current policies	Slow change	Low use	Low variation
Too-little-too-late	Fragmented World NEW	2.3 °C (2.3 °C)	Delayed and Fragmented	Slow/Fragmented change	Low-medium use	High variation

Input-output analysis

- The input-output (IO) model was first introduced by **Leontief** (1936, 1941)
- It quantifies the interdependencies between different sectors in a single or multi-regional economy, based on the product flows between sectors
- The underlying idea is to model the linkages between sectors and to describe the relationships from each of the producer/seller sectors to each of the purchaser/buyer sectors

The demand-pull quantity model

- n different sectors
- $Z_{i,j}$ is the value of transactions from sector i to sector j:
  1. It is the output that sector i sells to sector j
  2. It is the input of sector i required by sector j for its production (or output)
- y_i is the final demand for products sold by sector i
- x_i is the total production of sector i

The demand-pull quantity model

- We have the following accounting identity:

$$\underbrace{x_i}_{\text{Supply}} = \underbrace{\sum_{j=1}^{n} Z_{i,j} + y_i}_{\text{Demand}}$$

- $z_i = \sum_{j=1}^{n} Z_{i,j}$ represents intermediate demand
- The interdependence relation between sectors is usually expressed as a ratio between $Z_{i,j}$ and x_j:

$$A_{i,j} = \frac{Z_{i,j}}{x_j}$$

- $A = (A_{i,j}) = Z \operatorname{diag}(x)^{-1}$ is the input-output matrix of the **technical coefficients** $A_{i,j}$

The demand-pull quantity model

- In a matrix form, we have $x = Z\mathbf{1}_n + y$ and $Z \equiv A \operatorname{diag}(x) = A \odot x^\top$, and we deduce that:

$$x = Ax + y$$

  where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$

- Assuming that final demand is exogenous, technical coefficients are fixed and output is endogenous, we obtain:

$$\boldsymbol{x = (I_n - A)^{-1}\, y}$$

- $\mathcal{L} = (I_n - A)^{-1}$ is known as the **Leontief inverse** (or multiplier) matrix and represents the amount of total output from sector i that is required by sector j to satisfy its final demand

The demand-pull quantity model

Example #2

		To				Final Demand	Total Output
		Energy	Materials	Industrials	Services		
				Z		y	x
From	Energy	500	800	1 600	1 250	850	5 000
	Materials	500	400	1 600	625	875	4 000
	Industrials	250	800	2 400	1 250	3 300	8 000
	Services	100	200	800	4 375	7 025	12 500

This economy has four sectors: energy, materials, industrials and services. In this economy, businesses in the energy sector buy \$500 of goods and services from other businesses in the energy sector, \$500 of goods and services from the materials sector, \$250 of goods and services from the industrials sector, and \$100 of goods and services from the services sector. The final demand for goods and services produced in the energy sector is equal to \$850, while the total output of this sector is equal to \$5 000.

The demand-pull quantity model

We deduce that the matrix of technical coefficients is equal to:

$$A = Z \operatorname{diag}(x)^{-1} = \begin{pmatrix} 10\% & 20\% & 20\% & 10\% \\ 10\% & 10\% & 20\% & 5\% \\ 5\% & 20\% & 30\% & 10\% \\ 2\% & 5\% & 10\% & 35\% \end{pmatrix}$$

It follows that the multiplier matrix is equal to:

$$\mathcal{L} = (I_4 - A)^{-1} = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix}$$

We verify that:

$$x = \mathcal{L}y = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix} \begin{pmatrix} 850 \\ 875 \\ 3\,300 \\ 7\,025 \end{pmatrix} = \begin{pmatrix} 5\,000 \\ 4\,000 \\ 8\,000 \\ 12\,500 \end{pmatrix}$$

The demand-pull quantity model

Suppose we have a variation in final demand. We obtain $\Delta x = \mathcal{L}\Delta y$. For instance, an increase of \$10 in the final demand for services implies:

$$\begin{aligned}\Delta x &= \mathcal{L}\Delta y = \begin{pmatrix} 1.1881 & 0.3894 & 0.4919 & 0.2884 \\ 0.1678 & 1.2552 & 0.4336 & 0.1891 \\ 0.1430 & 0.4110 & 1.6303 & 0.3044 \\ 0.0715 & 0.1718 & 0.2993 & 1.6087 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 10 \end{pmatrix} \\ &= \begin{pmatrix} 2.8842 \\ 1.8907 \\ 3.0444 \\ 16.0872 \end{pmatrix}\end{aligned}$$

This means that energy production increases by \$2.88, materials production increases by \$1.89, and so on

The cost-push price model

- m is number of primary inputs (*e.g.*, labor, capital, etc.)
- $V = (V_{k,j})$ is the **value added** matrix where $V_{k,j}$ represents the amount of primary input k required to produce the output of sector j
- Since the total input of each sector is equal to its total output, we have:

$$x_j = \sum_{i=1}^{n} Z_{i,j} + \sum_{k=1}^{m} V_{k,j}$$

- Therefore, $v_j = \sum_{k=1}^{m} V_{k,j} = x_j - \sum_{i=1}^{n} Z_{i,j}$ represents the other expenditure of sector j or the total primary inputs used in sector j
- We have $v = (v_1, \ldots, v_n) = V^\top \mathbf{1}_m$

The cost-push price model

- Let $p = (p_1, \ldots, p_n)$ and $\psi = (\psi_1, \ldots, \psi_m)$ be the vector of sector prices and primary inputs
- p_j and ψ_k are the prices per unit of sector j and primary input k
- The interdependence relationship between primary inputs and sectors is expressed as the ratio between $V_{k,j}$ and x_j:

$$B_{k,j} = \frac{V_{k,j}}{x_j}$$

- $B = (B_{k,j}) \equiv V \operatorname{diag}(x)^{-1}$ is the input-output matrix of the **technical coefficients** $B_{k,j}$
- The value of the output must be equal to the value of its inputs:

$$\underbrace{p_j x_j}_{\text{Value of the output}} = \underbrace{\sum_{i=1}^{n} Z_{i,j} p_i + \sum_{k=1}^{m} V_{k,j} \psi_k}_{\text{Value of the inputs}}$$

The cost-push price model

- We deduce that:

$$\begin{aligned} p_j &= \sum_{i=1}^{n} \frac{Z_{i,j}}{x_j} p_i + \sum_{k=1}^{m} \frac{V_{k,j}}{x_j} \psi_k \\ &= \sum_{i=1}^{n} A_{i,j} p_i + \sum_{k=1}^{m} B_{k,j} \psi_k \end{aligned}$$

- In a matrix form, we get:

$$p = A^\top p + B^\top \psi$$

- $\upsilon = B^\top \psi$ is the vector of value added ratios
- Finally, the output prices are equal to:

$$\boldsymbol{p = \left(I_n - A^\top\right)^{-1} \upsilon}$$

- $\tilde{\mathcal{L}} = \left(I_n - A^\top\right)^{-1}$ is known as the **dual inverse matrix** and represents the amount of costs from sector j that are passed on to sector i

Summary

Comprehensive input-output model

- Demand-pull quantity model + cost-push price model:

$$\begin{cases} x = (I_n - A)^{-1} y \\ v = V^\top \mathbf{1}_m \\ \upsilon = B^\top \psi \\ p = \left(I_n - A^\top\right)^{-1} \upsilon \\ x^\top \upsilon = y^\top p \end{cases}$$

- Equilibrium $\Rightarrow$ the total value of the revenues $y^\top p$ is equal to the total value of costs $x^\top \upsilon$

Summary

Remark

In this model, A, B and V are the model parameters, ψ, υ and y are the exogenous variables, and x and p are the endogenous variables. By changing the model parameters or the exogenous variables, we can measure the impacts Δy and $\Delta \upsilon$ on the quantities and prices in the economy.

Demand-pull quantity model

- We have:

$$x = (I_n - A)^{-1} y = \mathcal{L}\, y$$

where $\mathcal{L} = (I_n - A)^{-1}$ is the Leontief inverse matrix

Cost-push price model

- We have:

$$p = \left(I_n - A^\top\right)^{-1} \upsilon = \tilde{\mathcal{L}} \upsilon$$

where $\tilde{\mathcal{L}} = \left(I_n - A^\top\right)^{-1} = \mathcal{L}^\top$ is the dual inverse matrix

The cost-push price model

Example #3

		To				Final Demand	Total Output
		Energy	Materials	Industrials	Services		
		Z				y	x
From	Energy	500	800	1 600	1 250	850	5 000
	Materials	500	400	1 600	625	875	4 000
	Industrials	250	800	2 400	1 250	3 300	8 000
	Services	100	200	800	4 375	7 025	12 500
Value added	Labour	3 000	800	1 000	3 000		
	Capital	650	1 000	600	2 000		
	Income	5 000	4 000	8 000	12 000		

The energy sector has a labour consumption of \$3 000 and a total output of \$5 000. By construction, the income of the sector is equal to the output of the sector. We deduce that the capital item (capital interest and net profit) is equal to \$650.

The cost-push price model

We have:

$$V = \begin{pmatrix} 3\,000 & 800 & 1\,000 & 3\,000 \\ 650 & 1\,000 & 600 & 2\,000 \end{pmatrix}$$

and:

$$v = V^\top \mathbf{1}_2 = x - Z^\top \mathbf{1}_4 = \begin{pmatrix} 3\,650 \\ 1\,800 \\ 1\,600 \\ 5\,000 \end{pmatrix}$$

We deduce that:

$$B = \begin{pmatrix} 0.60 & 0.20 & 0.125 & 0.24 \\ 0.13 & 0.25 & 0.075 & 0.16 \end{pmatrix}$$

The cost-push price model

Since we have a monetary input-output table, the labour and capital costs are equal to the monetary unit, *i.e.* one dollar ($\psi_1 = \psi_2 = 1$). It follows that:

$$\upsilon = B^\top \mathbf{1}_2 = \begin{pmatrix} 0.73 \\ 0.45 \\ 0.20 \\ 0.40 \end{pmatrix}$$

The interpretation is as follows. For the energy sector, intermediate consumption is 27% and value added is 73%. For the other three sectors, the value added ratios are 45%, 20% and 40% respectively

The Cost-push price model

Finally, we obtain:

$$\tilde{\mathcal{L}} = \left(I_n - A^\top\right)^{-1} = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix}$$

and:

$$p = \tilde{\mathcal{L}} \upsilon = \mathbf{1}_4$$

We check that the prices in a monetary input-output table are normalized to one dollar. In this basic economy, the total final demand $y^\top p$ is equal to \$12 050, which is equal to the total value added $x^\top \upsilon$

The Cost-push price model

Suppose we have a variation in the labour/capital costs. We obtain $\Delta p = \tilde{\mathcal{L}} \Delta v$. For example, a 10% increase in costs in the energy sector means that the price of energy increases by 11.88%, the price of materials by 3.89%, and so on:

$$\begin{aligned} \Delta p &= \tilde{\mathcal{L}} \Delta v = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix} \begin{pmatrix} 0.10 \\ 0.00 \\ 0.00 \\ 0.00 \end{pmatrix} \\ &= \begin{pmatrix} 11.88\% \\ 3.89\% \\ 4.92\% \\ 2.88\% \end{pmatrix} \end{aligned}$$

Inflation

- The definition of a price index is:

$$\mathcal{PI} = \sum_{j=1}^{n} \alpha_j p_j = \alpha^\top p$$

where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is the weights of the basket of items

1. Producer price index (PPI): $\alpha_j \propto x_j$
2. Consumer price index (CPI): $\alpha_j \propto y_j$

- The inflation rate between two dates t_0 and t_1 is:

$$\pi = \frac{\mathcal{PI}(t_1) - \mathcal{PI}(t_0)}{\mathcal{PI}(t_0)} = \frac{\alpha^\top \left(I_n - A^\top\right)^{-1} \Delta\upsilon(t_0, t_1)}{\alpha^\top \left(I_n - A^\top\right)^{-1} \upsilon(t_0)}$$

- We can simplify this formula because $p(t_0) = \left(I_n - A^\top\right)^{-1} \upsilon(t_0) = \mathbf{1}_n$ and $\mathbf{1}_n^\top \alpha = 1$:

$$\pi = \alpha^\top \left(I_n - A^\top\right)^{-1} \Delta\upsilon$$

Inflation

Looking at the previous example, a 10% increase in energy costs will cause the producer price index to rise by 5.10% and the consumer price index by 4.15%.

Mathematical properties

Neumann series

A Neumann series is $S := \sum_{k=0}^{\infty} T^k$ where T is a bounded linear operator and $T^k = T^{k-1} \circ T = T \circ T^{k-1}$. If the Neumann series converges, then $\mathrm{Id} - T$ is invertible and its inverse is the Neumann series:

$$(\mathrm{Id} - T)^{-1} = S = \sum_{k=0}^{\infty} T^k$$

If A is an invertible matrix, we conclude that:

$$(I_n - A)^{-1} = \sum_{k=0}^{\infty} A^k$$

and $\lim_{k=0} \left\| A^k \right\| = 0$. This result generalizes the geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots \qquad \text{where } |x| < 1$$

Mathematical properties

- We have:

$$\left(I_n - A^\top\right)^{-1} = \left(\left(I_n - A\right)^\top\right)^{-1} = \left(\left(I_n - A\right)^{-1}\right)^\top$$

- The matrix $\mathcal{L}$ admits the following Neumann series:

$$\begin{aligned}\mathcal{L} &= \left(I_n - A\right)^{-1} \\ &= I_n + A + A^2 + A^3 + \ldots \\ &= \sum_{k=0}^{\infty} A^k\end{aligned}$$

- The multiplier matrix $\mathcal{L}$ is nonsingular
- $\mathcal{L} \succeq I_n$ because $A^k \succeq \mathbf{0}_{n,n}$ for $k \geq 1$

Mathematical properties

- We also get the following decomposition:

$$x = \sum_{k=0}^{\infty} A^k y = y + Ay + A^2 y + \ldots = \sum_{k=0}^{\infty} y_{(k)}$$

  where:

  - $y_{(0)} = y$ is the final demand (or zero-tier intermediate demand)
  - $y_{(1)} = Ay$ is the first-tier intermediate demand
  - $y_{(2)} = A^2 y$ is the second-tier intermediate demand
  - $y_{(k)} = A^k y$ is the k^{th}-tier intermediate demand

- We have:

$$\frac{\partial\, x}{\partial\, y} = (I_n - A)^{-1} \equiv \boldsymbol{\mathcal{L}} \succeq I_n$$

Mathematical properties

Remark

The matrix $\mathcal{L}$ is also called the multiplier matrix because it is an analogy to Keynesian consumption theory and the effect of a change in aggregate demand on the output

Mathematical properties

Table 10: Tier decomposition of the output (Example #2)

	k	0	1	2	3	4	5
$y_{(k)}$	Energy	850.0	1 622.5	1 066.0	630.8	362.1	205.2
	Materials	875.0	1 183.8	805.1	486.3	282.1	160.7
	Industrials	3 300.0	1 910.0	1 175.8	695.1	400.0	227.1
	Services	7 025.0	2 849.5	1 280.0	627.1	325.9	175.4
$y_{(0:k)}$	Energy	850.0	2 472.5	3 538.5	4 169.2	4 531.3	4 736.5
	Materials	875.0	2 058.8	2 863.9	3 350.1	3 632.2	3 792.9
	Industrials	3 300.0	5 210.0	6 385.8	7 080.9	7 480.9	7 708.0
	Services	7 025.0	9 874.5	11 154.5	11 781.6	12 107.5	12 283.0

	k	10	20	25
$y_{(k)}$	Energy	11.45	0.03	0.00
	Materials	9.01	0.03	0.00
	Industrials	12.69	0.04	0.00
	Services	9.29	0.03	0.00
$y_{(0:k)}$	Energy	4 985.42	4 999.96	5 000.00
	Materials	3 988.52	3 999.97	4 000.00
	Industrials	7 983.83	7 999.95	8 000.00
	Services	12 488.18	12 499.96	12 500.00

Multi-regional input-output analysis

- A **multi-regional input-output** table (MRIO) involves several regions
- The two best known MRIO databases are:
  1. GTAP (global trade analysis project):
     `www.gtap.agecon.purdue.edu`
  2. WIOD (world input-output database):
     `www.rug.nl/ggdc/valuechain/wiod`
- Other MRIO databases: OECD (`https://stats.oecd.org/Index.aspx?DataSetCode=IOTS`), Eora, Exiobase, etc.

Multi-regional input-output analysis

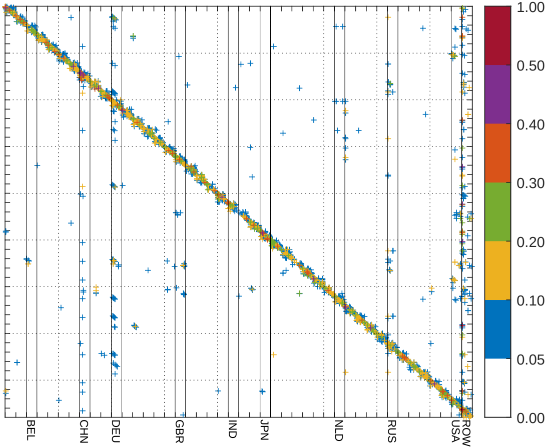
The structure of the WIOD database is:

	Z $(56 \times 44) \times (56 \times 44)$	y $(56 \times 44) \times (5 \times 44)$	x $(56 \times 44) \times 1$
Sum	$1 \times (56 \times 44)$	$1 \times (5 \times 44)$	$\mathbf{0}_{1,1}$
Value added	V $6 \times (56 \times 44)$	$6 \times (5 \times 44)$	$\mathbf{0}_{6,1}$
Output	w $1 \times (56 \times 44)$	$\mathbf{0}_{6,5\times 44}$	$\mathbf{0}_{1,1}$

- 44 regions
- 56 sectors

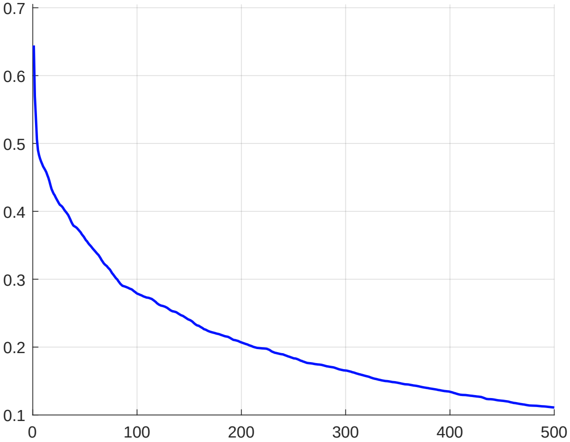
The WIOD database

Figure 58: Sparsity pattern of the input-output matrix A



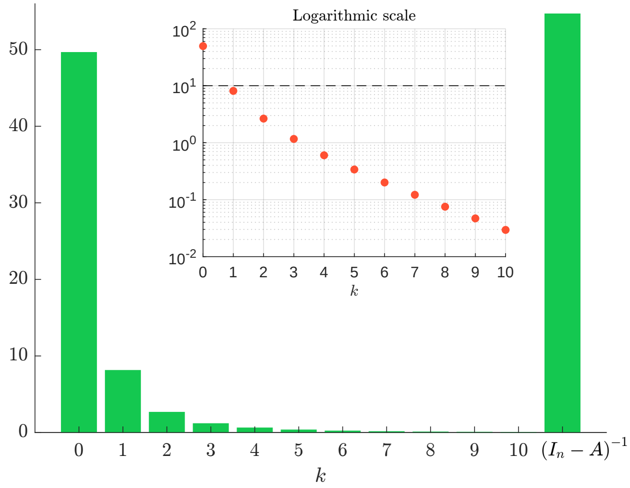
The WIOD database

Figure 59: Spectrum of the matrix A



The WIOD database

Figure 60: Frobenious norm of the matrix A^k



Application to environmental problems

$$\text{EEIO} = \text{MRIO} + \text{GHG emissions}$$

Production-based vs. consumption-based inventory

- $C^{(x)} = \left(C^{(x)}_{g,j}\right)$ is the **pollution output matrix** where $C^{(x)}_{g,j}$ is the total amount of the g^{th} pollutant generated by the output of the j^{th} sector
- $D^{(y)} = C^{(x)} \operatorname{diag}(x)^{-1} = \left(D^{(y)}_{g,j}\right)$ is the matrix of **direct impact coefficients** where $D^{(y)}_{g,j} = c^{(y)}_{g,j}/x_j$ is the amount of the g^{th} pollutant generated by 1\$ of the output of the j^{th} sector
- $\varpi = (\varpi_1, \ldots, \varpi_m)$ is the vector of pollution level:

$$\varpi = D^{(x)}x = D^{(x)} (I_n - A)^{-1} y = D^{(y)}y$$

  where $D^{(y)} = D^{(x)} (I_n - A)^{-1}$ is the **pollutant multiplier matrix** with respect to the final demand y
- $D^{(y)}$ also measures the **product carbon footprint** (PCF)

Production-based vs. consumption-based inventory

- Because $\varpi_g = \left(D^{(y)} y\right)_g = \sum_{j=1}^{n} D^{(y)}_{g,j} y_j$, we deduce that the total contribution of sector j to the g^{th} pollutant is equal to:

$$\mathcal{C}^{(x)}_{g,j} = \frac{\partial\, \varpi_g}{\partial\, y_j} y_j = D^{(y)}_{g,j} y_j$$

- Again, we can decompose the pollutant level according to the k^{th} tier:

$$\varpi = D^{(y)} y = \sum_{k=0}^{\infty} D^{(x)} A^k y = \sum_{k=0}^{\infty} \varpi_{(k)}$$

  where:

  1. $\varpi_{(0)} = D^{(x)} y$ is the pollutant level due to the final demand (or the zero-tier pollutant level)
  2. $\varpi_{(1)} = D^{(x)} A y$ is the pollutant level due to the first-tier supply chain
  3. $\varpi_{(k)} = D^{(x)} A^k y$ is the k^{th}-tier pollutant level

- The matrix $D^{(y)}_{(k)} = D^{(x)} A^k$ is called the k^{th}-tier multiplier matrix and satisfies the identity $D^{(y)} \equiv \sum_{k=0}^{\infty} D^{(y)}_{(k)}$

Production-based vs. consumption-based inventory

Example #4

We consider three products, whose input-output table is given below:

		To			Final demand	Total output
		P_1	P_2	P_3	y	x
From	P_1	100	300	100	500	1 000
	P_2	250	150	200	1 600	2 000
	P_3	25	200	75	200	500
	Value added	625	1 350	125		
	Total outlays	1 000	2 000	500		
GHG	CO_2	50	20	5	75	
	CH_4	3	1	0	4	

Intermediate production of \$100 of P_1, \$300 of P_2, and \$100 of P_3 is required to produce \$500 of P_1. This environmentally-extended input-output table has two additional rows corresponding to the GHG emissions. For instance, the production of P_1 causes 50 $\mathrm{kgCO_2}$ and 3 $\mathrm{kgCH_4}$.

Production-based vs. consumption-based inventory

The matrix of technical coefficients is equal to:

$$A = Z \operatorname{diag}(x)^{-1} = \begin{pmatrix} 10.0\% & 15.0\% & 20.0\% \\ 25.0\% & 7.5\% & 40.0\% \\ 2.5\% & 10.0\% & 15.0\% \end{pmatrix}$$

It follows that the matrix of multipliers is equal to:

$$\mathcal{L} = (I_3 - A)^{-1} = \begin{pmatrix} 1.1871 & 0.2346 & 0.3897 \\ 0.3539 & 1.2090 & 0.6522 \\ 0.0766 & 0.1491 & 1.2647 \end{pmatrix}$$

Production-based vs. consumption-based inventory

The direct impact matrix is equal to the GHG emissions divided by the output:

$$D^{(x)} = \begin{pmatrix} 50/1000 & 20/2000 & 5/500 \\ 3/1000 & 1/2000 & 0/500 \end{pmatrix} = \begin{pmatrix} 0.05 & 0.01 & 0.01 \\ 0.003 & 0.0005 & 0 \end{pmatrix}$$

The unit of $D^{(x)}$ is expressed in kilogram of the gas per dollar

For instance, the GHG intensities of the product P_1 are equal to 0.05 $\mathrm{kgCO}_2/\$$ and 0.003 $\mathrm{kgCH}_4/\$$

Production-based vs. consumption-based inventory

Finally, we obtain:

$$D^{(y)} = D^{(x)}\mathcal{L} = \begin{pmatrix} 0.0637 & 0.0253 & 0.0387 \\ 0.0037 & 0.0013 & 0.0015 \end{pmatrix}$$

- $D^{(x)}$ corresponds to the production-based inventory
- $D^{(y)}$ corresponds to the consumption-based inventory

This gives us the following decomposition:

$$C^{(y)} = \begin{pmatrix} 31.83 & 35.44 & 7.73 \\ 1.87 & 1.83 & 0.30 \end{pmatrix} \neq \begin{pmatrix} 50 & 20 & 5 \\ 3 & 1 & 0 \end{pmatrix} = C^{(x)}$$

Remark

The two contribution matrices are different. For instance, while the production of P_1 is responsible of 50 $\mathrm{kgCO_2}$, the final consumption of P_1 is responsible of only 31.83 $\mathrm{kgCO_2}$, meaning that 18.17 $\mathrm{kgCO_2}$ are emitted by P_1 for the other two products

Estimation of first-tier and indirect emissions

- The carbon footprint is evaluated in $\mathrm{CO_2e}$
- We have:

$$\begin{aligned} \mathcal{CI}_{\mathrm{total}} &= \mathcal{L}^\top \mathcal{CI}_1 \\ &= \left(I_n - A^\top\right)^{-1} \mathcal{CI}_1 \\ &= \tilde{\mathcal{L}} \mathcal{CI}_1 \end{aligned}$$

where:
  - $\mathcal{CI}_1 = \mathcal{CI}_{\mathrm{direct}}$ is the vector of **direct carbon intensities**
  - $\mathcal{CI}_{\mathrm{total}}$ is the vector of **direct plus indirect carbon intensities**

Estimation of first-tier and indirect emissions

- The indirect carbon intensities are given by:

$$\begin{aligned}\mathcal{CI}_{\mathrm{indirect}} &= \mathcal{CI}_{\mathrm{total}} - \mathcal{CI}_1 \\ &= \left(\left(I_n - A^\top\right)^{-1} - I_n\right)\mathcal{CI}_{\mathrm{direct}}\end{aligned}$$

- We can decompose $\mathcal{CI}_{\mathrm{indirect}}$ using the Neumann series:

$$\mathcal{CI}_{\mathrm{indirect}} = \underbrace{A^\top \mathcal{CI}_1}_{\text{First-tier}} + \underbrace{\left(A^\top\right)^2 \mathcal{CI}_1}_{\text{Second-tier}} + \ldots + \underbrace{\left(A^\top\right)^k \mathcal{CI}_1}_{k^{th}\text{-tier}} + \ldots$$

- We have:

$$\mathcal{CI}_{\mathrm{total}} = \underbrace{\underbrace{\mathcal{CI}_1}_{\text{Scope 1}}}_{\text{Direct intensities}} + \underbrace{\underbrace{A^\top \mathcal{CI}_1}_{\text{First-tier}} + \underbrace{\left(A^\top\right)^2 \mathcal{CI}_1}_{\text{Second-tier}} + \ldots + \underbrace{\left(A^\top\right)^k \mathcal{CI}_1}_{k^{th}\text{-tier}} + \ldots}_{\text{Indirect intensities}}$$

Estimation of first-tier and indirect emissions

Example #5

		To				Final	Total
		Energy	Materials	Industrials	Services	Demand	Output
		Z				y	x
From	Energy	500	800	1 600	1 250	850	5 000
	Materials	500	400	1 600	625	875	4 000
	Industrials	250	800	2 400	1 250	3 300	8 000
	Services	100	200	800	4 375	7 025	12 500

The carbon emissions, expressed in $\mathrm{ktCO_2e}$, are as follows: 500 for the energy sector, 200 for the materials sector, 200 for the industrials sector and 125 for the services sector

Estimation of first-tier and indirect emissions

The vector of Scope 1 carbon intensities is equal to:

$$\mathcal{CI}_1 = \operatorname{diag}(x)^{-1} \mathcal{CE}_1 = \begin{pmatrix} 500/5\,000 \\ 200/4\,000 \\ 200/8\,000 \\ 125/12\,500 \end{pmatrix} \times 10^3 = \begin{pmatrix} 100 \\ 50 \\ 25 \\ 10 \end{pmatrix}$$

We have:

$$\begin{aligned} \mathcal{CI}_{\text{total}} &= \tilde{\mathcal{L}} \, \mathcal{CI}_1 \\ &= \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix} \begin{pmatrix} 100 \\ 50 \\ 25 \\ 10 \end{pmatrix} \\ &= \begin{pmatrix} 131.49 \\ 113.69 \\ 114.62 \\ 61.99 \end{pmatrix} \end{aligned}$$

Estimation of first-tier and indirect emissions

Table 11: Direct and indirect carbon intensities (Example #5)

Sector	$\mathcal{CI}_1$	$\mathcal{CI}_{\text{total}}$ (in $\text{tCO}_2\text{e}/\$ \text{ mn}$)	$\mathcal{CI}_{\text{direct}}$	$\mathcal{CI}_{\text{indirect}}$	$\mathcal{CI}_{\text{direct}}$ (in %)	$\mathcal{CI}_{\text{indirect}}$	$\frac{\mathcal{CI}_{\text{total}}}{\mathcal{CI}_1}$
Energy	100.00	131.49	100.00	31.49	76.05%	23.95%	1.31
Materials	50.00	113.69	50.00	63.69	43.98%	56.02%	2.27
Industrials	25.00	114.62	25.00	89.62	21.81%	78.19%	4.58
Services	10.00	61.99	10.00	51.99	16.13%	83.87%	6.20

Estimation of first-tier and indirect emissions

Table 12: Tier decomposition of carbon intensities (Example #5)

	Sector	1	2	3	4	5	10	15	∞
$\mathcal{CI}_{(k)}$	Energy	16.45	6.99	3.60	1.97	1.09	0.06	0.00	0.00
	Materials	30.50	14.97	8.13	4.47	2.48	0.14	0.01	0.00
	Industrials	38.50	22.79	12.58	6.96	3.88	0.21	0.01	0.00
	Services	18.50	13.50	8.45	4.98	2.86	0.16	0.01	0.00
$\mathcal{CI}_{(1-k)}$	Energy	16.45	23.44	27.04	29.02	30.11	31.41	31.48	31.49
	Materials	30.50	45.47	53.59	58.06	60.55	63.52	63.68	63.69
	Industrials	38.50	61.29	73.87	80.83	84.71	89.35	89.61	89.62
	Services	18.50	32.00	40.44	45.43	48.29	51.79	51.98	51.99

We denote by $\mathcal{CI}_{(k)} = \left(A^\top\right)^k \mathcal{CI}_1$ the indirect carbon intensity when considering the k^{th} tier, and $\mathcal{CI}_{(1-k)} = \sum_{h=1}^{k} \left(A^\top\right)^h \mathcal{CI}_1$ the cumulative indirect carbon intensity for the first k tiers

Estimation of first-tier and indirect emissions

The estimation of total emissions use the following identities:

$$\frac{\mathcal{CE}_{\mathrm{total}}}{\mathcal{CE}_{1}} = \frac{\mathcal{CI}_{\mathrm{total}}}{\mathcal{CI}_{1}} \Leftrightarrow \mathcal{CE}_{\mathrm{total}} = \mathcal{CI}_{\mathrm{total}} \odot \frac{\mathcal{CE}_{1}}{\mathcal{CI}_{1}} = x \odot \mathcal{CI}_{\mathrm{total}}$$

Therefore, the indirect emissions are given by:

$$\mathcal{CE}_{\mathrm{indirect}} = \mathcal{CE}_{\mathrm{total}} - \mathcal{CE}_{\mathrm{direct}} = \left(\mathcal{CI}_{\mathrm{total}} - \mathcal{CI}_{1}\right) \odot \frac{\mathcal{CE}_{1}}{\mathcal{CI}_{1}}$$

Table 13: Decomposition of carbon emissions (Example #5)

Sector	$\mathcal{CE}_{\mathrm{direct}}$	$\mathcal{CE}_{\mathrm{indirect}}$ (in $\mathrm{ktCO_2e}$)	$\mathcal{CE}_{\mathrm{total}}$	$\mathcal{CE}_{\mathrm{direct}}$	$\mathcal{CE}_{\mathrm{indirect}}$ (in %)	$\mathcal{CE}_{\mathrm{total}}$
Energy	500	157.44	657.44	48.78	8.85	23.45
Materials	200	254.76	454.76	19.51	14.32	16.22
Industrials	200	716.97	916.97	19.51	40.30	32.70
Services	125	649.92	774.92	12.20	36.53	27.64
Total	1 025	1 779.10	2 804.10	100.00	100.00	100.00

Summary of formulas

- We have:

$$\left\{\begin{array}{l} \mathcal{CI}_{\mathrm{total}} = \left(I_n - A^\top\right)^{-1} \mathcal{CI}_1 \\ \mathcal{CE}_{\mathrm{total}} = x \odot \mathcal{CI}_{\mathrm{total}} \\ \mathcal{CI}_{(k)} = \left(A^\top\right)^k \mathcal{CI}_1 \\ \mathcal{CE}_{(k)} = x \odot \mathcal{CI}_{(k)} \end{array}\right.$$

- If we want to aggregate the results such that $i \in \Omega$, we have:

$$\mathcal{CE}_{\mathrm{total}}(\Omega) = \sum_{i \in \Omega} \mathcal{CE}_{\mathrm{total},i} = \omega^\top \mathcal{CE}_{\mathrm{total}}$$

where $\omega = (\omega_i)$ is a vector of dimension $n \times 1$ with $\omega_i = 1$ if $i \in \Omega$ and $\omega_i = 0$ otherwise

- The carbon intensity (WACI) of Ω is equal to:

$$\mathcal{CI}_{\mathrm{total}}(\Omega) = \frac{\sum_{i \in \Omega} \mathcal{CE}_{\mathrm{total},i}}{\sum_{i \in \Omega} x_i} = \frac{\sum_{i \in \Omega} x_i \mathcal{CI}_{\mathrm{total},i}}{\sum_{i \in \Omega} x_i} = \sum_{i \in \Omega} w_i \mathcal{CI}_{\mathrm{total},i}$$

where $w_i = \left(\sum_{j \in \Omega} x_j\right)^{-1} x_i$ is the weight of item i in the set Ω

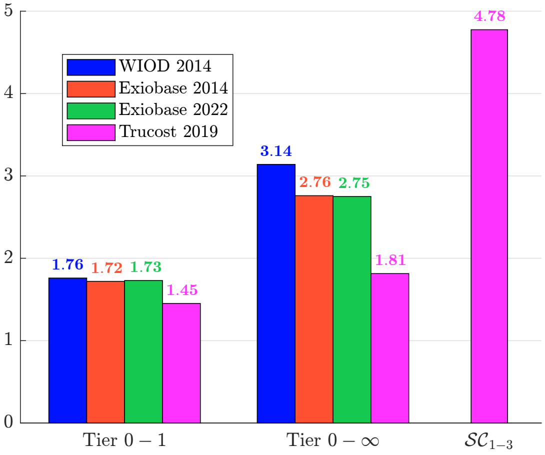
Comparison of upstream emissions between Exiobase, Trucost and WIOD

Table 14: Ratio of upstream carbon emissions (global analysis)

Tier	WIOD 2014			Exiobase 2014			Exiobase 2022		
	$m_{(k)}$	$m_{(0-k)}$	$c_{(0-k)}$	$m_{(k)}$	$m_{(0-k)}$	$c_{(0-k)}$	$m_{(k)}$	$m_{(0-k)}$	$c_{(0-k)}$
0	1.00	1.00	31.8%	1.00	1.00	36.2%	1.00	1.00	36.4%
1	0.77	1.76	56.1%	0.72	1.72	62.5%	0.73	1.73	62.9%
2	0.50	2.26	71.9%	0.43	2.15	78.0%	0.42	2.15	78.3%
3	0.32	2.58	82.1%	0.25	2.40	87.0%	0.25	2.40	87.3%
4	0.20	2.78	88.6%	0.15	2.55	92.3%	0.14	2.54	92.5%
5	0.13	2.91	92.7%	0.09	2.63	95.5%	0.08	2.62	95.5%
6	0.08	3.00	95.4%	0.05	2.69	97.3%	0.05	2.67	97.3%
7	0.05	3.05	97.0%	0.03	2.72	98.4%	0.03	2.70	98.4%
8	0.03	3.08	98.1%	0.02	2.73	99.0%	0.02	2.72	99.0%
9	0.02	3.11	98.8%	0.01	2.74	99.4%	0.01	2.73	99.4%
10	0.01	3.12	99.2%	0.01	2.75	99.7%	0.01	2.74	99.7%
∞	0.00	3.14	100.0%	0.00	2.76	100.0%	0.00	2.75	100.0%

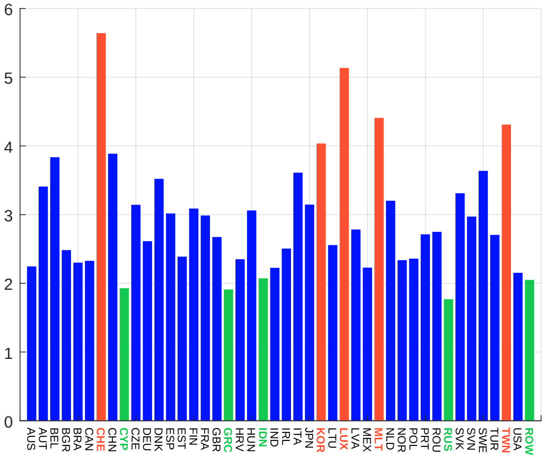
Comparison of Exiobase, Trucost and WIOD

Figure 61: Multiplication coefficient $m_{(0-1)}$ and $m_{(0-\infty)}$ (global analysis)



Comparison of Exiobase, Trucost and WIOD

Figure 62: Multiplying coefficient $m_{(0-\infty)}$ (country analysis, WIOD 2014)



Comparison of upstream emissions between Exiobase, Trucost and WIOD

Table 15: Direct + indirect carbon intensities of GICS sectors (MSCI World index, May 2023)

Sector	Exiobase 2022	Trucost 2021	WIOD 2014
Communication Services	66	78	102
Consumer Discretionary	168	209	219
Consumer Staples	437	387	277
Energy	1 373	796	757
Financials	83	55	83
Health Care	108	120	167
Industrials	276	277	307
Information Technology	110	138	131
Materials	791	973	747
Real Estate	128	134	138
Utilities	1 872	1 833	1 889
MSCI World	299	281	278

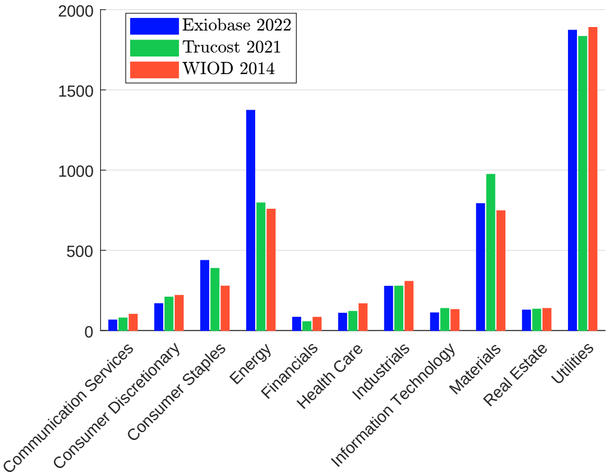
Comparison of upstream emissions between Exiobase, Trucost and WIOD

Table 16: Breakdown of the portfolio intensity by GICS sector (MSCI World Index, May 2023)

Sector	Exiobase 2022	Trucost 2021	WIOD 2014
Communication Services	1.5%	1.9%	2.5%
Consumer Discretionary	5.9%	7.8%	8.4%
Consumer Staples	11.6%	10.9%	7.9%
Energy	22.9%	14.1%	13.6%
Financials	4.2%	2.9%	4.5%
Health Care	4.8%	5.7%	8.0%
Industrials	10.2%	10.8%	12.1%
Information Technology	7.5%	10.0%	9.6%
Materials	11.7%	15.3%	11.9%
Real Estate	1.1%	1.2%	1.2%
Utilities	18.6%	19.4%	20.2%

Comparison of Exiobase, Trucost and WIOD

Figure 63: Total carbon intensity $\mathcal{CI}_{\text{total}}$ by GICS sector (MSCI World Index, May 2023)



Imported and exported carbon emissions

- Let $\mathcal{CI}_1 = (\mathcal{CI}_{1,1}, \ldots, \mathcal{CI}_{n,1})$ be the vector of carbon intensities evaluated in $\mathrm{CO_2e}$, where $\mathcal{CI}_{j,1}$ measures the direct emission intensity of sector j
- The vector of consumption-based carbon intensities is equal to:

$$\mathcal{CI}^{(y)} = \tilde{\mathcal{L}}\,\mathcal{CI}_1 := \mathcal{CI}_{\mathrm{total}}$$

Imported and exported carbon emissions

- Let $y = (y_{j,r})$ be the $n \times p$ matrix, where $y_{j,r}$ is the final demand of the j^{th} sector and the r^{th} region
- We have:

$$\mathcal{CE}^{(y)} = \mathcal{CI}^{(y)} \odot y$$

where $\mathcal{CE}^{(y)}$ is the $n \times p$ matrix of carbon emissions

Imported and exported carbon emissions

- The consumption-based carbon emissions of the r^{th} region is then equal to:

$$\mathcal{CE}^{(y,r)} = \mathbf{1}_n^\top \left(\mathcal{CE}^{(y)} \mathbf{e}_r\right)$$

while the imported and exported carbon emissions of the r^{th} region are:

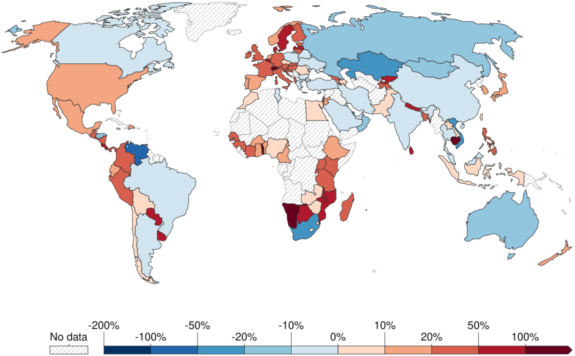
$$\mathcal{CE}_{\mathrm{imported}}^{(y,r)} = \sum_{j \notin r} \left(\mathcal{CE}^{(y)} \mathbf{e}_r\right)_j$$

and:

$$\mathcal{CE}_{\mathrm{exported}}^{(y,r)} = \sum_{j \in r} \sum_{k \neq r} \left(\mathcal{CE}^{(y)} \mathbf{e}_r\right)_j$$

Imported and exported carbon emissions

Figure 64: CO_2 emissions embedded in trade, 2020



This is measured as emissions exported or imported as a percentage of domestic production emissions. Positive values (red) represent net importers of CO_2. Negative values (blue) represent net exporters of CO_2.

Source: `https://ourworldindata.org/consumption-based-co2`.

Imported and exported carbon emissions

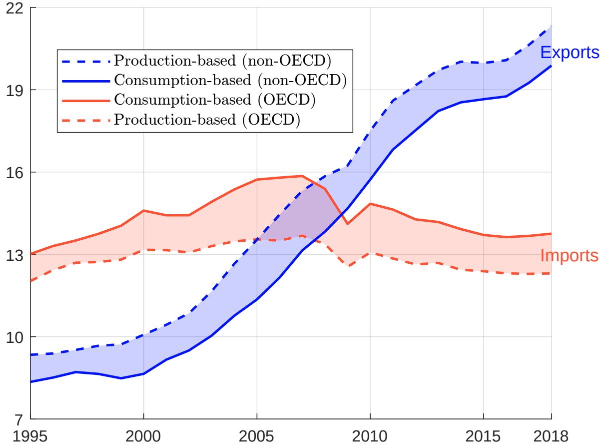
Table 17: Top importing and exporting countries by carbon emissions (in $\mathrm{MtCO_2e}$, 2018)

		Top importers				Top exporters	
Rank	ISO	Country	Balance	Rank	ISO	Country	Balance
1	USA	United States	−752.10	1	CHN	China	895.45
2	JPN	Japan	−160.62	2	RUS	Russian Federation	343.48
3	DEU	Germany	−128.73	3	ZAF	South Africa	122.47
4	GBR	United Kingdom	−123.77	4	IND	India	106.10
5	FRA	France	−111.65	5	TWN	Chinese Taipei	77.03
6	ITA	Italy	−80.09	6	SGP	Singapore	62.19
7	HKG	Hong Kong, China	−70.14	7	KOR	Korea	54.35
8	CHE	Switzerland	−44.53	8	CAN	Canada	53.12
9	PHL	Philippines	−40.49	9	VNM	Viet Nam	52.31
10	SWE	Sweden	−29.67	10	MYS	Malaysia	46.52

Source: Yamano and Guilhoto (2020), `https://stats.oecd.org` & Author's calculations.

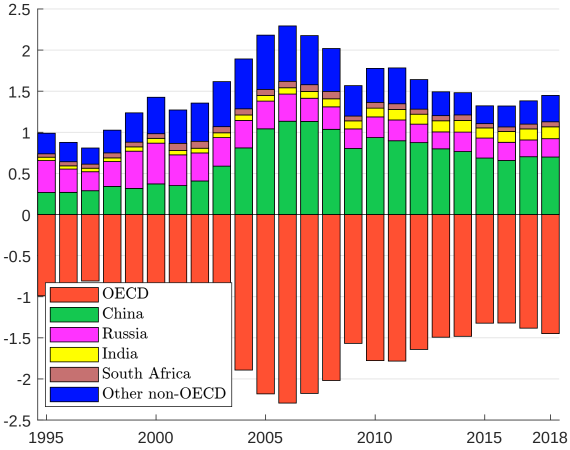
Imported and exported carbon emissions

Figure 65: Total production- and consumption-based CO_2 emitted by OECD and non-OECD countries (in $\mathrm{GtCO_2e}$)



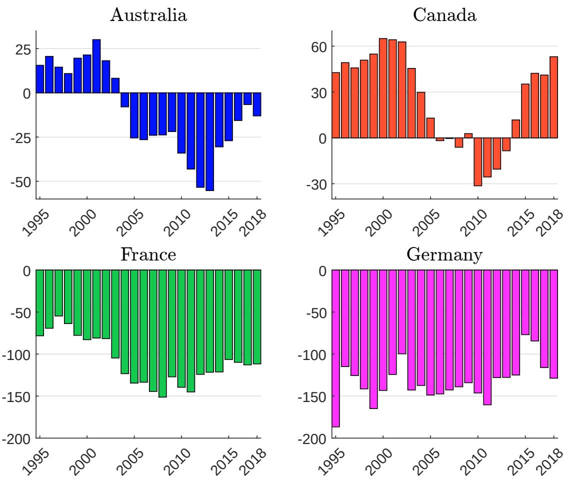
Imported and exported carbon emissions

Figure 66: Decomposition of OECD imported emissions (in $\mathrm{GtCO_2e}$)



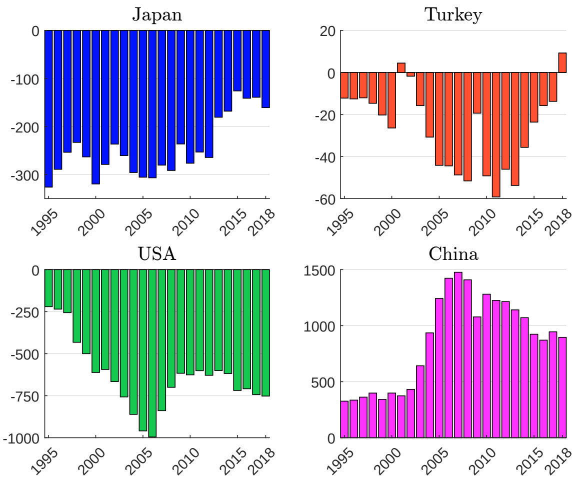
Imported and exported carbon emissions

Figure 67: Net exported emissions (in $\mathrm{MtCO_2e}$)



Imported and exported carbon emissions

Figure 68: Net exported emissions (in $\mathrm{MtCO_2e}$)



Value added approach

Impact on production prices

- The **absolute amount of the carbon tax** for sector j is equal to:

$$T_{\mathrm{direct},j} = \boldsymbol{\tau}_j \mathcal{CE}_{1,j}$$

where $\boldsymbol{\tau}_j$ is the nominal carbon tax expressed in $\$/\mathrm{tCO_2e}$ and $\mathcal{CE}_{1,j}$ is the Scope 1 emissions of the sector

- We deduce that the **carbon tax rate** is equal to:

$$t_{\mathrm{direct},j} = \frac{T_{\mathrm{direct},j}}{x_j} = \frac{\boldsymbol{\tau}_j \mathcal{CE}_{1,j}}{x_j} = \boldsymbol{\tau}_j \mathcal{CI}_{1,j}$$

- Note that $t_{\mathrm{direct},j}$ has no unit and is equal to the product of the tax and the Scope 1 carbon intensity
- The input-output model implies that:

$$p_j x_j = \sum_{i=1}^{n} Z_{i,j} p_i + \sum_{k=1}^{m} V_{k,j} \psi_k + T_{\mathrm{direct},j}$$

Value added approach
Impact on production prices

- We deduce that:

$$p_j = \sum_{i=1}^{n} A_{i,j} p_i + \sum_{k=1}^{m} B_{k,j} \psi_k + t_{\mathrm{direct},j} = \sum_{i=1}^{n} A_{i,j} p_i + \upsilon_j + t_{\mathrm{direct},j}$$

- It follows that:

$$p = \left(I_n - A^\top\right)^{-1} \left(\upsilon + t_{\mathrm{direct}}\right)$$

where $t_{\mathrm{direct}} = \left(t_{\mathrm{direct},1}, \ldots, t_{\mathrm{direct},n}\right)$ is the vector of direct tax rates

Remark

We recover the **cost-push price model**, where the vector υ of value added ratios is replaced by $\upsilon + t_{\mathrm{direct}}$. Because $\Delta \upsilon = t_{\mathrm{direct}}$, the vector of price changes due to the carbon tax is equal to:

$$\Delta p = \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}$$

Value added approach
Impact on the price index

- The definition of a price index is:

$$\mathcal{PI} = \sum_{i=1}^{n} \alpha_i p_i = \alpha^\top p$$

where $\alpha = (\alpha_1, \ldots, \alpha_n)$ are the weights of the items in the basket

- We deduce that the **inflation rate** is:

$$\pi = \frac{\Delta \mathcal{PI}}{\mathcal{PI}^-} = \frac{\mathcal{PI} - \mathcal{PI}^-}{\mathcal{PI}^-} = \frac{\alpha^\top \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}}{\alpha^\top \left(I_n - A^\top\right)^{-1} \upsilon}$$

- We can simplify this formula because $p^- = \left(I_n - A^\top\right)^{-1} \upsilon = \mathbf{1}_n$ and $\mathbf{1}_n^\top \alpha = 1$. Finally, we have:

$$\pi = \alpha^\top \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}$$

Value added approach

Computation of the total tax amount

- The **total tax cost** is equal to:

$$\mathcal{T}_{\mathrm{total}} = x \odot \Delta p = x \odot \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}$$

  while the direct tax cost is $\mathcal{T}_{\mathrm{direct}} = x \odot t_{\mathrm{direct}}$

- We can show that the total tax cost is greater than the direct tax cost for all the sectors:

$$\mathcal{T}_{\mathrm{total},j} \geq \mathcal{T}_{\mathrm{direct},j}$$

- Since the total cost to the economy is equal to $\mathcal{C}ost_{\mathrm{total}} = \sum_{j=1}^{n} \mathcal{T}_{\mathrm{total},j} = x^\top \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}$, the **tax incidence** is then equal to:

$$\mathcal{TI} = \frac{\mathcal{C}ost_{\mathrm{total}}}{\mathbf{1}_n^\top x} = \frac{x^\top \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}}{\mathbf{1}_n^\top x}$$

Value added approach

Common mistakes in calculating total tax costs

Remark

In some research papers, we can find two formulas that seem to be intuitive:

$$T'_{\mathrm{total}} = \left(I_n - A^\top\right)^{-1} T_{\mathrm{direct}}$$

and:

$$T''_{\mathrm{total}} = \boldsymbol{\tau} \odot \boldsymbol{\mathcal{CE}}_{\mathrm{total}}$$

The two previous equations are generally wrong

Value added approach
Mathematical properties

- Let us denote by $f(\boldsymbol{\tau})$ the function f that depends on the vector $\boldsymbol{\tau} = (\boldsymbol{\tau}_1, \ldots, \boldsymbol{\tau}_n)$ of carbon taxes
- The functions Δp, π, T_{total}, $\mathcal{C}ost_{\mathrm{total}}$ and $\mathcal{TI}$ are homogeneous[3] and additive[4]
- Let $\lambda \geq 0$ be a positive scalar. We have:

$$\begin{aligned}
\Delta p(\lambda\boldsymbol{\tau}) &= \left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}(\lambda\boldsymbol{\tau}) \\
&= \lambda\left(I_n - A^\top\right)^{-1} t_{\mathrm{direct}}(\boldsymbol{\tau}) \\
&= \lambda\Delta p(\boldsymbol{\tau})
\end{aligned}$$

[3] This means that $f(\lambda\boldsymbol{\tau}) = \lambda f(\boldsymbol{\tau})$.
[4] We have $f(\boldsymbol{\tau} + \boldsymbol{\tau}') = f(\boldsymbol{\tau}) + f(\boldsymbol{\tau}')$.

Value added approach
Mathematical properties

- If the tax is uniform $\boldsymbol{\tau} = \tau \mathbf{1}_n$, the vector of total tax amount is the product of the tax by the total emissions:

$$T_{\text{total}}\left(\tau \mathbf{1}_n\right) = \tau\, \boldsymbol{\mathcal{CE}}_{\text{total}}$$

- The **tax incidence for a given sector is then proportional to the direct plus indirect carbon emissions** of the sector
- At the global level, the tax incidence is equal to the carbon tax multiplied by the total carbon intensity of the world:

$$\mathcal{TI}\left(\tau \mathbf{1}_n\right) = \frac{\mathbf{1}_n^\top \tau\, \boldsymbol{\mathcal{CE}}_{\text{total}}}{\mathbf{1}_n^\top x} = \tau\, \boldsymbol{\mathcal{CI}}_{\text{total}}$$

Value added approach

Example #6

Table 18: Environmentally extended monetary input-output table

Sector	Z				y	x	$\mathcal{CE}_1$	$\mathcal{CI}_1$
Energy	500	800	1 600	1 250	850	5 000	500	100
Materials	500	400	1 600	625	875	4 000	200	50
Industrials	250	800	2 400	1 250	3 300	8 000	200	25
Services	100	200	800	4 375	7 025	12 500	125	10
Value added	3 650	1 800	1 600	5 000				
Income	5 000	4 000	8 000	12 500				

The values of $Z_{i,j}$, y_j, x_j and $V_{1,j}$ are in \$ mn. The carbon emissions are expressed in $\mathrm{ktCO_2e}$, while the carbon intensities are expressed in $\mathrm{tCO_2e/\$ \ mn}$.

Value added approach

- We have:

$$A = Z \operatorname{diag}^{-1}(x) = \begin{pmatrix} 0.10 & 0.20 & 0.20 & 0.10 \\ 0.10 & 0.10 & 0.20 & 0.05 \\ 0.05 & 0.20 & 0.30 & 0.10 \\ 0.02 & 0.05 & 0.10 & 0.35 \end{pmatrix}$$

  and:

$$\tilde{\mathcal{L}} = \left(I_4 - A^\top\right)^{-1} = \begin{pmatrix} 1.1881 & 0.1678 & 0.1430 & 0.0715 \\ 0.3894 & 1.2552 & 0.4110 & 0.1718 \\ 0.4919 & 0.4336 & 1.6303 & 0.2993 \\ 0.2884 & 0.1891 & 0.3044 & 1.6087 \end{pmatrix}$$

- Then, we calculate the vector υ of value added ratios:

$$\upsilon = \begin{pmatrix} 3\,650/5\,000 \\ 1\,800/4\,000 \\ 1\,600/8\,000 \\ 5\,000/12\,500 \end{pmatrix} = \begin{pmatrix} 0.73 \\ 0.45 \\ 0.20 \\ 0.40 \end{pmatrix}$$

Value added approach

- We introduce a differentiated carbon tax: $\boldsymbol{\tau}_1 = \$200/\mathrm{tCO_2e}$ and $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_3 = \boldsymbol{\tau}_4 = \$100/\mathrm{tCO_2e}$
- The direct tax costs are 100, 20, 20 and 12.5 million dollars for Energy, Materials, Industrials and Services respectively
- We deduce that the vector of carbon tax rates is $t_{\mathrm{direct}} = (2.00\%, 0.50\%, 0.25\%, 0.10\%)$
- It follows that:

$$p = \left(I_n - A^\top\right)^{-1} \left(\upsilon + t_{\mathrm{direct}}\right) = \begin{pmatrix} 1.0250 \\ 1.0153 \\ 1.0164 \\ 1.0091 \end{pmatrix}$$

- If we assume that the basket of goods and services is $\alpha = (10\%, 20\%, 30\%, 40\%)$, the price index $\mathcal{PI}$ is 1.0141
- The inflation rate π is 1.410%

Value added approach

Table 19: Total carbon costs (in $ mn) (differentiated tax)

Sector	T_{direct}	T_{total}	T'_{total}	T''_{total}	$\mathcal{CE}_{\mathrm{direct}}$	$\mathcal{CE}_{\mathrm{total}}$
Energy	100.00	125.15	125.92	131.49	500.00	657.44
Materials	20.00	61.05	74.41	45.48	200.00	454.76
Industrials	20.00	131.05	94.21	91.70	200.00	916.97
Services	12.50	113.54	58.82	77.49	125.00	774.92
Sum	152.50	430.79	353.36	346.15	1 025.00	2 804.10

Table 20: Total carbon costs (in $ mn) (uniform tax of $\$100/\mathrm{tCO_2e}$)

Sector	T_{direct}	T_{total}	T'_{total}	T''_{total}	$\mathcal{CE}_{\mathrm{direct}}$	$\mathcal{CE}_{\mathrm{total}}$
Energy	50.00	65.74	66.51	65.74	500.00	657.44
Materials	20.00	45.48	54.94	45.48	200.00	454.76
Industrials	20.00	91.70	69.62	91.70	200.00	916.97
Services	12.50	77.49	44.40	77.49	125.00	774.92
Sum	102.50	280.41	235.47	280.41	1 025.00	2 804.10

Pass-through rate

> *"[...] cost pass-through describes what happens when a business changes the price of the production or services it sells following a change in the cost of producing them" RBB Economics (2014).*

A pass-through rate is related to the **supply and demand elasticity**:

- In the case of competition, the pass-through rate $\phi \in [0, 100\%]$ is:

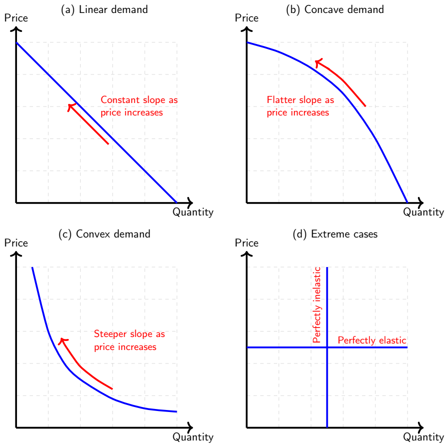
$$\phi = \frac{\mathrm{d}\, p}{\mathrm{d}\, \tau} = \frac{\text{price sensitivity of supply}}{\text{price sensitivity of supply} - \text{price sensitivity of demand}}$$

- In a monopolistic situation, the previous formula becomes ($\phi \geq 50\%$):

$$\phi = \frac{1}{2 + \text{elasticity of the slope of inverse demand}}$$

Pass-through rate

Figure 69: Demand curvature



Source: RBB Economics (2014, Figure 2, page 16).

Pass-through rate

Table 21: Pass-through rates (in %) for intensive sectors

Sector	Rate
Electricity, gas and steam	100%
Petroleum refining	100%
Base metals	78%
Mining	78%
Waste/wastewater	78%
Land transport	78%
Fishery	75%
Non-metallic minerals	60%
Agriculture	50%
Chemicals	40%
Maritime transport	30%
Aviation	30%
Paper	10%

Source: Sautel *et al.* (2022, page 35).

Pass-through integration

Recurrence formula without pass-through

- We have:

$$\Delta p = \tilde{\mathcal{L}} \Delta v = \sum_{k=0}^{\infty} \left(A^\top \right)^k \Delta v = \sum_{k=0}^{\infty} \Delta p_{(k)}$$

  where $\Delta p_{(k)} = \left(A^\top \right)^k \Delta v$ is the price impact at the k^{th} tier

- In fact, $\Delta p_{(k)}$ satisfies the following recurrence relation:

$$\left\{ \begin{array}{l} \Delta p_{(k)} = A^\top \Delta p_{(k-1)} \\ \Delta p_{(0)} = \Delta v \end{array} \right.$$

- If we consider the price p_j of sector j, we have $\Delta p_{(0),j} = \Delta v_j$ and:

$$\Delta p_{(k),j} = \sum_{i=1}^{n} A_{i,j} \Delta p_{(k-1),i}$$

Pass-through integration

Cascading effect of the carbon tax

- In the zeroth round, it induces an additional cost $\Delta\upsilon_j$, which is fully passed on to the price p_j of the sector
- The new price is then $p_j + \Delta p_{(0),j} = p_j + \Delta\upsilon_j$
- In the first round, sector j faces new additional costs due to the price increase of intermediate consumption:

$$\Delta p_{(1),j} = \sum_{i=1}^{n} A_{i,j} \Delta p_{(0),i} = \sum_{i=1}^{n} A_{i,j} \Delta\upsilon_i$$

- The iteration process continues and we have at the second round:

$$\Delta p_{(2),j} = \sum_{i=1}^{n} A_{i,j} \Delta p_{(1),i} = \sum_{i=1}^{n} \sum_{k=1}^{n} A_{i,j} A_{k,i} \Delta v_k$$

Pass-through integration

Recurrence formula with pass-through

- We introduce the pass-through mechanism
- We have $\Delta p_{(0),j} = \phi_j \Delta \upsilon_j$ where ϕ_j denotes the pass-through rate of sector j
- In the first round, we have:

$$\Delta p_{(1),j} = \sum_{i=1}^{n} A_{i,j} \left(\phi_i \Delta p_{(0),i} \right) = \sum_{i=1}^{n} A_{i,j} \left(\phi_i \Delta \upsilon_i \right)$$

- More generally, the recurrence relation is:

$$\Delta p_{(k),j} = \sum_{i=1}^{n} A_{i,j} \phi_i \Delta p_{(k-1),i}$$

Pass-through integration

Recurrence formula with pass-through

- Let $\phi = (\phi_1, \ldots, \phi_n)$ and $\Phi = \operatorname{diag}(\phi)$ be the pass-through vector and matrix
- The recurrence matrix form is:

$$\begin{cases} \Delta p_{(k)} = A^\top \Phi \Delta p_{(k-1)} \\ \Delta p_{(0)} = \Phi \Delta v \end{cases}$$

- We deduce that:

$$\begin{aligned} \Delta p &= \sum_{k=0}^{\infty} \left(A^\top \Phi\right)^k \Phi \Delta v \\ &= \left(I_n - A^\top \Phi\right)^{-1} \Phi \Delta v \\ &= \tilde{\mathcal{L}}(\phi) \Delta v \end{aligned}$$

where $\tilde{\mathcal{L}}(\phi) = \left(I_n - A^\top \Phi\right)^{-1} \Phi$

Pass-through integration

Analytical formula

- Finally, we have:

$$\boldsymbol{\Delta p} = \tilde{\mathcal{L}}(\phi)\,\boldsymbol{\Delta \upsilon} := \left(I_n - A^\top \Phi\right)^{-1} \Phi \boldsymbol{\Delta \upsilon}$$

- Since A is a substochastic matrix and Φ is a positive diagonal matrix, we verify that

$$\phi' \succeq \phi \Rightarrow \tilde{\mathcal{L}}(\phi') \succeq \tilde{\mathcal{L}}(\phi)$$

- The lower bound is then reached when $\phi = \mathbf{0}_n$
- The upper bound is reached when $\phi = \mathbf{1}_n$

Application to the carbon tax

- We have $\Delta \upsilon = t_{\mathrm{direct}}$
- The cost paid by producers is:

$$\begin{array}{rcl} T_{\mathrm{producer}} & = & x \odot (I_n - \Phi)\, t_{\mathrm{direct}} \\ & = & x \odot (\mathbf{1}_n - \phi) \odot t_{\mathrm{direct}} \\ & = & (\mathbf{1}_n - \phi) \odot T_{\mathrm{direct}} \end{array}$$

- The cost paid by consumers (downstream of the value chain) is:

$$T_{\mathrm{consumer}} = T_{\mathrm{downstream}} = x \odot \tilde{\mathcal{L}}(\phi)\, t_{\mathrm{direct}}$$

- We deduce that:

$$T_{\mathrm{total}} = T_{\mathrm{producer}} + T_{\mathrm{consumer}} = x \odot \left(I_n - \Phi + \tilde{\mathcal{L}}(\phi) \right) t_{\mathrm{direct}}$$

- If $\phi_j = 100\%$, we have $\tilde{\mathcal{L}}(\mathbf{1}_n) = \tilde{\mathcal{L}}$ and $\Delta p = \tilde{\mathcal{L}}\, t_{\mathrm{direct}}$
- If $\phi_j = 0\%$, we have $\tilde{\mathcal{L}}(\mathbf{0}_n) = \mathbf{0}_{n,n}$, $\Delta p = \mathbf{0}_n$, $T_{\mathrm{producer}} = T_{\mathrm{direct}}$ but $T_{\mathrm{consumer}} = \mathbf{0}_n$

Application to the carbon tax

Remark

The functions Δp, π, T_{total}, $\mathcal{C}ost_{\mathrm{total}}$ and $\mathcal{TI}$ remain homogeneous and additive with respect to $\boldsymbol{\tau}$. We can also show that:

$$\boldsymbol{\phi}' \succeq \boldsymbol{\phi} \Rightarrow T_{\mathrm{total}}\left(\boldsymbol{\tau}, \boldsymbol{\phi}'\right) \succeq T_{\mathrm{total}}\left(\boldsymbol{\tau}, \boldsymbol{\phi}\right)$$

The effects of the tax is maximum when $\boldsymbol{\phi} = \mathbf{1}_n$ and minimum when $\boldsymbol{\phi} = \mathbf{0}_n$. If we consider a uniform pass-through, the total cost of the carbon tax is an increasing function of the pass-through rate

Application to the carbon tax

Example #7

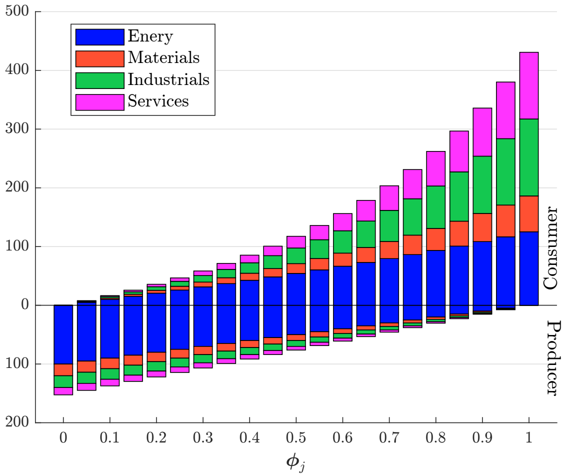
Table 22: Environmentally extended monetary input-output table

Sector	Z				y	x	$\mathcal{CE}_1$	$\mathcal{CI}_1$
Energy	500	800	1 600	1 250	850	5 000	500	100
Materials	500	400	1 600	625	875	4 000	200	50
Industrials	250	800	2 400	1 250	3 300	8 000	200	25
Services	100	200	800	4 375	7 025	12 500	125	10
Value added	3 650	1 800	1 600	5 000				
Income	5 000	4 000	8 000	12 500				

- The values of $Z_{i,j}$, y_j, x_j and $V_{1,j}$ are in \$ mn
- The carbon emissions are expressed in $\mathrm{ktCO_2e}$, while the carbon intensities are expressed in $\mathrm{tCO_2e}/\$\ \mathrm{mn}$
- The carbon tax is differentiated: $\boldsymbol{\tau}_1 = \$200/\mathrm{tCO_2e}$ and $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_3 = \boldsymbol{\tau}_4 = \$100/\mathrm{tCO_2e}$

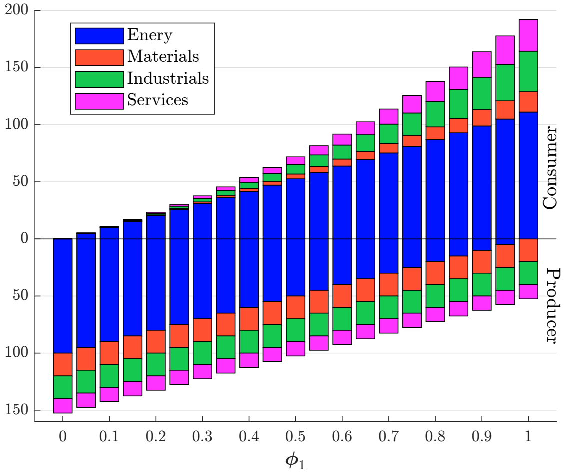
Application to the carbon tax

Figure 70: Producer and consumer cost contributions (uniform pass-through)



Application to the carbon tax

Figure 71: Producer and consumer cost contributions ($\phi_2 = \phi_3 = \phi_4 = 0\%$)



Empirical results (Desnos *et al.*, 2023)

- The direct cost is:

$$\mathcal{C}ost_{\text{direct}} = \sum_{j=1}^{n} T_{\text{direct}}$$

- The total cost is:

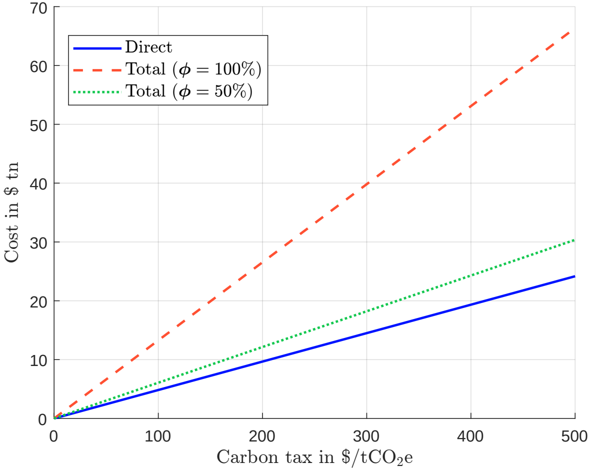
$$\mathcal{C}ost_{\text{total}} = \sum_{j=1}^{n} T_{\text{total}}$$

- If the carbon tax is set at $\$100/\mathrm{tCO_2e}$, the direct cost is \$4.8 tn, while the total cost is \$6.1 tn if $\phi = 50\%$ and \$13.3 tn if $\phi = 100\%$
- These correspond to 2.8%, 3.6% and 7.8% of the world GDP respectively
- If we apply a carbon tax of $\$500/\mathrm{tCO_2e}$, these costs become \$24.2, \$30.4 and \$66.4 tn respectively
- The relationship between total costs and the pass-through parameter is cubic:

$$\frac{\mathcal{C}ost_{\text{total}}\left(\boldsymbol{\tau}, \phi\mathbf{1}_n\right)}{\mathcal{C}ost_{\text{direct}}\left(\boldsymbol{\tau}, \phi\mathbf{1}_n\right)} \approx 1 + m_{(1-\infty)}\phi^3$$

Empirical results (Desnos *et al.*, 2023)

Figure 72: World economic cost in $ tn (global analysis, uniform tax, Exiobase 2022)



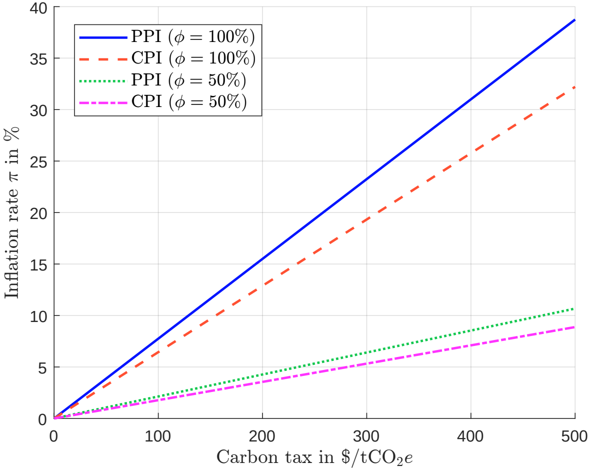
Empirical results (Desnos *et al.*, 2023)

- Producer price index (PPI): the basket weights are proportional to the output ($\alpha_j \propto x_j$)
- Consumer price index (CPI): the basket weights are proportional to the final demand ($\alpha_j \propto y_j$)
- For a carbon tax of \$500/$\mathrm{tCO_2e}$ and a pass-through rate of 100%, the PPI inflation rate is close to 40%, while the CPI inflation rate reaches 30%
- The dispersion of the inflation rates explained by three factors (basket composition, value chain impact and direct carbon emissions of the country):

$$\pi = \underbrace{\alpha^\top}_{\text{Basket}} \cdot \underbrace{\tilde{\mathcal{L}}(\phi)}_{\text{Value chain}} \cdot \underbrace{t_{\mathrm{direct}}}_{\text{Scope 1}}$$

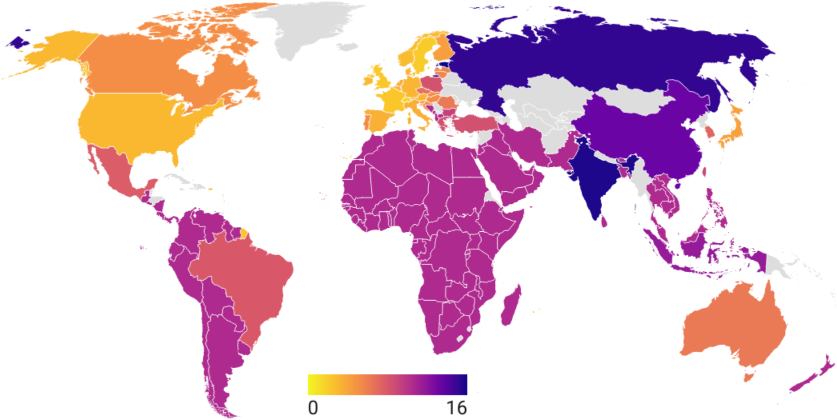
Empirical results (Desnos *et al.*, 2023)

Figure 73: World inflation rate in % (global analysis, uniform tax, Exiobase 2022)



Empirical results (Desnos *et al.*, 2023)

Figure 74: Production inflation rate in % (global analysis, uniform tax, $\tau = \$100/\mathrm{tCO_2e}$, $\phi = 100\%$, Exiobase 2022)

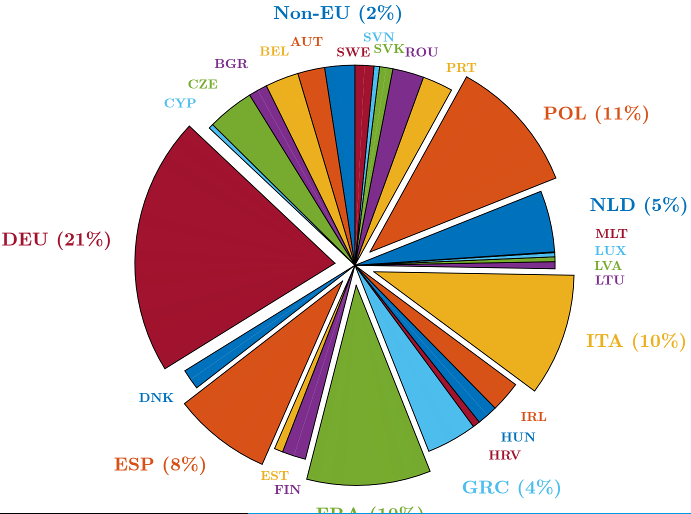


Empirical results (Desnos *et al.*, 2023)

Regional taxation

Empirical results (Desnos *et al.*, 2023)

Figure 75: Cost breakdown (EU, uniform tax, $\phi = 50\%$, Exiobase 2022)



Empirical results (Desnos *et al.*, 2023)

Table 23: Domestic and foreign impacts (in $ bn) of a regional tax (uniform taxation, $\phi = 100\%$, Exiobase 2022)

Carbon tax	Domestic impact			Foreign impact		
	EU	USA	China	EU	USA	China
$\$100/\mathrm{tCO_2e}$	792	886	4 710	104	118	257
$\$250/\mathrm{tCO_2e}$	1 979	2 215	11 774	261	296	643
$\$500/\mathrm{tCO_2e}$	3 959	4 430	23 549	521	592	1 287

Empirical results (Desnos *et al.*, 2023)

Table 24: Fifteen most affected foreign countries (uniform tax, $\tau = \$100/\mathrm{tCO_2e}$, $\phi = 100\%$, Exiobase 2022)

Rank	EU tax		US tax		Chinese tax	
1	ROW	25.25%	CHN	24.74%	ROW	36.89%
2	CHN	23.62%	ROW	18.60%	USA	12.95%
3	USA	11.45%	CAN	9.35%	KOR	8.87%
4	GBR	8.77%	MEX	8.51%	IND	6.91%
5	CHE	4.32%	KOR	6.89%	JPN	6.44%
6	KOR	4.05%	JPN	5.05%	DEU	3.61%
7	IND	3.67%	IND	4.28%	MEX	2.19%
8	JPN	3.31%	DEU	2.80%	FRA	1.88%
9	TUR	2.62%	BRA	2.51%	GBR	1.83%
10	TWN	2.08%	GBR	2.34%	BRA	1.75%
11	CAN	2.06%	FRA	1.63%	IDN	1.74%
12	RUS	1.96%	TWN	1.59%	CAN	1.62%
13	BRA	1.90%	IRL	1.47%	ITA	1.59%
14	MEX	1.70%	ITA	1.43%	AUS	1.31%
15	NOR	1.46%	NLD	1.23%	TUR	1.11%

Empirical results (Sautel *et al.*, 2022)

"The total additional cost of introducing a price of €250 per tonne of CO_2 to be paid by French emitting installations is €57.6 billion, or about 2.5 points of GDP. Of this total, €7 billion corresponds to purchases by foreign operators and investments by French and foreign operators. [...] Of this €50.3 billion, French companies would ultimately bear 57% of the additional costs, or about €28.7 billion. The rest would be passed on to final demand, i.e. 21.6 billion euros." (Sautel et al., 2022, page 39).

Pass-through modeling

It is common to assume that the pass-through rate follows a beta distribution, as it is a parameter between 0 and 1:

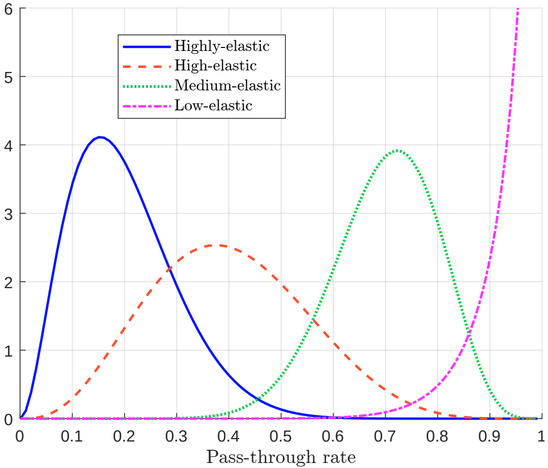
$$\phi \sim \mathcal{B}(\alpha, \beta)$$

Table 25: Probabilistic characterization of the four pass-through types

Statistic		Highly-elastic	High-elastic	Medium-elastic	Low-elastic
Parameters	α	3.0	4.0	14.0	12.0
	β	12.0	6.0	6.0	0.6
Moments	μ_ϕ	20%	40%	70%	95%
	σ_ϕ	10%	15%	10%	6%
Range	$Q_\phi(2.5\%)$	5%	14%	49%	79%
	$Q_\phi(97.5\%)$	43%	70%	87%	100%

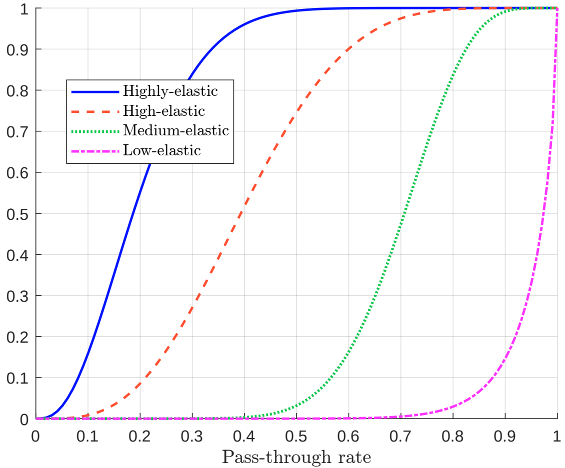
Pass-through modeling

Figure 76: Probability density function of pass-through rates



Pass-through modeling

Figure 77: Probability density function of pass-through rates



Impact of a global carbon tax

- The total cost vector is $\mathcal{T}_{\mathrm{total}} = x \odot \left(I_n - \mathrm{diag}\left(\phi\right) + \tilde{\mathcal{L}}\left(\phi\right)\right) t_{\mathrm{direct}}$
- The pass-through rate of the sector is constant and equal to the mean of the corresponding beta distribution: $\phi = \mu_\phi$
- We consider two decompositions:

$$\left\{\begin{array}{l} \mathcal{T}_{\mathrm{producer}} = x \odot \left(\mathbf{1}_n - \phi\right) \odot t_{\mathrm{direct}} \\ \mathcal{T}_{\mathrm{downstream}} = x \odot \tilde{\mathcal{L}}\left(\phi\right) t_{\mathrm{direct}} \end{array}\right.$$

and:

$$\left\{\begin{array}{l} \mathcal{T}_{\mathrm{direct}} = x \odot t_{\mathrm{direct}} \\ \mathcal{T}_{\mathrm{indirect}} = \mathcal{T}_{\mathrm{total}} - \mathcal{T}_{\mathrm{direct}} = x \odot \left(\tilde{\mathcal{L}}\left(\phi\right) - \mathrm{diag}\left(\phi\right)\right) t_{\mathrm{direct}} \end{array}\right.$$

- Government revenue is equal to the direct cost of the carbon tax:

$$\mathcal{R}_{\mathrm{government}} = \mathcal{T}_{\mathrm{direct}} = x \odot t_{\mathrm{direct}}$$

- Inflation rates are calculated using the following formula:

$$\pi = \alpha^\top \Delta p = \alpha^\top \tilde{\mathcal{L}}\left(\phi\right) t_{\mathrm{direct}}$$

- $\alpha_{\mathrm{ppi}} = x / \left(\mathbf{1}_n^\top x\right)$ and $\alpha_{\mathrm{cpi}} = y / \left(\mathbf{1}_n^\top y\right)$

Empirical results (Roncalli and Semet, 2024)

Table 26: Economic impact of a global carbon tax ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Region	Cost						Revenue
	$\mathcal{C}_{\text{total}}$	$\mathcal{C}_{\text{direct}}$	$\mathcal{C}_{\text{indirect}}$	$\mathcal{C}_{\text{producer}}$	$\mathcal{C}_{\text{downstream}}$	$\mathcal{C}_{\text{net}}$	$\mathcal{R}_{\text{government}}$
World	5.01%	2.82%	2.18%	0.93%	4.08%	2.18%	2.82%
RUS	12.79%	8.55%	4.24%	1.44%	11.34%	4.24%	8.55%
IND	11.38%	6.83%	4.55%	2.28%	9.11%	4.55%	6.83%
IDN	7.85%	5.53%	2.31%	2.08%	5.77%	2.31%	5.53%
CHN	7.47%	3.44%	4.03%	1.21%	6.26%	4.03%	3.44%
BGR	7.07%	3.94%	3.12%	0.89%	6.18%	3.12%	3.94%
DNK	1.47%	0.98%	0.49%	0.54%	0.93%	0.49%	0.98%
FRA	1.39%	0.79%	0.60%	0.35%	1.04%	0.60%	0.79%
SWE	1.21%	0.59%	0.62%	0.21%	1.00%	0.62%	0.59%
LUX	1.15%	0.51%	0.64%	0.35%	0.80%	0.64%	0.51%
CHE	0.75%	0.30%	0.45%	0.16%	0.59%	0.45%	0.30%

Only 20% of the costs are borne by producers

Empirical results (Roncalli and Semet, 2024)

Table 27: Economic impact of a EU carbon tax ($\$100/\text{tCO}_2\text{e}$, Exiobase 2022)

Region	Cost						Revenue
	$\mathcal{C}_{\text{total}}$	$\mathcal{C}_{\text{direct}}$	$\mathcal{C}_{\text{indirect}}$	$\mathcal{C}_{\text{producer}}$	$\mathcal{C}_{\text{downstream}}$	$\mathcal{C}_{\text{net}}$	$\mathcal{R}_{\text{government}}$
World	0.36%	0.22%	0.14%	0.07%	0.28%	0.14%	0.22%
BGR	6.30%	3.94%	2.35%	0.89%	5.41%	2.35%	3.94%
GRC	5.64%	4.61%	1.03%	2.52%	3.12%	1.03%	4.61%
POL	5.21%	3.44%	1.77%	0.98%	4.24%	1.77%	3.44%
CYP	4.86%	3.94%	0.92%	2.49%	2.37%	0.92%	3.94%
CZE	3.90%	2.13%	1.76%	0.44%	3.46%	1.76%	2.13%
ROU	3.60%	2.19%	1.41%	0.69%	2.91%	1.41%	2.19%
PRT	3.28%	2.13%	1.15%	0.70%	2.58%	1.15%	2.13%
LTU	3.22%	2.41%	0.82%	1.00%	2.22%	0.82%	2.41%
LVA	3.11%	2.15%	0.96%	1.07%	2.05%	0.96%	2.15%
HRV	2.88%	2.18%	0.70%	0.89%	1.99%	0.70%	2.18%
SVK	2.72%	1.62%	1.09%	0.42%	2.30%	1.09%	1.62%
HUN	2.70%	1.83%	0.87%	0.61%	2.08%	0.87%	1.83%
SVN	2.38%	1.51%	0.87%	0.47%	1.91%	0.87%	1.51%
FIN	2.27%	1.36%	0.91%	0.36%	1.91%	0.91%	1.36%
ESP	1.82%	1.15%	0.68%	0.41%	1.41%	0.68%	1.15%

95% of the costs fall on European countries

Empirical results (Roncalli and Semet, 2024)

Table 28: Economic impact of a US carbon tax ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Region	Cost						Revenue
	$\mathcal{C}_{\text{total}}$	$\mathcal{C}_{\text{direct}}$	$\mathcal{C}_{\text{indirect}}$	$\mathcal{C}_{\text{producer}}$	$\mathcal{C}_{\text{downstream}}$	$\mathcal{C}_{\text{net}}$	$\mathcal{R}_{\text{government}}$
World	0.44%	0.29%	0.14%	0.07%	0.37%	0.14%	0.29%
USA	1.96%	1.40%	0.57%	0.34%	1.62%	0.57%	1.40%
CAN	0.18%	0.00%	0.18%	0.00%	0.18%	0.18%	0.00%
MEX	0.18%	0.00%	0.18%	0.00%	0.18%	0.18%	0.00%
KOR	0.07%	0.00%	0.07%	0.00%	0.07%	0.07%	0.00%
IRL	0.06%	0.00%	0.06%	0.00%	0.06%	0.06%	0.00%
BRA	0.05%	0.00%	0.05%	0.00%	0.05%	0.05%	0.00%
TWN	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
ROW	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
IND	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
NLD	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%
GBR	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%
NOR	0.02%	0.00%	0.02%	0.00%	0.02%	0.02%	0.00%
BEL	0.02%	0.00%	0.02%	0.00%	0.02%	0.02%	0.00%
JPN	0.02%	0.00%	0.02%	0.00%	0.02%	0.02%	0.00%
CHN	0.02%	0.00%	0.02%	0.00%	0.02%	0.02%	0.00%

Empirical results (Roncalli and Semet, 2024)

Table 29: Economic impact of a carbon tax in China ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Region	Cost						Revenue
	$\mathcal{C}_{\text{total}}$	$\mathcal{C}_{\text{direct}}$	$\mathcal{C}_{\text{indirect}}$	$\mathcal{C}_{\text{producer}}$	$\mathcal{C}_{\text{downstream}}$	$\mathcal{C}_{\text{net}}$	$\mathcal{R}_{\text{government}}$
World	1.66%	0.81%	0.85%	0.29%	1.38%	0.85%	0.81%
CHN	6.89%	3.44%	3.45%	1.21%	5.68%	3.45%	3.44%
ROW	0.13%	0.00%	0.13%	0.00%	0.13%	0.13%	0.00%
KOR	0.12%	0.00%	0.12%	0.00%	0.12%	0.12%	0.00%
MEX	0.06%	0.00%	0.06%	0.00%	0.06%	0.06%	0.00%
IND	0.05%	0.00%	0.05%	0.00%	0.05%	0.05%	0.00%
IDN	0.05%	0.00%	0.05%	0.00%	0.05%	0.05%	0.00%
JPN	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
POL	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
CZE	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
HUN	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
TUR	0.04%	0.00%	0.04%	0.00%	0.04%	0.04%	0.00%
CAN	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%
BEL	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%
SVK	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%
AUS	0.03%	0.00%	0.03%	0.00%	0.03%	0.03%	0.00%

Empirical results (Roncalli and Semet, 2024)

Table 30: Producer price index estimates ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Rank	Global tax		EU tax		US tax		China tax	
	World	4.08%	World	0.28%	World	0.37%	World	1.38%
1	RUS	11.34%	BGR	5.41%	USA	1.62%	CHN	5.68%
2	IND	9.11%	POL	4.24%	CAN	0.18%	ROW	0.13%
3	CHN	6.26%	CZE	3.46%	MEX	0.18%	KOR	0.12%
4	BGR	6.18%	GRC	3.12%	KOR	0.07%	MEX	0.06%
5	IDN	5.77%	ROU	2.91%	IRL	0.06%	IND	0.05%
6	ROW	5.68%	PRT	2.58%	BRA	0.05%	IDN	0.05%
7	POL	4.86%	CYP	2.37%	TWN	0.04%	JPN	0.04%
8	MEX	4.57%	SVK	2.30%	ROW	0.04%	POL	0.04%
9	TWN	4.41%	LTU	2.22%	IND	0.04%	CZE	0.04%
10	TUR	4.39%	HUN	2.08%	NLD	0.03%	HUN	0.04%
11	CZE	4.03%	LVA	2.05%	GBR	0.03%	TUR	0.04%
12	GRC	3.87%	HRV	1.99%	NOR	0.02%	CAN	0.03%
13	KOR	3.85%	SVN	1.91%	BEL	0.02%	BEL	0.03%
14	AUS	3.82%	FIN	1.91%	JPN	0.02%	SVK	0.03%
15	ROU	3.42%	AUT	1.50%	CHN	0.02%	AUS	0.03%

Empirical results (Roncalli and Semet, 2024)

Table 31: Consumer price index estimates ($\$100/\mathrm{tCO_2e}$, Exiobase 2022)

Rank	Global tax		EU tax		US tax		China tax	
	World	3.53%	World	0.48%	World	0.27%	World	1.15%
1	IDN	6.75%	FRA	5.95%	USA	1.06%	CHN	5.88%
2	CHN	6.35%	CZE	4.07%	MEX	0.16%	ROW	0.16%
3	FRA	6.29%	HRV	3.83%	CAN	0.16%	KOR	0.08%
4	IND	5.98%	GRC	3.59%	IRL	0.05%	AUS	0.07%
5	RUS	5.72%	POL	3.49%	BRA	0.04%	IND	0.07%
6	CZE	4.63%	CYP	3.32%	GBR	0.04%	CAN	0.07%
7	HRV	4.42%	BGR	3.16%	ROW	0.04%	MEX	0.06%
8	GRC	4.35%	SVK	2.80%	KOR	0.03%	TUR	0.05%
9	POL	4.14%	MLT	2.69%	IND	0.03%	IDN	0.04%
10	BGR	3.89%	PRT	2.58%	NOR	0.03%	BRA	0.04%
11	ROW	3.82%	LUX	2.30%	NLD	0.03%	JPN	0.04%
12	TWN	3.73%	HUN	2.20%	LUX	0.03%	BEL	0.04%
13	CYP	3.57%	LTU	2.11%	TWN	0.02%	RUS	0.04%
14	MLT	3.38%	NLD	2.11%	BEL	0.02%	GRC	0.04%
15	SVK	3.36%	SVN	1.90%	TUR	0.02%	POL	0.04%

Empirical results (Roncalli and Semet, 2024)

PPI

- Producer inflation: 4.08%
- Emerging markets are the most affected
- Regional taxation penalize domestic economies

CPI

- Consumer inflation: 3.53%
- Consumption inflation $\neq$ production inflation
- Global value chain (supply and downstream)