Course 2022-2023 in Sustainable Finance

Thierry Roncalli*

*Amundi Asset Management[1]

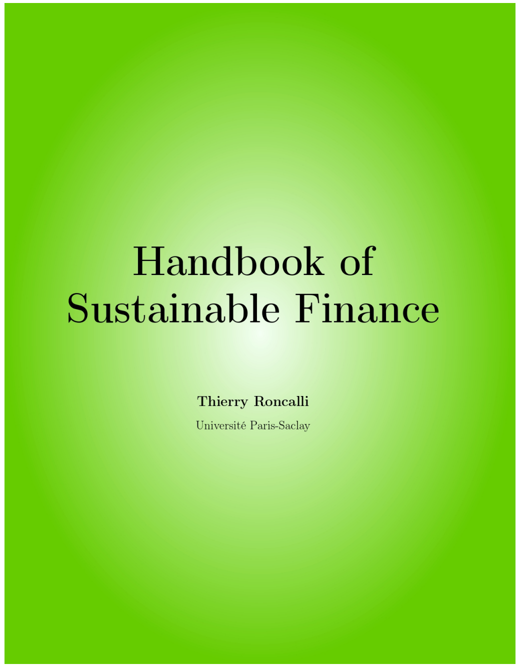
*University of Paris-Saclay

March 2023

[1] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

General information

1. **Overview**
   The objective of this course is to understand the concepts of sustainable finance from the viewpoint of asset owners and managers
2. **Textbook**



General information

1. **Textbook**

SSRN: https://ssrn.com/abstract=4277875

ResearchGate: https://www.researchgate.net/publication/365355205

2. **Slides**

SSRN: https://ssrn.com/abstract=4339823

ResearchGate: https://www.researchgate.net/publication/367479551

3. **Additional materials** (LaTeX + Figures + Matlab programs)

http://www.thierry-roncalli.com/SustainableFinanceBook.html

http://www.thierry-roncalli.com/SustainableFinanceCourse.html

Part 1. Introduction

Part 2. ESG Scoring

Part 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

Part 4. Exercise — Equity Portfolio Optimization with ESG Scores

Part 5. Sustainable Financial Products, Impact Investing & Engagement

Part 6. Global Warning & Climate Change

Part 7. Economic Modeling of Climate Change

Part 8. Climate Risk Measures

Part 9. Transition Risk Modeling

Part 10. Climate Portfolio Construction

Part 11. Exercise — Equity and Bond Portfolio Optimization with Green Preferences

Part 12. Physical Risk Modeling

Part 13. Climate Stress Testing and Risk Management

Course 2022-2023 in Sustainable Finance
Lecture 1. Introduction

Thierry Roncalli*

*Amundi Asset Management[2]

*University of Paris-Saclay

March 2023

[2]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Definition

"Sustainable finance refers to the process of taking **environmental, social and governance (ESG) considerations** *into account when making investment decisions in the financial sector, leading to more long-term investments in sustainable economic activities and projects.* **Environmental considerations** *might include climate change mitigation and adaptation, as well as the environment more broadly, for instance the preservation of biodiversity, pollution prevention and the circular economy.* **Social considerations** *could refer to issues of inequality, inclusiveness, labour relations, investment in human capital and communities, as well as human rights issues. The* **governance** *of public and private institutions — including management structures, employee relations and executive remuneration — plays a fundamental role in ensuring the inclusion of social and environmental considerations in the decision-making process." (European Commission).*

Many words, one concept

Figure 1: Many words, one concept

RI, SI, SRI, ESG, etc.

Responsible investment (RI)

Responsible investment is an approach to investment that explicitly acknowledges the relevance to the investor of environmental, social and governance factors, and of the long-term health of the market as a whole

Sustainable investing (SI)

Sustainable investing is an investment approach that considers environmental, social and governance factors in portfolio selection

Socially responsible investing (SRI)

SRI is an investment strategy that is considered socially responsible, because it invests in companies that have ethical practices

Environmental, Social and Governance (ESG)

Environmental, Social, and Corporate Governance (ESG) refers to the factors that measure the sustainability of an investment

Definition

Sustainable Investing

$\approx$

Socially Responsible Investing (SRI)

$\approx$

Environmental, Social, and Governance (ESG)

Remark

Blue Finance $\subset$ **Green Finance**, Climate Finance $\subset$ Sustainable Finance

Historical perspective

- Responsible investment (RI): 2000's
- ESG investing (ESG): 2010's
- Sustainable finance (SF): 2020's

Why?

Historical perspective

- At the beginning, sustainable finance mainly concerns final investors and asset owners (ethics) $\Rightarrow$ **responsible investment**
- Then, it gains momentum in asset management $\Rightarrow$ **ESG investing**
- Finally, it spreads across all financial actors (e.g. issuers, banks, central banks, etc.) $\Rightarrow$ **Sustainable finance**

ESG motivations

Figure 2: The raison d'être of ESG investing

A myriad of acronyms

How many acronyms do you know?

A myriad of acronyms

CAT: Cap-And-Trade, CBI: Climate Bonds Initiative, **CDP: Carbon Disclosure Project**, CDR: Carbon Dioxide Removal, CDSB: Climate Disclosure Standards Board, CI: Carbon Intensity, **COP: Conference of the Parties**, **CTB: Climate Transition Benchmark**, DAC: Direct Air Capture, DICE: Dynamic Integrated Climate-Economy Model, ETS: Emissions Trading Scheme, Eurosif: European Sustainable Investment Forum, ESG: Environmental, Social and Governance, GB: Green Bond, **GBP: Green Bonds Principles**, : Greenhouse gas Emissions per unit of Value Added, **GHG: Greenhouse Gaz**, **GIIN: Global Impact Investing Network**, GLP: Green Loans Principles, GQE: Green Quantitative Easing, GRI: Global Reporting Initiative, **GSIA: Global Sustainable Investment Alliance**, HLEG: High Level Expert Group on Sustainable Finance, IAM: Integrated Assessment Model (economic model of climate risk), IIRC: International Integrated Reporting Council, **IPCC: Intergovernmental Panel on Climate Change**, **NDC: Nationally Determined Contribution**, NFRD: Non-financial Reporting Directive, NGFS: Network for Greening the Financial System, OPS: One Planet Summit, **PAB: Paris Aligned Benchmark**, PBOC: People's Bank of China (China green bonds), **PRI: Principles for Responsible Investment**, **RCP: Representative Concentration Pathway (climate scenario)**, SASB: Sustainability Accounting Standards Board, SB: Social Bond, SBP: Social Bonds Principles, SBT: Science-Based Target, SCC: Social Cost of Carbon (= optimal carbon tax), SDA: Sectoral Decarbonisation Approach **SDG: Sustainable Development Goals**, **SFDR: Sustainable Finance Disclosure Reporting**, SIB: Social Impact Bond, SRI: Socially Responsible Investing, SSB: Sustainabilty Standards Board (IFRS), SSP: Shared Socioeconomic Pathway, **TCFD: Task Force on Climate-Related Financial Disclosures**, TEG: Technical Expert Group on Sustainable Finance, UNPRI: Principles for Responsible Investment (PRI)

Many financial actors

ESG financial ecosystem

- Asset owners (pension funds, sovereign wealth funds (SWF), insurance and institutional investors, retail investors, etc.)
- Asset managers
- ESG rating agencies
- ESG index sponsors
- Banks
- ESG associations (GSIA, UNPRI, etc.)
- Regulators and international bodies (governments, financial and industry regulators, central banks, etc.)
- **Issuers** (equities, bonds, loans, etc.)
- Society and people

ESG Investing $\Leftrightarrow$ ESG Financing (= Sustainable Finance)

The issuer point of view of ESG

Corporate financial performance (CFP)

- Friedman (1970)
- Shareholder theory
- Corporations have no social responsibility to the public or society
- Their only responsibility is to its shareholders (profit maximization)

Corporate social responsibility (CSR)

- Freeman (2010)
- Stakeholder theory
- Corporations create negative externalities
- They must have social and moral responsibilities
- Impact on the cost-of-capital and business risk

Sustainable investment forums

GSIA members

- The European Sustainable Investment Forum (Eurosif), `http://www.eurosif.org`
- Responsible Investment Association Australasia (RIAA), `https://responsibleinvestment.org`
- Responsible Investment Association Canada (RIA Canada), `https://www.riacanada.ca`
- UK Sustainable Investment & Finance Association (UKSIF), `https://www.uksif.org`
- The Forum for Sustainable & Responsible Investment (US SIF), `https://www.ussif.org`
- Dutch Association of Investors for Sustainable Development (VBDO), `https://www.vbdo.nl/en/`
- Japan Sustainable Investment Forum (JSIF), `https://japansif.com/english`

Sustainable investment forums

Figure 3: 2018 GSIA report

Figure 4: 2020 GSIA report

Initiatives

Initiatives

- Principles for responsible investment (PRI)
- Climate Action 100+
- Net zero alliances: (NZAOA, NZAM, PAII, NZBA, NZIA, etc) $\Rightarrow$ GFANZ

PRI (or UNPRI)

Figure 5: Principles for Responsible Investment (PRI)

https://www.unpri.org

PRI

PRI (or UNPRI)

- Early 2005: UN Secretary-General Kofi Annan invited a group of the world's largest institutional investors to join a process to develop the Principles for Responsible Investment
- April 2006: The Principles were launched at the New York Stock Exchange
- 6 ESG principles
- The 63 founding signatories are 32 asset owners[a] and 31 asset managers[b] and data providers[c]

[a] AP2, CDC, CDPQ, CalPERS, ERAFP, FRR, IFC, NZSF, NGPF, PGGM, UNJSPF, USS, etc.

[b] Amundi (CAAM), Sumitomo Trust, BNP PAM, Mitsubishi Trust, Threadneedle, Aviva, Candriam, etc.

[c] Trucost, Vigeo, etc.

PRI

Signatories' commitment

"As institutional investors, we have a duty to act in the best long-term interests of our beneficiaries. In this fiduciary role, we believe that environmental, social, and corporate governance (ESG) issues can affect the performance of investment portfolios (to varying degrees across companies, sectors, regions, asset classes and through time).
We also recognise that applying these Principles may better align investors with broader objectives of society. Therefore, where consistent with our fiduciary responsibilities, we commit to the following:

- Principle 1: We will incorporate ESG issues into investment analysis and decision-making processes.
- Principle 2: We will be active owners and incorporate ESG issues into our ownership policies and practices.
- Principle 3: We will seek appropriate disclosure on ESG issues by the entities in which we invest.
- Principle 4: We will promote acceptance and implementation of the Principles within the investment industry.
- Principle 5: We will work together to enhance our effectiveness in implementing the Principles.
- Principle 6: We will each report on our activities and progress towards implementing the Principles.

The Principles for Responsible Investment were developed by an international group of institutional investors reflecting the increasing relevance of environmental, social and corporate governance issues to investment practices. The process was convened by the United Nations Secretary-General.
In signing the Principles, we as investors publicly commit to adopt and implement them, where consistent with our fiduciary responsibilities. We also commit to evaluate the effectiveness and improve the content of the Principles over time. We believe this will improve our ability to meet commitments to beneficiaries as well as better align our investment activities with the broader interests of society.
We encourage other investors to adopt the Principles."

Source: `https://www.unpri.org`

PRI

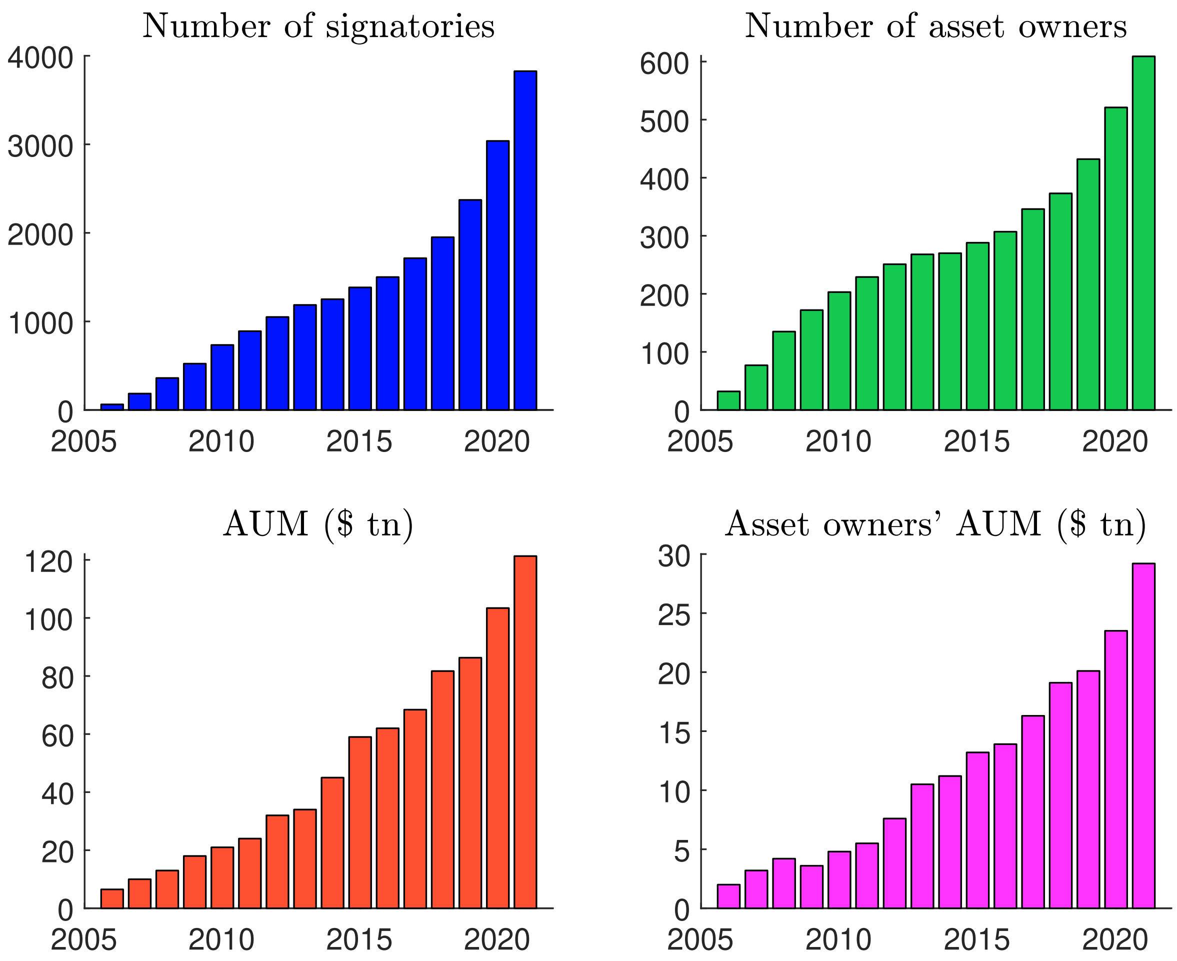


Figure 6: PRI Signatory growth

Source: `https://www.unpri.org`

Rating agencies

- Early stage (1990-2010): Eiris (1985, UK), KLD (1988, US), Jantzi Research (1992, Canada), GES (1992, Sweden), Innovest (1995, US), SAM (1995, Switzerland), RepRisk (1998, Switzerland), Oekom (1999, Germany), Ethix (1999, Sweden), Trucost (2000, UK), Inrate (2001, Switzerland), Vigeo (2002, France), DSR (2002, Netherlands), EthiFinance (2004, France), etc.
- Consolidation of the industry (2010-2020): ISS ESG, Moody's, MSCI, Refinitiv, Reprisk, S&P Global, Sustainalytics.

Rating agencies

1. ESG scores and ratings
2. ESG data
3. ESG indices

Regulators: Who? Why?

Table 1: The supervision institutions in finance

	Banks	Insurers	Markets	All sectors
Global	BCBS	IAIS	IOSCO	FSB
EU	EBA/ECB	EIOPA	ESMA	ESFS
US	FDIC/FRB	FIO	SEC	FSOC

- Greenwashing
  - Explicit & deliberate greenwashing;
  - Unintentional greenwashing.
- Fiduciary duties

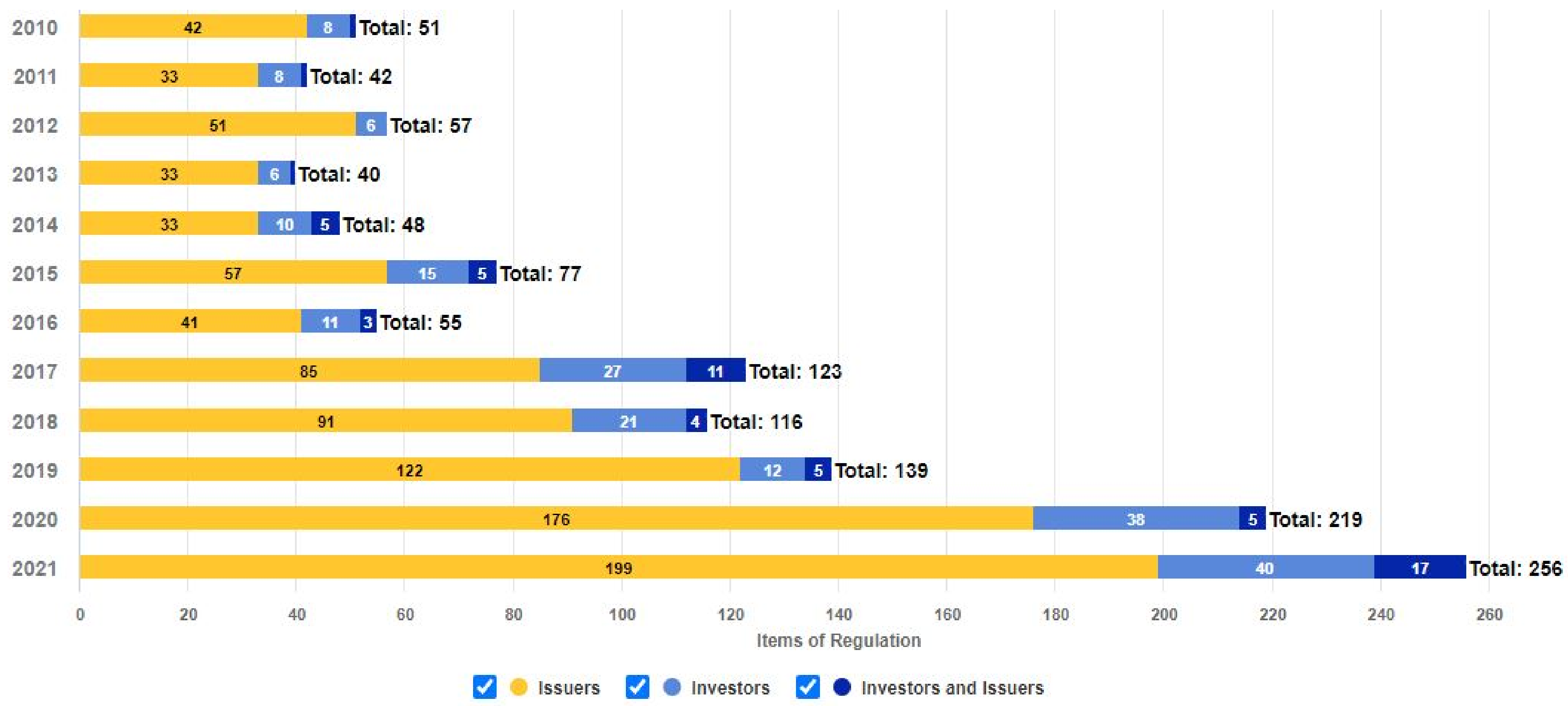
ESG regulations

Figure 7: Who will regulate ESG? — The regulators viewpoint (MSCI, 2022)

Source: https://www.msci.com/who-will-regulate-esg.

ESG regulations

Figure 8: Who will regulate ESG? — The regulated viewpoint (MSCI, 2022)



Source: `https://www.msci.com/who-will-regulate-esg`.

ESG regulations

Visit the MSCI website

`https://www.msci.com/who-will-regulate-esg`

and obtain the detailed list of regulations

by year, country, regulator, regulated investors, etc.

The example of central banks

Figure 9: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)

- Launched at the Paris One Planet Summit (OPS) on December 2017
- 8 founding members: Banco de Mexico, BoE, BdF, Dutch Central Bank, Buba, Swedish FSA, HKMA, MAS and PBOC
- As of March 19th 2021, the NGFS consists of 89 members (CBs, EBA, EIOPA, ESMA) and 13 observers (BCBS, IMF, IAIS, IOSCO)

The example of central banks

Go the NGFS website (`https://www.ngfs.net`) and download the NGFS climate scenarios: `https://www.ngfs.net/en/publications/ngfs-climate-finance-research-portal`

See also `https://data.ene.iiasa.ac.at/ngfs` (NGFS scenario explorer hosted by IIASA[3])

[3] International Institute for Applied Systems Analysis

Reporting frameworks

Table 2: List of the main reporting frameworks

Perimeter	Acronym	Name	Dates
General	GC	UN Global Compact Initiative	2000/2000
	GRI	Global Reporting Initiative	1997/2000
	IIRC	International Integrated Reporting Council	2010/2013
	ISSB	International SustainabilityStandards Board	2021/2023
	SASB	Sustainability Accounting Standards Board	2011/2016
	SDGs	UN Sustainable Development Goals	2015/2016
Climate	CDP	Carbon Disclosure Project	2000/2000
	CDSB	Climate Disclosure Standards Board	2007/2015
	GHG Protocol	Greenhouse Gas Protocol	1998/2001
	PCAF	Partnership for Carbon Accounting Financials	2019/2020
	SBTi	Science Based Targets initiative	2015/2015
	TCFD	Task Force on Climate-Related Financial Disclosures	2015/2017

Sustainable Development Goals

Figure 10: The SDGs icons

Source: https://sdgs.un.org/goals#icons.

Sustainable Development Goals

Table 3: The 17 SDGs

#	Name	Description	E	S	G
1	No poverty	End poverty in all its forms everywhere		✓	
2	Zero hunger	End hunger, achieve food security and improved nutrition and promote sustainable agriculture		✓	
3	Good health and well-being	Ensure healthy lives and promote well-being for all at all ages		✓	
4	Quality education	Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all		✓	
5	Gender equality	Achieve gender equality and empower all women and girls		✓	✓
6	Clean water and sanitation	Ensure availability and sustainable management of water and sanitation for all	✓	✓	
7	Affordable and clean energy	Ensure access to affordable, reliable, sustainable and modern energy for all	✓		
8	Decent work and economic growth	Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all		✓	✓
9	Industry, innovation and infrastructure	Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation	✓	✓	✓

Source: `https://sdgs.un.org/goals`.

Sustainable Development Goals

Table 4: The 17 SDGs

#	Name	Description	E	S	G
10	Reduced inequality	Reduce inequality within and among countries		✓	
11	Sustainable cities and communities	Make cities and human settlements inclusive, safe, resilient and sustainable	✓		✓
12	Responsible consumption and production	Ensure sustainable consumption and production patterns	✓	✓	✓
13	Climate action	Take urgent action to combat climate change and its impacts	✓		✓
14	Life below water	Conserve and sustainably use the oceans, seas and marine resources for sustainable development	✓		
15	Life on land	Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss	✓		
16	Peace, justice, and strong institutions	Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels		✓	✓
17	Partnerships for the goals	Strengthen the means of implementation and revitalize the Global Partnership for Sustainable Development			✓

Source: `https://sdgs.un.org/goals`.

GHG Protocol

The GHG Protocol corporate standard classifies a company's greenhouse gas emissions in three scopes[4]:

- **Scope 1**: Direct GHG emissions (○)
- **Scope 2**: Consumption of purchased energy (○ ○)
- **Scope 3**: Other indirect GHG emissions (● ●)
  - **Scope 3 upstream**: emissions associated to the supply side
    1. First tier direct (●)
    2. Tier 2 and 3 suppliers (● ●)
  - **Scope 3 downstream**: emissions associated with the product sold by the entity
    1. Use of the product (● ● ●)
    2. Waste disposal & recycling (● ● ● ●)

[4] Measurement robustness: from ○ ○ ○ ○ (very high) to ● ● ● ● (very low)

Carbon Disclosure Project (CDP)

Each year, CDP sends a questionnaire to organizations and collects information on three environmental dimensions:

1. Climate change (based on the GHG Protocol)
2. Forest management
3. Water security

Carbon Disclosure Project (CDP)

Table 5: Examples of 2019 carbon emissions and intensity

Company	Emission (in tCO_2e)				Revenue	Intensity (in tCO_2e/\$ mn)			
	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{up}$	$\mathcal{SC}_3^{down}$	(in \$ mn)	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{up}$	$\mathcal{SC}_3^{down}$
Amazon	5 760 000	5 500 000	20 054 722	10 438 551	280 522	20.5	19.6	71.5	37.2
Apple	50 549	862 127	27 624 282	5 470 771	260 174	**0.2**	**3.3**	**106.2**	21.0
BNP Paribas	64 829	280 789	1 923 307	1 884	78 244	0.8	3.6	24.6	0.0
BP	49 199 999	5 200 000	103 840 194	582 639 687	276 850	177.7	18.8	375.1	2 104.5
Caterpillar	905 000	926 000	15 197 607	401 993 744	53 800	16.8	17.2	282.5	7 472.0
Danone	722 122	944 877	28 969 780	4 464 773	28 308	25.5	33.4	**1 023.4**	**157.7**
Exxon	111 000 000	9 000 000	107 282 831	594 131 943	255 583	434.3	35.2	419.8	2 324.6
JPMorgan Chase	81 655	692 299	3 101 582	15 448 469	115 627	0.7	6.0	26.8	133.6
LVMH	67 613	262 609	11 853 749	942 520	60 083	1.1	4.4	197.3	15.7
Microsoft	113 414	3 556 553	5 977 488	4 003 770	125 843	0.9	28.3	47.5	31.8
Nestle	3 291 303	3 206 495	61 262 078	33 900 606	93 153	35.3	34.4	**657.6**	**363.9**
Pfizer	734 638	762 840	4 667 225	133 468	51 750	14.2	14.7	90.2	2.6
Samsung Electronics	5 067 000	10 998 000	33 554 245	60 978 947	197 733	**25.6**	**55.6**	169.7	308.4
Volkswagen	4 494 066	5 973 894	65 335 372	354 913 446	282 817	15.9	21.1	231.0	1 254.9
Walmart	6 101 641	13 057 352	40 651 079	32 346 229	514 405	11.9	25.4	79.0	62.9

Source: Trucost (2022) & Authors' calculations.

TCFD

Table 6: The 11 recommended disclosures (TCFD, 2017)

Recommendation	#	Recommended Disclosure
Governance	1	Board oversight
	2	Management's role
Strategy	3	Risks and opportunities
	4	Impact on organization
	5	Resilience of strategy
Risk management	6	Risk ID and assessment processes
	7	Risk management processes
	8	Integration into overall risk management
Metrics and targets	9	Climate-related metrics
	10	Scope 1, 2, 3 GHG emissions
	11	Climate-related targets

Source: `https://www.fsb-tcfd.org`.

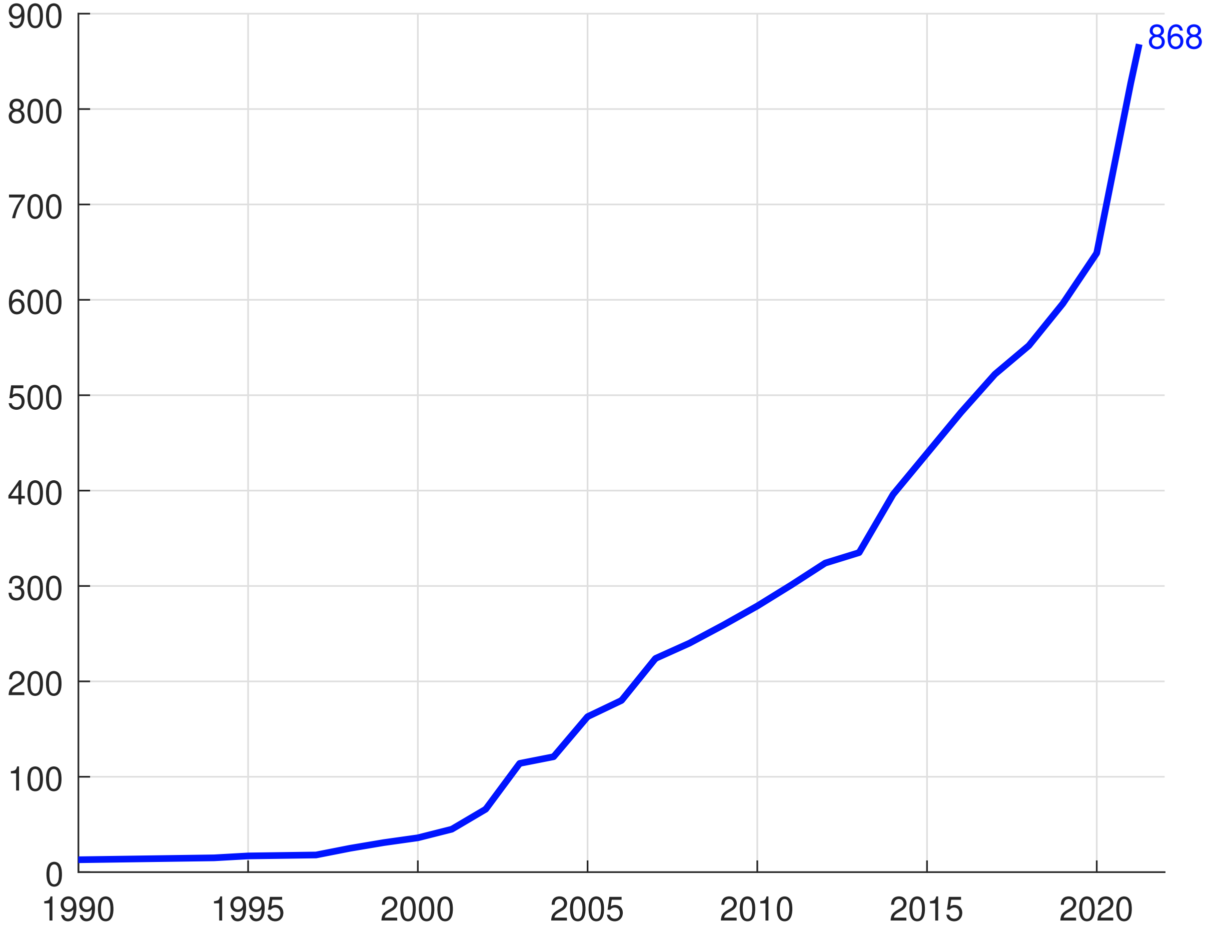
TCFD

Examples of recommended metrics

- GHG emissions (absolute scope 1, scope 2, and scope 3 GHG emissions; financed emissions by asset class; weighted average carbon intensity)
- Transition risks (volume of real estate collaterals highly exposed to transition risk; concentration of credit exposure to carbon-related assets; percent of revenue from coal mining)
- Physical risks (number and value of mortgage loans in 100-year flood zones; proportion of real assets exposed to 1:100 or 1:200 climate-related hazards)
- Climate-related opportunities (proportion of green buildings, green revenues)
- Capital deployment (green CAPEX)
- Internal carbon prices (internal carbon price, shadow carbon price)
- Remuneration

Regulatory framework

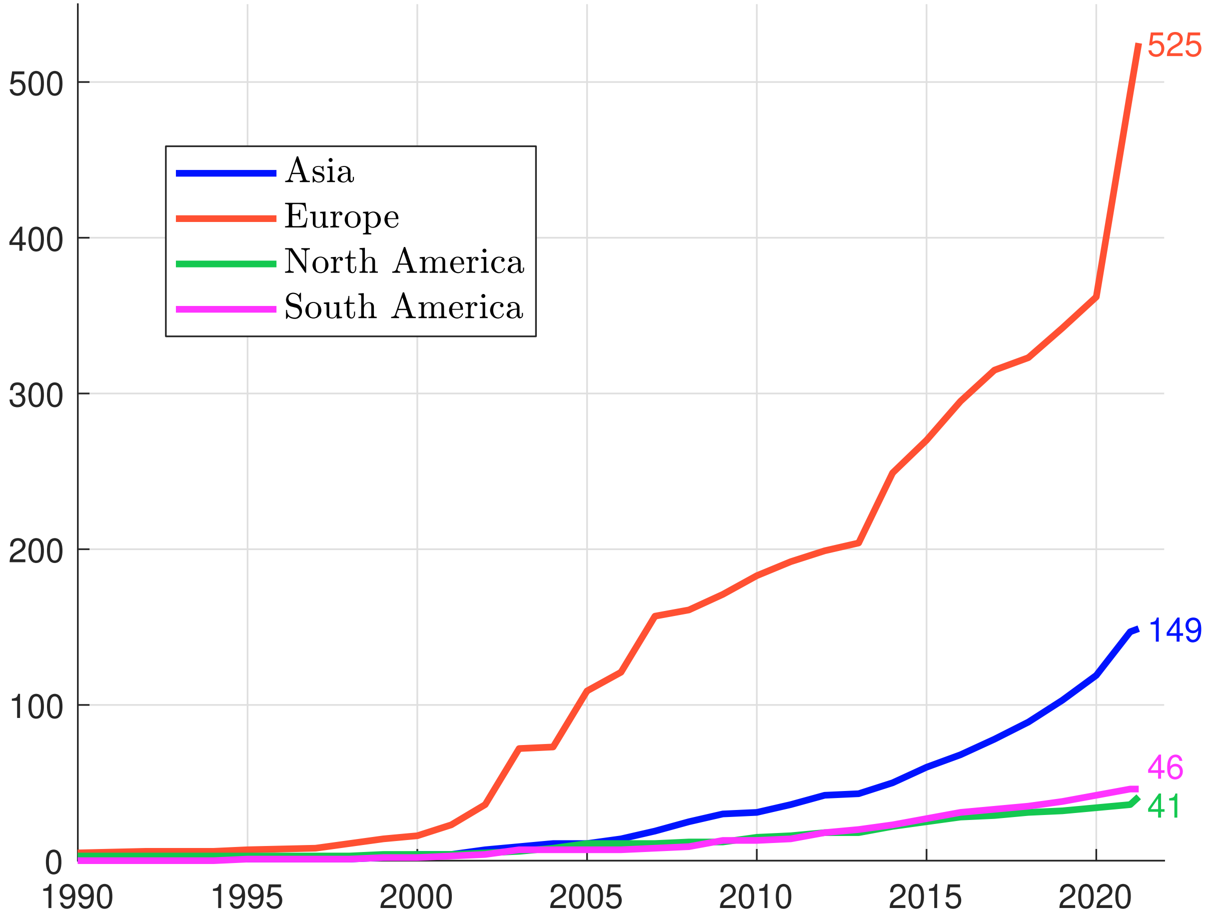
Figure 11: Total number of ESG regulations



Source: PRI (2022), `https://www.unpri.org/policy/regulation-database`.

Regulatory framework

Figure 12: Number of ESG regulations per region



Source: PRI (2022), `https://www.unpri.org/policy/regulation-database`.

European Union

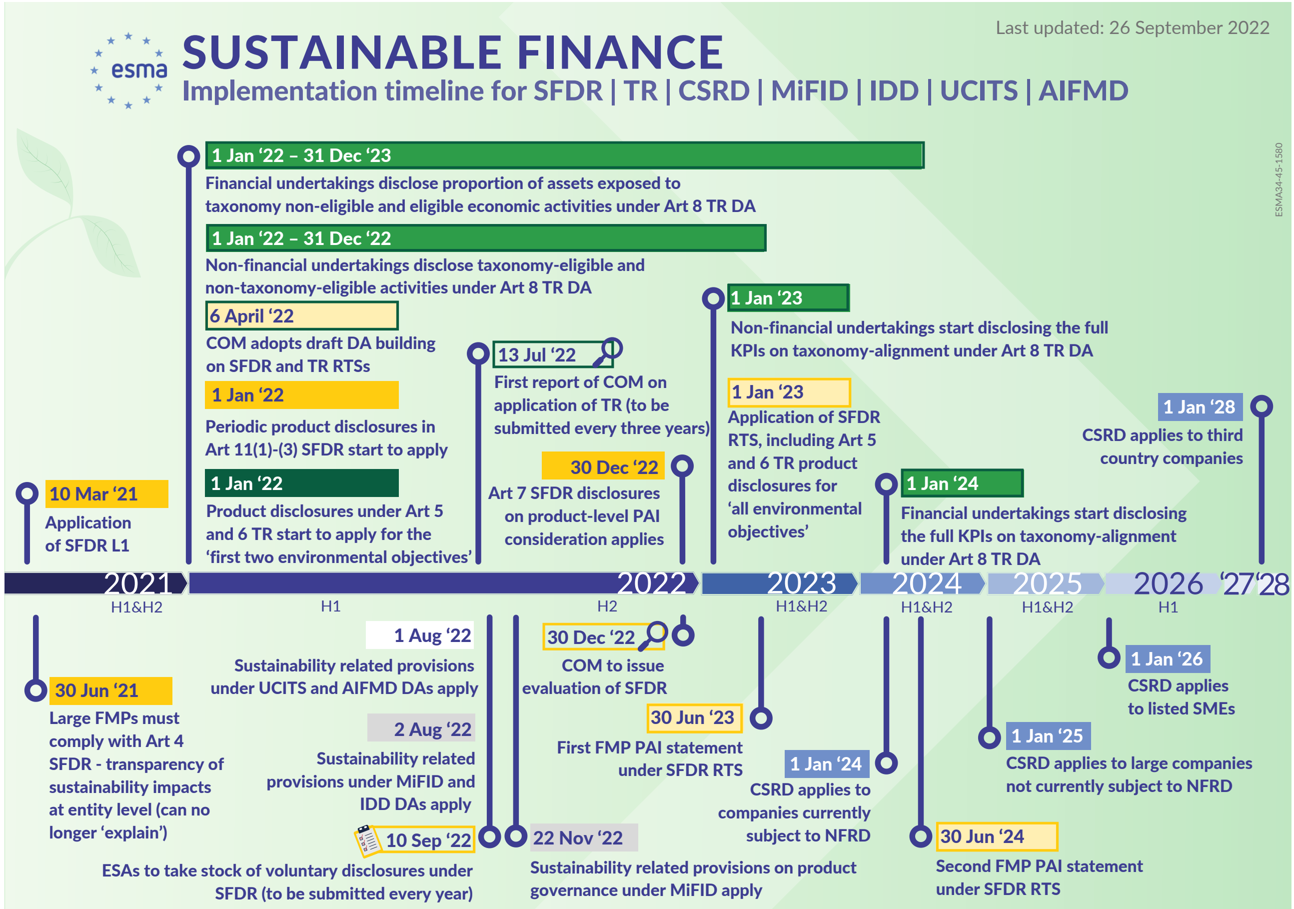
- The action plan on sustainable finance (May 2018)
- The European Green Deal (December 2019)
- The Fit-for-55 package (July 2021)
- The REPowerEU plan or energy security package (May 2022)

European Union

- EU taxonomy regulation
- Climate benchmarks (PAB)
- Sustainable finance disclosure regulation (SFDR)
- MiFID II & IDD
- Corporate sustainability reporting directive (CSRD)

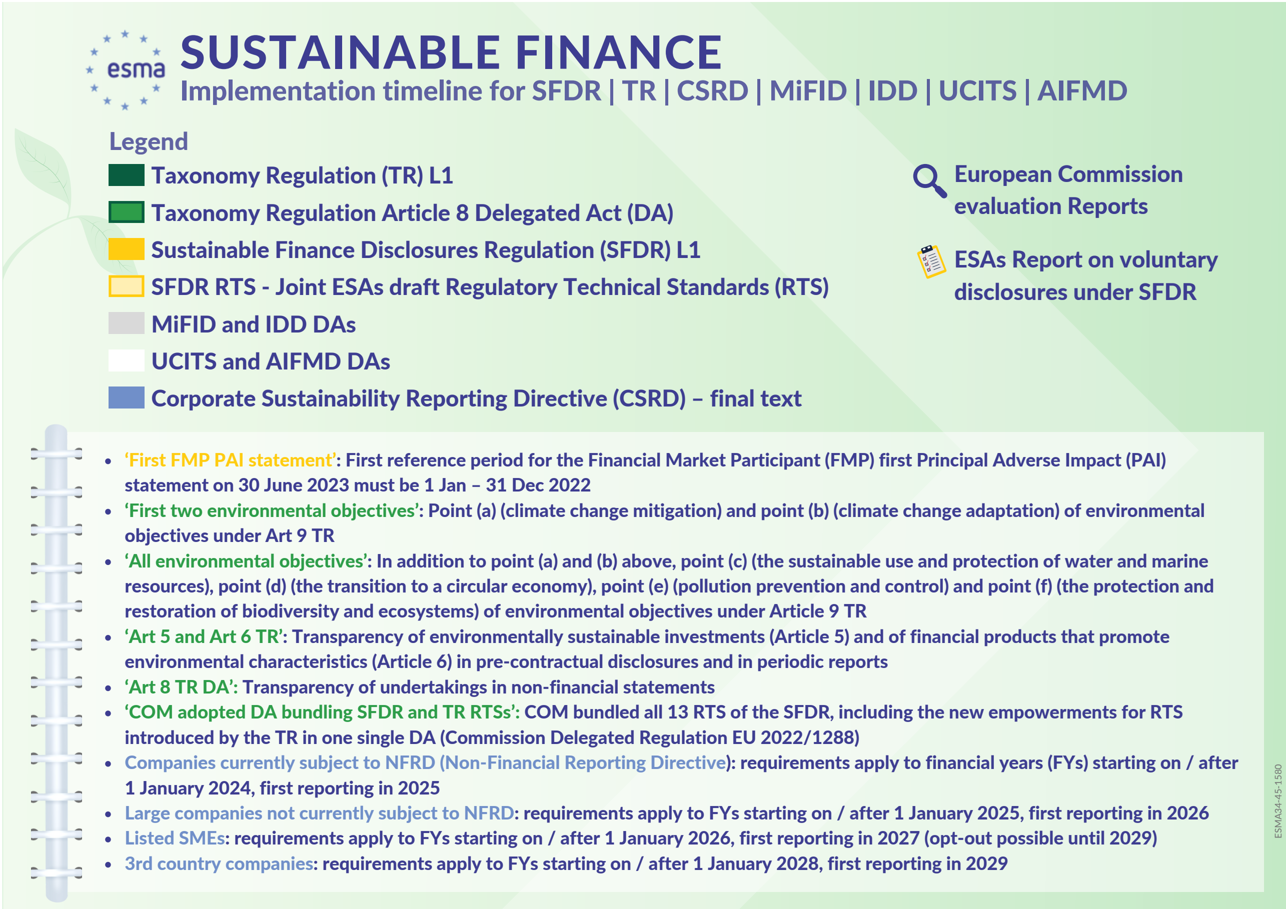
European Union

Figure 13: Sustainable finance — implementation timeline



European Union

Figure 14: Sustainable finance — implementation timeline



EU taxonomy regulation

1. Climate change mitigation
2. Climate change adaptation
3. Sustainable use and protection of water and marine resources
4. Transition to a circular economy
5. Pollution prevention and control
6. Protection and restoration of biodiversity and ecosystem

Climate benchmarks

The common principles are:

- A year-on-year self-decarbonization of 7% on average per annum, based on scope 1, 2 and 3 emissions
- A minimum carbon intensity reduction $\mathcal{R}^-$ compared to the investable universe
- A minimum exposure to sectors highly exposed to climate change

Two labels:

1. CTB: (climate transition benchmark) $\Rightarrow \mathcal{R}^- = 30\%$
2. PAB: (Paris aligned benchmark) $\Rightarrow \mathcal{R}^- = 50\%$

SFDR

- Article 6 (or non-ESG products)
  It covers standard financial products that cannot be Article 8 or Article 9
- Article 8 (or ESG products)
  It corresponds to financial products which "*promote, among other characteristics, environmental or social characteristics, or a combination of those characteristics, provided that the companies in which the investments are made follow good governance practices*"
- Article 9 (or sustainable products)
  In addition to the points covered by Article 8, these financial products have a sustainable investment objective

+ SI, PAI, etc.

MiFID II & IDD

$\Rightarrow$ sustainable preferences

CSRD

- **E**nvironmental factors: (1) climate change mitigation; (2) climate change adaptation; (3) water and marine resources; (4) resource use and circular economy; (5) pollution; (6) biodiversity and ecosystems.
- **S**ocial factors: (1) equal opportunities for all; (2) working conditions; (3) respect for human rights.
- **G**overnance factors: (1) role and composition of administrative, management and supervisory bodies; (2) business ethics and corporate culture, including anti-corruption and anti-bribery; (3) political engagements of the undertaking, including its lobbying activities; (4) management and quality of relationships with business partners.

single materiality $\neq$ double materiality

ESG strategies

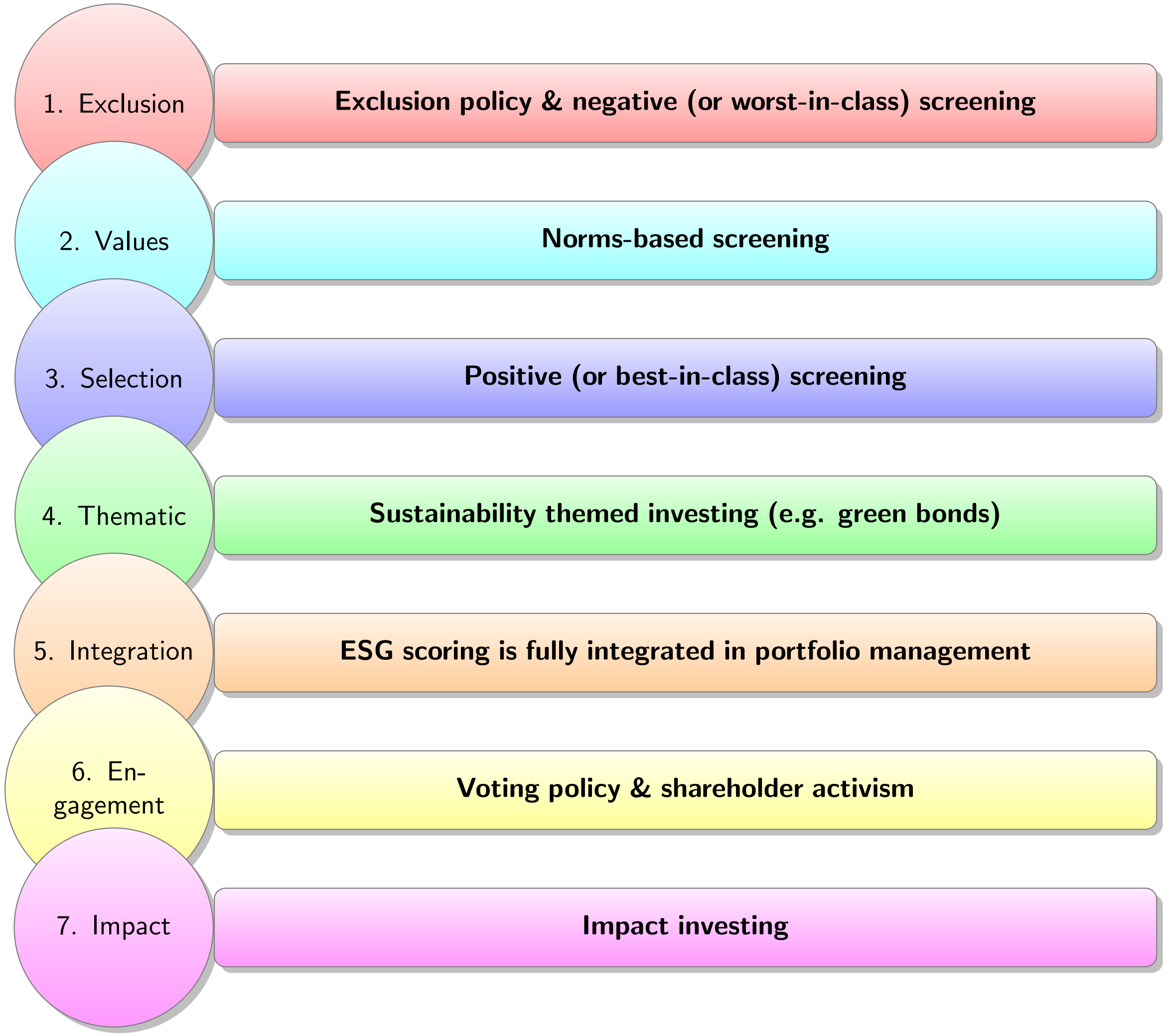


Figure 15: Categorisation of ESG strategies (Eurosif, 2019)

ESG strategies

Exclusion/Negative Screening

The exclusion from a fund or portfolio of certain sectors, companies or practices based on specific ESG criteria (worst-in-class)

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Systematic exclusion of issuers rated **CCC**
- Exclusion of issuers rated **BB**, **B** and **CCC**
- Sector exclusion (e.g., Energy)
- Sub-industry exclusion (e.g. Coal & Consumable Fuels)
- Exclusion list of individual issuers

ESG strategies

Values/Norms-based Screening (and Red Flags)

Screening of investments against minimum standards of business practice based on international norms, such as those issued by the OECD, ILO, UN (Global Compact) and UNICEF[a]

[a] In Europe, the top exclusion criteria are (1) controversial weapons (Ottawa and Oslo treaties), (2), tobacco, (3) all weapons, (4) gambling, (5) pornography, (6) nuclear energy, (7) alcohol, (8) GMO and (9) animal testing (Eurosif, 2019)

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Controversial sectors: controversial weapons, conventional weapons, civilian firearms, nuclear weapons, nuclear power, thermal coal, tobacco, alcohol, gambling, adult entertainment, genetically modified, fossil fuels production & reserves
- Many ETF funds

ESG strategies

Selection/Positive Screening

Investment in sectors, companies or projects selected for positive ESG performance relative to industry peers (best-in-class)

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Selection of issuers rated **AAA**, **AA** and **A**
- Selection of issuers that have improved their rating (Momentum ESG strategy)

ESG strategies

Thematic/Sustainability Themed Investing

Investment in themes or assets specifically related to sustainability (for example clean energy, green technology or sustainable agriculture)

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Funds invested in Green Bonds
- Funds invested in Social Bonds
- Funds invested in Sustainable Infrastructure
- Funds invested in Natural Ressources

ESG strategies

ESG Integration

The systematic and explicit inclusion by investment managers of environmental, social and governance factors into financial analysis

Source: Global Sustainable Investment Alliance (2019)

Examples:

- The stock picking score is a mix (50/50) of a fundamental score and an ESG score
- The fund must have an ESG score greater than the score of its benchmark

ESG strategies

Corporate Engagement/Shareholder Action

The use of shareholder power to influence corporate behavior, including through direct corporate engagement (i.e., communicating with senior management and/or boards of companies), filing or co-filing shareholder proposals, and proxy voting that is guided by comprehensive ESG guidelines.

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Voting policy
- Public divestment
- Biodiversity and deforestation financing
- Engagement with target companies on a specific subject (e.g., pay ratio or living wage)
- Escalated engagement: concerns public, proposing shareholder resolutions & litigation

ESG strategies

Impact Investing

Targeted investments aimed at solving social or environmental problems, and including community investing, where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose

Source: Global Sustainable Investment Alliance (2019)

Examples:

- Funds with a Social Impact objective
- Funds invested in Green Bonds
- PAB and CTB ETFs

ESG strategies

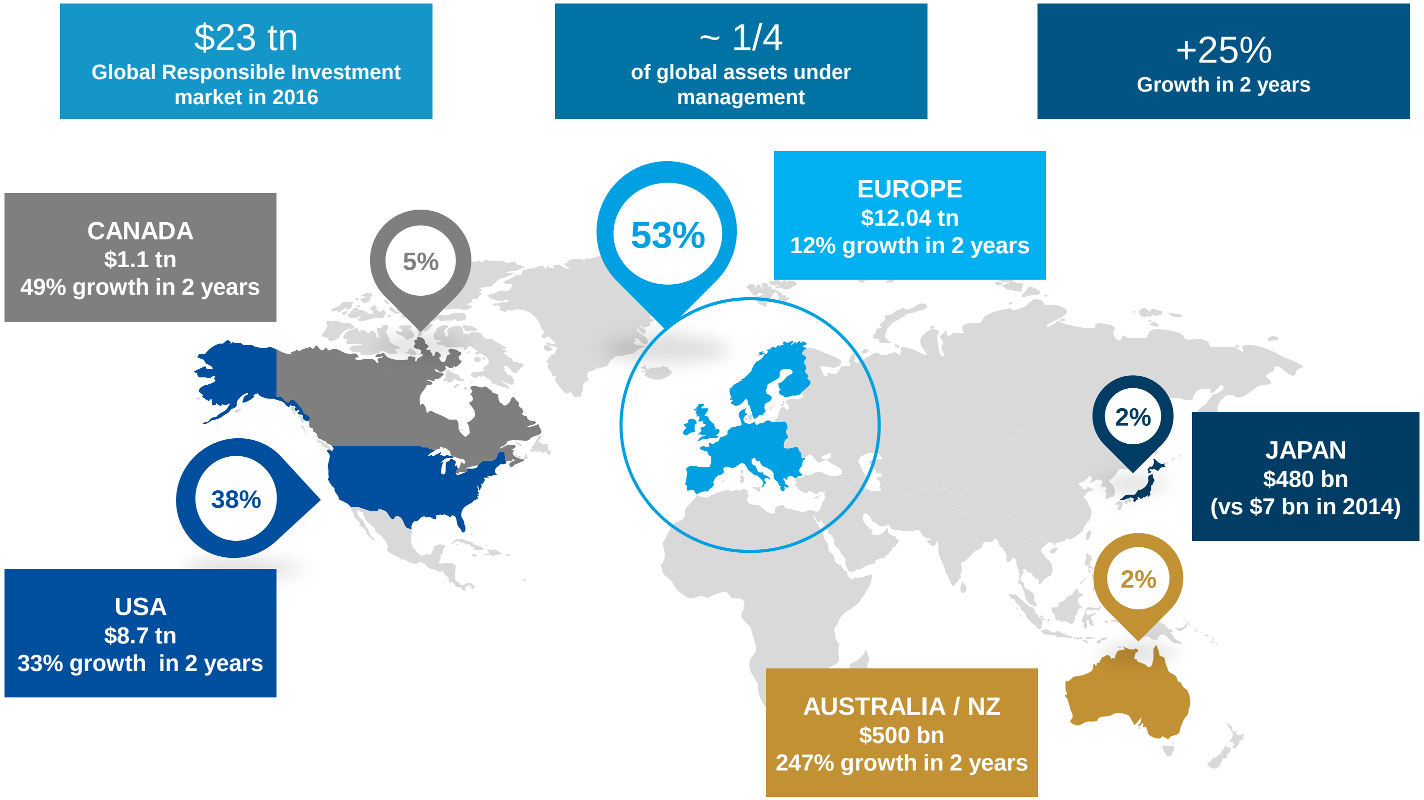
Impact Investing/Community Investing

- Impact Investing
  Investing to achieve positive, social and environmental impacts – requires measuring and reporting against these impacts, demonstrating the intentionality of investor and underlying asset/investee, and demonstrating the investor contribution
- Community Investing
  Where capital is specifically directed to traditionally underserved individuals or communities, as well as financing that is provided to businesses with a clear social or environmental purpose. Some community investing is impact investing, but community investing is broader and considers other forms of investing and targeted lending activities.

Source: Global Sustainable Investment Alliance (2021)

The market of ESG investing

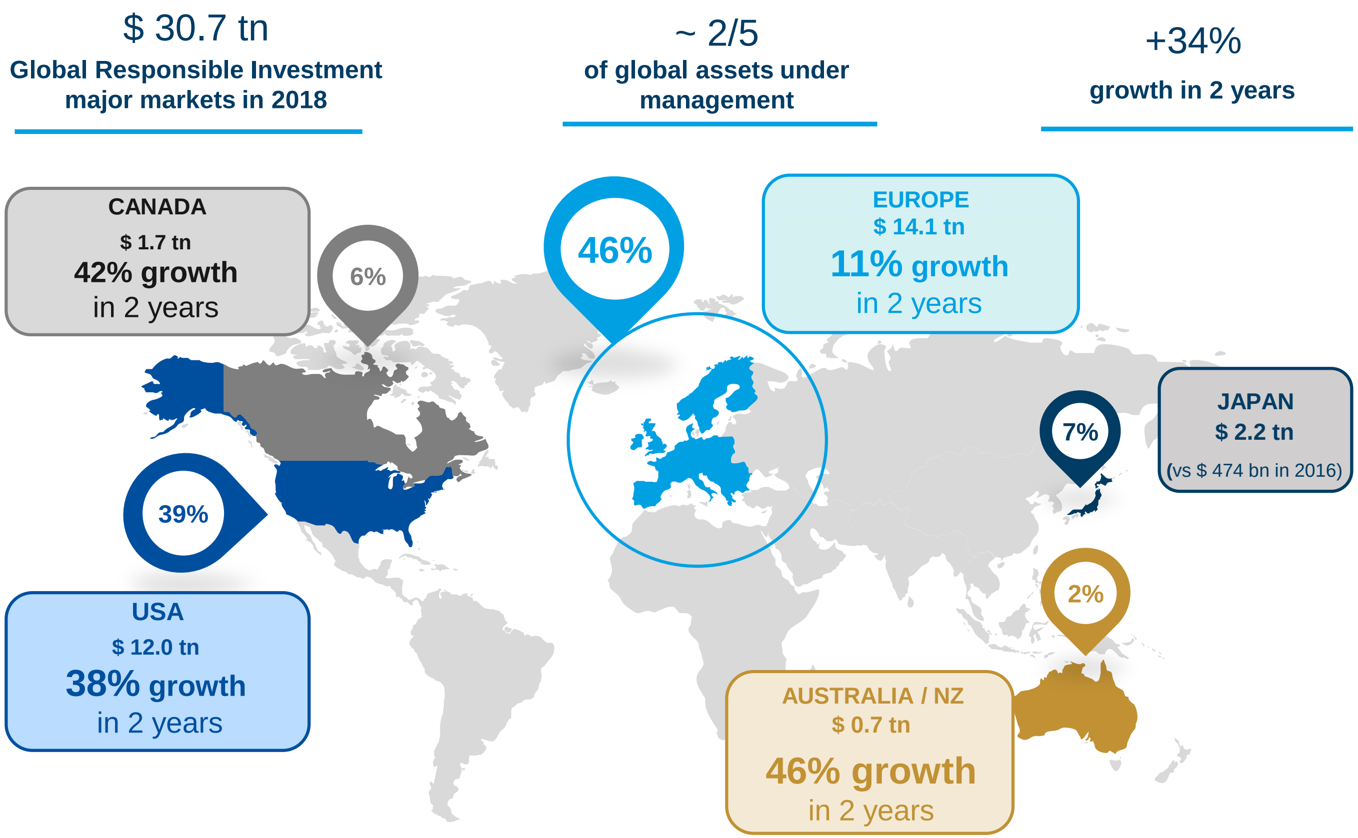
Figure 16: Sustainable investment assets at the start of 2016



Source: GSIA (2016).

The market of ESG investing

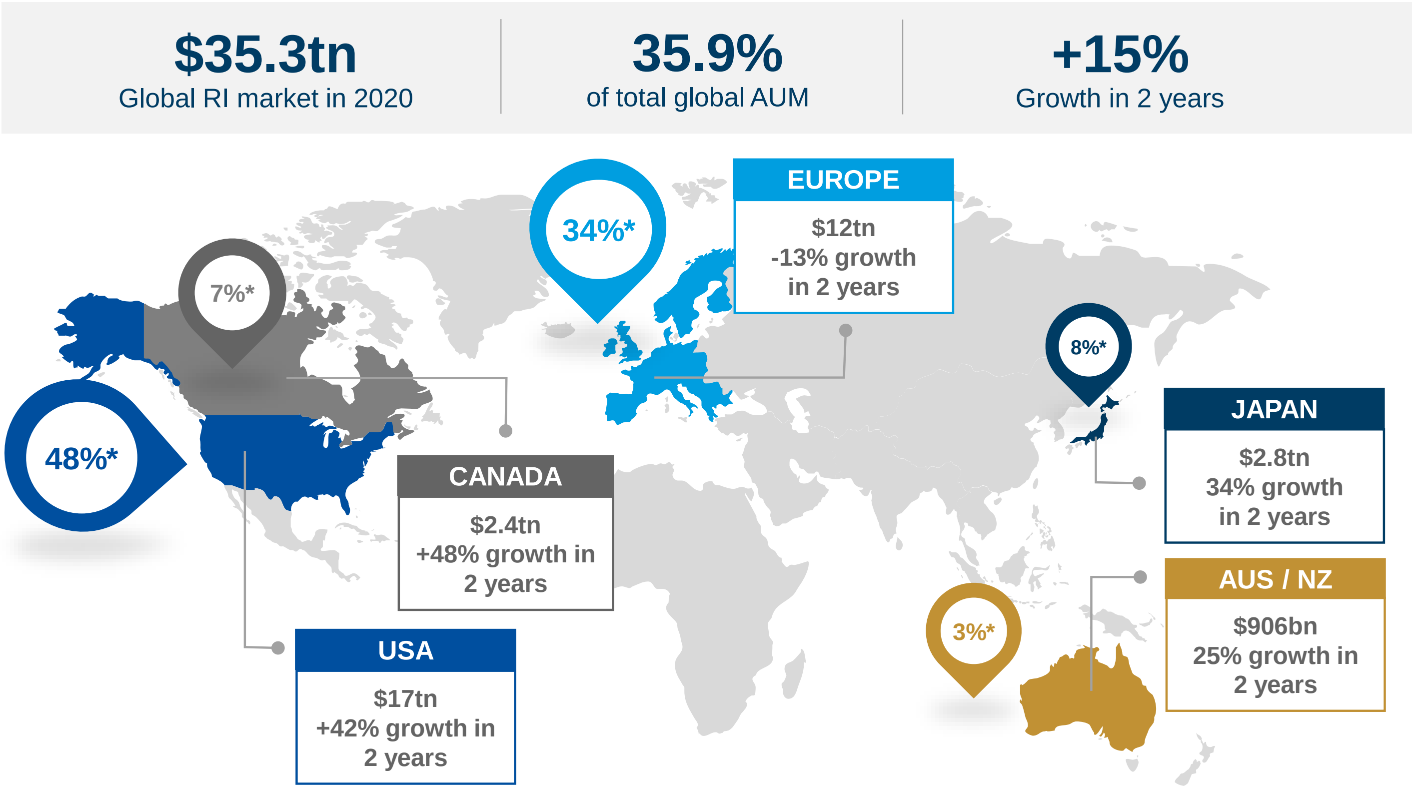
Figure 17: Sustainable investment assets at the start of 2018



Source: GSIA (2018).

The market of ESG investing

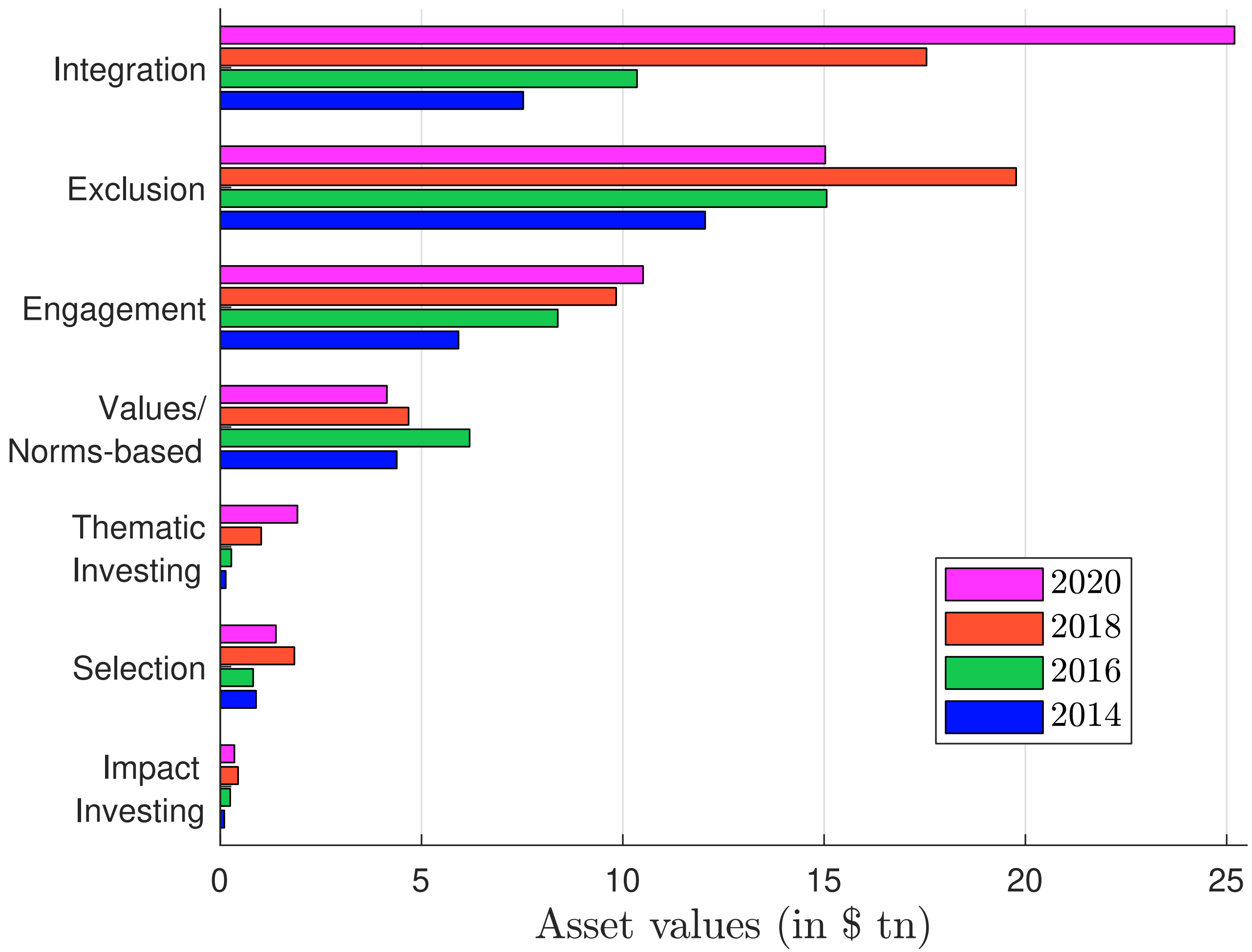
Figure 18: Sustainable investment assets at the start of 2020



Source: GSIA (2020).

The market of ESG investing

Figure 19: Asset values of ESG strategies between 2014 and 2018



Source: GSIA (2012, 2014, 2016, 2018, 2020) & Author's calculations.

The market of ESG investing

Table 7: ESG asset growth

#	ESG strategy	Asset growth 2014-2016	2016-2018	2018-2020	2020 AUM (in $ bn)
1	Exclusion	11.7%	14.6%	−24.0%	15 030
2	Values/Norms-based	19.0%	−13.1%	−11.5%	4 140
3	Selection	7.6%	50.1%	−24.9%	1 384
4	Thematic Investing	55.1%	92.0%	91.4%	1 948
5	Integration	17.4%	30.2%	43.6%	25 195
6	Engagement	18.9%	8.3%	6.8%	10 504
7	Impact Investing	56.8%	33.7%	−20.8%	352

Source: GSIA (2012, 2014, 2016, 2018, 2020) & Author's calculations.

Course 2022-2023 in Sustainable Finance
Lecture 2. ESG Scoring

Thierry Roncalli*

*Amundi Asset Management[5]

*University of Paris-Saclay

March 2023

[5]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

ESG data

Several issues:

- **E**: climate change mitigation, climate change adaptation, preservation of biodiversity, pollution prevention, circular economy
- **S**: inequality, inclusiveness, labor relations, investment in human capital and communities, human rights
- **G**: management structure, employee relations, executive remuneration

$\Rightarrow$ requires a lot of alternative data

Sovereign ESG data

Sovereign ESG framework

- World Bank
- Data may be download at the following webpage:

  `https://datatopics.worldbank.org/esg/framework.html`
- E: 27 variables
- S: 22 variables
- G: 18 variables

Sovereign ESG data

Table 8: The World Bank database of sovereign ESG indicators

Environmental	Social	Governance
Emissions & pollution (5)	Education & skills (3)	Human rights (2)
Natural capital endowment and management (6)	Employment (3)	Government effectiveness (2)
Energy use & security (7)	Demography (3)	Stability & rule of law (4)
Environment/ climate risk & resilience (6)	Poverty & inequality (4)	Economic environment (3)
Food security (3)	Health & nutrition (5)	Gender (4)
	Access to services (4)	Innovation (3)

Sovereign ESG data

Table 9: Indicators of the environmental pillar (World Bank database)

- **Emissions & pollution** (1) CO2 emissions (metric tons per capita); (2) GHG net emissions/removals by LUCF (Mt of CO2 equivalent); (3) Methane emissions (metric tons of CO2 equivalent per capita); (4) Nitrous oxide emissions (metric tons of CO2 equivalent per capita); (5) PM2.5 air pollution, mean annual exposure (micrograms per cubic meter);
- **Natural capital endowment & management**: (1) Adjusted savings: natural resources depletion (% of GNI); (2) Adjusted savings: net forest depletion (% of GNI); (3) Annual freshwater withdrawals, total (% of internal resources); (4) Forest area (% of land area); (5) Mammal species, threatened; (6) Terrestrial and marine protected areas (% of total territorial area);
- **Energy use & security**: (1) Electricity production from coal sources (% of total); (2) Energy imports, net (% of energy use); (3) Energy intensity level of primary energy (MJ/$2011 PPP GDP); (4) Energy use (kg of oil equivalent per capita); (5) Fossil fuel energy consumption (% of total); (6) Renewable electricity output (% of total electricity output); (7) Renewable energy consumption (% of total final energy consumption);
- **Environment/climate risk & resilience**: (1) Cooling degree days (projected change in number of degree Celsius); (2) Droughts, floods, extreme temperatures (% of population, average 1990-2009); (3) Heat Index 35 (projected change in days); (4) Maximum 5-day rainfall, 25-year return level (projected change in mm); (5) Mean drought index (projected change, unitless); (6) Population density (people per sq. km of land area)
- **Food security**: (1) Agricultural land (% of land area); (2) Agriculture, forestry, and fishing, value added (% of GDP); (3) Food production index (2004-2006 = 100);

Source: `https://datatopics.worldbank.org/esg/framework.html`.

Sovereign ESG data

Table 10: Indicators of the social pillar (World Bank database)

- **Education & skills**: (1) Government expenditure on education, total (% of government expenditure); (2) Literacy rate, adult total (% of people ages 15 and above); (3) School enrollment, primary (% gross);
- **Employment**: (1) Children in employment, total (% of children ages 7-14); (2) Labor force participation rate, total (% of total population ages 15-64) (modeled ILO estimate); (3) Unemployment, total (% of total labor force) (modeled ILO estimate);
- **Demography**: (1) Fertility rate, total (births per woman); (2) Life expectancy at birth, total (years); (3) Population ages 65 and above (% of total population);
- **Poverty & inequality**: (1) Annualized average growth rate in per capita real survey mean consumption or income, total population (%); (2) Gini index (World Bank estimate); (3) Income share held by lowest 20%; (4) Poverty headcount ratio at national poverty lines (% of population);
- **Health & nutrition**: (1) Cause of death, by communicable diseases and maternal, prenatal and nutrition conditions (% of total); (2) Hospital beds (per 1,000 people); (3) Mortality rate, under-5 (per 1,000 live births); (4) Prevalence of overweight (% of adults); (5) Prevalence of undernourishment (% of population);
- **Access to services**: (1) Access to clean fuels and technologies for cooking (% of population); (2) Access to electricity (% of population); (3) People using safely managed drinking water services (% of population); (4) People using safely managed sanitation services (% of population);

Source: `https://datatopics.worldbank.org/esg/framework.html`.

Sovereign ESG data

Table 11: Indicators of the governance pillar (World Bank database)

- **Human rights**: (1) Strength of legal rights index (0 = weak to 12 = strong); (2) Voice and accountability (estimate);
- **Government effectiveness**: (1) Government effectiveness (estimate); (2) Regulatory quality (estimate);
- **Stability & rule of law**: (1) Control of corruption (estimate); (2) Net migration; (3) Political stability and absence of violence/terrorism (estimate); (4) Rule of law (estimate)
- **Economic environment**: (1) Ease of doing business index (1 = most business-friendly regulations); (2) GDP growth (annual %); (3) Individuals using the internet (% of population);
- **Gender**: (1) Proportion of seats held by women in national parliaments (%); (2) Ratio of female to male labor force participation rate (%) (modeled ILO estimate); (3) School enrollment, primary and secondary (gross), gender parity index (GPI); (4) Unmet need for contraception (% of married women ages 15-49);
- **Innovation**: (1) Patent applications, residents; (2) Research and development expenditure (% of GDP); (3) Scientific and technical journal articles;

Source: `https://datatopics.worldbank.org/esg/framework.html`.

Where to find the data?

- National accounts statistics collected by OECD, United Nations Statistics Division (UNSD), etc.
- Internal departments and specialized databases of the World Bank: World Bank Open Data, Business Enabling Environment (BEE), Climate Change Knowledge Portal (CCKP), Global Electrification Database (GEP), etc.
- International organizations: Emission Database for Global Atmospheric Research (EDGAR), Food and Agriculture Organization FAO, International Energy Agency (IEA), International Labour Organization (ILO), World Health Organization (WHO), etc.
- NGOs: Climate Watch, etc.;
- Academic resources: International disasters database (EM-DAT) of the Centre for Research on the Epidemiology of Disasters (Université Catholique de Louvain), etc.

Other frameworks

The most known are FTSE (Beyond Ratings), Moody's (Vigeo-Eiris), MSCI, Sustainalytics, RepRisk and Verisk Mapplecroft.

$\Rightarrow$ The average cross-correlation between data providers is equal to 85% for the ESG score, 42% for the environmental score, 85% for the social score and 71% for the governance score

Bias towards richest countries

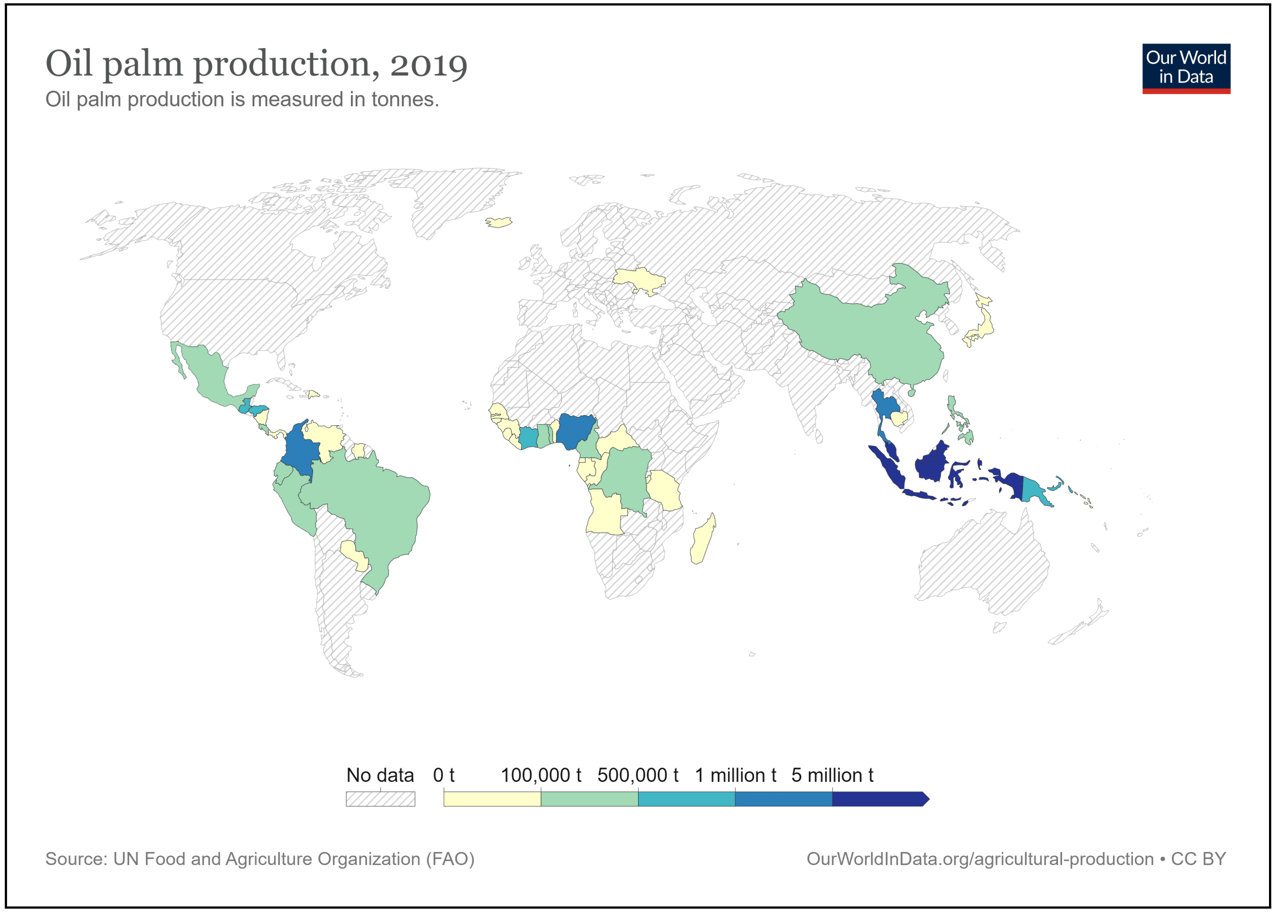
Table 12: Correlation of ESG scores with countrys national income (GNI per capita)

Factor	ESG	E	S	G
ISS	68%	7%	86%	77%
FTSE (Beyond Ratings)	91%	74%	88%	84%
MSCI	84%	10%	90%	77%
RepRisk	78%	79%	75%	37%
RobecoSAM	89%	82%	85%	85%
Sustainalytics	95%	83%	94%	93%
V.E	60%	23%	79%	39%
Total	81%	51%	85%	70%

Source: Gratcheva *et al.* (2020).

The mushrooming growth of data

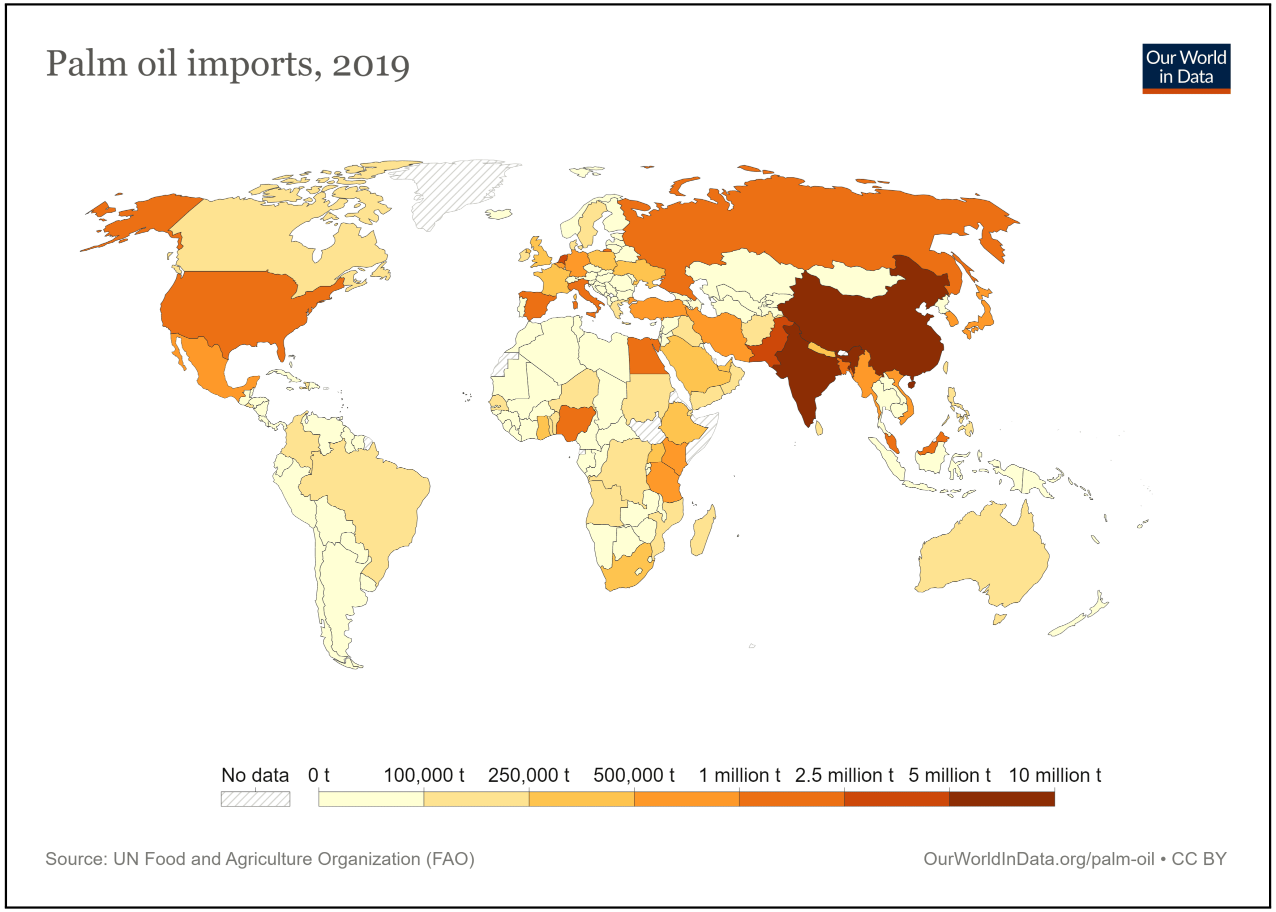
Figure 20: Palm oil production (2019)



Source: Our World in Data, `https://ourworldindata.org/palm-oil`.

The mushrooming growth of data

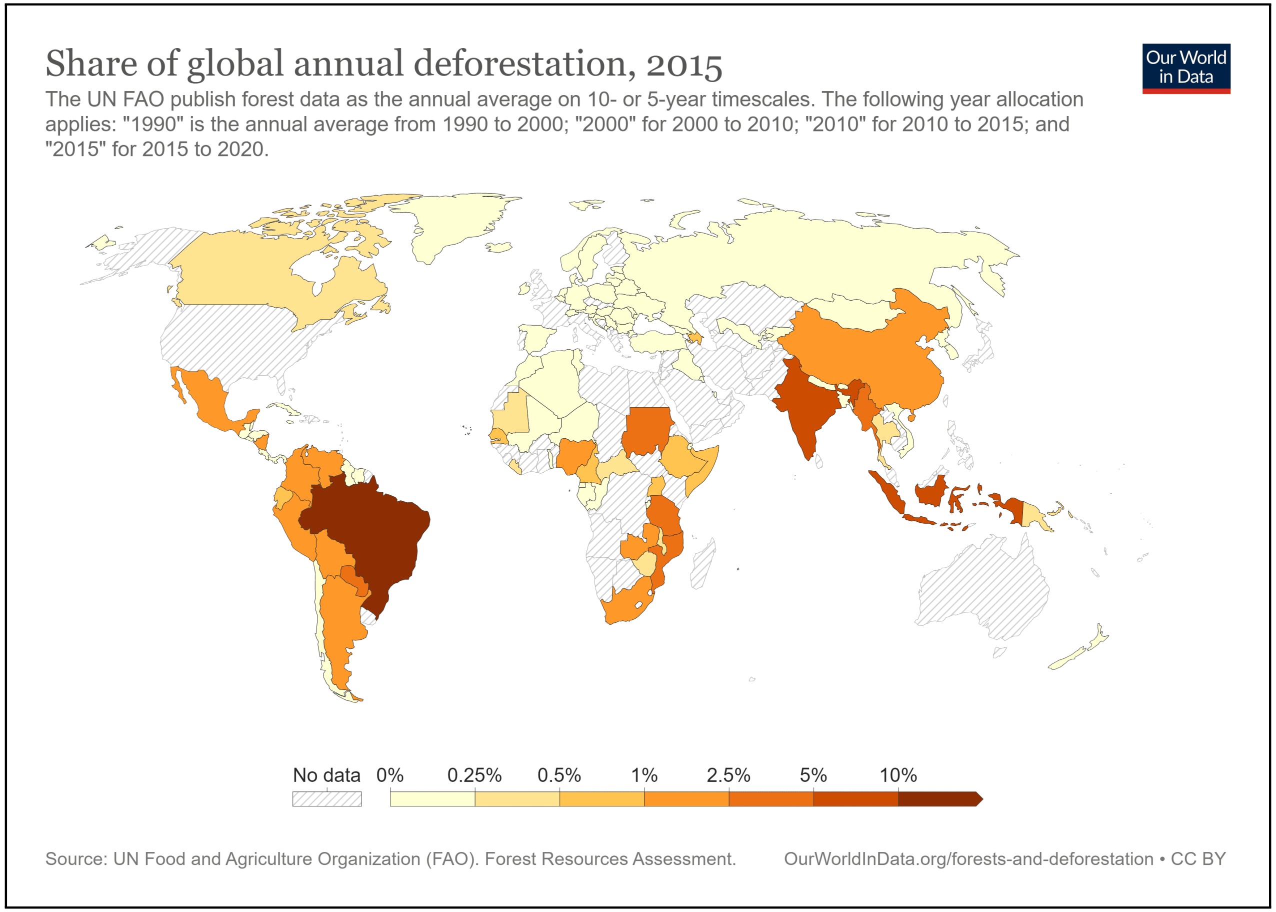
Figure 21: Palm oil imports (2019)



Source: Our World in Data, `https://ourworldindata.org/palm-oil`.

The mushrooming growth of data

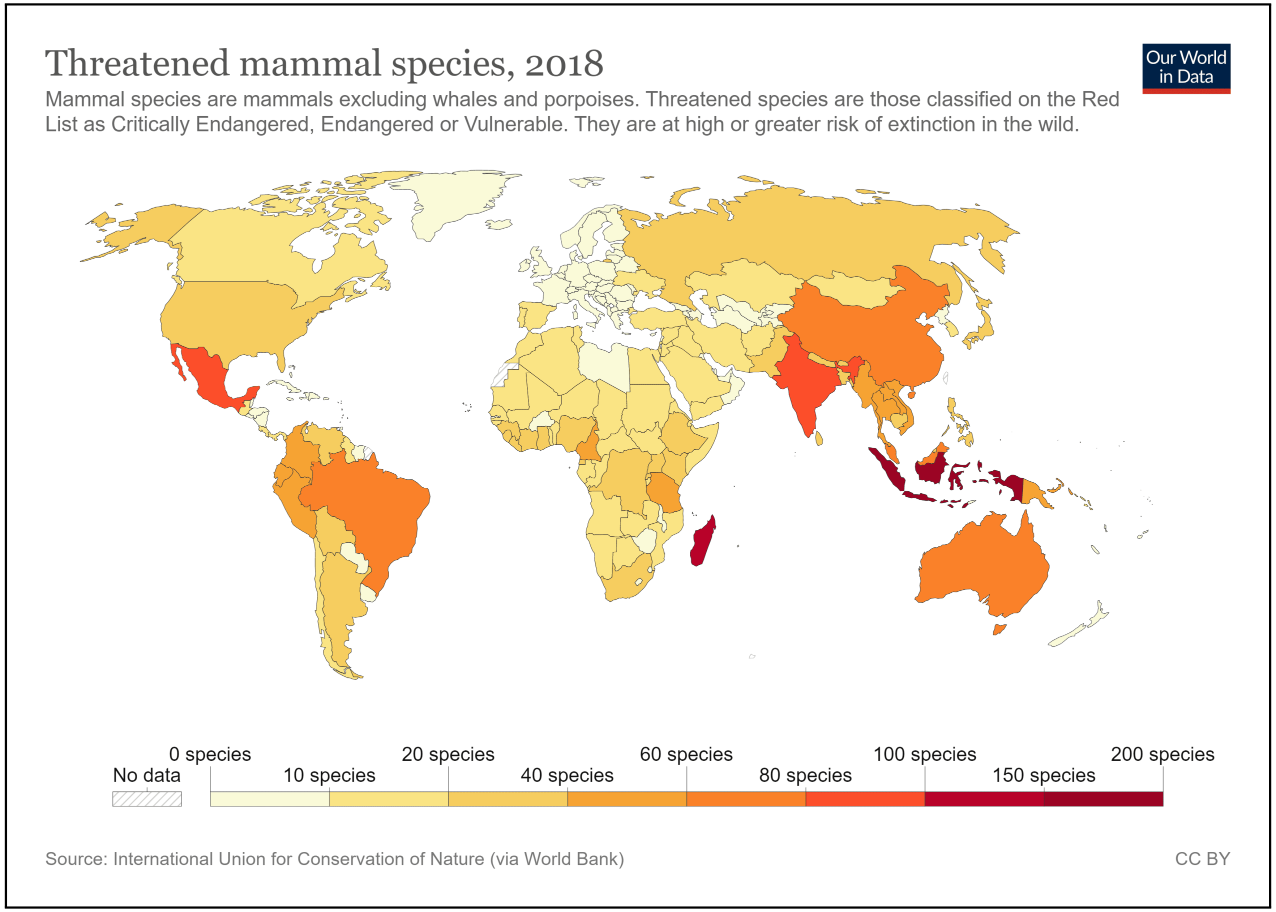
Figure 22: Share of global annual deforestation (2015)



Source: Our World in Data, `https://ourworldindata.org/deforestation`.

The mushrooming growth of data

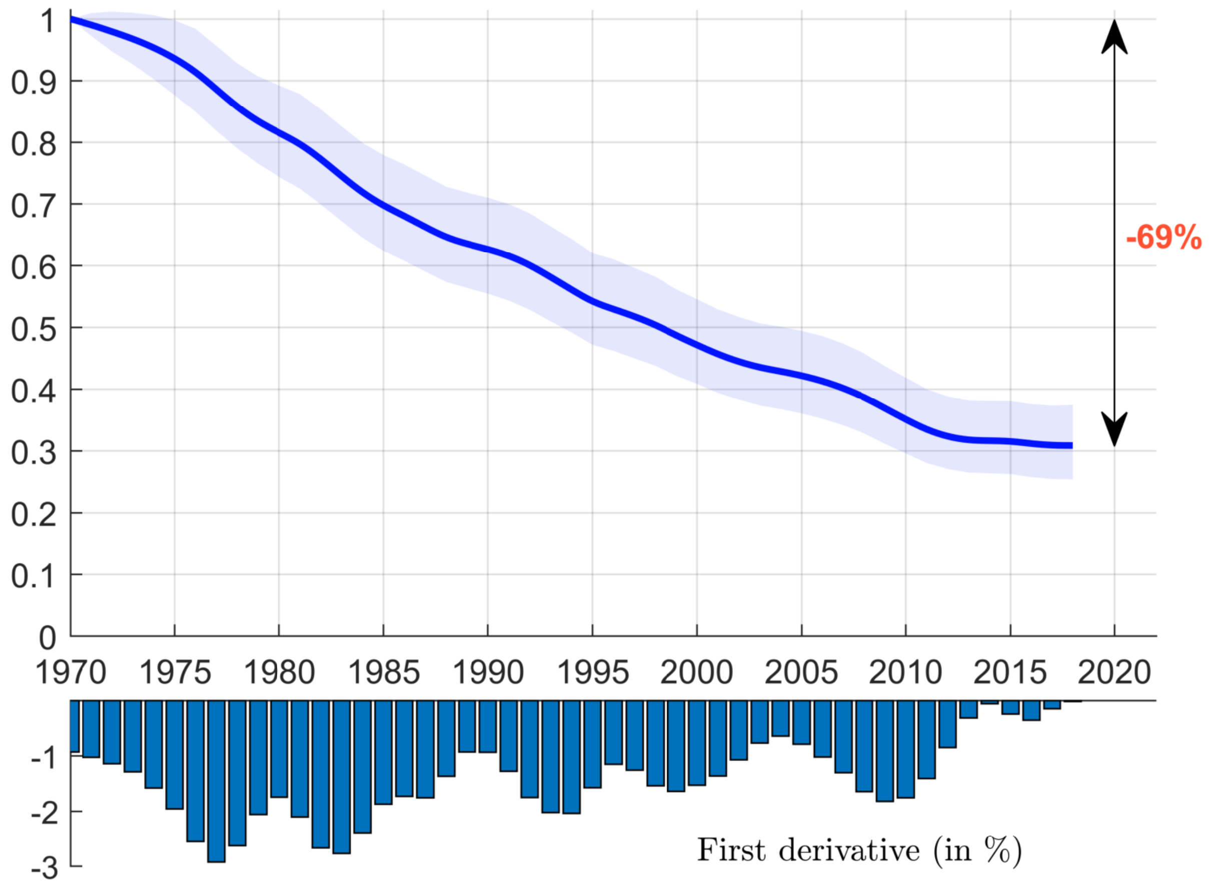
Figure 23: Threatened mammal species (2018)



Source: Our World in Data, `https://ourworldindata.org/biodiversity`.

An example with the biodiversity risk

Figure 24: Global living planet index



Source: `https://livingplanetindex.org/latest_results` & Author's calculation.

An example with the biodiversity risk

Some databases:

- the Red List Index (RLI)
- World Database on Protected Areas (WDPA)
- Integrated Biodiversity Assessment Tool (IBAT)
- Exploring Natural Capital Opportunities, Risks and Exposure (ENCORE)
- Etc.

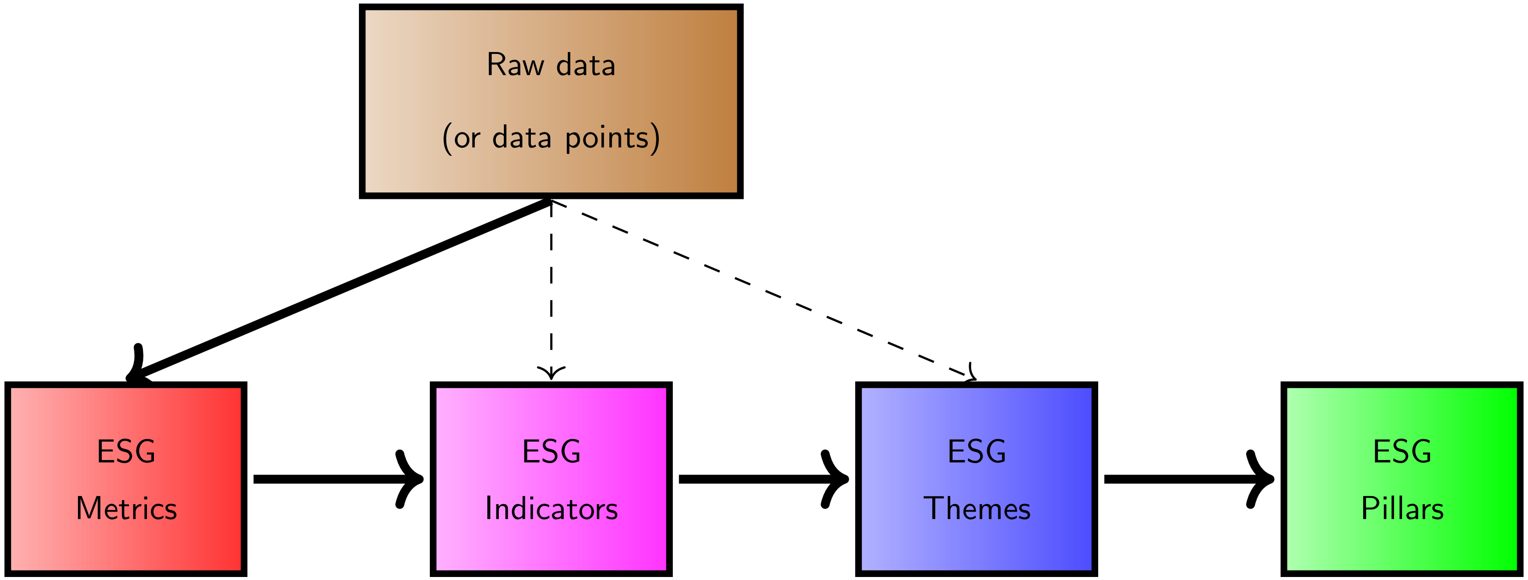
Corporate ESG data

Data sources:

1. Corporate publications (self-reporting)
   1. Annual reports
   2. Corporate sustainability reports
2. Financial and regulatory filings (standardized reporting)
   1. Mandatory reports (SFDR, CSRD, EUTR, etc.)
   2. Non-mandatory frameworks (PRI, TCFD, CDP, etc.)
3. News and other media
4. NGO reports and websites
5. Company assessment and due diligence questionnaire (DDQ)
6. Internal models

Corporate ESG data

Figure 25: From raw data to ESG pillars



Corporate ESG data

Table 13: An example of ESG criteria (corporate issuers)

Environmental	Social	Governance
• Carbon emissions	• Employment conditions	• Board independence
• Energy use	• Community involvement	• Corporate behaviour
• Pollution	• Gender equality	• Audit and control
• Waste disposal	• Diversity	• Executive compensation
• Water use	• Stakeholder opposition	• Shareholder' rights
• Renewable energy	• Access to medicine	• CSR strategy
• Green cars*		
• Green financing*		

$^{(\star)}$means a specific criterion related to one or several sectors (Green cars $\Rightarrow$ Automobiles, Green financing $\Rightarrow$ Financials)

Corporate ESG data

Some examples:

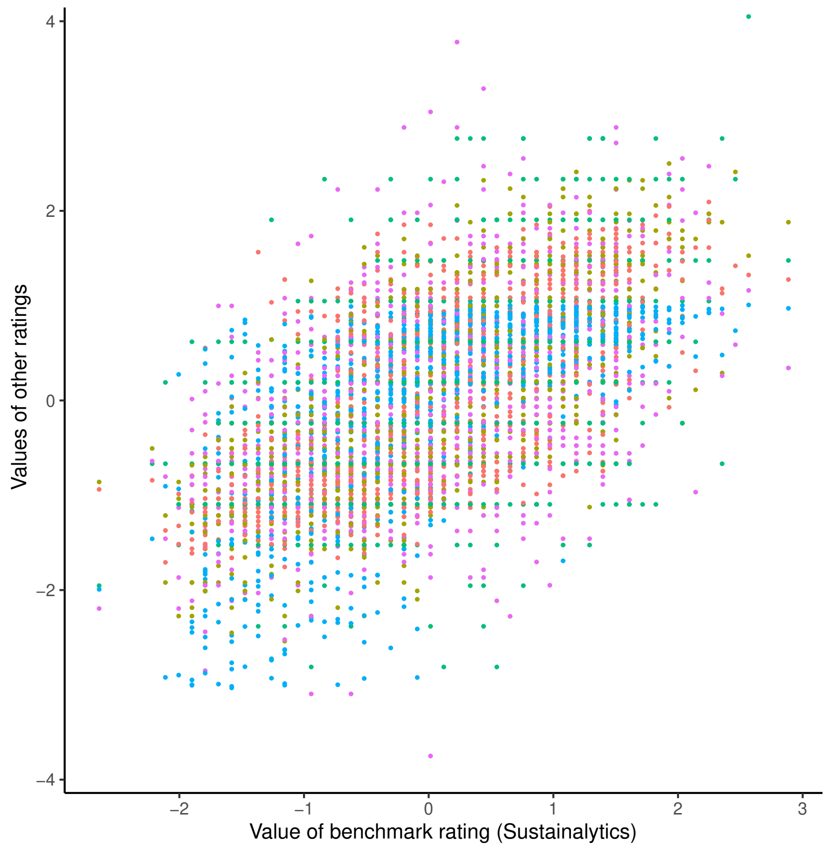
- Bloomberg rates 11 800 public companies. They use more than 120 ESG indicators and 2 000+ data points.
- ISS ESG rates about 10 000 issuers. They use more than 800 indicators and applies approximately 100 indicators per company.
- FTSE Russell rates about 7 200 securities. They use more than 300 indicators and 14 themes.
- MSCI rates 10 000 companies (14 000 issuers including subsidiaries) and 680 000 securities globally. They use 10 themes, 1000+ data points, 80 exposure metrics and 250+ management metrics.
- Refinitiv rates 12 000 public and private companies. They consider 10 themes. These themes are built using 186 metrics and 630+ data points.
- S&P Dow Jones Indices uses between 16 to 27 criteria scores, a questionnaire and 1 000 data points.
- Sustainalytics rates more than 16 300 companies. They consider 20 material ESG issues, based on 350+ indicators.

The race for alternative data

- Controversies ⇒ NLP (RepRisk, daily basis: 500 000+ documents, 100 000+ sources, 23 languages)
- Geospatial data ⇒ Physical risk

The divergence of corporate ESG ratings

Figure 26: ESG rating disagreement



Source: Berg *et al.* (2022).

The divergence of corporate ESG ratings

Berg *et al.* (2022) identify three sources of divergence:

1. "**Measurement** *divergence refers to situation where rating agencies measure the same indicator using different ESG metrics* (56%)
2. **Scope** *divergence refers to situation where ratings are based on different set of ESG indicators* (38%)
3. **Weight** *divergence emerges when rating agencies take different views on the relative importance of ESG indicators*" (6%)

The divergence of corporate ESG ratings

Table 14: Rank correlation among ESG ratings

	MSCI	Refinitiv	S&P Global	
MSCI	100%			
Refinitiv	43%	100%		
S&P Global	45%	69%	100%	
Sustainalytics	53%	64%	69%	100%

Source: Billio *et al.* (2021).

One-level tree structure

- $X_1, \ldots, X_m$ are m features
- The score is linear:

$$\mathcal{S} = \sum_{j=1}^{m} \omega_j X_j$$

- ω_j is the weight of the j^{th} metric

One-level tree structure

- $X_1, \ldots, X_m$ are m features
- The score is linear:

$$\mathcal{S} = \sum_{j=1}^{m} \omega_j X_j$$

- ω_j is the weight of the j^{th} metric

One-level tree structure

The Altman Z score is equal to:

$$Z = 1.2 \cdot X_1 + 1.4 \cdot X_2 + 3.3 \cdot X_3 + 0.6 \cdot X_4 + 1.0 \cdot X_5$$

where the variables X_j represent the following financial ratios:

X_j	Ratio
X_1	Working capital / Total assets
X_2	Retained earnings / Total assets
X_3	Earnings before interest and tax / Total assets
X_4	Market value of equity / Total liabilities
X_5	Sales / Total assets

$$Z_i \Rightarrow Z_i^\star = (Z_i - m_z) / \sigma_z \Rightarrow \text{Decision rule}$$

Two-level tree structure

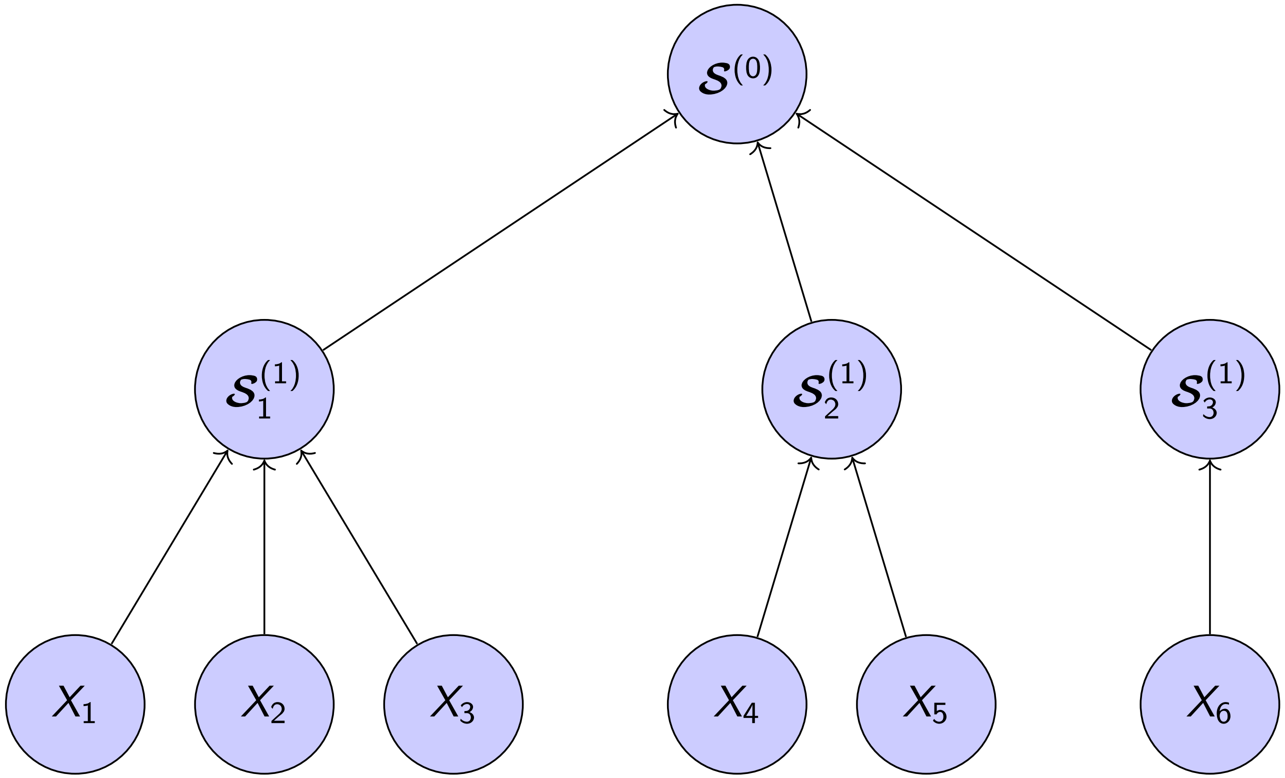
The intermediary scores are equal to:

$$\mathcal{S}_k^{(1)} = \sum_{j=1}^{m} \omega_{j,k}^{(1)} X_j$$

whereas the expression of the final score is:

$$\mathcal{S} := \mathcal{S}_1^{(0)} = \sum_{k=1}^{m_{(1)}} \omega_k^{(0)} \mathcal{S}_k^{(1)}$$

Figure 27: A two-level non-overlapping tree



- Level 1: $\omega_{1,1}^{(1)} = 50\%$; $\omega_{2,1}^{(1)} = 25\%$; $\omega_{3,1}^{(1)} = 25\%$; $\omega_{4,2}^{(1)} = 50\%$; $\omega_{5,2}^{(1)} = 50\%$; $\omega_{6,3}^{(1)} = 100\%$;
- Level 0: $\omega_1^{(0)} = \omega_2^{(0)} = \omega_3^{(0)} = 33.33\%$;

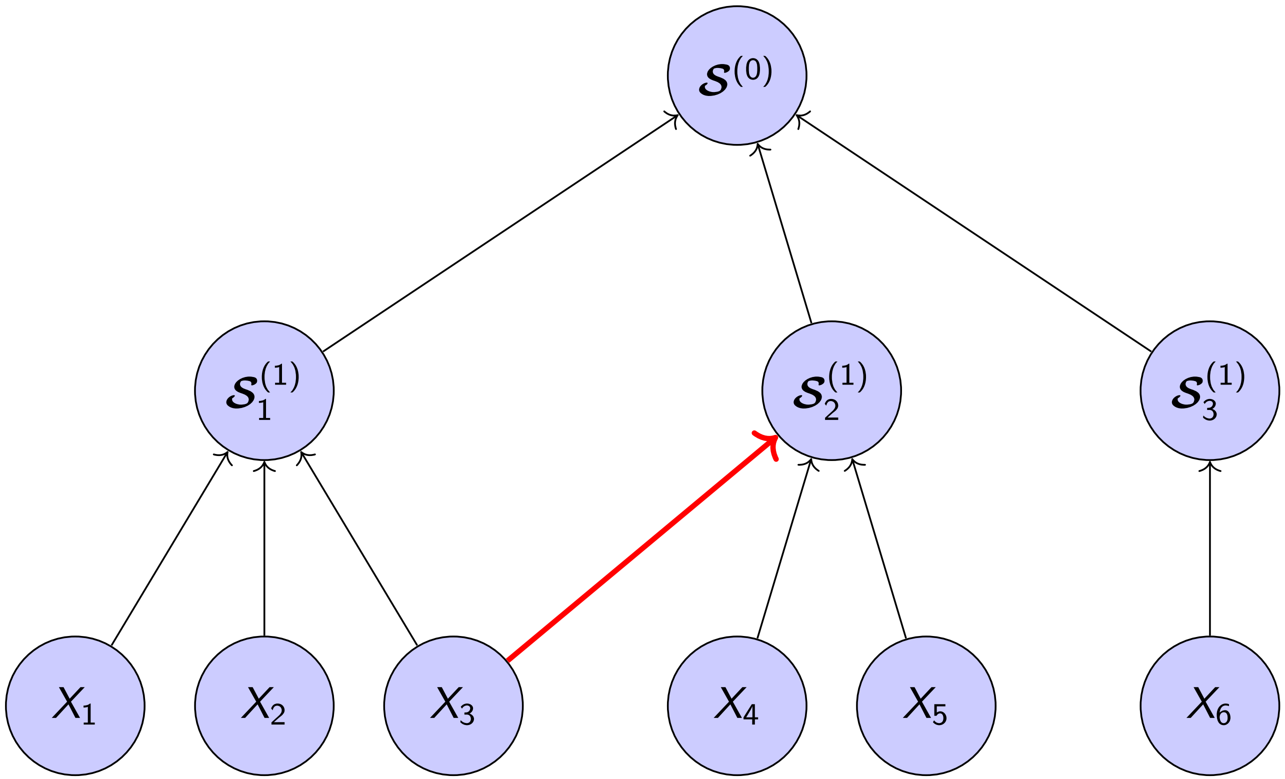
Two-level tree structure

$$\left\{ \begin{array}{l} \mathcal{S}_1^{(1)} = 0.5 \cdot X_1 + 0.25 \cdot X_2 + 0.25 \cdot X_3 \\ \mathcal{S}_2^{(1)} = 0.5 \cdot X_4 + 0.5 \cdot X_5 \\ \mathcal{S}_3^{(1)} = X_6 \end{array} \right.$$

$$\mathcal{S} = \frac{\mathcal{S}_1^{(1)} + \mathcal{S}_2^{(1)} + \mathcal{S}_3^{(1)}}{3}$$

Two-level tree structure

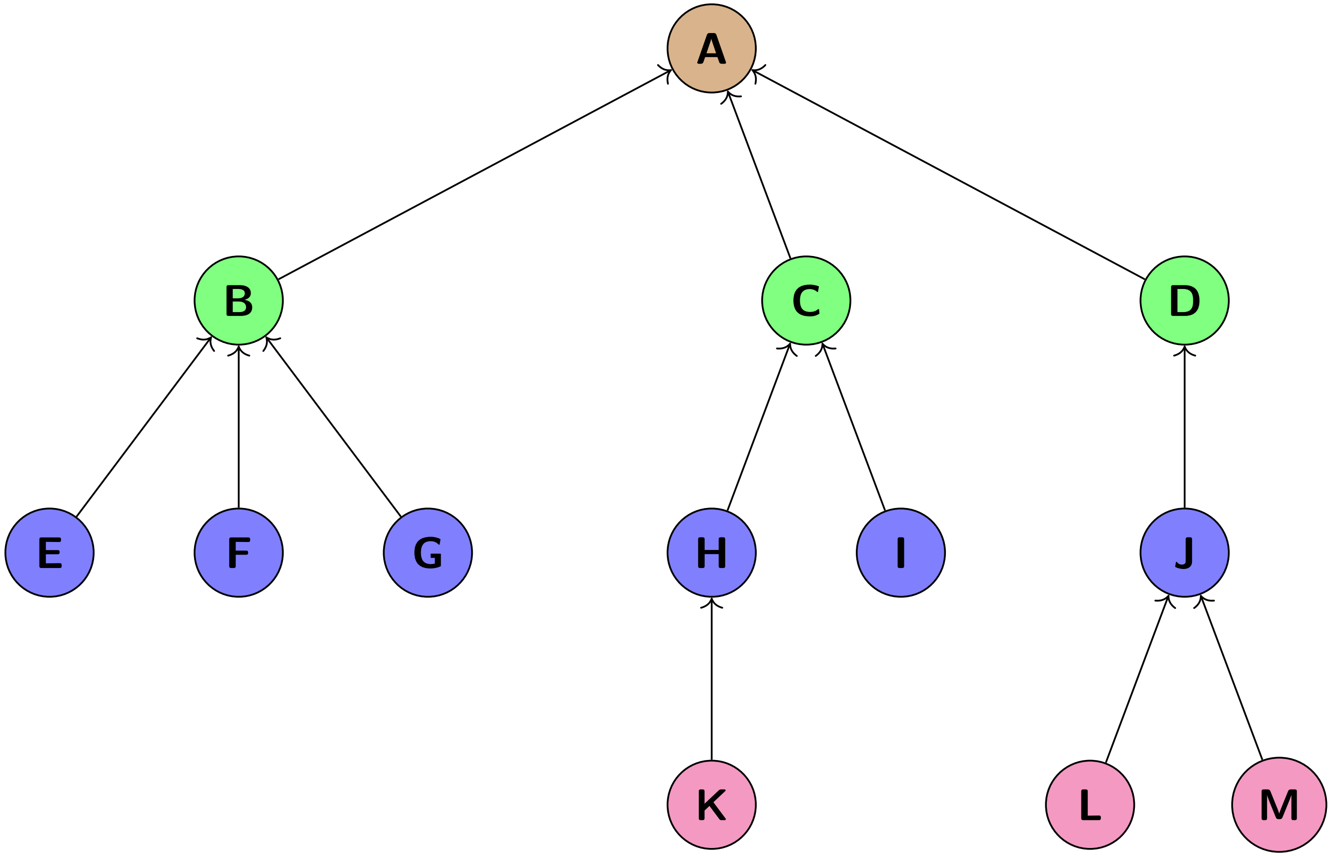
Figure 28: A two-level overlapping ~~tree~~ graph



- Level 1: $\omega_{1,1}^{(1)} = 50\%$; $\omega_{2,1}^{(1)} = 25\%$; $\omega_{3,1}^{(1)} = 25\%$; $\omega_{3,2}^{(1)} = 25\%$; $\omega_{4,2}^{(1)} = 25\%$; $\omega_{5,2}^{(1)} = 50\%$; $\omega_{6,3}^{(1)} = 100\%$;
- Level 0: $\omega_1^{(0)} = \omega_2^{(0)} = \omega_3^{(0)} = 33.33\%$;

Tree and graph theory

Figure 29: Tree data structure



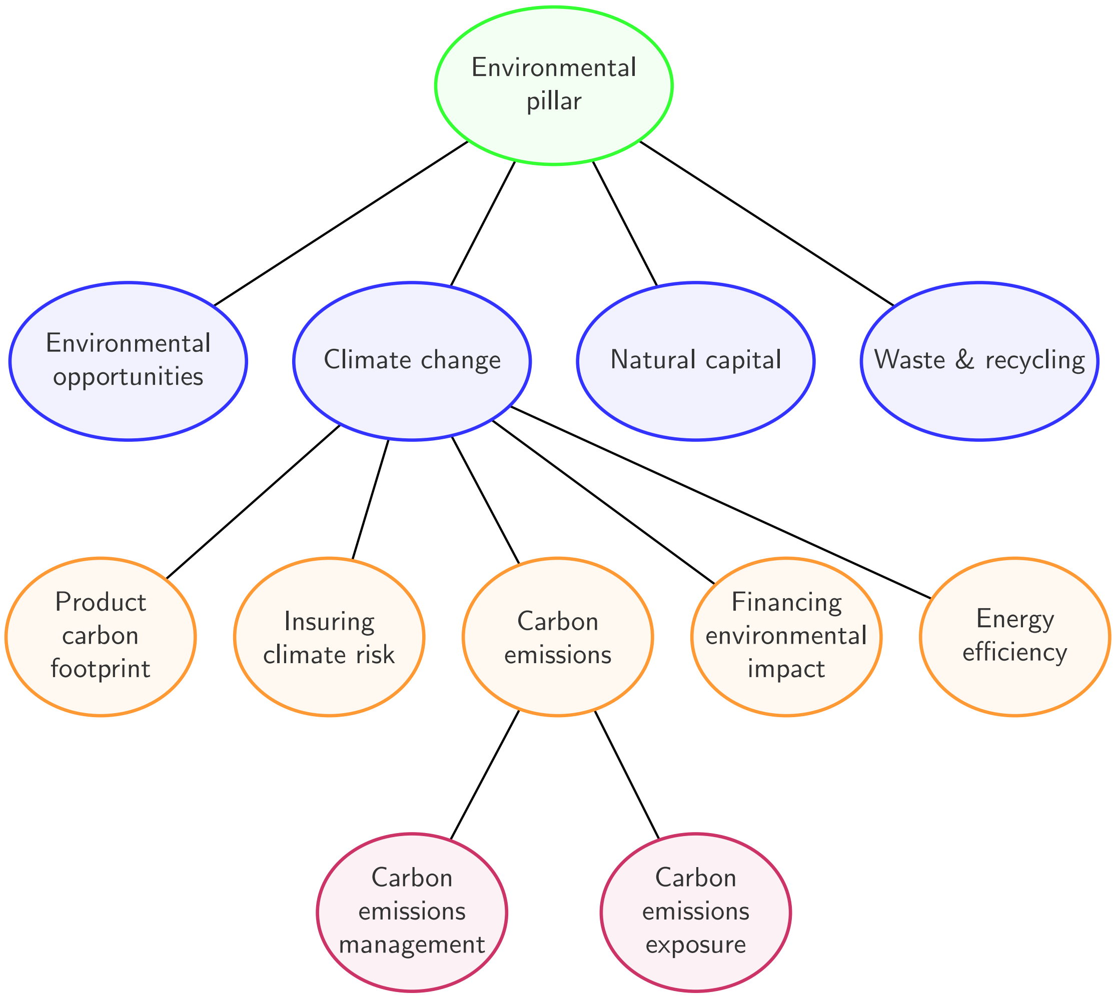
Tree and graph theory

- L is the number of levels
- We have $\boldsymbol{\mathcal{S}}_j^{(L)} = X_j$
- The value of the k^{th} node at level ℓ is given by:

$$\boldsymbol{\mathcal{S}}_k^{(\ell)} = \sum_{j=1}^{m_{(\ell+1)}} \omega_{j,k}^{(\ell)} \boldsymbol{\mathcal{S}}_j^{(\ell+1)}$$

An example of ESG scoring tree

Figure 30: An example of ESG scoring tree (MSCI methodology)



Source: MSCI (2020)

Score normalization

Let $\omega_{(\ell)}$ be the $m_{(\ell+1)} \times m_{(\ell)}$ matrix, whose elements are $\omega_{j,k}^{(\ell)}$ for $j = 1, \ldots, m_{(\ell+1)}$ and $k = 1, \ldots, m_{(\ell)}$

The final score is equal to:

$$\mathcal{S} = \omega^\top X$$

where:

$$\omega = \omega_{(L-1)} \cdots \omega_{(1)} \omega_{(0)}$$

Score normalization

If $X \sim \mathbf{F}$, we obtain:

$$\begin{aligned}
\mathbf{G}(s) &= \Pr\{\boldsymbol{\mathcal{S}} \leq s\} \\
&= \Pr\{\omega^\top X \leq s\} \\
&= \int \cdots \int \mathbb{1}\{\omega^\top x \leq s\} \,\mathrm{d}\mathbf{F}(x) \\
&= \int \cdots \int \mathbb{1}\left\{\sum_{j=1}^m \omega_j x_j \leq s\right\} \mathrm{d}\mathbf{F}(x_1, \ldots, x_m) \\
&= \int \cdots \int \mathbb{1}\left\{\sum_{j=1}^m \omega_j x_j \leq s\right\} \mathrm{d}\mathbf{C}(\mathbf{F}_1(x_1), \ldots, \mathbf{F}_m(x_m))
\end{aligned}$$

Therefore, the distribution $\mathbf{G}$ depends on the copula function $\mathbf{C}$ and the marginals $(\mathbf{F}_1, \ldots, \mathbf{F}_m)$ of $\mathbf{F}$

$$\mathbf{F}_1 \equiv \mathbf{F}_1 \equiv \ldots \equiv \mathbf{F}_m \Rightarrow \mathbf{G} \equiv \mathbf{F}_1?$$

Score normalization

In the independent case, we obtain a a convolution probability distribution:

$$\mathbf{G}(s) = \int \cdots \int \mathbb{1}\left\{\sum_{j=1}^{m} \omega_j x_j \leq s\right\} \prod_{j=1}^{m} \mathrm{d}\mathbf{F}_j(x_j)$$

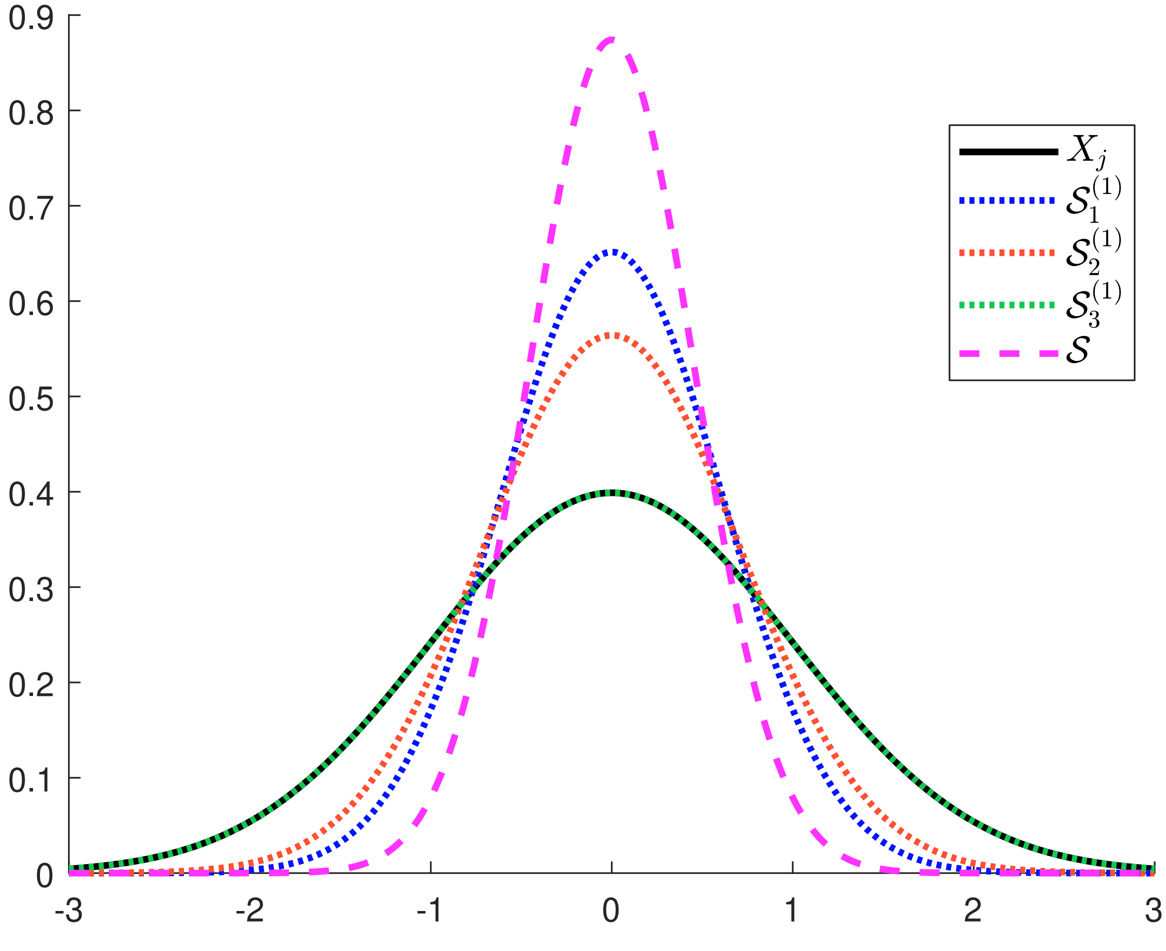
If $X_j \sim \mathcal{N}(\mu_j, \sigma_j^2)$, we have $\omega_j X_j \sim \mathcal{N}(\omega_j \mu_j, \omega_j^2 \sigma_j^2)$. We deduce that:

$$\boldsymbol{\mathcal{S}} \sim \mathcal{N}\left(\sum_{j=1}^{m} \omega_j \mu_j, \sum_{j=1}^{m} \omega_j^2 \sigma_j^2\right) \equiv \mathcal{N}\left(\omega^\top \mu, \omega^\top \Sigma \omega\right)$$

where $\mu = (\mu_1, \ldots, \mu_m)$ and $\Sigma = \operatorname{diag}(\sigma_1^2, \ldots, \sigma_m^2)$.

Score normalization

Figure 31: Probability distribution of the scores based on the previous tree



Score normalization

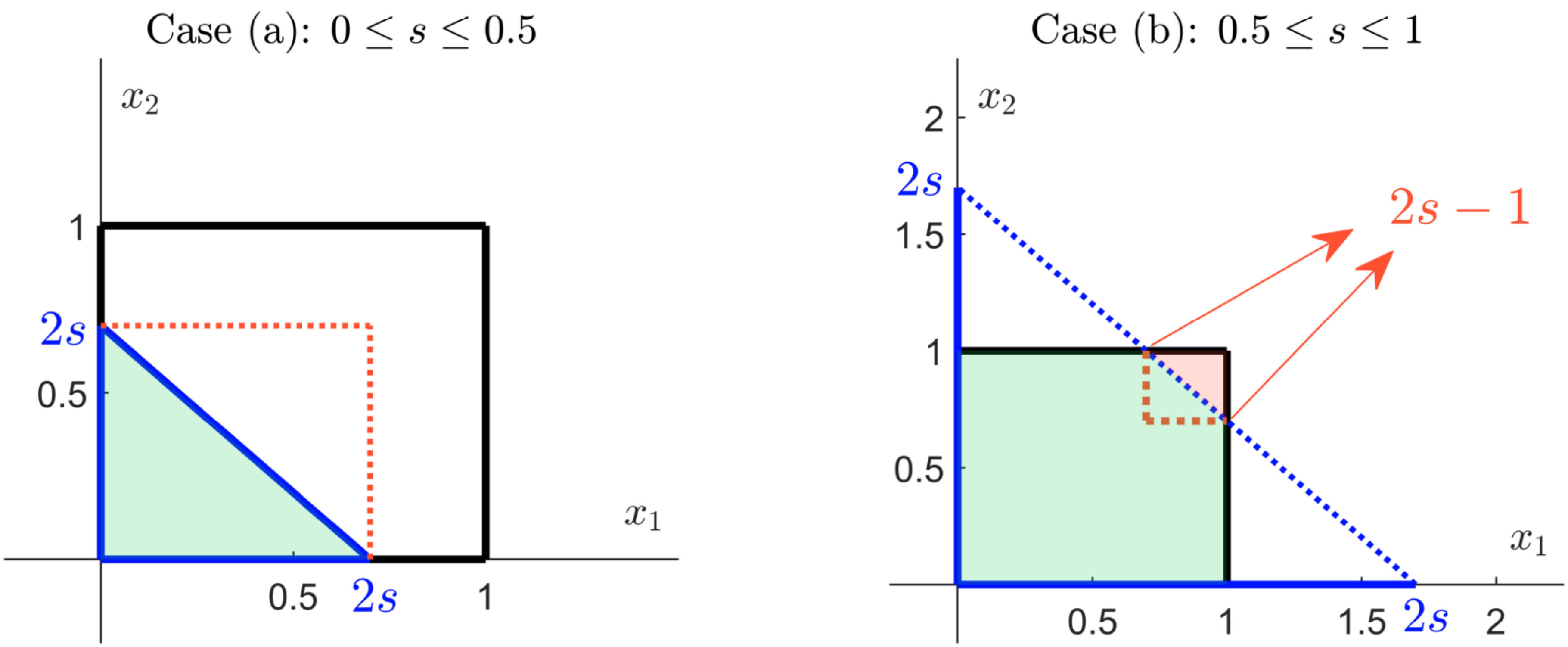
Exercise

We assume that $X_1 \sim \mathcal{U}_{[0,1]}$ and $X_2 \sim \mathcal{U}_{[0,1]}$ are two independent random variables. We consider the score $\boldsymbol{\mathcal{S}}$ defined as:

$$\boldsymbol{\mathcal{S}} = \frac{X_1 + X_2}{2}$$

Score normalization

Figure 32: Geometric interpretation of the probability mass function



Score normalization

We deduce that:

$$\Pr\left\{\boldsymbol{\mathcal{S}} \leq s\right\} = \begin{cases} \frac{1}{2}\left(2s\right)^2 = 2s^2 & \text{if} \quad 0 \leq s \leq \frac{1}{2} \\ 1 - \frac{1}{2}\left(2 - 2s\right)^2 = -1 + 4s - 2s^2 & \text{if} \quad \frac{1}{2} \leq s \leq 1 \end{cases}$$

The density function is then:

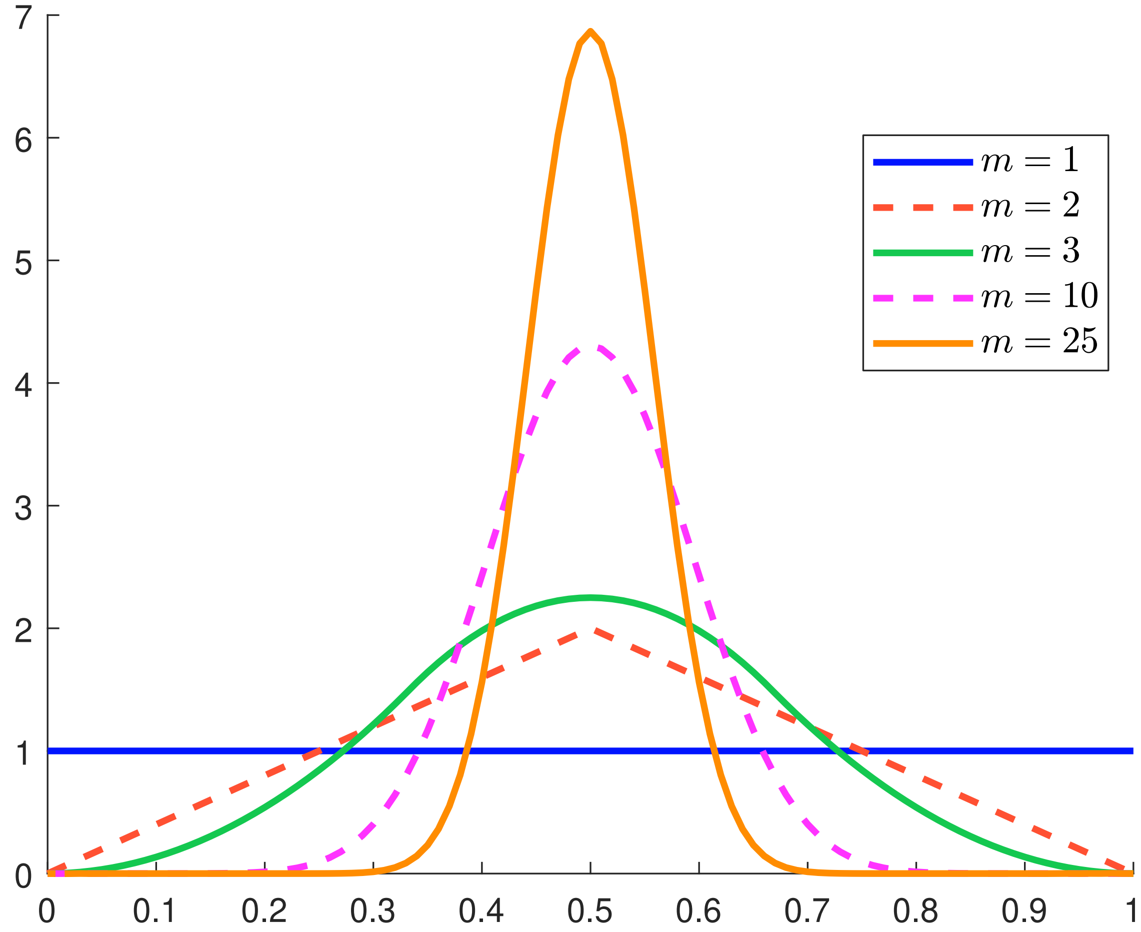
$$g\left(s\right) = \begin{cases} 4s & \text{if} \quad 0 \leq s \leq \frac{1}{2} \\ 4 - 4s & \text{if} \quad \frac{1}{2} \leq s \leq 1 \end{cases}$$

In the general case, we have:

$$\boldsymbol{\mathcal{S}} = \frac{X_1 + X_2 + \cdots + X_m}{m} \sim \mathfrak{B}ates\left(m\right)$$

Score normalization

Figure 33: Probability density function of $\mathcal{S}$ (uniform distribution)



Score normalization

Exercise

We assume that $X \sim \mathcal{N}(\mu, \Sigma)$ with $\mu_j = 0$, $\sigma_j = 1$ and $\rho_{j,k} = \rho$ for $j \neq k$. Show that:

$$\mathbb{E}[\boldsymbol{\mathcal{S}}] = 0$$

and

$$\operatorname{var}(\boldsymbol{\mathcal{S}}) = \rho \mathcal{S}^2(w) + (1 - \rho)\mathcal{H}(\omega)$$

where $\mathcal{S}(w) = \sum_{j=1}^{m} \omega_j$ is the sum index and $\mathcal{H}(\omega) = \sum_{j=1}^{m} \omega_j^2$ is the Herfindahl index. Deduce that:

$$\sigma_{\boldsymbol{\mathcal{S}}} = \sqrt{\rho + (1 - \rho)\mathcal{H}(\omega)}$$

Score normalization

How to normalize?

$$\mathcal{S}_k^{(\ell)} = \varphi \left(\sum_{j=1}^{m_{(\ell+1)}} \omega_{j,k}^{(\ell)} \mathcal{S}_j^{(\ell+1)} \right)$$

Score normalization

1. m-score normalization:

$$m_i = \frac{x_i - x^-}{x^+ - x^-}$$

where $x^- = \min x_i$ and $x^+ = \max x_i$

2. q-score normalization:

$$q_i = \mathbf{H}(x_i)$$

where $\mathbf{H}$ is the distribution function of X

3. z-score normalization:

$$z_i = \frac{x_i - \mu}{\sigma}$$

where μ and σ are the mathematical expectation and standard deviation of X

4. b-score normalization:

$$b_i = \mathfrak{B}^{-1}\left(\mathbf{H}(x_i); \alpha, \beta\right)$$

where $\mathcal{B}(\alpha, \beta)$ is the beta distribution

Score normalization

Probability integral transform (PIT)

If $X \sim \mathbf{H}$ and is continuous, $Y = \mathbf{H}(X)$ is a uniform random variable.

We have $Y \in [0, 1]$ and:

$$
\begin{aligned}
\Pr\{Y \leq y\} &= \Pr\{\mathbf{H}(X) \leq y\} \\
&= \Pr\left\{X \leq \mathbf{H}^{-1}(y)\right\} \\
&= \mathbf{H}\left(\mathbf{H}^{-1}(y)\right) \\
&= y
\end{aligned}
$$

Score normalization

Computing the empirical distribution $\hat{\mathbf{H}}$

- Let $\{x_1, x_2, \ldots, x_n\}$ be the sample
- We have:

$$q_i = \hat{\mathbf{H}}(x_i) = \Pr\{X \leq x_i\} = \frac{\#\{x_j \leq x_i\}}{n_q}$$

- $n_q = n$ or $n_q = n + 1$?

Score normalization

Exercise

What is the normalization shape of this transformation?

$$\mathcal{S} = \frac{2}{1 + e^{-z}} - 1$$

Hint: compute the density function.

Score normalization

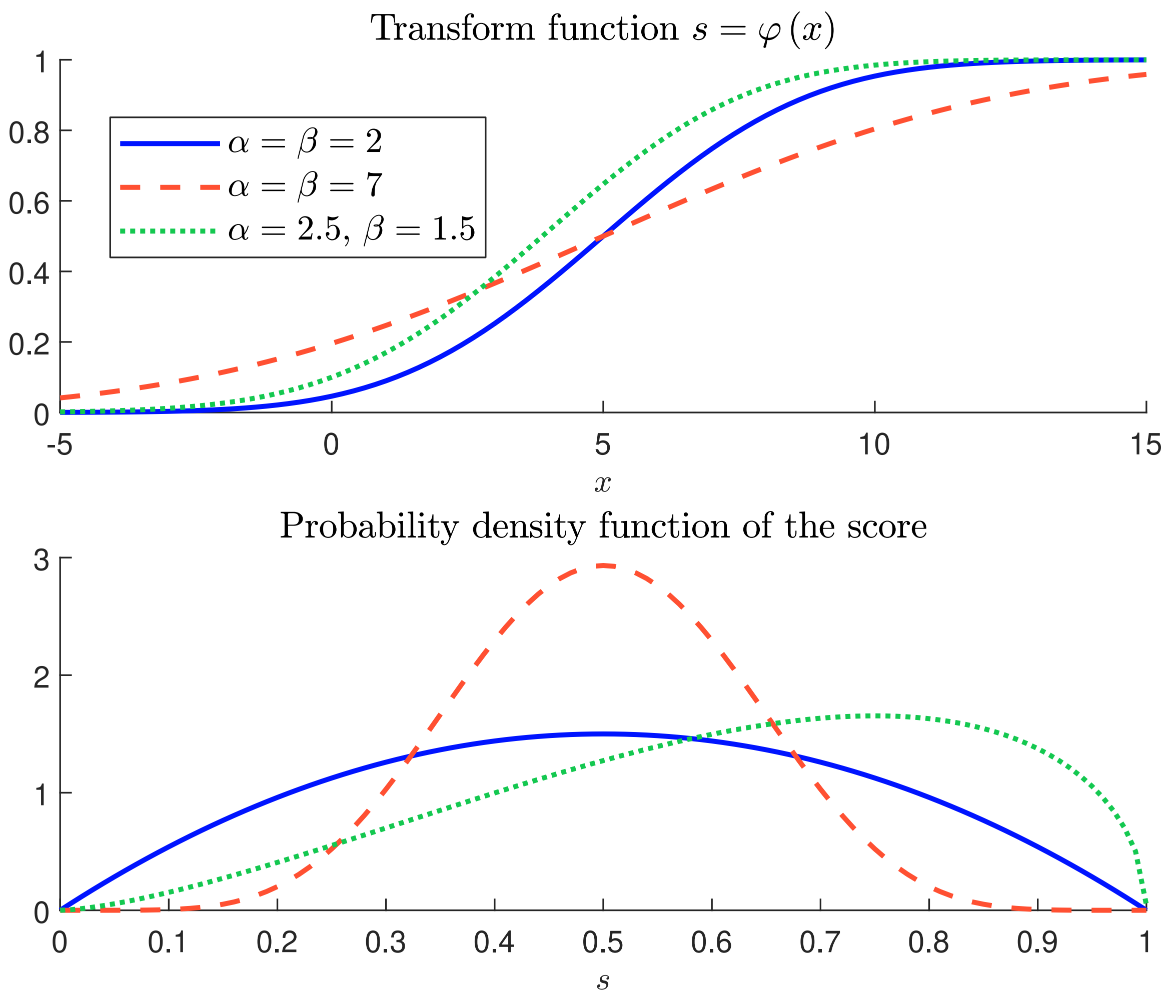
Example

The data are normally distributed with mean $\mu = 5$ and standard deviation $\sigma = 2$. To map these data into a 0/1 score, we consider the following transform:

$$s := \varphi(x) = \mathfrak{B}^{-1}\left(\Phi\left(\frac{x-5}{2}\right); \alpha, \beta\right)$$

Score normalization

Figure 34: Transforming data into $\mathscr{b}$-score



Score normalization

Example

We consider the raw data of 9 companies that belong to the same industry. The first variable measures the carbon intensity of the scope 1 + 2 in 2020, while the second variable is the variation of carbon emissions between 2015 and 2020. We would like to create the score $\mathcal{S} \equiv 70\% \cdot X_1 + 30\% \cdot X_2$.

Firm	Carbon intensity in tCO_2e/\$ mn)	Carbon momentum (in %)
1	94.0	-3.0
2	38.6	-5.5
3	30.6	5.6
4	74.4	-1.3
5	97.1	-16.8
6	57.1	-3.5
7	132.4	8.5
8	92.5	-9.1
9	64.9	-4.6

Score normalization

- q-score $0/100$
- z-score
- $qz = 100 \cdot \Phi(z)$
- $zq = \Phi^{-1}\left(\frac{q}{100}\right)$
- $bz = \mathfrak{B}^{-1}(\Phi(z); \alpha, \beta)$ where $\alpha = \beta = 2$
- $bz^{\star} = \mathfrak{B}^{-1}(\Phi(z); \alpha, \beta)$ where $\alpha = 2.5$ and $\beta = 1.5$.

Score normalization

Table 15: Computation of the score $\mathcal{S} \equiv 70\% \cdot X_1 + 30\% \cdot X_2$ (q-score 0/100 normalization)

#	X_1	q_1	X_2	q_2	s	$\mathcal{S}$	$\mathfrak{R}$
1	94.00	70.00	−3.00	60.00	67.00	80.00	8
2	38.60	20.00	−5.50	30.00	23.00	10.00	1
3	30.60	10.00	5.60	80.00	31.00	20.00	2
4	74.40	50.00	−1.30	70.00	56.00	60.00	6
5	97.10	80.00	−16.80	10.00	59.00	70.00	7
6	57.10	30.00	−3.50	50.00	36.00	30.00	3
7	132.40	90.00	8.50	90.00	90.00	90.00	9
8	92.50	60.00	−9.10	20.00	48.00	50.00	5
9	64.90	40.00	−4.60	40.00	40.00	40.00	4
Mean	75.73	50.00	−3.30	50.00	50.00	50.00	
Std-dev.	31.95	27.39	7.46	27.39	20.60	27.39	

Score normalization

Table 16: Computation of the score $\mathcal{S} \equiv 70\% \cdot X_1 + 30\% \cdot X_2$ (z-score normalization)

#	X_1	z_1	X_2	z_2	s	$\mathcal{S}$	$\mathfrak{R}$
1	94.00	0.572	−3.00	0.040	0.412	0.543	8
2	38.60	−1.162	−5.50	−0.295	−0.902	−1.188	1
3	30.60	−1.413	5.60	1.193	−0.631	−0.831	2
4	74.40	−0.042	−1.30	0.268	0.051	0.067	6
5	97.10	0.669	−16.80	−1.810	−0.075	−0.099	5
6	57.10	−0.583	−3.50	−0.027	−0.416	−0.548	3
7	132.40	1.774	8.50	1.582	1.716	2.261	9
8	92.50	0.525	−9.10	−0.778	0.134	0.177	7
9	64.90	−0.339	−4.60	−0.174	−0.290	−0.382	4
Mean	75.73	0.000	−3.30	0.000	0.000	0.000	
Std-dev.	31.95	1.000	7.46	1.000	0.759	1.000	

Score normalization

Table 17: Comparison of the different scoring methods

#	q		z		qz		zq		bz		$bz^{\star}$	
	$\mathcal{S}$	$\mathfrak{R}$	$\mathcal{S}$	$\mathfrak{R}$	$\mathcal{S}$	$\mathfrak{R}$	$\mathcal{S}$	$\mathfrak{R}$	$\mathcal{S}$	$\mathfrak{R}$	$\mathcal{S}$	$\mathfrak{R}$
1	80.00	8	0.54	8	76.27	8	0.84	8	0.66	8	0.81	8
2	10.00	1	−1.19	1	9.19	1	−1.28	1	0.20	1	0.30	1
3	20.00	2	−0.83	2	21.37	2	−0.84	2	0.29	2	0.38	2
4	60.00	6	0.07	6	54.13	5	0.25	6	0.52	6	0.70	6
5	70.00	7	−0.10	5	56.65	6	0.52	7	0.51	5	0.64	5
6	30.00	3	−0.55	3	24.42	3	−0.52	3	0.34	3	0.50	3
7	90.00	9	2.26	9	98.04	9	1.28	9	0.93	9	0.96	9
8	50.00	5	0.18	7	60.39	7	0.00	5	0.56	7	0.72	7
9	40.00	4	−0.38	4	30.96	4	−0.25	4	0.39	4	0.56	4
Mean	50.00		0.00		47.94		0.00		0.49		0.62	
Std-dev.	27.39		1.00		28.79		0.82		0.22		0.21	

An example with the CEO pay ratio

The CEO pay ratio is calculated by dividing the CEO's compensation by the pay of the median employee. It is one of the key metrics for the **G** pillar. It has been imposed by the Dodd-Frank Act, which requires that publicly traded companies disclose:

1. the median total annual compensation of all employees other than the CEO;
2. the ratio of the CEO's annual total compensation to that of the median employee;
3. the wage ratio of the CEO to the median employee.

⇒ the average S&P 500 company's CEO-to-worker pay ratio was 324-to-1 in 2021 (AFL-CIO)

An example with the CEO pay ratio

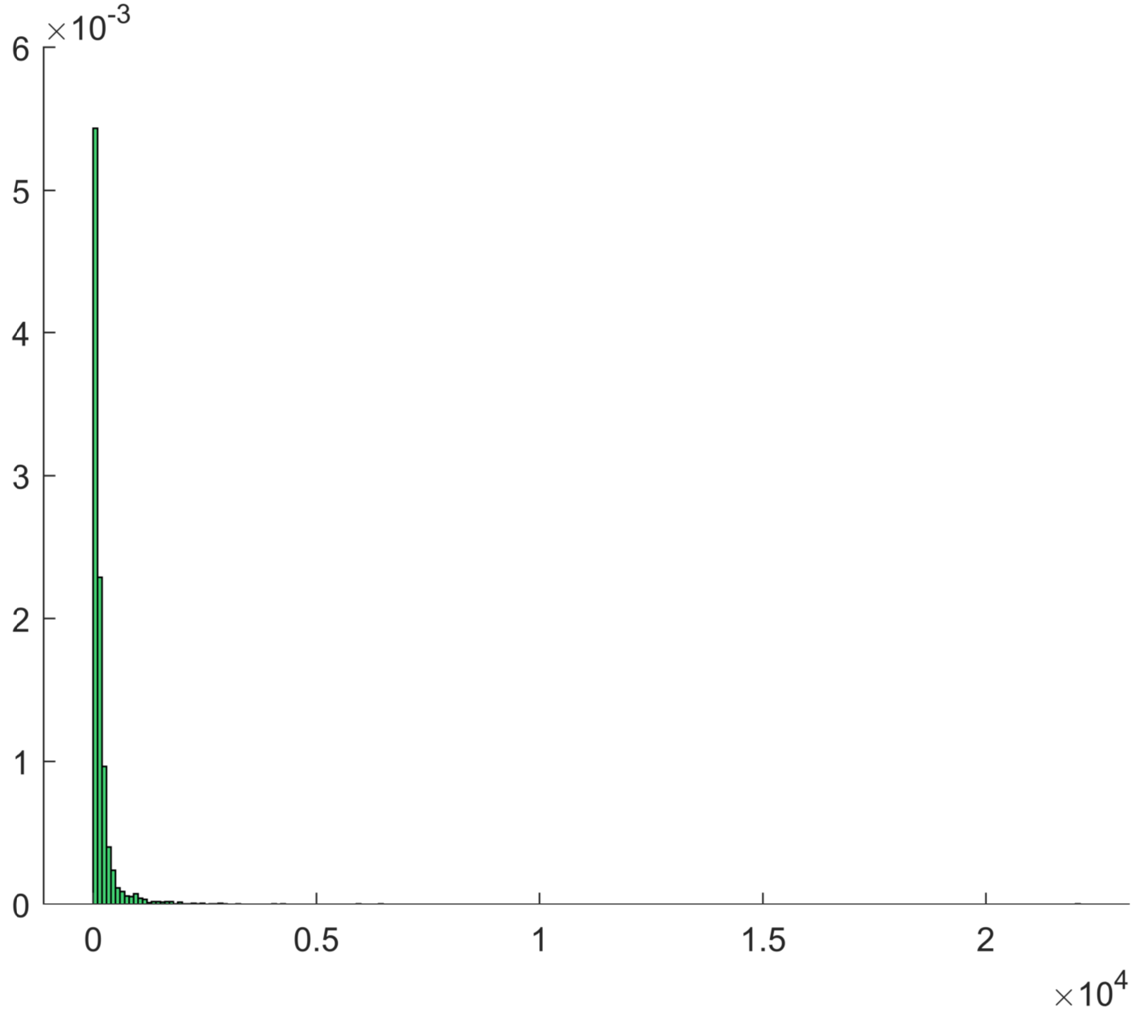
Table 18: Examples of CEO pay ratio (June 2021)

Company name	P	R	Company name	P	R
Abercrombie & Fitch	1 954	4,293	Netflix	202 931	190
McDonald's	9 291	1,939	BlackRock	133 644	182
Coca-Cola	11 285	1,657	Pfizer	98 972	181
Gap	6 177	1,558	Goldman Sachs	138 854	178
Alphabet	258 708	1,085	MSCI	55 857	165
Walmart	22 484	983	Verisk Analytics	77 055	117
Estee Lauder	30 733	697	Facebook	247 883	94
Ralph Lauren	21 358	570	Invesco	125 282	92
NIKE	25 386	550	Boeing	158 869	90
Citigroup	52 988	482	Citrix Systems	181 769	80
PepsiCo	45 896	368	Harley-Davidson	187 157	59
Microsoft	172 512	249	Amazon.com	28 848	58
Apple	57 596	201	Berkshire Hathaway	65 740	6

Source: `https://aflcio.org` (June 2021)

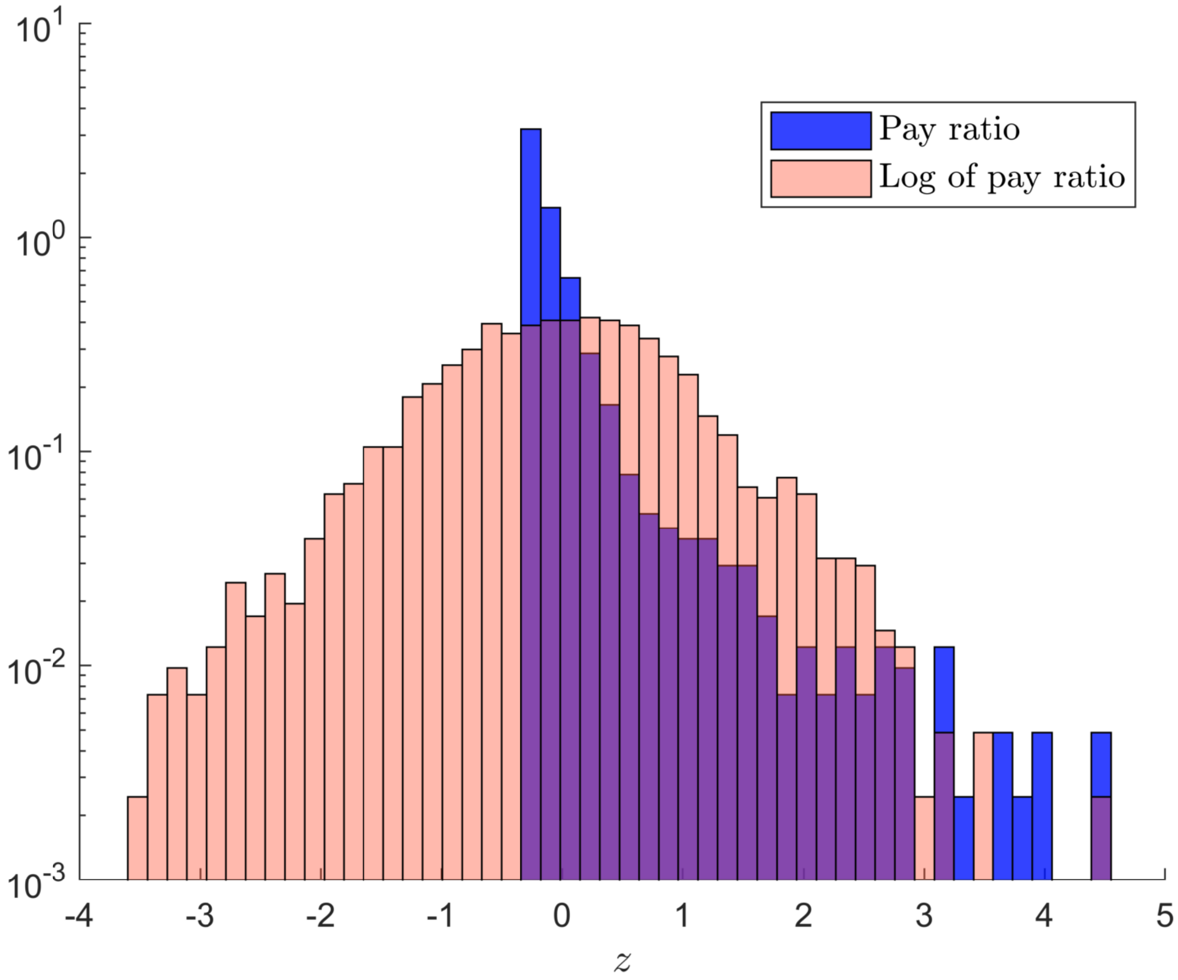
An example with the CEO pay ratio

Figure 35: Histogram of the CEO pay ratio



An example with the CEO pay ratio

Figure 36: Histogram of z-score applied to the CEO pay ratio



An example with the CEO pay ratio

What is the solution? Give the transform function $y = \varphi(x)$.

Hint: use the beta distribution.

Other statistical methods

Unsupervised learning

- Clustering (K-means, hierarchical clustering)
- Dimension reduction (PCA, NMF)

Other statistical methods

Supervised learning

- Discriminant analysis (LDA, QDA)
- Binary choice models (logistic regression, probit model)
- Regression models (OLS, lasso)

$\Rightarrow$ Advanced learning models (k-NN, neural networks and support vector machines) are not relevant in the case of ESG scoring

We need to define the response variable Y

Other statistical methods

Example with credit scoring models

- Let $\mathcal{S}_i(t)$ be the credit score of individual i at time t
- We have:

$$Y_i(t) = \mathbb{1}\{\tau_i \leq t + \delta\} = \mathbb{1}\{D_i(t + \delta) = 1\}$$

  where τ_i and D_i are the default time and the default indicator function, and δ is the time horizon (e.g., one year)
- The calibration problem of the credit scoring model is:

$$\Pr\{Y_i(t) = 0\} = f(\mathcal{S}_i(t))$$

  where f is an increasing function

Application to ESG scoring models

- Let $\mathcal{S}_i(t)$ be the ESG score of company i at time t
- Endogenous response variable:
  (a) Best-in-class oriented scoring system:
  $$Y_i(t) = \mathbb{1}\left\{\mathcal{S}_i(t+h) \geq s^\star\right\}$$
  where $s^\star$ is the best-in-class threshold
  (b) Worst-in-class oriented scoring system: $Y_i(t) = \mathbb{1}\left\{\mathcal{S}_i(t+h) \leq s^\star\right\}$
  where $s^\star$ is the worst-in-class threshold
- Exogenous response variable
  (c) Binary response:
  $$Y_i(t) = \mathbf{1}\left\{\mathcal{C}_i(t+h) \geq 0\right\}$$
  where $\mathcal{C}_i(t)$ is the controversy index
  d Continuous response:
  $$Y_i(t) = \mathcal{C}_i(t+h)$$
- The calibration problem of the ESG scoring model is $\Pr\left\{Y_i(t) = 0\right\} = f\left(\mathcal{S}_i(t)\right)$ or $Y_i(t) = f\left(\mathcal{S}_i(t)\right)$ where the function f is increasing for case (a) and decreasing for cases (b), (c) and (d)

Performance evaluation criteria

- ESG scoring and rating
  - Shannon entropy
  - Confusion matrix
  - Binary classification ratios (TPR, FNR, TNR, FPR, PPV, ACC, F_1)
- ESG scoring
  - Performance, selection and discriminant curves
  - ROC curve
  - Gini coefficient

Definition

Table 19: Credit rating system of S&P, Moody's and Fitch

	Prime Maximum Safety	High Grade High Quality			Upper Medium Grade		
S&P/Fitch	AAA	AA+	AA	AA−	A+	A	A−
Moody's	Aaa	Aa1	Aa2	Aa3	A1	A2	A3

	Lower Medium Grade			Non Investment Grade Speculative		
S&P/Fitch	BBB+	BBB	BBB−	BB+	BB	BB−
Moody's	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3

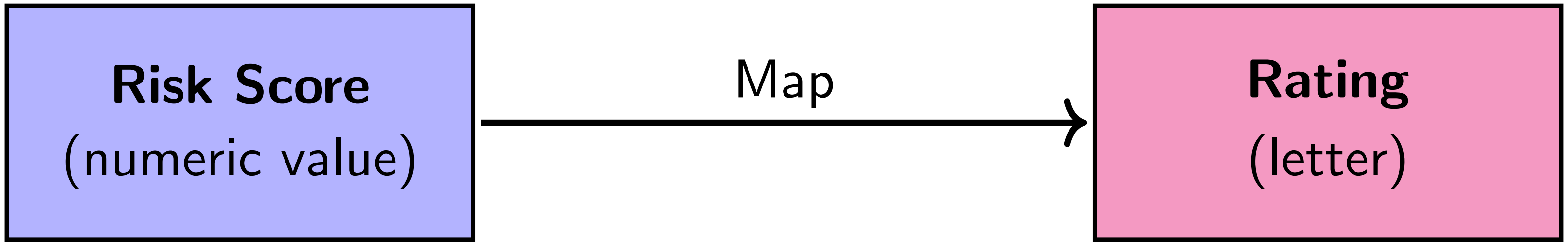
	Highly Speculative			Substantial Risk	In Poor Standing		Extremely Speculative
S&P/Fitch	B+	B	B−	CCC+	CCC	CCC−	CC
Moody's	B1	B2	B3	Caa1	Caa2	Caa3	Ca

Definition

- Amundi: A (high), B,... to G (low) — 7-grade scale
- FTSE Russell: 0 (low), 1,... to 5 (high) — 6-grade scale
- ISS ESG: 1 (high), 2,... to 10 (low) — 10-grade scale
- MSCI: AAA (high), AA,... to CCC (low) — 7-grade scale
- Refinitiv: A+ (high), A, A-, B+,... to D- (low) — 12-grade scale
- RepRisk: AAA (high), AA,... to D (low) — 8-grade scale
- Sustainanalytics: 1 (low), 2,... to 5 (high) — 5-grade scale

ESG rating process

Figure 37: From ESG score to ESG rating



Two-step approach:

1. Specification of the map function:

$$\begin{array}{rccc} \mathcal{M}ap: & \Omega_{\boldsymbol{\mathcal{S}}} & \longrightarrow & \Omega_{\mathcal{R}} \\ & \boldsymbol{\mathcal{S}} & \longmapsto & \mathcal{R} = \mathcal{M}ap(\boldsymbol{\mathcal{S}}) \end{array}$$

   where $\Omega_{\boldsymbol{\mathcal{S}}}$ is the support of ESG scores, $\Omega_{\mathcal{R}}$ is the ordered state space of ESG ratings and $\mathcal{R}$ is the ESG rating

2. Validation (and the possible *forcing*) of the rating by the analyst

ESG rating process

Example with the MSCI ESG rating system

- $\Omega_{\mathcal{S}} = [0, 10]$
- $\Omega_{\mathcal{R}} = \{\text{CCC}, \text{B}, \text{BB}, \text{BBB}, \text{A}, \text{AA}, \text{AAA}\}$
- The map function is defined as

$$\mathcal{M}ap\,(s) = \begin{cases} \text{CCC} & \text{if } \boldsymbol{\mathcal{S}} \in [0, {}^{10}/_{7}] & (0 - 1.429) \\ \text{B} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{10}/_{7}, {}^{20}/_{7}] & (1.429 - 2.857) \\ \text{BB} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{20}/_{7}, {}^{30}/_{7}] & (2.857 - 4.286) \\ \text{BBB} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{30}/_{7}, {}^{40}/_{7}] & (4.286 - 5.714) \\ \text{A} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{40}/_{7}, {}^{50}/_{7}] & (5.714 - 7.143) \\ \text{AA} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{50}/_{7}, {}^{60}/_{7}] & (7.143 - 8.571) \\ \text{AAA} & \text{if } \boldsymbol{\mathcal{S}} \in [{}^{60}/_{7}, 10] & (8.571 - 10) \end{cases}$$

ESG rating process

- The map function is an increasing piecewise function
- $\boldsymbol{\mathcal{S}} \sim \mathbf{F}$ and $\boldsymbol{\mathcal{S}} \in (s^-, s^+)$
- $\left\{s_0^\star = s^-, s_1^\star, \ldots, s_{K-1}^\star, s_K^\star = s^+\right\}$ are the knots of the piecewise function
- $\Omega_{\mathcal{R}} = \{R_1, \ldots, R_K\}$ is the set of grades

$\Rightarrow$ The frequency distribution of the ratings is given by:

$$\begin{aligned} p_k &= \Pr\{\mathcal{R} = R_k\} \\ &= \Pr\left\{s_{k-1}^\star \leq \boldsymbol{\mathcal{S}} < s_k^\star\right\} \\ &= \mathbf{F}(s_k^\star) - \mathbf{F}\left(s_{k-1}^\star\right) \end{aligned}$$

ESG rating process

If we would like to build a rating system with pre-defined frequencies $(p_1, \ldots, p_K)$, we have to solve the following equation:

$$\mathbf{F}\left(s_k^\star\right) - \mathbf{F}\left(s_{k-1}^\star\right) = p_k$$

We deduce that:

$$\begin{aligned}
\mathbf{F}\left(s_k^\star\right) &= p_k + \mathbf{F}\left(s_{k-1}^\star\right) \\
&= p_k + p_{k-1} + \mathbf{F}\left(s_{k-2}^\star\right) \\
&= \left(\sum_{j=1}^{k} p_j\right) + \mathbf{F}\left(s_0^\star\right)
\end{aligned}$$

and:

$$s_k^\star = \mathbf{F}^{-1}\left(\sum_{j=1}^{k} p_j\right)$$

ESG rating process

Exercise

- We assume that $\mathcal{S} \sim \mathcal{U}_{[a,b]}$
- Show that $p_k = K^{-1}$ If the rating system consists in K equally-sized intervals
- Show that the knots of the map function are equal to:

$$s_k^\star = a + (b - a)\left(\sum_{j=1}^{k} p_j\right)$$

  when we impose pre-defined frequencies $(p_1, \ldots, p_K)$
- If we consider a 0/100 uniform score and $\Omega_{\mathcal{R}} \times \mathbb{P} = (\text{CCC}, 5\%), (\text{B}, 10\%), (\text{BB}, 20\%), (\text{BBB}, 30\%), (\text{A}, 20\%), (\text{AA}, 10\%), (\text{AAA}, 5\%)$, show that $s_{\text{CCC}}^\star = 5$, $s_{\text{B}}^\star = 15$, $s_{\text{BB}}^\star = 35$, $s_{\text{BBB}}^\star = 65$, $s_{\text{A}}^\star = 85$ and $s_{\text{AA}}^\star = 95$

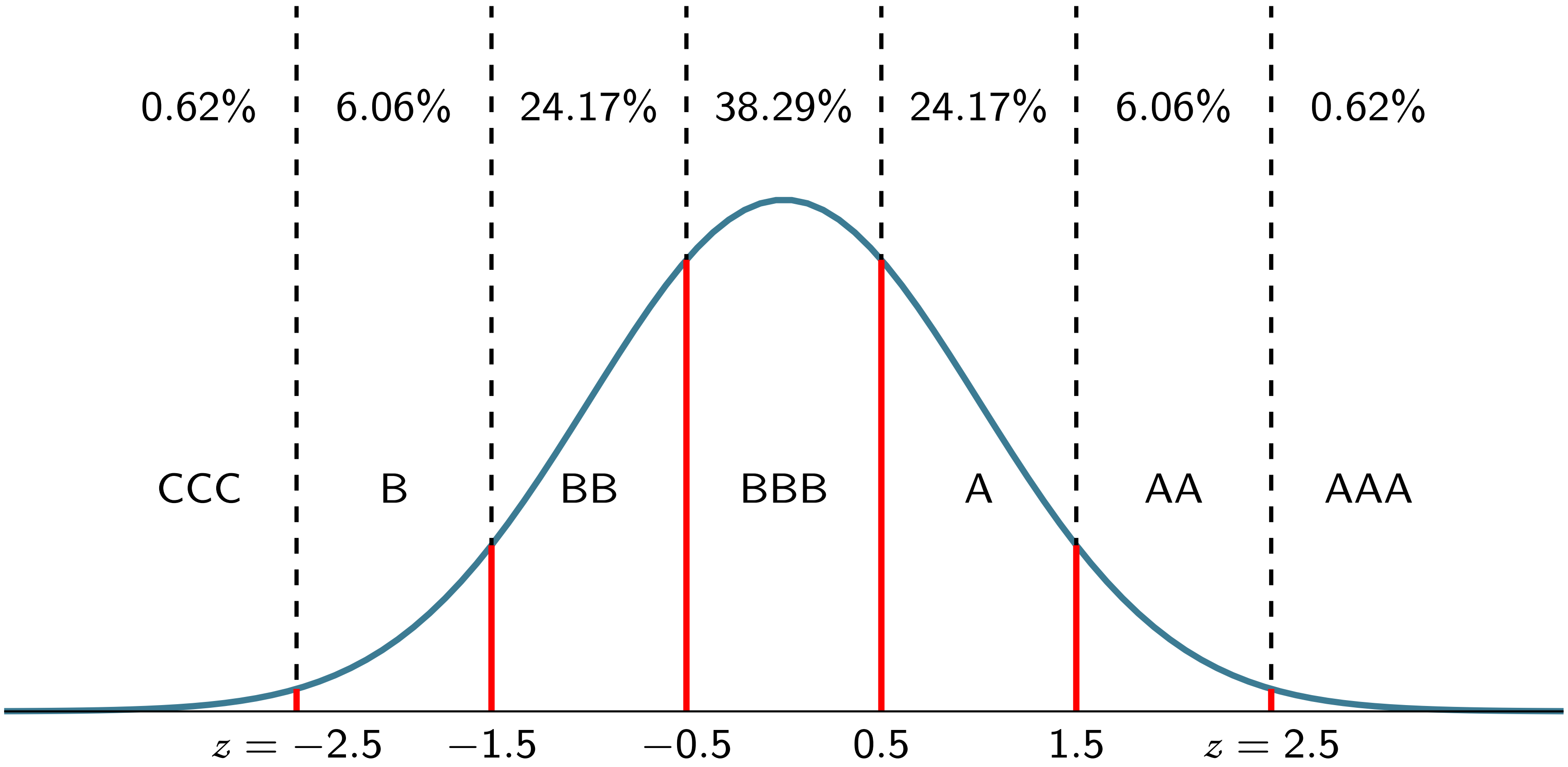
ESG rating process

For a z-score system ($\boldsymbol{\mathcal{S}} \sim \mathcal{N}(0,1)$), we obtain:

$$p_k = \Phi\left(s_k^{\star}\right) - \Phi\left(s_{k-1}^{\star}\right)$$

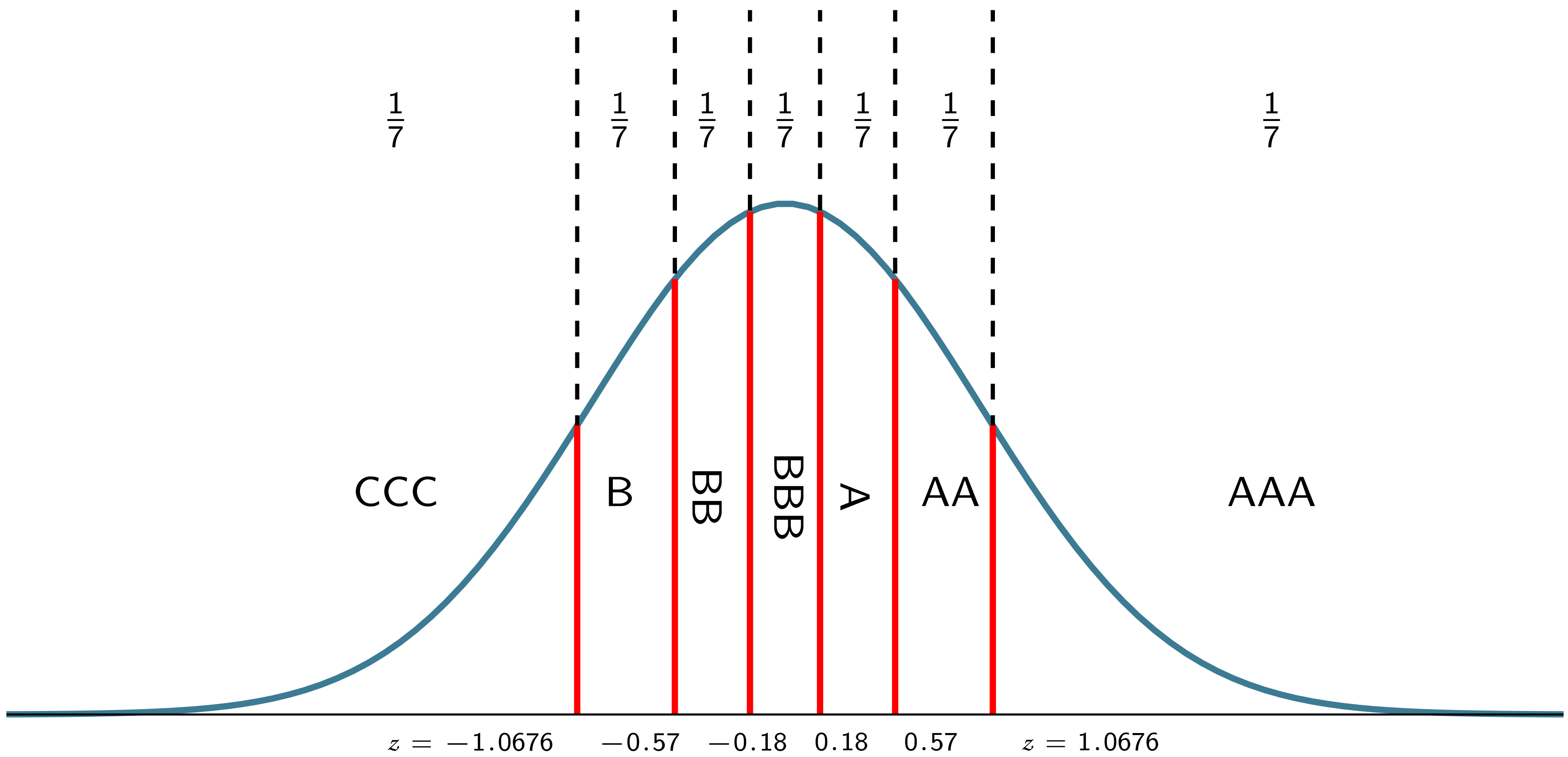
ESG rating process

Figure 38: Map function of a z-score (equal-space ratings)



ESG rating process

Figure 39: Map function of a z-score (equal-frequency ratings)



Rating migration matrix

Which rating model do you prefer? This one...

Table 20: ESG migration matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
AA	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
A	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
BBB	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
BB	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
B	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%
CCC	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%	14.3%

$\Rightarrow \mathcal{I}\left(\mathcal{R}\left(t\right) \mid \mathcal{R}\left(s\right)\right) = \ln 7$

Rating migration matrix

Which rating model do you prefer? Or this one...

Table 21: ESG migration matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	100%	0%	0%	0%	0%	0%	0%
AA	0%	100%	0%	0%	0%	0%	0%
A	0%	0%	100%	0%	0%	0%	0%
BBB	0%	0%	0%	100%	0%	0%	0%
BB	0%	0%	0%	0%	100%	0%	0%
B	0%	0%	0%	0%	0%	100%	0%
CCC	0%	0%	0%	0%	0%	0%	100%

$\Rightarrow \mathcal{I}\left(\mathcal{R}\left(t\right) \mid \mathcal{R}\left(s\right)\right) = 0$

Rating migration matrix

Which rating model do you prefer? Or this one?

Table 22: ESG migration matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	96%	4%	0%	0%	0%	0%	0%
AA	2%	96%	2%	0%	0%	0%	0%
A	0%	2%	96%	2%	0%	0%	0%
BBB	0%	0%	2%	96%	2%	0%	0%
BB	0%	0%	0%	2%	96%	2%	0%
B	0%	0%	0%	0%	2%	96%	2%
CCC	0%	0%	0%	0%	0%	4%	96%

$$\Rightarrow 0 < \mathcal{I}\left(\mathcal{R}\left(t\right) \mid \mathcal{R}\left(s\right)\right) \ll \ln 7$$

Rating migration matrix

A good reference on Markov chains is:

NORRIS, J. R. (1997).
Markov Chains.
Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press.

Rating migration matrix

Discrete time modeling

Definition

- $\mathcal{R}$ is a time-homogeneous Markov chain
- $\Omega_{\mathcal{R}} = \{R_1, \ldots, R_K\}$ is the state space of the chain
- $\mathbb{K} = \{1, \ldots, K\}$ is the corresponding index set
- The transition matrix is defined as $P = (p_{i,j})$
- $p_{i,j}$ is the probability that the entity migrates from rating R_i to rating R_j
- The matrix P satisfies the following properties:
  - $\forall i, j \in \mathbb{K},\ p_{i,j} \geq 0$
  - $\forall i \in \mathbb{K},\ \sum_{j=1}^{K} p_{i,j} = 1$

Rating migration matrix

Discrete time modeling

Table 23: ESG migration matrix #1 (one-year transition probability in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	92.76	5.66	0.90	0.45	0.23	0.00	0.00
AA	4.15	82.73	11.86	0.89	0.30	0.07	0.00
A	0.18	15.47	72.98	10.46	0.82	0.09	0.00
BBB	0.07	1.32	19.60	69.49	9.03	0.42	0.07
BB	0.04	0.19	1.55	19.36	70.88	7.75	0.23
B	0.00	0.05	0.24	1.43	21.54	74.36	2.38
CCC	0.00	0.00	0.22	0.44	2.21	13.24	83.89

Rating migration matrix

Discrete time modeling

The probability that the entity reaches the state R_j at time t given that it has reached the state R_i at time s is equal to:

$$p(s,i;t,j) = \Pr\{\mathcal{R}(t) = R_j \mid \mathcal{R}(s) = R_i\} = p_{i,j}^{(t-s)}$$

We note $p_{i,j}^{(n)}$ the n-step transition probability:

$$p_{i,j}^{(n)} = \Pr\{\mathcal{R}(t+n) = R_j \mid \mathcal{R}(t) = R_i\}$$

and the associated n-step transition matrix $P^{(n)} = \left(p_{i,j}^{(n)}\right)$

Rating migration matrix

Discrete time modeling

For $n = 2$, we obtain:

$$
\begin{aligned}
p_{i,j}^{(2)} &= \Pr\left\{\mathcal{R}(t+2) = R_j \mid \mathcal{R}(t) = R_i\right\} \\
&= \sum_{k=1}^{K} \Pr\left\{\mathcal{R}(t+2) = R_j, \mathfrak{R}(t+1) = R_k \mid \mathcal{R}(t) = R_i\right\} \\
&= \sum_{k=1}^{K} \Pr\left\{\mathcal{R}(t+2) = R_j \mid \mathcal{R}(t+1) = R_k\right\} \cdot \Pr\left\{\mathcal{R}(t+1) = R_k \mid \mathcal{R}(t) = R_i\right\} \\
&= \sum_{k=1}^{K} p_{i,k} \cdot p_{k,j}
\end{aligned}
$$

Rating migration matrix

Discrete time modeling

- The forward Chapman-Kolmogorov equation is :

$$p_{i,j}^{(n+m)} = \sum_{k=1}^{K} p_{i,k}^{(n)} \cdot p_{k,j}^{(m)} \qquad \forall n, m > 0$$

or $P^{(n+m)} = P^{(n)} \cdot P^{(m)}$ with $P^{(0)} = I$

- We have:

$$\begin{aligned} P^{(n)} &= P^{(n-1)} \cdot P^{(1)} \\ &= P^{(n-2)} \cdot P^{(1)} \cdot P^{(1)} \\ &= \prod_{t=1}^{n} P^{(1)} \\ &= P^{n} \end{aligned}$$

- We deduce that:

$$p\left(t, i; t+n, j\right) = p_{i,j}^{(n)} = \mathbf{e}_i^\top P^n \mathbf{e}_j$$

Rating migration matrix

Discrete time modeling

Table 24: Two-year transition probability in % (migration matrix #1)

	AAA	AA	A	BBB	BB	B	CCC
AAA	86.28	10.08	2.25	0.92	0.44	0.02	0.00
AA	7.30	70.52	18.68	2.67	0.66	0.15	0.00
A	0.95	24.24	57.16	15.20	2.19	0.25	0.01
BBB	0.21	5.06	28.22	52.11	12.93	1.33	0.14
BB	0.09	0.79	6.07	27.45	53.68	11.37	0.55
B	0.01	0.18	0.98	6.26	31.47	57.28	3.82
CCC	0.00	0.05	0.50	1.32	6.31	21.13	70.70

Rating migration matrix
Discrete time modeling

We have:

$$\begin{aligned}
p^{(2)}_{\mathrm{AAA,AAA}} &= p_{\mathrm{AAA,AAA}} \times p_{\mathrm{AAA,AAA}} + p_{\mathrm{AAA,AA}} \times p_{\mathrm{AA,AAA}} + p_{\mathrm{AAA,A}} \times p_{\mathrm{A,AAA}} + \\
&\quad p_{\mathrm{AAA,BBB}} \times p_{\mathrm{BBB,AAA}} + p_{\mathrm{AAA,BB}} \times p_{\mathrm{BB,AAA}} + \\
&\quad p_{\mathrm{AAA,B}} \times p_{\mathrm{B,AAA}} + p_{\mathrm{AAA,CCC}} \times p_{\mathrm{CCC,AAA}} \\
&= 0.9276^2 + 0.0566 \times 0.0415 + 0.0090 \times 0.0018 + \\
&\quad 0.0045 \times 0.0007 + 0.0023 \times 0.0004 \\
&= 86.28\%
\end{aligned}$$

Rating migration matrix

Discrete time modeling

Table 25: Five-year transition probability in % (migration matrix #1)

	AAA	AA	A	BBB	BB	B	CCC
AAA	70.45	18.69	6.97	2.61	1.08	0.18	0.01
AA	13.13	50.21	26.03	7.90	2.22	0.48	0.03
A	4.35	33.20	37.78	17.99	5.52	1.08	0.09
BBB	1.50	16.49	32.49	30.90	14.61	3.63	0.38
BB	0.50	5.98	17.83	30.10	31.35	12.85	1.39
B	0.15	1.90	7.40	18.95	35.11	31.26	5.23
CCC	0.05	0.64	2.55	6.93	17.96	38.54	43.33

Rating migration matrix

Discrete time modeling

Stationary distribution

- $\pi_k^{(n)} = \Pr\{\mathcal{R}(n) = R_k\}$ is the probability of the state R_k at time n:
- $\pi^{(n)} = \left(\pi_1^{(n)}, \ldots, \pi_K^{(n)}\right)$ satisfies $\pi^{(n+1)} = P^\top \pi^{(n)}$
- The Markov chain $\mathcal{R}$ has a stationary distribution $\pi^\star$ if $\pi^\star = P^\top \pi^\star$
- $\mathcal{T}_k = \inf\{n : \mathcal{R}(n) = R_k \mid \mathcal{R}(0) = R_k\}$ is the return period of state R_k
- The average return period is then equal to:

$$\boldsymbol{\tau}_k := \mathbb{E}[\mathcal{T}_k] = \frac{1}{\pi_k^\star}$$

Rating migration matrix

Discrete time modeling

- We obtain:

$$\pi^{\star} = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%)$$

- The average return periods are then equal to 5.6, 3.4, 4.0, 6.6, 12.0, 30.4 and 149.0 years

$\Rightarrow$ Best-in-class (or winning-) oriented system

Rating migration matrix

Discrete time modeling

Table 26: ESG migration matrix #2 (one-month transition probability in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	93.50	5.00	0.50	0.50	0.50	0.00	0.00
AA	2.00	93.00	4.00	0.50	0.50	0.00	0.00
A	0.00	3.00	93.00	3.90	0.10	0.00	0.00
BBB	0.00	0.10	2.80	94.00	3.00	0.10	0.00
BB	0.00	0.00	0.10	3.50	94.50	1.80	0.10
B	0.00	0.00	0.00	0.10	3.70	96.00	0.20
CCC	0.00	0.00	0.00	0.40	0.50	0.60	98.50

$\Rightarrow$ The stationary distribution is $\pi^{\star} = (3.11\%, 10.10\%, 17.46\%, 27.76\%, 25.50\%, 12.68\%, 3.39\%)$ and the average return periods are equal to 32.2, 9.9, 5.7, 3.6, 3.9, 7.9 and 29.5 years

$\Rightarrow$ balanced rating system

Rating migration matrix
Discrete time modeling

Table 27: One-year probability transition in % (migration matrix #2)

	AAA	AA	A	BBB	BB	B	CCC
AAA	48.06	29.71	10.34	6.42	4.95	0.49	0.03
AA	11.65	49.25	24.10	9.60	4.87	0.49	0.03
A	2.02	17.51	49.67	24.72	5.52	0.54	0.03
BBB	0.27	3.53	17.46	55.50	20.21	2.88	0.16
BB	0.03	0.60	4.21	23.43	57.45	13.27	1.01
B	0.00	0.08	0.74	5.94	27.10	64.18	1.96
CCC	0.00	0.07	0.57	4.22	5.77	5.85	83.51

Rating migration matrix
Discrete time modeling

Table 28: One-month probability transition in % (migration matrix #1)

	AAA	AA	A	BBB	BB	B	CCC
AAA	99.36	0.53	0.05	0.04	0.02	0.00	0.00
AA	0.39	98.31	1.26	0.01	0.03	0.01	0.00
A	**−0.02**	1.65	97.14	1.21	0.02	0.01	0.00
BBB	0.01	**−0.07**	2.28	96.72	1.06	**−0.01**	0.01
BB	0.00	0.02	**−0.12**	2.29	96.92	0.88	0.01
B	0.00	0.00	0.04	**−0.15**	2.45	97.42	0.25
CCC	0.00	0.00	0.02	0.04	0.05	1.37	98.53

$\Rightarrow$ Negative probabilities

The ESG rating system is not Markovian!

Rating migration matrix
Discrete time modeling

Mean hitting time

- Let $\mathcal{A} \subset \mathbb{K}$ be a given subset. The first hitting time of $\mathcal{A}$ is given by:

$$\mathcal{T}(\mathcal{A}) = \inf \{n : \mathcal{R}(n) \in \mathcal{A}\}$$

- The mean first hitting time to target $\mathcal{A}$ from state k is defined as:

$$\boldsymbol{\tau}_k(\mathcal{A}) = \mathbb{E}[\mathcal{T}(\mathcal{A}) \mid \mathcal{R}(0) = R_k]$$

- We can show that $\boldsymbol{\tau}_k(\mathcal{A}) = 1 + \sum_{j=1}^K p_{k,j} \boldsymbol{\tau}_j(\mathcal{A})$
- The solution is given by the LP problem:

$$\boldsymbol{\tau}(\mathcal{A}) = \arg\min \sum_{k=1}^K x_k \quad \text{s.t.} \quad \begin{cases} x_k = 0 & \text{if } k \in \mathcal{A} \\ x_k = 1 + \sum_{j=1}^K p_{k,j} x_j & \text{if } k \notin \mathcal{A} \\ x_k \geq 0 & \end{cases}$$

Rating migration matrix
Discrete time modeling

- $\mathcal{B} = \{\text{AAA}, \text{AA}, \text{A}\}$
- $\mathcal{W} = \{\text{BB}, \text{B}, \text{CCC}\}$

Rating system	$\mathcal{W}$-target				$\mathcal{B}$-target			
	AAA	AA	A	BBB	BBB	BB	B	CCC
#1	79.21	70.04	62.34	46.54	7.50	13.28	17.58	22.68
#2	10.24	9.92	9.13	6.68	8.68	11.99	14.26	17.54

Rating migration matrix

Estimation of the transition matrix

Theoretical approach:

- Bayes theorem:

$$\begin{aligned} p_{i,j} &= \Pr\left\{\mathcal{R}(t+1) = R_j \mid \mathcal{R}(t) = R_i\right\} \\ &= \frac{\Pr\left\{\mathcal{R}(t+1) = R_j, \mathcal{R}(t) = R_i\right\}}{\Pr\left\{\mathcal{R}(t) = R_i\right\}} \end{aligned}$$

- We have seen that:

$$\Pr\left\{\mathcal{R}(t) = R_k\right\} = \mathbf{F}\left(s_k^\star\right) - \mathbf{F}\left(s_{k-1}^\star\right) = p_k$$

- We deduce that:

$$p_{i,j} = \frac{\mathbf{C}\left(\mathbf{F}\left(s_i^\star\right), \mathbf{F}\left(s_j^\star\right)\right) - \mathbf{C}\left(\mathbf{F}\left(s_{i-1}^\star\right), \mathbf{F}\left(s_j^\star\right)\right) - \mathbf{C}\left(\mathbf{F}\left(s_i^\star\right), \mathbf{F}\left(s_{j-1}^\star\right)\right) + \mathbf{C}\left(\mathbf{F}\left(s_{i-1}^\star\right), \mathbf{F}\left(s_{j-1}^\star\right)\right)}{\mathbf{F}\left(s_i^\star\right) - \mathbf{F}\left(s_{i-1}^\star\right)}$$

  where $\mathbf{C}$ is the copula function of the random vector $\left(\boldsymbol{\mathcal{S}}(t), \boldsymbol{\mathcal{S}}(t+1)\right)$

Rating migration matrix

Estimation of the transition matrix

Non-parametric approach:

$$\hat{p}_{i,j}(t) = \frac{\#\{\mathcal{R}(t+1) = R_j, \mathcal{R}(t) = R_i\}}{\#\{\mathcal{R}(t) = R_i\}} = \frac{n_{i,j}(t)}{n_{i,\cdot}(t)}$$

$\Rightarrow$ cohort method vs. pooling method

Rating migration matrix

Estimation of the transition matrix

Table 29: Number of observations $n_{i,j}$ (migration matrix #1)

$n_{i,j}$	AAA	AA	A	BBB	BB	B	CCC	$n_{i,\cdot}(t)$	$\hat{p}_{i,\cdot}(t)$
AAA	2 050	125	20	10	5	0	0	2 210	3.683%
AA	280	5 580	800	60	20	5	0	6 745	11.242%
A	20	1 700	8 020	1 150	90	10	0	10 990	18.317%
BBB	10	190	2 820	10 000	1 300	60	10	14 390	23.983%
BB	5	25	200	2 500	9 150	1 000	30	12 910	21.517%
B	0	5	25	150	2 260	7 800	250	10 490	17.483%
CCC	0	0	5	10	50	300	1 900	2 265	3.775%
$n_{\cdot,j}(t)$	2 365	7 625	11 890	13 850	12 875	9 175	2 190	60 000	
$\hat{p}_{\cdot,j}(t)$	3.942%	12.708%	19.817%	23.133%	21.458%	15.292%	3.650%		100.00%

Rating migration matrix

Estimation of the transition matrix

- For the migration matrix #1, we have:

$$\pi^\star = (17.78\%, 29.59\%, 25.12\%, 15.20\%, 8.35\%, 3.29\%, 0.67\%)$$

- The initial empirical distribution of ratings is:

$$\hat{\pi}^{(0)} = (3.683\%, 11.242\%, 18.317\%, 23.983\%, 21.517\%, 17.483\%, 3.775\%)$$

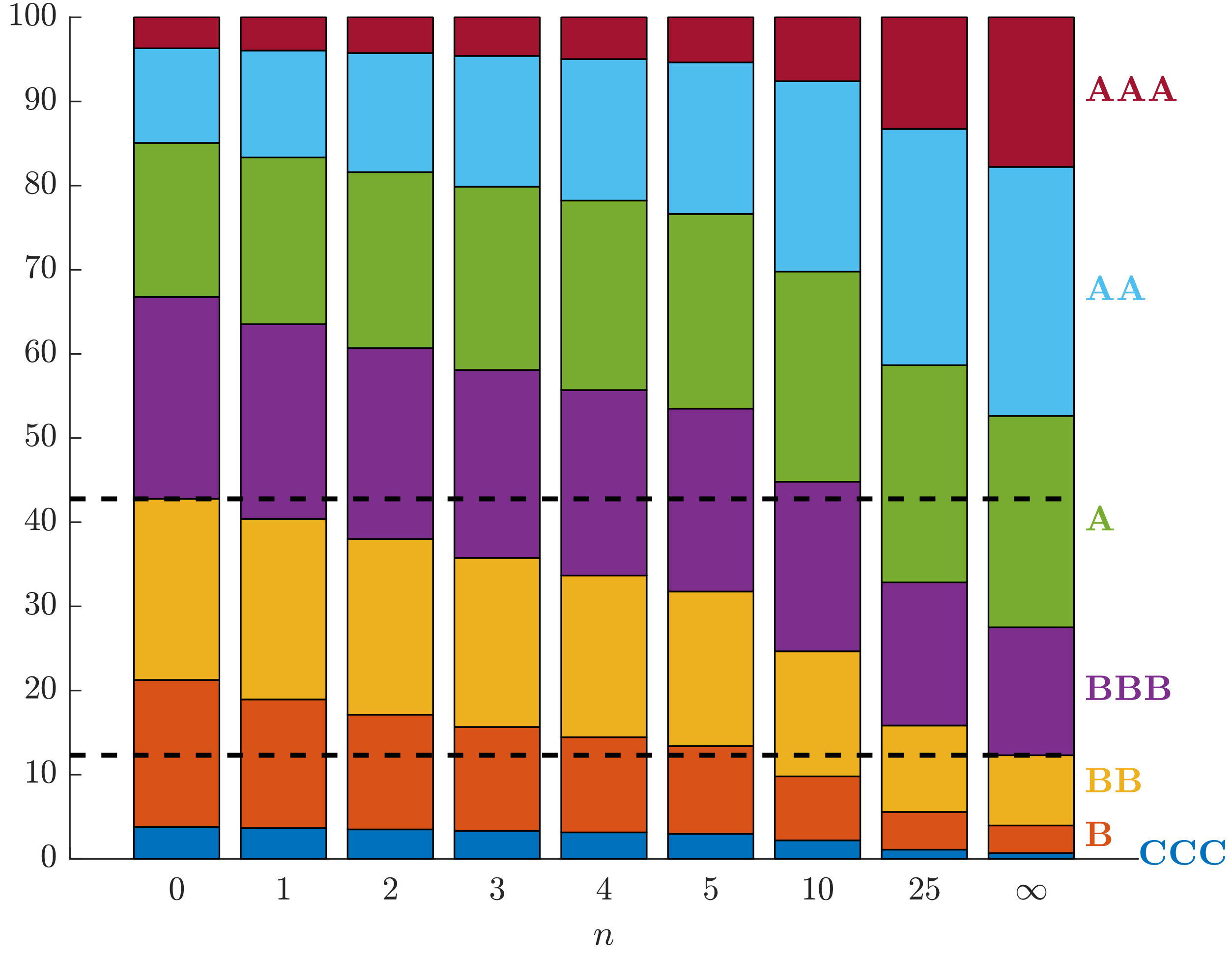
- We have:

$$\begin{aligned}\hat{\pi}^{(1)} &= \hat{P}^\top \hat{\pi}^{(0)} \\ &= (3.942\%, 12.708\%, 19.817\%, 23.133\%, 21.458\%, 15.290\%, 3.650\%)\end{aligned}$$

Rating migration matrix

Estimation of the transition matrix

Figure 40: Dynamics of the probability distribution $\pi^{(n)}$ (migration matrix #1)



Rating migration matrix

Continuous-time modeling

Markov generator

- $t \in \mathbb{R}_+$
- The transition matrix is defined as follows:

$$P_{i,j}(s;t) = p(s,i;t,j) = \Pr\{\mathcal{R}(t) = R_j \mid \mathcal{R}(s) = R_i\}$$

- If $\mathcal{R}$ is a time-homogenous Markov, we have:

$$P(t) = P(0;t) = \exp(t\Lambda)$$

- $\Lambda = (\lambda_{i,j})$ is the Markov generator matrix $\Lambda = (\lambda_{i,j})$ where $\lambda_{i,j} \geq 0$ for all $i \neq j$ and $\lambda_{i,i} = -\sum_{j \neq i}^{K} \lambda_{i,j}$

Rating migration matrix

Continuous-time modeling

An example

- Rating system with three states: A (good rating), B (average rating) and C (bad rating)
- The Markov generator is equal to:

$$\Lambda = \begin{pmatrix} -0.30 & 0.20 & 0.10 \\ 0.15 & -0.40 & 0.25 \\ 0.10 & 0.15 & -0.25 \end{pmatrix}$$

Rating migration matrix

Continuous-time modeling

- The one-year transition probability matrix is equal to:

$$P(1) = e^{\Lambda} = \begin{pmatrix} 75.63\% & 14.84\% & 9.53\% \\ 11.63\% & 69.50\% & 18.87\% \\ 8.52\% & 11.73\% & 79.75\% \end{pmatrix}$$

- For the two-year maturity, we get:

$$P(2) = e^{2\Lambda} = \begin{pmatrix} 59.74\% & 22.65\% & 17.61\% \\ 18.49\% & 52.24\% & 29.27\% \\ 14.60\% & 18.76\% & 66.63\% \end{pmatrix}$$

- We verify that $P(2) = P(1) \cdot P(1)$ because:

$$P(t) = e^{t\Lambda} = \left(e^{\Lambda}\right)^{t} = P(1)^{t}$$

- We have:

$$P\left(\frac{1}{12}\right) = e^{\frac{1}{12}\Lambda} = \begin{pmatrix} 97.54\% & 1.62\% & 0.83 \\ 1.22\% & 96.74\% & 2.03 \\ 0.82\% & 1.22\% & 97.95 \end{pmatrix}$$

Rating migration matrix

Matrix function

Matrix function

We consider the matrix function in the space $\mathbb{M}$ of square matrices:

$$\begin{aligned} f: \quad & \mathbb{M} \longrightarrow \mathbb{M} \\ & A \longmapsto B = f(A) \end{aligned}$$

For instance, if $f(x) = \sqrt{x}$ and A is positive, we can define the matrix B such that:

$$BB^* = B^* B = A$$

B is called the square root of A and we note $B = A^{1/2}$

Rating migration matrix

Matrix function

- We consider the following Taylor expansion:

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \ldots$$

- We can show that if the series converge for $|x - x_0| < \alpha$, then the matrix $f(A)$ defined by the following expression:

$$f(A) = f(x_0) + (A - x_0 I) f'(x_0) + \frac{(A - x_0 I)^2}{2!} f''(x_0) + \ldots$$

  converges to the matrix B if $|A - x_0 I| < \alpha$ and we note $B = f(A)$

Rating migration matrix

Matrix function

- In the case of the exponential function, we have:

$$f(x) = e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

- We deduce that the exponential of the matrix A is equal to:

$$B = e^A = \sum_{k=0}^{\infty} \frac{A^k}{k"!}$$

- The logarithm of A is the matrix B such that $e^B = A$ and we note $B = \ln A$

Rating migration matrix
Matrix function

- Let A and B be two $n \times n$ square matrices. We have the properties:

$$\begin{aligned} f\left(A^\top\right) &= f\left(A\right)^\top \\ Af\left(A\right) &= f\left(A\right)A \\ f\left(B^{-1}AB\right) &= B^{-1}f\left(A\right)B \end{aligned}$$

- It follows that:

$$\left\{\begin{array}{ll} e^{A^\top} = \left(e^A\right)^\top & \\ e^{B^{-1}AB} = B^{-1}e^A B & \\ Ae^B = e^B A & \text{if } AB = BA \\ e^{A+B} = e^A e^B = e^B e^A & \text{if } AB = BA \end{array}\right.$$

Rating migration matrix

Matrix function

Definition

The Schur decomposition of the $n \times n$ matrix A is equal to:

$$A = QTQ^*$$

where Q is a unitary matrix and T is an upper triangular matrix

For transcendental functions, we have:

$$f(A) = Qf(T)Q^*$$

where $A = QTQ^*$ is the Schur decomposition of A

Rating migration matrix

Continuous-time modeling

Estimation of the Markov generator

We have:

$$\hat{\Lambda} = \frac{1}{t} \ln \left(\hat{P}(t) \right)$$

$\Rightarrow$ $\hat{\Lambda}$ may not verify the Markov conditions: $\hat{\lambda}_{i,j} \geq 0$ for all $i \neq j$ and $\sum_{j=1}^{K} \lambda_{i,j} = 0$

Rating migration matrix

Continuous-time modeling

Table 30: Non-Markov generator $\Lambda' = \ln(P)$ of the migration matrix #1 (in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	−7.663	6.427	0.542	0.466	0.245	**−0.016**	**−0.000**
AA	4.770	−20.604	15.451	**−0.001**	0.318	0.066	**−0.001**
A	**−0.267**	20.259	−35.172	14.953	0.152	0.083	**−0.008**
BBB	0.102	**−1.051**	28.263	−40.366	13.100	**−0.128**	0.080
BB	0.032	0.307	**−1.762**	28.351	−37.889	10.832	0.129
B	**−0.005**	**−0.008**	0.503	**−2.240**	30.227	−31.482	3.006
CCC	0.000	**−0.024**	0.194	0.469	0.365	16.806	−17.810

Rating migration matrix

Continuous-time modeling

Israel-Rosenthal-Wei estimators

1. The first approach consists in adding the negative values back into the diagonal values:

$$\begin{cases} \bar{\lambda}_{i,j} = \max\left(\hat{\lambda}_{i,j}, 0\right) & i \neq j \\ \bar{\lambda}_{i,i} = \hat{\lambda}_{i,i} + \sum_{j \neq i} \min\left(\hat{\lambda}_{i,j}, 0\right) \end{cases}$$

2. The second estimator carries forward the negative values on the matrix entries which have the correct sign:

$$\begin{cases} G_i = \left|\hat{\lambda}_{i,i}\right| + \sum_{j \neq i} \max\left(\hat{\lambda}_{i,j}, 0\right), B_i = \sum_{j \neq i} \max\left(-\hat{\lambda}_{i,j}, 0\right) \\ \tilde{\lambda}_{i,j} = \begin{cases} 0 & \text{if } i \neq j \text{ and } \hat{\lambda}_{i,j} < 0 \\ \hat{\lambda}_{i,j} - B_i \left|\hat{\lambda}_{i,j}\right| / G_i & \text{if } G_i > 0 \\ \hat{\lambda}_{i,j} & \text{if } G_i = 0 \end{cases} \end{cases}$$

Rating migration matrix

Continuous-time modeling

Table 31: Markov generator of the migration matrix #1 (in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	−7.679	6.427	0.542	0.466	0.245	0.000	0.000
AA	4.770	−20.606	15.451	0.000	0.318	0.066	0.000
A	0.000	20.259	−35.447	14.953	0.152	0.083	0.000
BBB	0.102	0.000	28.263	−41.545	13.100	0.000	0.080
BB	0.032	0.307	0.000	38.351	−39.651	10.832	0.129
B	0.000	0.000	0.503	0.000	30.227	−33.735	3.006
CCC	0.000	0.000	0.194	0.469	0.365	16.806	−17.834

Rating migration matrix

Continuous-time modeling

Table 32: ESG migration Markov matrix #1 (one-year transition probability in %)

	AAA	AA	A	BBB	BB	B	CCC
AAA	92.75	5.66	0.90	0.45	0.23	0.01	0.00
AA	4.17	82.73	11.85	0.89	0.30	0.07	0.00
A	0.40	15.51	72.79	10.39	0.81	0.10	0.01
BBB	0.12	2.11	19.60	68.69	8.91	0.50	0.07
BB	0.04	0.43	2.79	19.25	69.65	7.61	0.23
B	0.01	0.09	0.65	2.98	21.21	72.71	2.35
CCC	0.00	0.02	0.25	0.58	2.19	13.09	83.87

Rating migration matrix

Continuous-time modeling

Table 33: **Original** migration matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	92.76	5.66	0.90	0.45	0.23	0.00	0.00
AA	4.15	82.73	11.86	0.89	0.30	0.07	0.00
A	**0.18**	15.47	72.98	10.46	0.82	0.09	0.00
BBB	0.07	**1.32**	19.60	69.49	9.03	0.42	0.07
BB	0.04	0.19	**1.55**	19.36	70.88	7.75	0.23
B	0.00	0.05	0.24	**1.43**	21.54	74.36	2.38
CCC	0.00	0.00	0.22	0.44	2.21	13.24	83.89

Table 34: **New** migration matrix

	AAA	AA	A	BBB	BB	B	CCC
AAA	92.75	5.66	0.90	0.45	0.23	0.01	0.00
AA	4.17	82.73	11.85	0.89	0.30	0.07	0.00
A	**0.40**	15.51	72.79	10.39	0.81	0.10	0.01
BBB	0.12	**2.11**	19.60	68.69	8.91	0.50	0.07
BB	0.04	0.43	**2.79**	19.25	69.65	7.61	0.23
B	0.01	0.09	0.65	**2.98**	21.21	72.71	2.35
CCC	0.00	0.02	0.25	0.58	2.19	13.09	83.87

Rating migration matrix

Continuous-time modeling

Why it is important that ESG ratings satisfy the Markov property

- Lack of memory:

$t-2$		$t-1$		t		$t+1$
AAA	$\longrightarrow$	BBB	$\longrightarrow$	BBB	$\longrightarrow$	?
BBB	$\longrightarrow$	BBB	$\longrightarrow$	BBB	$\longrightarrow$	?
BB	$\longrightarrow$	BB	$\longrightarrow$	BBB	$\longrightarrow$	?

- Non-Markov property:

$$\Pr\left\{\mathcal{R}_{c_1}(t+1) = R_j \mid \mathcal{R}_{c_1}(t) = R_i\right\} \neq \Pr\left\{\mathcal{R}_{c_2}(t+1) = R_j \mid \mathcal{R}_{c_2}(t) = R_i\right\}$$

for two different companies c_1 and c_2

Rating migration matrix

Continuous-time modeling

How to perform a dynamic analysis?

- We deduce that:

$$\pi_k(t,\mathcal{A}) = \Pr\{\mathcal{R}(t) \in \mathcal{A} \mid \mathcal{R}(0) = k\} = \sum_{j \in \mathcal{A}} \mathbf{e}_k^\top e^{t\Lambda} \mathbf{e}_j$$

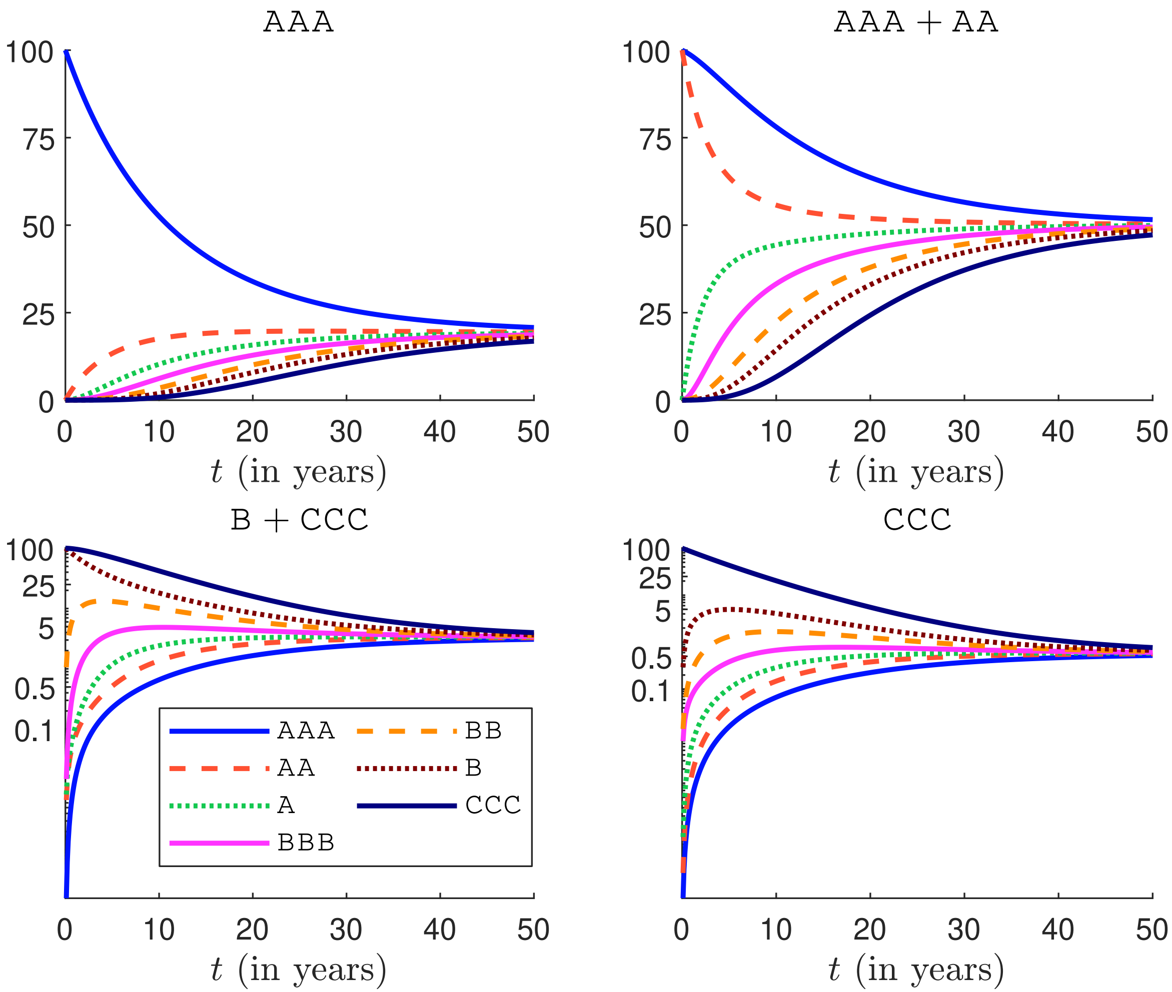
- Some properties
  - $\partial_t \exp(\Lambda t) = \Lambda \exp(\Lambda t)$
  - $\partial_t^m \exp(\Lambda t) = \Lambda^m \exp(\Lambda t)$
  - $\int_0^t e^{\Lambda s}\,\mathrm{d}s = \left(e^{\Lambda t} - I_K\right)\Lambda^{-1}$
- For example, the "*time density function*" is given by:

$$\pi_k^{(m)}(t,\mathcal{A}) := \frac{\partial\, \pi_k(t,\mathcal{A})}{\partial\, t^m} = \sum_{j \in \mathcal{A}} \mathbf{e}_k^\top \Lambda^m e^{t\Lambda} \mathbf{e}_j$$

Rating migration matrix

Continuous-time modeling

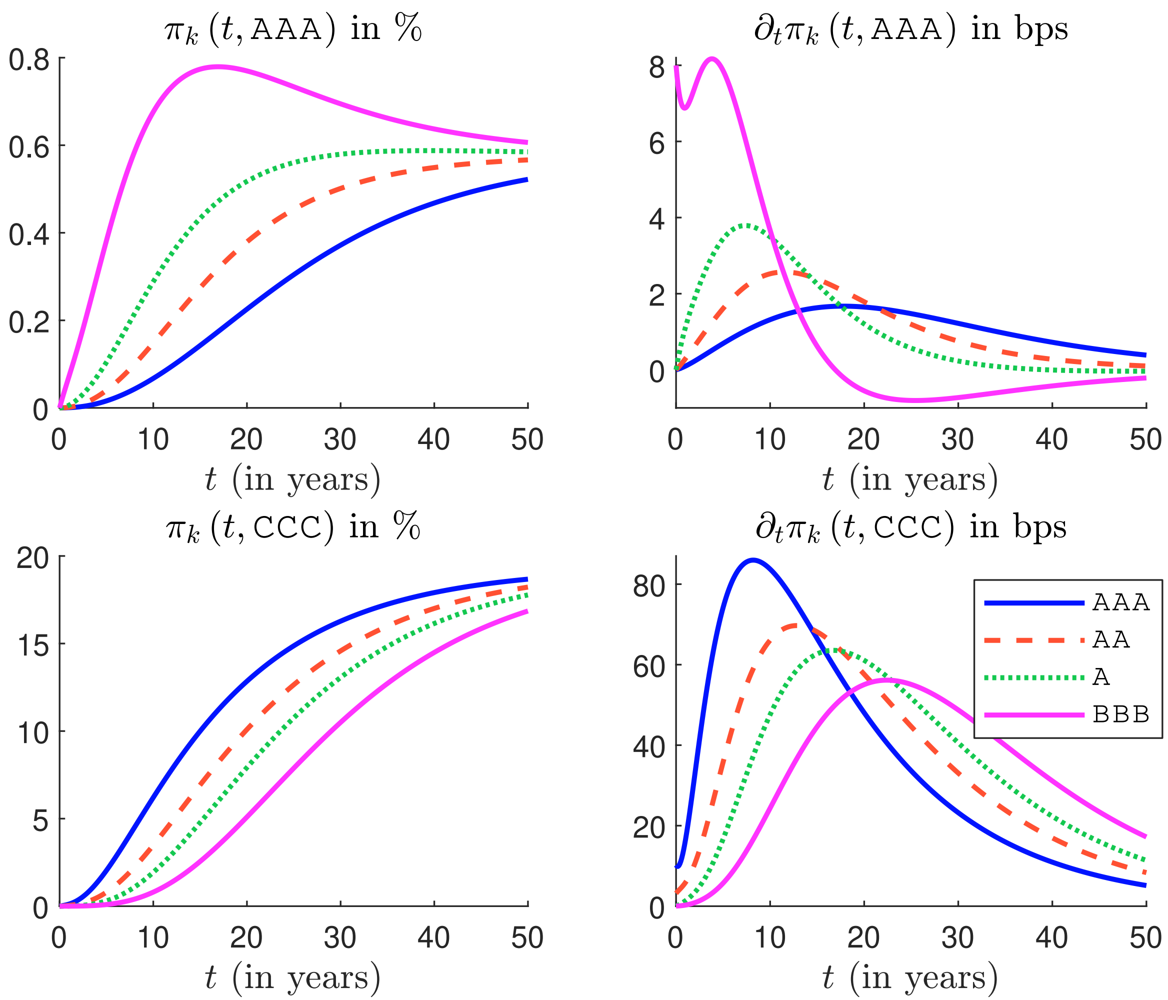
Figure 41: Probability $\pi_k(t, \mathcal{A})$ to reach $\mathcal{A}$ at time t (migration matrix #1)



Rating migration matrix

Continuous-time modeling

Figure 42: Dynamic analysis (migration matrix #1)



Rating migration matrix

Comparison with credit ratings

Table 35: Example of credit migration matrix (one-year probability transition in %)

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	92.82	6.50	0.56	0.06	0.06	0.00	0.00	0.00
AA	0.63	91.87	6.64	0.65	0.06	0.11	0.04	0.00
A	0.08	2.26	91.66	5.11	0.61	0.23	0.01	0.04
BBB	0.05	0.27	5.84	87.74	4.74	0.98	0.16	0.22
BB	0.04	0.11	0.64	7.85	81.14	8.27	0.89	1.06
B	0.00	0.11	0.30	0.42	6.75	83.07	3.86	5.49
CCC	0.19	0.00	0.38	0.75	2.44	12.03	60.71	23.50
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Source: Kavvathas (2001).

Rating migration matrix

Comparison with credit ratings

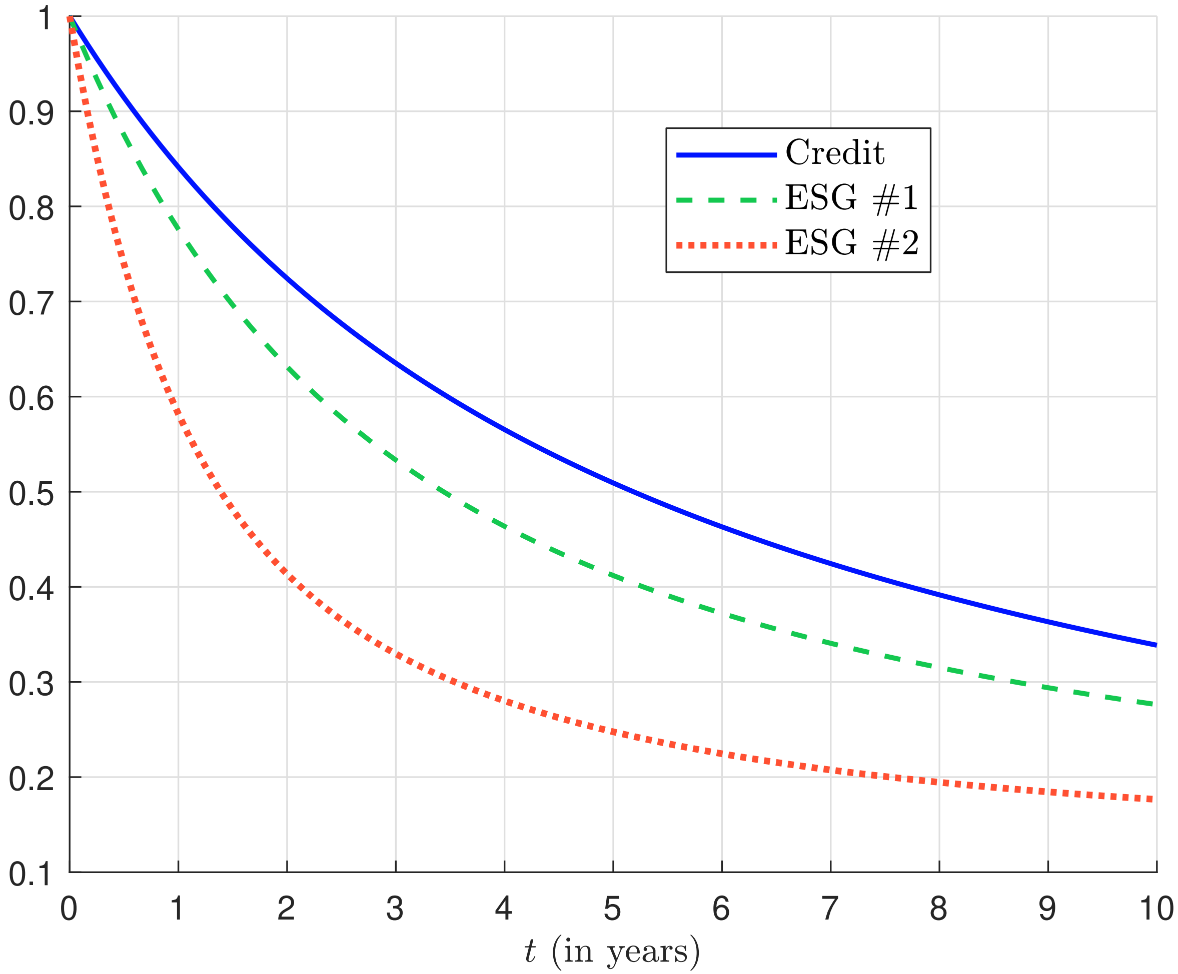
The trace statistics is equal to:

$$\lambda(t) = \frac{\operatorname{trace}\left(e^{t\Lambda}\right)}{K}$$

Rating migration matrix

Comparison with credit ratings

Figure 43: Trace statistics of credit and ESG migration matrices



Course 2022-2023 in Sustainable Finance
Lecture 3. Impact of ESG Investing on Asset Prices and Portfolio Returns

Thierry Roncalli*

*Amundi Asset Management[6]

*University of Paris-Saclay

March 2023

[6]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Mean-variance optimization problem

Model settings

- An investment universe of n assets
- $w = (w_1, \ldots, w_n)$ is the vector of weights in the portfolio
- The portfolio is fully invested meaning that $\sum_{i=1}^{n} w_i = \mathbf{1}^\top w = 1$
- $R = (R_1, \ldots, R_n)$ is the vector of asset returns
- We denote by $\mu = \mathbb{E}[R]$ and $\Sigma = \mathbb{E}\left[(R - \mu)(R - \mu)^\top\right]$ the vector of expected returns and the covariance matrix of asset returns

Mean-variance optimization problem

Model setup

We have:

$$R(w) = \sum_{i=1}^{n} w_i R_i = w^\top R$$

The expected return $\mu(w) := \mathbb{E}[R(w)]$ of the portfolio is equal to:

$$\mu(w) = \mathbb{E}\left[w^\top R\right] = w^\top \mathbb{E}[R] = w^\top \mu$$

whereas its variance $\sigma^2(w) := \operatorname{var}(R(w))$ is given by:

$$\begin{aligned}
\sigma^2(w) &= \mathbb{E}\left[(R(w) - \mu(w))(R(w) - \mu(w))^\top\right] \\
&= \mathbb{E}\left[w^\top (R - \mu)(R - \mu)^\top w\right] \\
&= w^\top \Sigma w
\end{aligned}$$

Mean-variance optimization problem

μ- and σ-problems

We can then formulate the investor's financial problem as follows:

1. Maximizing the expected return of the portfolio under a volatility constraint (σ-problem):

$$\max \mu(w) \quad \text{s.t.} \quad \sigma(w) \leq \sigma^{\star}$$

2. Or minimizing the volatility of the portfolio under a return constraint (μ-problem):

$$\min \sigma(w) \quad \text{s.t.} \quad \mu(w) \geq \mu^{\star}$$

$\Rightarrow$ The key idea of Markowitz was to transform the original non-linear optimization problems into a quadratic optimization problem

Mean-variance optimization problem

Introducing the quadratic utility function

- The mean-variance (or quadratic) utility function is:

$$\mathcal{U}(w) := \mathbb{E}[R(w)] - \frac{\bar{\gamma}}{2}\operatorname{var}(R(w)) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w$$

  where $\bar{\gamma}$ is the absolute risk-aversion parameter

- We obtain the following problem:

$$\begin{aligned} w^\star(\bar{\gamma}) &= \arg\max \left\{ \mathcal{U}(w) = w^\top \mu - \frac{\bar{\gamma}}{2} w^\top \Sigma w \right\} \\ &\text{s.t.} \quad \mathbf{1}^\top w = 1 \end{aligned}$$

- $\bar{\gamma} = 0 \Rightarrow$ maximum mean portfolio
- $\bar{\gamma} = \infty \Rightarrow$ minimum variance portfolio:

$$w^\star(\infty) = \arg\min \frac{1}{2} w^\top \Sigma w \quad \text{s.t.} \quad \mathbf{1}^\top w = 1$$

Mean-variance optimization problem

The engineering viewpoint

In practice, professionals formulate the optimization problem as follows:

$$\begin{aligned} w^\star(\gamma) &= \arg\min \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu \\ &\text{s.t.} \quad \mathbf{1}^\top w = 1 \end{aligned}$$

where $\gamma = \bar{\gamma}^{-1}$ is called the risk-tolerance

This is a standard QP problem

Quadratic programming problem

Definition

The formulation of a standard QP problem is:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2} w^{\top} Q w - w^{\top} R \\ & \text{u.c.} \quad \left\{ \begin{array}{l} Aw = B \\ Cw \leq D \\ w^{-} \leq w \leq w^{+} \end{array} \right. \end{aligned}$$

$\Rightarrow$ We have $Q = \Sigma$, $R = \gamma\mu$, $A = \mathbf{1}^{\top}$ and $B = 1$

Mean-variance optimization problem

Illustration

Example #1

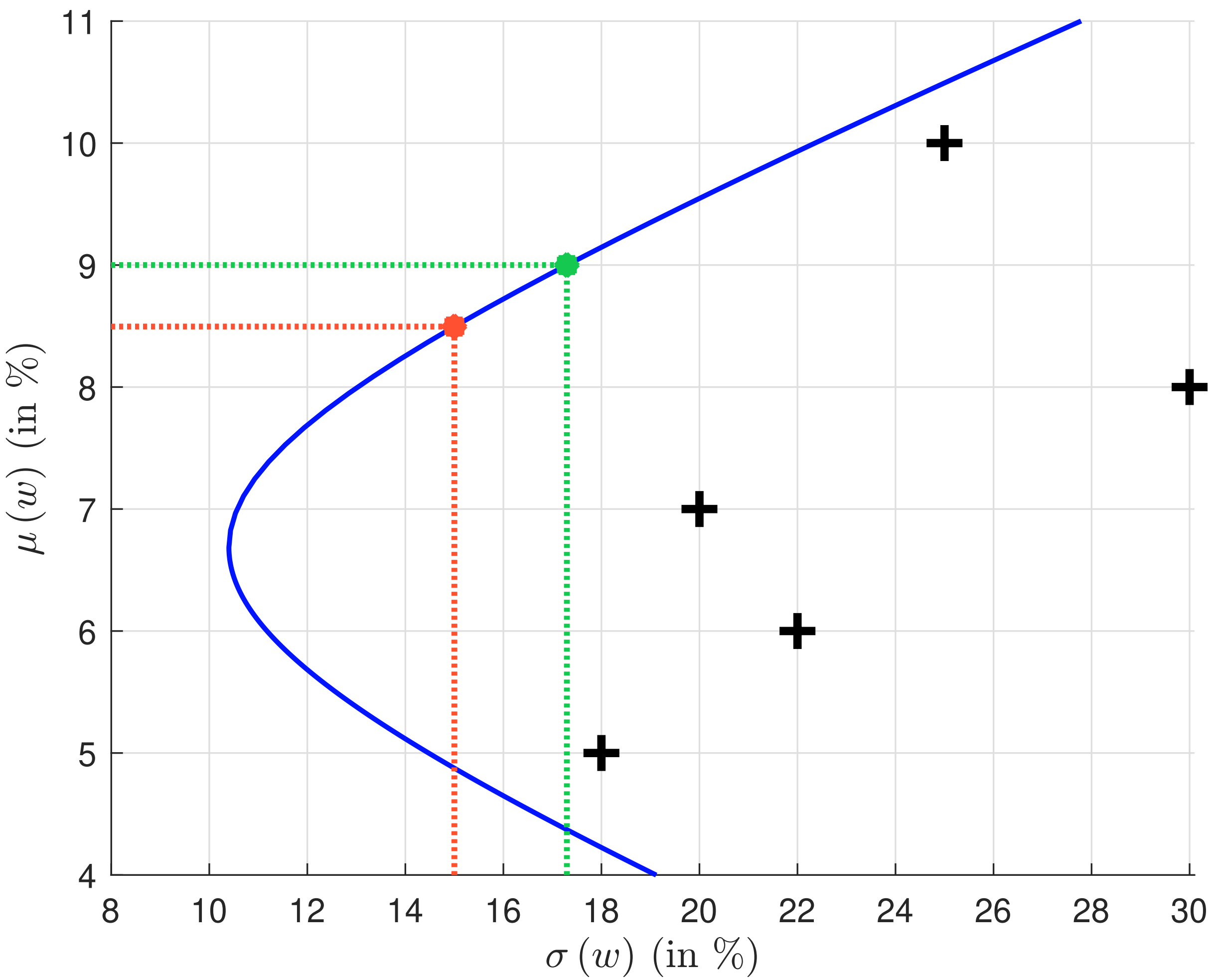
We consider an investment universe of five assets. Their expected returns are equal to 5%, 7%, 6%, 10% and 8% while their volatilities are equal to 18%, 20%, 22%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & & \\ 70\% & 100\% & & & \\ 20\% & 30\% & 100\% & & \\ -30\% & 20\% & 10\% & 100\% & \\ 0\% & 0\% & 0\% & 0\% & 100\% \end{pmatrix}$$

Mean-variance optimization problem

Illustration

Figure 44: Efficient frontier (Example #1)



Mean-variance optimization problem

Illustration

- The GMV portfolio is obtained with $\gamma = 0$
- The solution is:

$$w_{\mathrm{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

- We have:

$$\sigma(w) \geq \sigma(w_{\mathrm{gmv}}) = 10.40\% \qquad \forall w$$

Mean-variance optimization problem

Illustration

Table 36: Solution of the Markowitz optimization problem (in %)

γ	0.00	0.10	0.20	0.50	1.00	5.00
$w_1^\star(\gamma)$	66.35	58.25	50.14	25.84	−14.67	−338.72
$w_2^\star(\gamma)$	−28.52	−22.67	−16.82	0.74	30.00	264.12
$w_3^\star(\gamma)$	15.31	13.30	11.30	5.28	−4.74	−84.93
$w_4^\star(\gamma)$	34.85	37.65	40.44	48.82	62.78	174.50
$w_5^\star(\gamma)$	12.02	13.48	14.94	19.32	26.62	85.03
$\mu(w^\star(\gamma))$	6.69	6.97	7.25	8.09	9.49	20.71
$\sigma(w^\star(\gamma))$	10.40	10.53	10.93	13.35	19.71	84.38

Mean-variance optimization problem

How to solve the μ-problem and the σ-problem?

- We have to find the optimal value of γ such that $\mu\left(w^{\star}\left(\gamma\right)\right)=\mu^{\star}$ or $\sigma\left(w^{\star}\left(\gamma\right)\right)=\sigma^{\star}$
- We use the bisection algorithm
- If we target a portfolio with $\sigma^{\star}=15\%$, we know that $\gamma\in[0.5,1]$. The optimal solution $w^{\star}$ is $(14.06\%,9.25\%,2.37\%,52.88\%,21.44\%)$ and the bisection algorithm returns $\gamma=0.6455$. In this case, we obtain $\mu\left(w^{\star}\left(\gamma\right)\right)=8.50\%$
- If we consider a μ-problem with $\mu^{\star}=9\%$, we find $\gamma=0.8252$, $w^{\star}=(-0.50\%,19.77\%,-1.23\%,57.90\%,24.07)$ and $\sigma\left(w^{\star}\left(\gamma\right)\right)=17.30\%$

Mean-variance optimization problem

Adding some constraints

- The Lagrange function of the optimization problem is equal to:

$$\mathcal{L}(w;\lambda_0) = \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu + \lambda_0 \left(\mathbf{1}^\top w - 1\right)$$

where λ_0 is the Lagrange coefficients associated with the constraint $\mathbf{1}^\top w = 1$

- The solution $w^\star$ verifies the following first-order conditions:

$$\begin{cases} \partial_w \mathcal{L}(w;\lambda_0) = \Sigma w - \gamma \mu + \lambda_0 \mathbf{1} = \mathbf{0} \\ \partial_{\lambda_0} \mathcal{L}(w;\lambda_0) = \mathbf{1}^\top w - 1 = 0 \end{cases}$$

- We obtain $w = \Sigma^{-1}(\gamma\mu - \lambda_0 \mathbf{1})$. Because $\mathbf{1}^\top w - 1 = 0$, we have $\gamma \mathbf{1}^\top \Sigma^{-1} \mu - \lambda_0 \mathbf{1}^\top \Sigma^{-1} \mathbf{1} = 1$. It follows that:

$$\lambda_0 = \frac{\gamma \mathbf{1}^\top \Sigma^{-1} \mu - 1}{\mathbf{1}^\top \Sigma^{-1} \mathbf{1}}$$

Mean-variance optimization problem

Adding some constraints

- The solution is then:

$$
\begin{aligned}
w^{\star}(\gamma) &= \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}} + \gamma \frac{\left(\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}\right)\Sigma^{-1}\mu - \left(\mathbf{1}^{\top}\Sigma^{-1}\mu\right)\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}} \\
&= w_{\mathrm{gmv}} + \gamma w_{\mathrm{lsp}}
\end{aligned}
$$

where:

- $w_{\mathrm{gmv}} = \left(\Sigma^{-1}\mathbf{1}\right) / \left(\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}\right)$ is the global minimum variance portfolio
- w_{lsp} is a long/short cash-neutral portfolio such that $\mathbf{1}^{\top} w_{\mathrm{lsp}} = 0$

Mean-variance optimization problem

Adding some constraints

- We could think that a QP solver is not required
- The analytical calculus gives:

$$w_{\mathrm{gmv}} = (66.35\%, -28.52\%, 15.31\%, 34.85\%, 12.02\%)$$

and:

$$w_{\mathrm{lsp}} = (-81.01\%, 58.53\%, -20.05\%, 27.93\%, 14.60\%)$$

- In practice, professionals consider other constraints:

$$\begin{aligned} w^{\star}(\gamma) &= \arg\min \frac{1}{2} w^{\top} \Sigma w - \gamma w^{\top} \mu \\ & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top} w = 1 \\ w \in \Omega \end{array} \right. \end{aligned}$$

where $w \in \Omega$ corresponds to the set of restrictions

- No short-selling restriction ($w_i \geq 0$ and $\Omega = [0,1]^n$) and asset bounds ($w_i \leq w^+$) $\Rightarrow$ No analytical solution (because of the KKT conditions) $\Rightarrow$ **QP solver**

The tangency portfolio

Two-fund separation theorem

We consider a combination of the risk-free asset and a portfolio w:

$$R(\tilde{w}) = (1-\alpha) r + \alpha R(w)$$

where:

- r is the return of the risk-free asset
- $\tilde{w} = (\alpha w, 1-\alpha)$ is a vector of dimension $(n+1)$
- $\alpha \geq 0$ is the proportion of the wealth invested in the risky portfolio

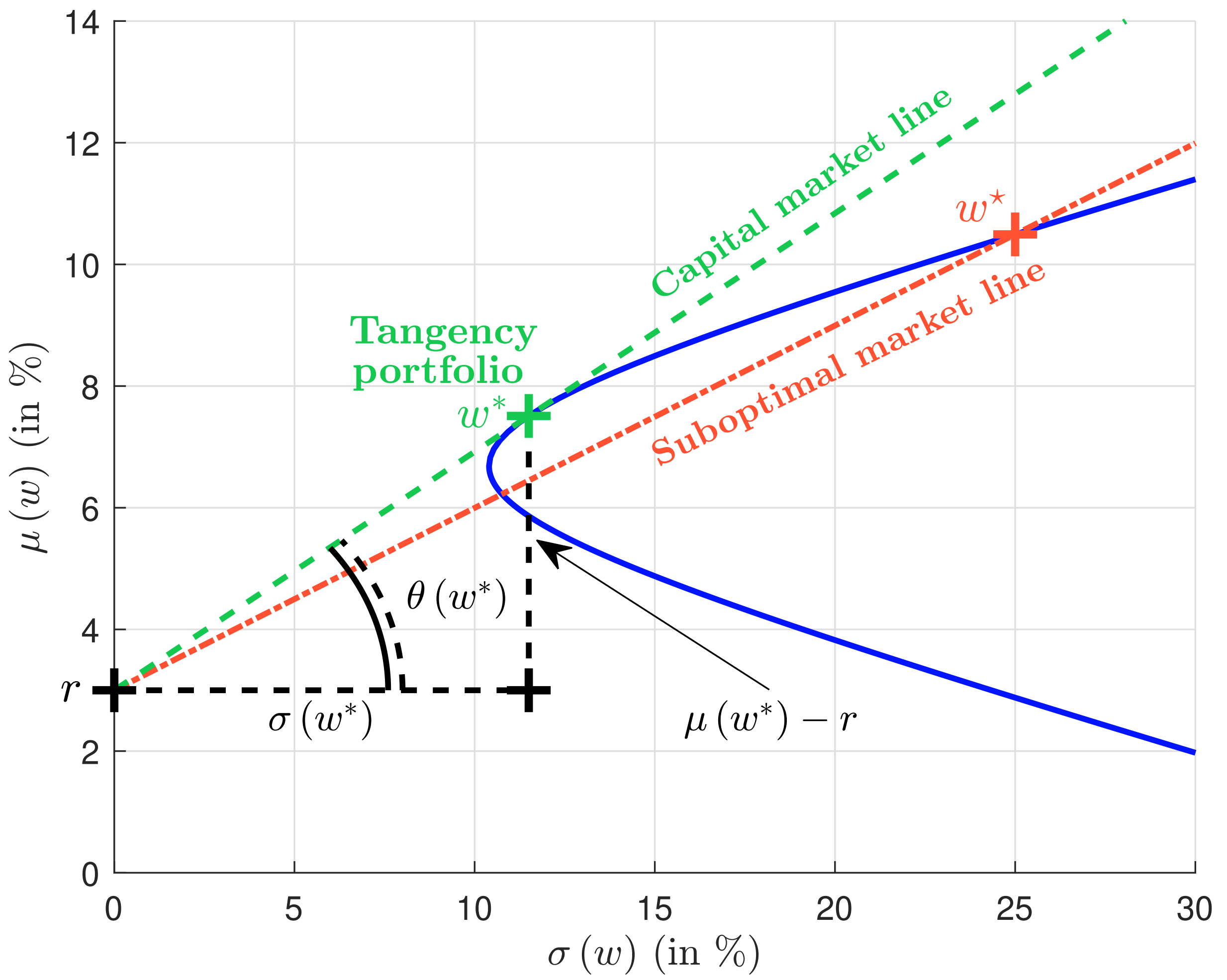
$\Rightarrow$ It follows that $\mu(\tilde{w}) = (1-\alpha) r + \alpha \mu(w) = r + \alpha(\mu(w) - r)$, $\sigma^2(\tilde{w}) = \alpha^2 \sigma^2(w)$ and:

$$\mu(\tilde{w}) = r + \frac{(\mu(w) - r)}{\sigma(w)} \sigma(\tilde{w})$$

The tangency portfolio

Two-fund separation theorem

Figure 45: Capital market line (Example #1)



The tangency portfolio

Two-fund separation theorem

- Let $\mathrm{SR}\left(w \mid r\right)$ be the Sharpe ratio of portfolio w:

$$\mathrm{SR}\left(w \mid r\right) = \frac{\mu\left(w\right) - r}{\sigma\left(w\right)}$$

- We have:

$$\frac{\mu\left(\tilde{w}\right) - r}{\sigma\left(\tilde{w}\right)} = \frac{\mu\left(w\right) - r}{\sigma\left(w\right)} \Leftrightarrow \mathrm{SR}\left(\tilde{w} \mid r\right) = \mathrm{SR}\left(w \mid r\right)$$

- The tangency portfolio w^* satisfies:

$$w^* = \arg\max \tan\theta\left(w\right)$$

The tangency portfolio

Two-fund separation theorem

If we consider our example with $r = 3\%$, the composition of the tangency portfolio is:

$$w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)$$

and we have:

$$\left\{ \begin{array}{l} \mu(w^*) = 7.51\% \\ \sigma(w^*) = 11.50\% \\ \mathrm{SR}(w^* \mid r) = 0.39 \\ \theta(w^*) = 21.40 \text{ degrees} \end{array} \right.$$

The tangency portfolio

Augmented optimization problem

- When the risk-free asset belongs to the investment universe, the optimization problem becomes:

$$\begin{aligned} \tilde{w}^{\star}(\gamma) &= \arg\min \frac{1}{2}\tilde{w}^{\top}\tilde{\Sigma}\tilde{w} - \gamma\tilde{w}^{\top}\tilde{\mu} \\ & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top}\tilde{w} = 1 \\ \tilde{w} \in \Omega \end{array} \right. \end{aligned}$$

  where $\tilde{w} = (w, w_r)$ is the augmented allocation vector of dimension $n+1$

- It follows that:

$$\tilde{\Sigma} = \left(\begin{array}{cc} \Sigma & \mathbf{0} \\ \mathbf{0} & 0 \end{array} \right) \quad \text{and} \quad \tilde{\mu} = \left(\begin{array}{c} \mu \\ r \end{array} \right)$$

The tangency portfolio

Augmented optimization problem

- In the case where $\Omega = \mathbb{R}^{n+1}$, we can show that the optimal solution is equal to:

$$\tilde{w}^{\star}(\gamma) = \underbrace{\alpha \cdot \begin{pmatrix} w^{*} \\ 0 \end{pmatrix}}_{\text{risky assets}} + \underbrace{(1-\alpha) \cdot \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}}_{\text{risk-free asset}}$$

where w^{*} is the tangency portfolio:

$$w^{*} = \frac{\Sigma^{-1}(\mu - r\mathbf{1})}{\mathbf{1}^{\top}\Sigma^{-1}(\mu - r\mathbf{1})}$$

- The proportion of risky assets is equal to

$$\alpha = \gamma \mathbf{1}^{\top}\Sigma^{-1}(\mu - r\mathbf{1})$$

- The risk-tolerance coefficient associated to the tangency portfolio is given by:

$$\gamma(w^{*}) = \frac{1}{\mathbf{1}^{\top}\Sigma^{-1}(\mu - r\mathbf{1})}$$

Market equilibrium and CAPM

Risk premium and beta

At the equilibrium, Sharpe (1964) showed that:

$$\pi_i := \mu_i - r = \beta_i \left(\mu\left(w^*\right) - r\right)$$

where π_i is the risk premium of the asset i and:

$$\beta_i = \frac{\operatorname{cov}\left(R_i, R\left(w^*\right)\right)}{\operatorname{var}\left(R\left(w^*\right)\right)}$$

We have:

$$\beta\left(x \mid w\right) = \frac{\sigma\left(x, w\right)}{\sigma^2\left(w\right)} = \frac{x^\top \Sigma w}{w^\top \Sigma w}$$

and:

$$\beta_i = \beta\left(\mathbf{e}_i \mid w\right) = \frac{\mathbf{e}_i^\top \Sigma w}{w^\top \Sigma w} = \frac{\left(\Sigma w\right)_i}{w^\top \Sigma w}$$

Market equilibrium and CAPM

Risk premium and beta

In the case of Example #1, we have:

- $w^* = (42.57\%, -11.35\%, 9.43\%, 43.05\%, 16.30\%)$
- $(\mu(w^*) = 7.51\%, r = 3\%) \Rightarrow \mu(w^*) = 4.51\%$

Table 37: Computation of the beta and risk premia (Example #1)

Portfolio	$\mu(w)$	$\mu(w) - r$	$\beta(w \mid w^*)$	$\pi(w \mid w^*)$
$\mathbf{e}_1$	5.00%	2.00%	0.444	2.00%
$\mathbf{e}_2$	7.00%	4.00%	0.887	4.00%
$\mathbf{e}_3$	6.00%	3.00%	0.665	3.00%
$\mathbf{e}_4$	10.00%	7.00%	1.553	7.00%
$\mathbf{e}_5$	8.00%	5.00%	1.109	5.00%
w_{ew}	7.20%	4.20%	0.932	4.20%
w_{gmv}	6.69%	3.69%	0.817	3.69%

Market equilibrium and CAPM

Risk premium and alpha return

- Jensen (1968) defined the alpha return as:

$$R_{j,t} - r = \alpha_j + \beta_j \left(R_t \left(w_m \right) - r \right) + \varepsilon_{j,t}$$

  where $R_{j,t}$ is the return of the mutual fund j at time t, $R_t \left(w_m \right)$ is the return of the market portfolio and $\varepsilon_{j,t}$ is an idiosyncratic risk
- More generally, the alpha is defined by the difference between the risk premium $\pi \left(w \right)$ of portfolio w and the beta $\beta \left(w \right)$ of the portfolio times the market risk premium π_m:

$$\begin{aligned} \alpha &= \left(\mu \left(w \right) - r \right) - \beta \left(w \mid w_m \right) \left(\mu \left(w_m \right) - r \right) \\ &= \pi \left(w \right) - \beta \left(w \right) \pi_m \end{aligned}$$

Market equilibrium and CAPM

Risk premium and alpha return

In the case of Example #1 & no short-selling constraint, we have:

- $w^{\star} = (33.62\%, 0\%, 8.79\%, 40.65\%, 16.95\%)$
- $(\mu(w^{*}) = 7.63\%, r = 3\%) \Rightarrow \mu(w^{*}) = 4.63\%$

Table 38: Computation of the alpha return (Example #1)

Portfolio	$\mu(w)$	$\mu(w) - r$	$\beta(w \mid w^{*})$	$\pi(w \mid w^{*})$	$\alpha(w \mid w^{*})$
$\mathbf{e}_1$	5.00%	2.00%	0.432	2.00%	0.00%
$\mathbf{e}_2$	7.00%	4.00%	0.970	4.49%	−0.49%
$\mathbf{e}_3$	6.00%	3.00%	0.648	3.00%	0.00%
$\mathbf{e}_4$	10.00%	7.00%	1.512	7.00%	0.00%
$\mathbf{e}_5$	8.00%	5.00%	1.080	5.00%	0.00%
w_{ew}	7.20%	4.20%	0.929	4.30%	−0.10%
w_{gmv}	6.69%	3.69%	0.766	3.55%	0.14%

Portfolio optimization in the presence of a benchmark

Utility function revisited

- b is the benchmark
- The tracking error is:

$$\epsilon = R(w) - R(b) = \sum_{i=1}^{n} w_i R_i - \sum_{i=1}^{n} b_i R_i = w^\top R - b^\top R = (w - b)^\top R$$

- The expected excess return is equal to:

$$\mu(w \mid b) := \mathbb{E}[\epsilon] = (w - b)^\top \mu$$

- The volatility of the tracking error is defined as:

$$\sigma(w \mid b) := \sigma(e) = \sqrt{(w - b)^\top \Sigma (w - b)}$$

Portfolio optimization in the presence of a benchmark
Utility function revisited

- The objective of the investor is then to maximize the expected tracking error with a constraint on the tracking error volatility:

$$w^{\star} = \arg\max \mu\left(w \mid b\right) \quad \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top} x = 1 \\ \sigma\left(w \mid b\right) \leq \sigma^{\star} \end{array} \right.$$

- We have:

$$\begin{array}{rcl} f\left(w \mid b\right) & = & \dfrac{1}{2}\sigma^{2}\left(w \mid b\right) - \gamma\mu\left(w \mid b\right) \\ & = & \dfrac{1}{2}\left(w - b\right)^{\top} \Sigma \left(w - b\right) - \gamma\left(w - b\right)^{\top} \mu \\ & = & \dfrac{1}{2} w^{\top}\Sigma w - w^{\top}\left(\gamma\mu + \Sigma b\right) + \underbrace{\dfrac{1}{2} b^{\top}\Sigma b + \gamma b^{\top}\mu}_{\text{constant}} \end{array}$$

Portfolio optimization in the presence of a benchmark

QP formulation

We have:

$$
\begin{aligned}
Q &= \Sigma \\
R &= \gamma\mu + \Sigma b \\
A &= \mathbf{1}^\top \\
B &= 1 \\
C &= \\
D &= \\
w^- &= \mathbf{0}_n \text{ (if no short-selling)} \\
w^+ &= \mathbf{1}_n \text{ (if no short-selling)}
\end{aligned}
$$

Portfolio optimization in the presence of a benchmark

Example #2

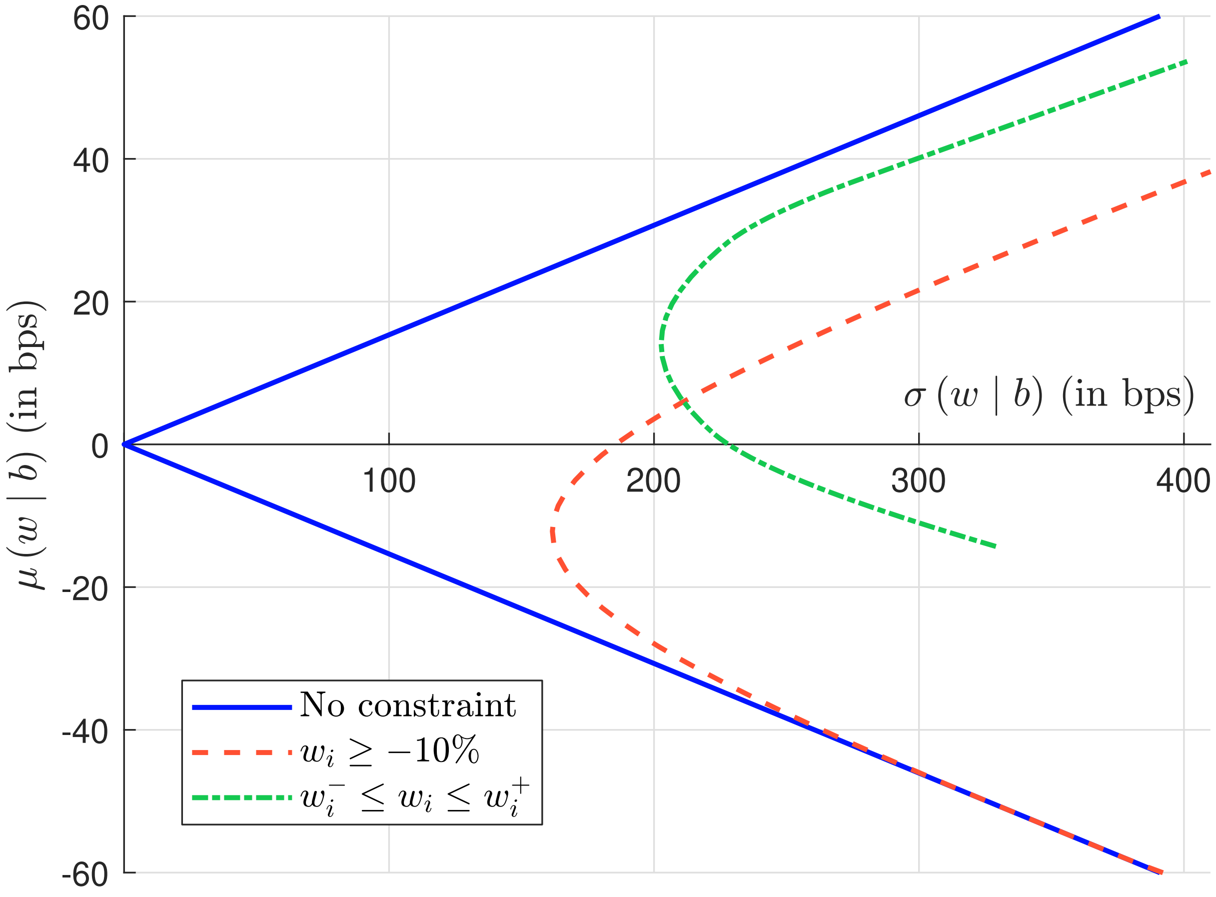
We consider an investment universe of four assets. Their expected returns are equal to 5%, 6.5%, 8% and 6.5% while their volatilities are equal to 15%, 20%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & \\ 10\% & 100\% & & \\ 40\% & 70\% & 100\% & \\ 50\% & 40\% & 80\% & 100\% \end{pmatrix}$$

The benchmark is $b = (60\%, 40\%, 20\%, -20\%)$.

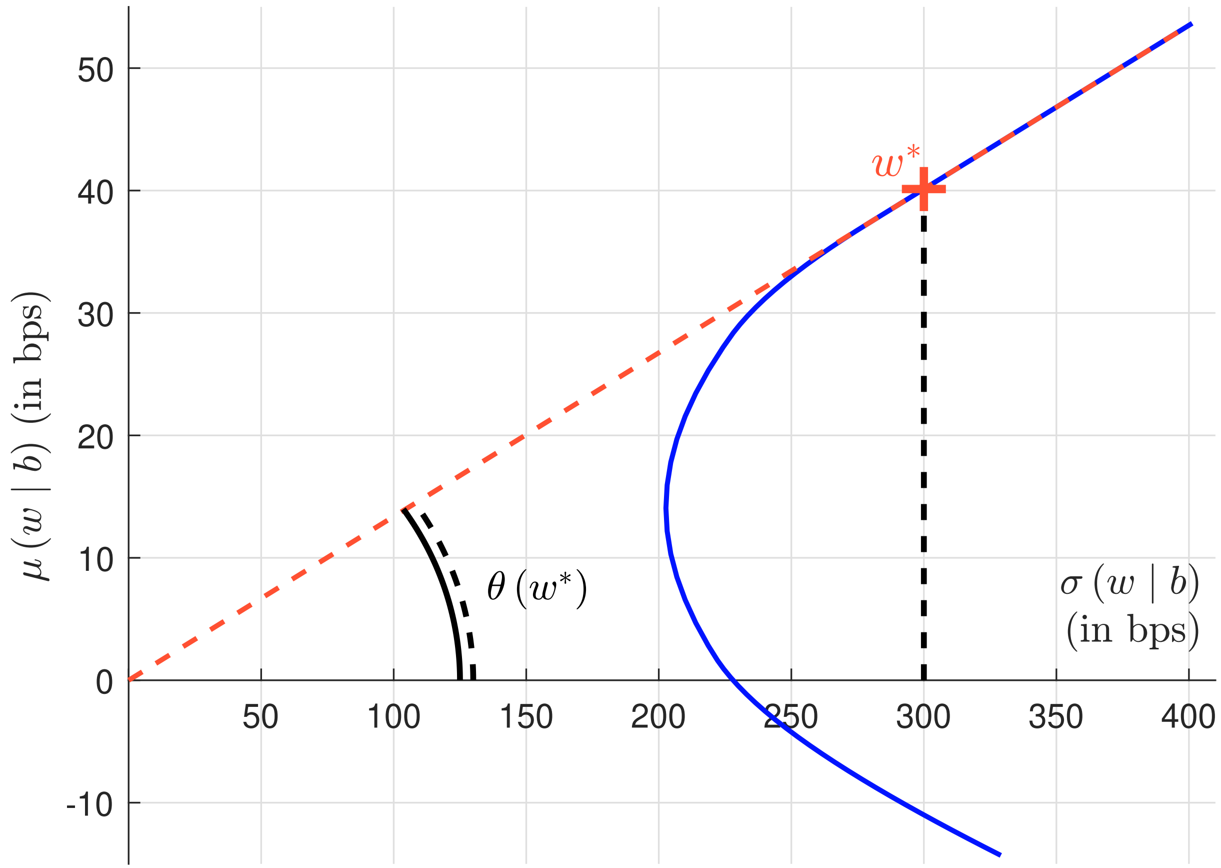
Portfolio optimization in the presence of a benchmark

Figure 46: Efficient frontier with a benchmark (Example #2)



Portfolio optimization in the presence of a benchmark

Figure 47: Tangency portfolio with respect to a benchmark (Example #2)



$\Rightarrow$ the tangency portfolio is equal to $(46.56\%, 33.49\%, 39.95\%, -20.00\%)$

Portfolio optimization in the presence of a benchmark

Information ratio

- We have:

$$\mathrm{IR}\left(w \mid b\right) = \frac{\mu\left(w \mid b\right)}{\sigma\left(w \mid b\right)} = \frac{\left(w-b\right)^\top \mu}{\sqrt{\left(w-b\right)^\top \Sigma \left(w-b\right)}}$$

- If we consider a combination of the benchmark b and the active portfolio w, the composition of the portfolio is:

$$x = \left(1-\alpha\right) b + \alpha w$$

where $\alpha \geq 0$ is the proportion of wealth invested in the portfolio w

- It follows that:

$$\mu\left(x \mid b\right) = \left(x-b\right)^\top \mu = \alpha\mu\left(w \mid b\right)$$

and:

$$\sigma^2\left(x \mid b\right) = \left(x-b\right)^\top \Sigma \left(x-b\right) = \alpha^2\sigma^2\left(w \mid b\right)$$

- We deduce that:

$$\mu\left(x \mid b\right) = \mathrm{IR}\left(w \mid b\right) \cdot \sigma\left(x \mid b\right)$$

ESG risk premium

- Expected (or required) returns $\neq$ historical (or realised) returns:

$$\pi_i \neq R_i$$

- Difference between the unconstrained risk premium and the implied risk premium:

$$\pi_i \neq \tilde{\pi}_i$$

The Pastor-Stambaugh-Taylor model

Model settings

- The asset excess returns $\tilde{R} = R - r = \left(\tilde{R}_1, \ldots, \tilde{R}_n\right)$ are normally distributed: $\tilde{R} \sim \mathcal{N}(\pi, \Sigma)$
- Each firm has an ESG characteristic $\boldsymbol{\mathcal{G}}_i$, which is positive for *esg-friendly* (or *green*) firms and negative for *esg-unfriendly* (or *brown*) firms
- $\boldsymbol{\mathcal{G}}_i > 0$ induces positive social impact, while $\boldsymbol{\mathcal{G}}_i < 0$ induces negative externalities on the society
- Economy with a continuum of agents $(j = 1, 2, \ldots, \infty)$
- $w_{i,j}$ is the fraction of the wealth invested by agent j in stock i
- $w_j = (w_{1,j}, \ldots, w_{n,j})$ is the allocation vector of agent j

The Pastor-Stambaugh-Taylor model

Model settings

- The relationship between the initial and terminal wealth W_j and $\tilde{W}_j$ is given by:

$$\tilde{W}_j = \left(1 + r + w_j^\top \tilde{R}\right) W_j$$

- Exponential CARA utility function:

$$\mathcal{U}\left(\tilde{W}_j, w_j\right) = -\exp\left(-\bar{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)$$

  where:

  - $\bar{\gamma}_j$ is the absolute risk-aversion
  - $b_j = \varphi_j \mathcal{G}$ is the vector of nonpecuniary benefits ($\varphi_j \geq 0$)

The Pastor-Stambaugh-Taylor model

Optimal portfolio

- The expected utility is equal to:

$$\begin{aligned}
\mathbb{E}\left[\mathcal{U}\left(\tilde{W}_j, w_j\right)\right] &= \mathbb{E}\left[-\exp\left(-\bar{\gamma}_j \tilde{W}_j - w_j^\top b_j W_j\right)\right] \\
&= \mathbb{E}\left[-\exp\left(-\bar{\gamma}_j\left(1 + r + w_j^\top \tilde{R}\right) W_j - w_j^\top b_j W_j\right)\right] \\
&= -e^{-\bar{\gamma}_j(1+r)W_j}\mathbb{E}\left[\exp\left(-\bar{\gamma}_j w_j^\top W_j\left(\tilde{R} + \bar{\gamma}_j^{-1} b_j\right)\right)\right] \\
&= e^{-\bar{\Gamma}_j(1+r)}\mathbb{E}\left[\exp\left(-\bar{\Gamma}_j w_j^\top\left(\tilde{R} + \bar{\gamma}_j^{-1} b_j\right)\right)\right]
\end{aligned}$$

where $\bar{\Gamma}_j = \bar{\gamma}_j W_j$ is the nominal risk aversion

- We notice that $\tilde{R} + \bar{\gamma}_j^{-1} b_j \sim \mathcal{N}\left(\pi + \bar{\gamma}_j^{-1} b_j, \Sigma\right)$ and:

$$-\bar{\Gamma}_j w_j^\top\left(\tilde{R} + \bar{\Gamma}_j^{-1} b_j\right) \sim \mathcal{N}\left(-\bar{\Gamma}_j w_j^\top\left(\pi + \bar{\gamma}_j^{-1} b_j\right), \bar{\Gamma}_j^2 w_j^\top \Sigma w_j\right)$$

The Pastor-Stambaugh-Taylor model

Optimal portfolio

- We deduce that:

$$\mathbb{E}\left[\mathcal{U}\left(\tilde{W}_j, w_j\right)\right] = e^{-\bar{\Gamma}_j(1+r)} \exp\left(-\bar{\Gamma}_j w_j^\top\left(\pi + \bar{\gamma}_j^{-1} b_j\right) + \frac{1}{2}\bar{\Gamma}_j^2 w_j^\top \Sigma w_j\right)$$

- The first-order condition is equal to:

$$-\bar{\Gamma}_j\left(\pi + \bar{\gamma}_j^{-1} b_j\right) + \bar{\Gamma}_j^2 \Sigma w_j = 0$$

Finally, Pastor et al. (2021) concluded that the optimal portfolio is:

$$w_j^\star = \Gamma_j \Sigma^{-1}\left(\pi + \gamma_j b_j\right)$$

where $\Gamma_j = \bar{\Gamma}_j^{-1}$ and $\gamma_j = \bar{\gamma}_j^{-1}$ are the relative nominal and unitary risk-tolerance

The Pastor-Stambaugh-Taylor model

Optimal portfolio

Maximizing the expected utility is equivalent to solve the classical Markowitz QP problem:

$$\begin{aligned} w_j^{\star}(\gamma_j) &= \arg\min \frac{1}{2} w_j^{\top} \Sigma w_j - \gamma_j w_j^{\top} \mu' \\ \text{s.t.} & \quad \mathbf{1}^{\top} w_j = 1 \end{aligned}$$

where

- $\gamma_j = \bar{\gamma}_j^{-1}$ is the relative risk tolerance
- $\mu' = \mu + \gamma_j b_j$ is the vector of modified expected returns

The Pastor-Stambaugh-Taylor model

Optimal portfolio

Example #3

We consider a universe of n risky assets, where n is an even number. The risk-free rate r is set to 3%. We assume that the Sharpe ratio of these assets is the same and is equal to 20%. The volatility of asset i is equal to $\sigma_i = 0.10 + 0.20 \cdot e^{-n^{-1}\lfloor 0.5i \rfloor}$. The correlation between asset returns is constant: $\mathbb{C} = \mathbb{C}_n(\rho)$. The social impact of the firms is given by the vector $\boldsymbol{\mathcal{G}}$. When $\boldsymbol{\mathcal{G}}$ is not specified, it is equal to the cyclic vector $(+1\%, -1\%, +1\%, \ldots, +1\%, -1\%)$. This implies that half of the firms (green firms) have a positive social impact while the others (brown firms) have a negative impact.

The Pastor-Stambaugh-Taylor model

Optimal portfolio

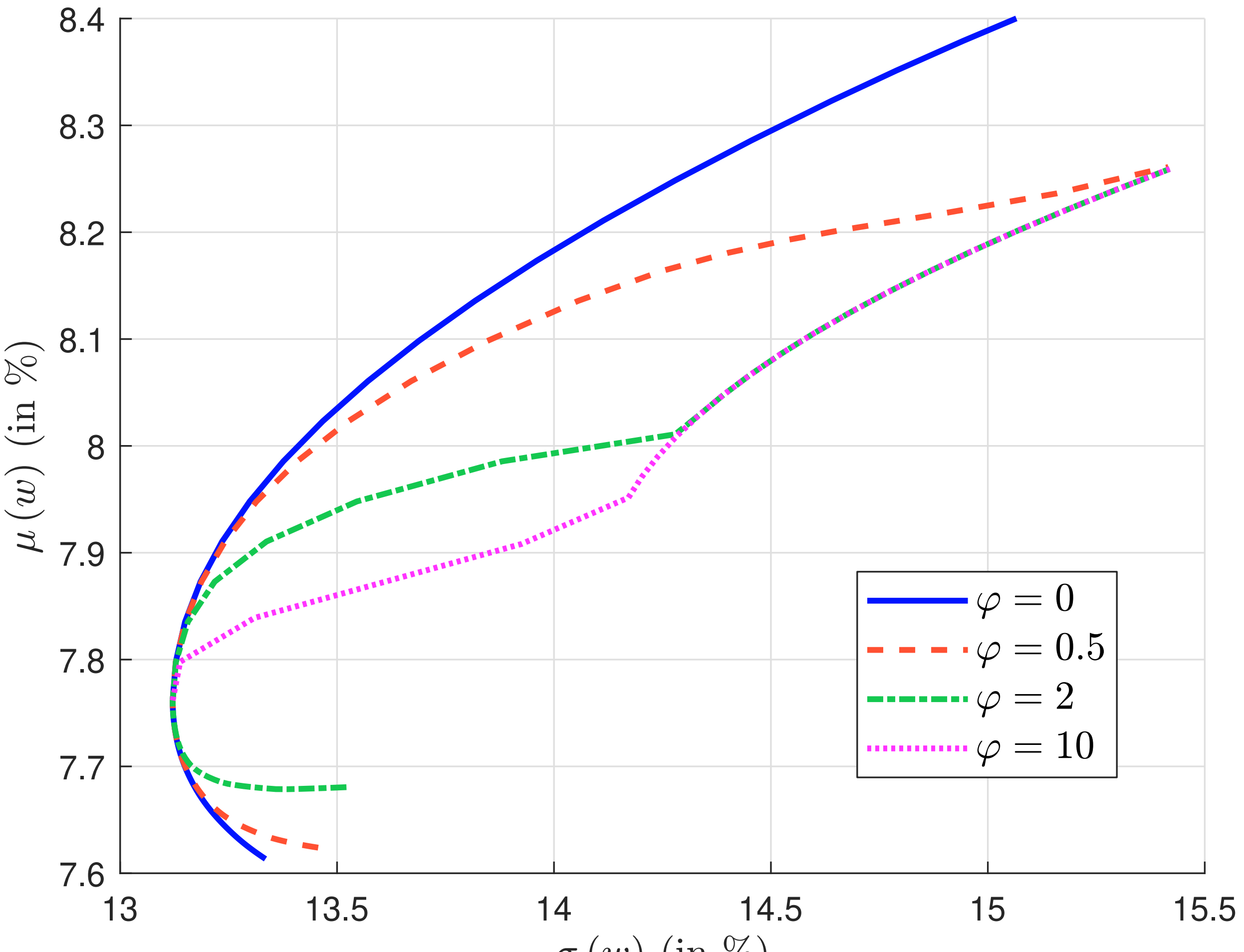
Table 39: Mean-variance optimized portfolios with ESG preferences (Example #3, $n = 6$, $\rho = 25\%$)

	$\mathcal{G} = (1\%, -1\%, 1\%, -1\%, 1\%, -1\%)$				$\mathcal{G} = (10\%, 5\%, 2\%, 3\%, 25\%, 30\%)$			
φ	0.00%	1.00%	5.00%	50.00%	0.00%	0.50%	1.00%	2.00%
$w_1^\star$	44.97%	48.87%	58.65%	67.48%	44.97%	46.83%	28.69%	0.00%
$w_2^\star$	44.97%	41.06%	19.60%	0.00%	44.97%	37.06%	9.17%	0.00%
$w_3^\star$	5.03%	9.82%	21.75%	32.52%	5.03%	0.00%	0.00%	0.00%
$w_4^\star$	5.03%	0.25%	0.00%	0.00%	5.03%	0.00%	0.00%	0.00%
$w_5^\star$	0.00%	0.00%	0.00%	0.00%	0.00%	0.83%	16.62%	21.09%
$w_6^\star$	0.00%	0.00%	0.00%	0.00%	0.00%	15.28%	45.53%	78.91%
$\mu(w^\star)$	8.33%	8.33%	8.27%	8.22%	8.33%	8.23%	7.79%	7.43%
$\sigma(w^\star)$	20.00%	20.09%	20.07%	21.56%	20.00%	19.33%	16.70%	19.17%
$\mathrm{SR}(w^\star \mid r)$	0.27	0.27	0.26	0.24	0.27	0.27	0.29	0.23

The Pastor-Stambaugh-Taylor model

Optimal portfolio

Figure 48: Efficient frontier with ESG preferences (Example #3, $n = 20$, $\rho = 25\%$)



The Pastor-Stambaugh-Taylor model

Risk premium

- $W = \int W_j \,\mathrm{d}j$
- $\omega_j = W_j / W$ is the market share of the economic agent j
- $W_{i,j} = w^{\star}_{i,j} W_j = w^{\star}_{i,j} \omega_j W$
- We have:

$$W_i = \int_j W_{i,j} \,\mathrm{d}j = \int_j w^{\star}_{i,j} \omega_j W \,\mathrm{d}j$$

- Let $w_m = (w_{1,m}, \ldots, w_{n,m})$ be the market portfolio. We have:

$$w_{i,m} = \frac{W_i}{W} = \int_j w^{\star}_{i,j} \omega_j \,\mathrm{d}j$$

and $\int_j \omega_j \,\mathrm{d}j = 1$

The Pastor-Stambaugh-Taylor model

Risk premium

- The market clearing condition satisfies:

$$\begin{aligned} w_m &= \int_j \omega_j w_j^\star \,\mathrm{d}j \\ &= \int_j \omega_j \Gamma_j \Sigma^{-1} \left(\pi + \gamma_j b_j\right) \mathrm{d}j \\ &= \int_j \omega_j \Gamma_j \Sigma^{-1} \left(\pi + \gamma_j \varphi_j \mathcal{G}\right) \mathrm{d}j \\ &= \left(\int_j \Gamma_j \omega_j \,\mathrm{d}j\right) \Sigma^{-1} \pi + \left(\int_j \omega_j \Gamma_j \psi_j \,\mathrm{d}j\right) \Sigma^{-1} \mathcal{G} \end{aligned}$$

  where $\psi_j = \gamma_j \varphi_j$

- It follows that:

$$w_m = \Gamma_m \Sigma^{-1} \pi + \Gamma_m \psi_m \Sigma^{-1} \mathcal{G}$$

  where $\Gamma_m = \int_j \Gamma_j \omega_j \,\mathrm{d}j$ and $\psi_m = \Gamma_m^{-1} \left(\int_j \omega_j \Gamma_j \psi_j \,\mathrm{d}j\right)$ are the average risk tolerance and the weighted average of ESG preferences

The Pastor-Stambaugh-Taylor model

Risk premium

- The asset risk premia are equal to:

$$\pi = \frac{1}{\Gamma_m} \Sigma w_m - \psi_m \mathcal{G}$$

while the market risk premium is defined as:

$$\begin{aligned} \pi_m &= w_m^\top \pi \\ &= \frac{1}{\Gamma_m} w_m^\top \Sigma w_m - \psi_m w_m^\top \mathcal{G} \\ &= \frac{1}{\Gamma_m} \sigma_m^2 - \psi_m \mathcal{G}_m \end{aligned}$$

where $\sigma_m = \sqrt{w_m^\top \Sigma w_m}$ and $\mathcal{G}_m = w_m^\top \mathcal{G}$ are the volatility and the green intensity (or greenness) of the market portfolio

The Pastor-Stambaugh-Taylor model

Risk premium

- The risk premium including the ESG sentiment is lower than the CAPM risk premium if the market ESG intensity is positive:

$$\mathcal{G}_m > 0 \Longrightarrow \pi_m \leq \pi_m^{\mathrm{capm}}$$

- It is greater than the CAPM risk premium if the market ESG intensity is negative:

$$\mathcal{G}_m < 0 \Longrightarrow \pi_m \geq \pi_m^{\mathrm{capm}}$$

- The gap $\Delta\pi_m^{\mathrm{esg}} := \left|\pi_m - \pi_m^{\mathrm{capm}}\right|$ is an increasing function of the market ESG sentiment ψ_m:

$$\psi_m \nearrow \Longrightarrow \Delta\pi_m^{\mathrm{esg}} \nearrow$$

The Pastor-Stambaugh-Taylor model

Risk premium

If we assume that $\boldsymbol{\mathcal{G}}_m \approx 0$, we have $\Gamma_m = \sigma_m^2 / \pi_m$,

$$\pi = \beta \pi_m - \psi_m \boldsymbol{\mathcal{G}}$$

and:

$$\alpha_i = \pi_i - \beta_i \pi_m = -\psi_m \boldsymbol{\mathcal{G}}_i$$

If $\psi_m > 0$, "green stocks have negative alphas, and brown stocks have positive alphas. Moreover, greener stocks have lower alphas" (Pastor et al., 2021).

The Pastor-Stambaugh-Taylor model

Risk premium

Example #4

We consider Example #3. The market is made up of two long-only investors ($j = 1, 2$): a non-ESG investor ($\varphi_1 = 0$) and an ESG investor ($\varphi_2 > 0$). We assume that they have the same risk tolerance γ. We note W_1 and W_2 their financial wealth, which is entirely invested in the risky assets. We assume that $W_1 = W_2 = 1$.

The Pastor-Stambaugh-Taylor model

Risk premium

- The tangency portfolio is equal to:

$$\begin{aligned} w^* &= \frac{\Sigma^{-1}(\mu - r\mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})} \\ &= (15.04\%, 15.04\%, 16.65\%, 16.65\%, 18.31\%, 18.31\%) \end{aligned}$$

- $w_1^\star = w^*$ and $\gamma_1 = 1/\left(\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})\right) = 0.4558$
- $\gamma_2 = \gamma_1$ and:

$$\begin{aligned} w_2^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - \gamma_2 w^\top (\mu + \gamma_2 \varphi_2 \boldsymbol{\mathcal{G}}) \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^\top w = 1 \\ w \geq \mathbf{0} \end{array} \right. \end{aligned}$$

- We obtain

$$w_2^\star = (18.86\%, 11.22\%, 21.33\%, 11.97\%, 23.96\%, 12.65\%)$$

The Pastor-Stambaugh-Taylor model
Risk premium

- The market portfolio is then equal to:

$$\begin{aligned} w_m &= \frac{W_1}{W} w_1^\star + \frac{W_2}{W} w_2^\star \\ &= \left(1 - \omega^{\text{esg}}\right) \cdot w_1^\star + \omega^{\text{esg}} \cdot w_2^\star \end{aligned}$$

- When $W_1 = W_2 = 1$, we obtain

$$\begin{aligned} w_m &= (16.95\%, 13.13\%, 18.99\%, 14.31\%, 21.13\%, 15.48\%) \\ \mu_m &= 7.86\% \\ \sigma_m &= 14.93\% \end{aligned}$$

- We deduce that:

$$\begin{aligned} \beta &= (1.15, 1.05, 1.04, 0.95, 0.95, 0.86) \\ \pi &= (5.58\%, 5.12\%, 5.06\%, 4.61\%, 4.62\%, 4.17\%) \\ \alpha &= (-19.09, 26.19, -19.43, 25.84, -19.72, 25.55) \quad \text{(in bps)} \end{aligned}$$

The Pastor-Stambaugh-Taylor model

Risk premium

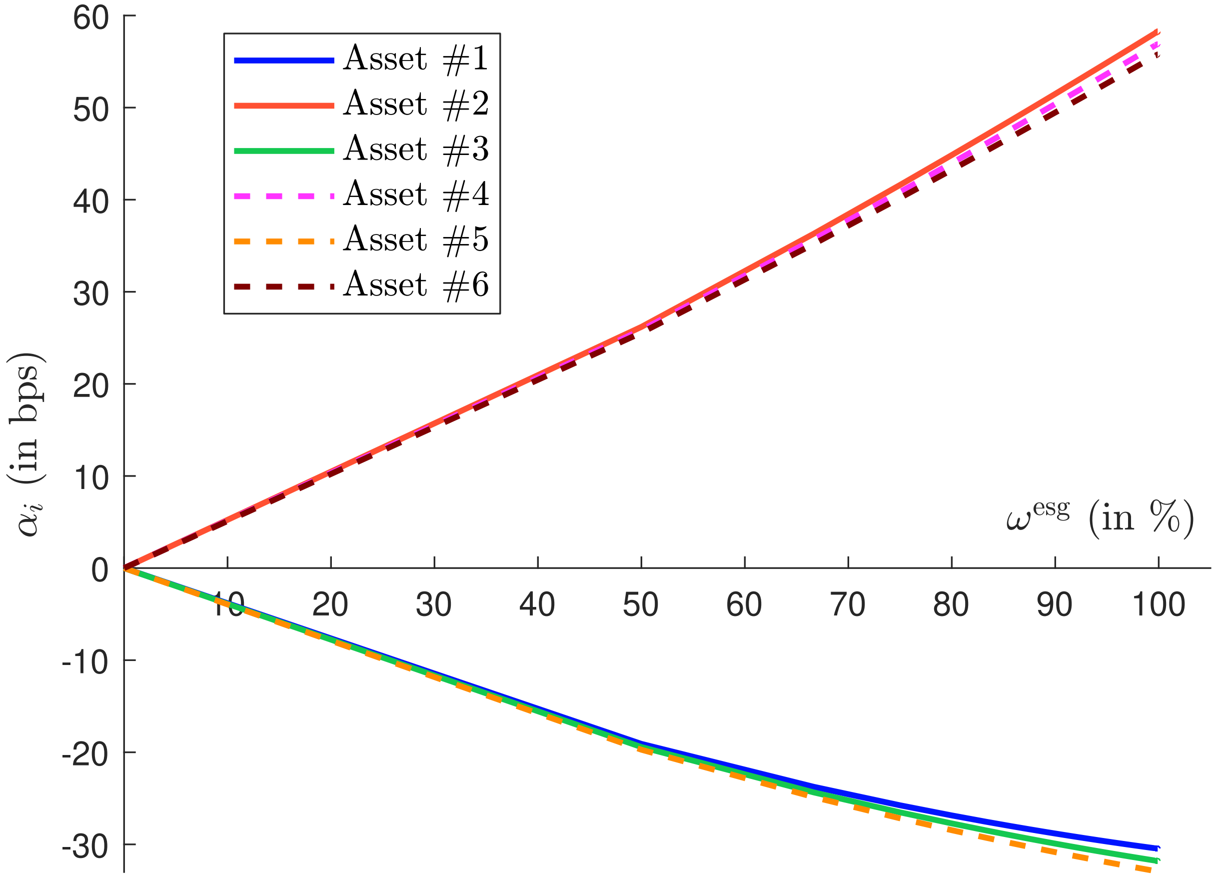
Table 40: Computation of alpha returns (Example #4, $n = 6$, $\rho = 25\%$)

i	Portfolio $w_1^\star$			Portfolio $w_2^\star$				Portfolio w_m			
	w_i (in %)	β_i	π_i (in %)	w_i (in %)	β_i	π_i (in %)	α_i (in bps)	w_i (in %)	β_i	π_i (in %)	α_i (in bps)
1	15.04	1.11	5.39	18.86	1.17	5.69	−30	16.95	1.15	5.58	−19
2	15.04	1.11	5.39	11.22	0.99	4.80	58	13.13	1.05	5.12	26
3	16.65	1.00	4.87	21.33	1.07	5.18	−32	18.99	1.04	5.06	−19
4	16.65	1.00	4.87	11.97	0.88	4.30	57	14.31	0.95	4.61	26
5	18.31	0.91	4.43	23.96	0.98	4.76	−33	21.13	0.95	4.62	−20
6	18.31	0.91	4.43	12.65	0.80	3.87	56	15.48	0.86	4.17	26

The Pastor-Stambaugh-Taylor model

Risk premium

Figure 49: Evolution of the alpha return with respect to the market share of ESG investors (Example #4, $n = 6$, $\rho = 25\%$)



The Pastor-Stambaugh-Taylor model

Risk premium

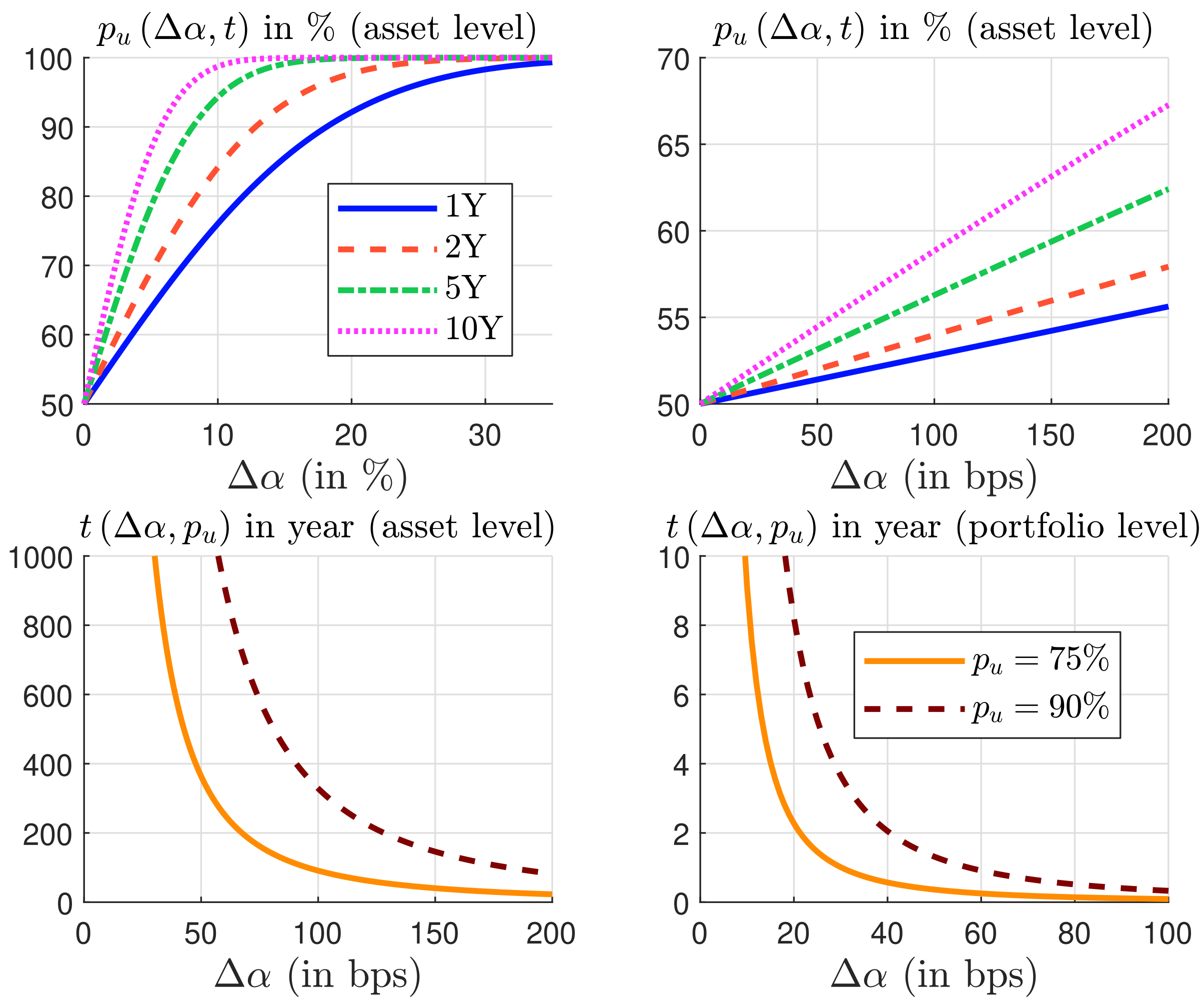
"In equilibrium, green assets have low expected returns because investors enjoy holding them and because green assets hedge climate risk. Green assets nevertheless outperform when positive shocks hit the ESG factor, which captures shifts in customers' tastes for green products and investors' tastes for green holdings." (Pastor et al., 2021).

- ESG risk premium?
- Green risk premium?

The Pastor-Stambaugh-Taylor model

What does equilibrium mean?

Figure 50: Impact of alpha returns on the underperformance probability



Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

- We have:

$$\begin{pmatrix} \tilde{R} \\ \boldsymbol{\mathcal{S}} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \pi \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma & \Sigma_{\pi,s} \\ \Sigma_{s,\pi} & \Sigma_s \end{pmatrix}\right)$$

- The optimal solution is:

$$w_j^\star = \underbrace{\Gamma_j \Sigma^{-1} \left(\pi + \psi_j \mu_s\right)}_{\text{PST solution}} + \underbrace{\Gamma_j^{-1} \Omega_j \left(\pi + \psi_j \mu_s\right)}_{\text{ESG uncertainty}}$$

Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

- If there is no ESG uncertainty ($\boldsymbol{\mathcal{S}} = \mu_s$ and $\Sigma_s = \mathbf{0}$), the vector of risk premia is given by:

$$\begin{aligned} \pi^{\mathrm{esg}} &= \beta \pi_m - \psi_m \left(\mu_s - \beta \bar{\boldsymbol{\mathcal{S}}}_m \right) \\ &= \pi^{\mathrm{capm}} - \psi_m \left(\mu_s - \beta \bar{\boldsymbol{\mathcal{S}}}_m \right) \end{aligned}$$

- If there is an uncertainty on ESG scores ($\boldsymbol{\mathcal{S}} \neq \mu_s$ and $\Sigma_s \neq \mathbf{0}$), the vector of risk premia becomes:

$$\begin{aligned} \breve{\pi}^{\mathrm{esg}} &= \breve{\beta} \breve{\pi}_m - \psi_m \left(\breve{\mu}_s - \breve{\beta} \breve{\boldsymbol{\mathcal{S}}}_m \right) \\ &= \beta \pi_m + \left(\breve{\beta} - \beta \right) \pi_m - \psi_m \left(\breve{\mu}_s - \breve{\beta} \breve{\boldsymbol{\mathcal{S}}}_m \right) \end{aligned}$$

Extension of the PST model

The Avromov-Cheng-Lioui-Tarelli model

"In equilibrium, the market premium increases and demand for stocks declines under ESG uncertainty. In addition, the CAPM alpha and effective beta both rise with ESG uncertainty and the negative ESG-alpha relation weakens." (Avramov et al., 2022).

Extension of the PST model

Risk factor model

The Pedersen-Fitzgibbons-Pomorski model

Model settings

- $\tilde{R} = R - r \sim \mathcal{N}(\pi, \Sigma)$
- $\boldsymbol{\mathcal{S}} = (\boldsymbol{\mathcal{S}}_1, \ldots, \boldsymbol{\mathcal{S}}_n)$
- The terminal wealth is $\tilde{W} = \left(1 + r + w^\top \tilde{R}\right) W$
- The model uses the mean-variance utility:

$$
\begin{aligned}
\boldsymbol{\mathcal{U}}\left(\tilde{W}, w\right) &= \mathbb{E}\left[\tilde{W}\right] - \frac{\bar{\gamma}}{2} \operatorname{var}\left(\tilde{W}\right) + \zeta\left(\boldsymbol{\mathcal{S}}(w)\right) W \\
&= \left(1 + r + w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta\left(w^\top \boldsymbol{\mathcal{S}}\right)\right) W
\end{aligned}
$$

where ζ is a function that depends on the investor

The Pedersen-Fitzgibbons-Pomorski model

Model settings

- Optimizing the utility function is equivalent to find the mean-variance-esg optimized portfolio:

$$\begin{aligned} w^\star &= \arg\max w^\top \pi - \frac{\bar{\gamma}}{2} w^\top \Sigma w + \zeta \left(w^\top \boldsymbol{\mathcal{S}} \right) \\ \text{s.t.} & \quad \mathbf{1}^\top w = 1 \end{aligned}$$

- $\sigma(w) = \sqrt{w^\top \Sigma w}$
- $\boldsymbol{\mathcal{S}}(w) = w^\top \boldsymbol{\mathcal{S}}$

The Pedersen-Fitzgibbons-Pomorski model

Model settings

- The optimization problem can be decomposed as follows:

$$w^{\star} = \arg \left\{ \max_{\bar{\boldsymbol{\mathcal{S}}}} \left\{ \max_{\bar{\sigma}} \left\{ \max_{w} \left\{ f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) \text{ s.t. } w \in \Omega\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) \right\} \right\} \right\} \right\}$$

where:

$$f\left(w; \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) = w^{\top}\pi - \frac{\bar{\gamma}}{2}\sigma^{2}\left(w\right) + \zeta\left(\boldsymbol{\mathcal{S}}\left(w\right)\right)$$

and:

$$\Omega = \left\{ w \in \mathbb{R}^{n} : \mathbf{1}^{\top} w = 1, \sigma\left(w\right) = \bar{\sigma}, \boldsymbol{\mathcal{S}}\left(w\right) = \bar{\boldsymbol{\mathcal{S}}} \right\}$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- We consider the $\sigma - \boldsymbol{\mathcal{S}}$ problem:

$$\begin{aligned} w^{\star}\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) & = & \arg\max w^{\top}\pi \\ & & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}^{\top}w = 1 \\ w^{\top}\Sigma w - \bar{\sigma}^{2} = 0 \\ w^{\top}\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = 0 \end{array} \right. \end{aligned}$$

- The Lagrange function is:

$$\mathcal{L}\left(w;\lambda_1,\lambda_2\right) = w^{\top}\pi + \lambda_1\left(w^{\top}\Sigma w - \bar{\sigma}^{2}\right) + \lambda_2\left(w^{\top}\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)$$

- The first-order condition is:

$$\frac{\partial\,\mathcal{L}\left(w;\lambda_1,\lambda_2\right)}{\partial\, w} = \pi + 2\lambda_1\Sigma w + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0}$$

- We deduce that the optimal portfolio is given by:

$$w = -\frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- The second constraint $w^\top \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = 0$ implies that:

$$
\begin{aligned}
(*) \quad \Leftrightarrow \quad & \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)^\top \frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right) = 0 \\
\Leftrightarrow \quad & \lambda_2 = -\frac{\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)^\top \Sigma^{-1}\pi}{\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)^\top \Sigma^{-1}\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)} \\
\Leftrightarrow \quad & \lambda_2 = \frac{\bar{\boldsymbol{\mathcal{S}}}\left(\mathbf{1}^\top\Sigma^{-1}\pi\right) - \boldsymbol{\mathcal{S}}^\top\Sigma^{-1}\pi}{\boldsymbol{\mathcal{S}}^\top\Sigma^{-1}\boldsymbol{\mathcal{S}} - 2\bar{\boldsymbol{\mathcal{S}}}\left(\mathbf{1}^\top\Sigma^{-1}\boldsymbol{\mathcal{S}}\right) + \bar{\boldsymbol{\mathcal{S}}}^2\left(\mathbf{1}^\top\Sigma^{-1}\mathbf{1}\right)} \\
\Leftrightarrow \quad & \lambda_2 = \frac{C_{1,\pi}\bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi}}{C_{s,s} - 2C_{1,s}\bar{\boldsymbol{\mathcal{S}}} + C_{1,1}\bar{\boldsymbol{\mathcal{S}}}^2}
\end{aligned}
$$

where $C_{x,y}$ is the compact notation for $x^\top\Sigma^{-1}y$ — $C_{1,\pi} = \mathbf{1}^\top\Sigma^{-1}\pi$, $C_{s,\pi} = \boldsymbol{\mathcal{S}}^\top\Sigma^{-1}\pi$, $C_{s,s} = \boldsymbol{\mathcal{S}}^\top\Sigma^{-1}\boldsymbol{\mathcal{S}}$, $C_{1,s} = \mathbf{1}^\top\Sigma^{-1}\boldsymbol{\mathcal{S}}$ and $C_{1,1} = \mathbf{1}^\top\Sigma^{-1}\mathbf{1}$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- Using the first constraint $w^\top \Sigma w - \bar{\sigma}^2 = 0$, we deduce that:

$$\begin{aligned}
\bar{\sigma}^2 &= -\frac{1}{2\lambda_1} w^\top \Sigma \Sigma^{-1} \left(\pi + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1}\right)\right) \\
&= -\frac{1}{2\lambda_1} \left(w^\top \pi + \lambda_2 w^\top \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1}\right)\right) \\
&= -\frac{1}{2\lambda_1} w^\top \pi \\
&= \frac{1}{4\lambda_1^2} \pi^\top \Sigma^{-1} \left(\pi + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \mathbf{1}\right)\right)
\end{aligned}$$

- The first Lagrange coefficient is then equal to ($C_{\pi,\pi} = \pi^\top \Sigma^{-1} \pi$):

$$\begin{aligned}
\lambda_1 &= -\frac{1}{2\bar{\sigma}} \sqrt{\pi^\top \Sigma^{-1} \pi + \lambda_2 \left(\pi^\top \Sigma^{-1} \boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}} \left(\pi^\top \Sigma^{-1} \mathbf{1}\right)\right)} \\
&= -\frac{1}{2\bar{\sigma}} \sqrt{C_{\pi,\pi} - \frac{\left(C_{1,\pi} \bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi}\right)^2}{C_{s,s} - 2 C_{1,s} \bar{\boldsymbol{\mathcal{S}}} + C_{1,1} \bar{\boldsymbol{\mathcal{S}}}^2}}
\end{aligned}$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- The optimal portfolio is the product of the volatility $\bar{\sigma}$ and the vector $\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)$:

$$\begin{aligned} w^{\star}\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) &= -\frac{1}{2\lambda_1}\Sigma^{-1}\left(\pi + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right) \\ &= \bar{\sigma} \cdot \varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right) \end{aligned}$$

where:

$$\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right) = \frac{1}{\lambda_1'}\Sigma^{-1}\left(\pi + \lambda_2\left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right)\right)$$

and:

$$\lambda_1' = \sqrt{C_{\pi,\pi} - \frac{\left(C_{1,\pi}\bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi}\right)^2}{C_{s,s} - 2C_{1,s}\bar{\boldsymbol{\mathcal{S}}} + C_{1,1}\bar{\boldsymbol{\mathcal{S}}}^2}}$$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

Example #5

We consider an investment universe of four assets. Their expected returns are equal to 6%, 7%, 8% and 10% while their volatilities are equal to 15%, 20%, 25% and 30%. The correlation matrix of asset returns is given by the following matrix:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & \\ 20\% & 100\% & & \\ 30\% & 50\% & 100\% & \\ 40\% & 60\% & 70\% & 100\% \end{pmatrix}$$

The risk-free rate is set to 2%. The ESG score vector is $\boldsymbol{\mathcal{S}} = (3\%, 2\%, -2\%, -3\%)$.

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- We obtain $\mathcal{C}_{1,\pi} = 2.4864$, $\mathcal{C}_{s,\pi} = 0.0425$, $\mathcal{C}_{s,s} = 0.1274$, $\mathcal{C}_{1,s} = 1.9801$, $\mathcal{C}_{1,1} = 64.1106$ and $\mathcal{C}_{\pi,\pi} = 0.1193$
- If we target $\bar{\sigma} = 20\%$ and $\bar{\mathcal{S}} = 1\%$, we deduce that $\lambda_1 = -0.8514$ and $\lambda_2 = -0.1870$
- The optimal portfolio is then:

$$w^{\star}\left(\bar{\sigma}, \bar{\mathcal{S}}\right) = \begin{pmatrix} 59.31\% \\ 29.52\% \\ 21.76\% \\ 20.72\% \end{pmatrix}$$

- It follows that the portfolio is leveraged since we have $w_r = 1 - \mathbf{1}^\top w = -31.31\%$

The Pedersen-Fitzgibbons-Pomorski model

Optimal portfolio

- We verify that $\sqrt{w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\Sigma w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)} = 20\%$ and $\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\boldsymbol{\mathcal{S}}\right) \Big/ \left(1^{\top} w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)\right) = 1\%$
- We also notice that:

$$\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right) = \begin{pmatrix} 2.9657 \\ 1.4759 \\ 1.0881 \\ 1.0358 \end{pmatrix}$$

  and verify that $w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right) = \bar{\sigma} \cdot \varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)$
- The portfolio is then leveraged when $\bar{\sigma} \geq 1/\left(\mathbf{1}^{\top}\varrho\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right) = 17.75\%$.

The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

- We rewrite the first-order condition as:

$$
\begin{aligned}
(*) \quad & \Leftrightarrow \quad \pi + 2\lambda_1 \Sigma w + \lambda_2 \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0} \\
& \Leftrightarrow \quad w^\top \pi + 2\lambda_1 w^\top \Sigma w + \lambda_2 w^\top \left(\boldsymbol{\mathcal{S}} - \bar{\boldsymbol{\mathcal{S}}}\mathbf{1}\right) = \mathbf{0} \\
& \Leftrightarrow \quad w^\top \pi + 2\lambda_1 \bar{\sigma}^2 = \mathbf{0} \\
& \Leftrightarrow \quad \lambda_1 = -\frac{1}{2}\frac{w^\top \pi}{\bar{\sigma}^2} = -\frac{1}{2}\frac{\mathrm{SR}\left(w \mid r\right)}{\bar{\sigma}}
\end{aligned}
$$

- We deduce that the Sharpe ratio of the optimal portfolio is:

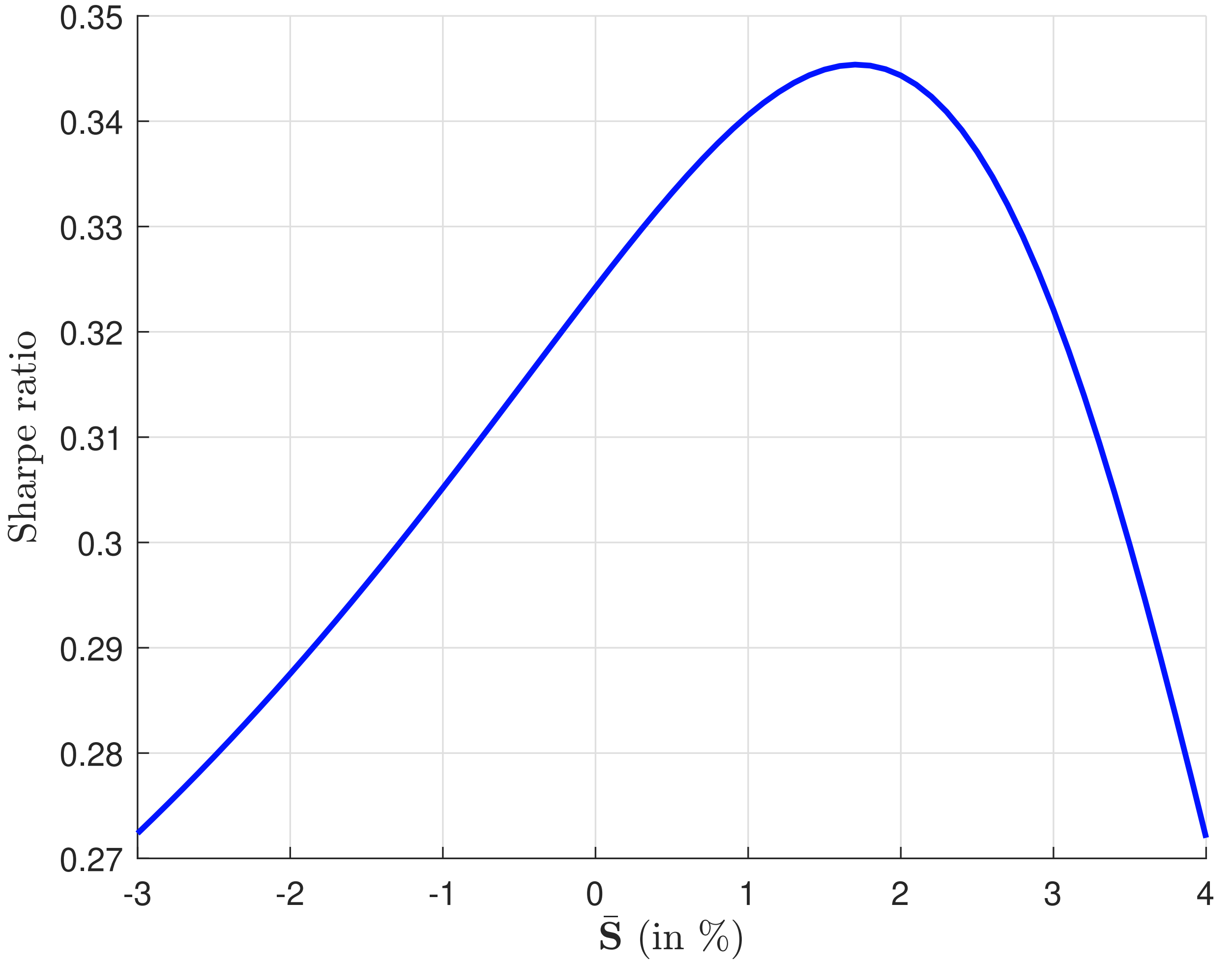
$$
\mathrm{SR}\left(w^\star\left(\bar{\sigma}, \bar{\boldsymbol{\mathcal{S}}}\right) \mid r\right) = \sqrt{C_{\pi,\pi} - \frac{\left(C_{1,\pi}\bar{\boldsymbol{\mathcal{S}}} - C_{s,\pi}\right)^2}{C_{s,s} - 2C_{1,s}\bar{\boldsymbol{\mathcal{S}}} + C_{1,1}\bar{\boldsymbol{\mathcal{S}}}^2}} = \mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}} \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right)
$$

- It depends on the asset parameters π, Σ, $\boldsymbol{\mathcal{S}}$, the ESG objective $\bar{\boldsymbol{\mathcal{S}}}$ of the investor, but not the volatility target $\bar{\sigma}$

The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

Figure 51: Relationship between $\bar{\mathcal{S}}$ and $\mathrm{SR}\left(\bar{\mathcal{S}} \mid \pi, \Sigma\right)$ (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The Sharpe ratio of the optimal portfolio

Using Example #5

- The Sharpe ratio of the optimal portfolio $w^\star(20\%, 1\%)$ is equal to 0.3406
- We have $\mathrm{SR}\left(w^\star(\bar{\sigma}, -3\%) \mid r\right) = 0.2724$, $\mathrm{SR}\left(w^\star(\bar{\sigma}, -2\%) \mid r\right) = 0.2875$, $\mathrm{SR}\left(w^\star(\bar{\sigma}, -1\%) \mid r\right) = 0.3052$, $\mathrm{SR}\left(w^\star(\bar{\sigma}, 0\%) \mid r\right) = 0.3242$, $\mathrm{SR}\left(w^\star(\bar{\sigma}, 1\%) \mid r\right) = 0.3406$, $\mathrm{SR}\left(w^\star(\bar{\sigma}, 2\%) \mid r\right) = 0.3443$, and $\mathrm{SR}\left(w^\star(\bar{\sigma}, 3\%) \mid r\right) = 0.3221$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- The objective function is equal to:

$$\begin{aligned} f\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right);\pi,\Sigma,\boldsymbol{\mathcal{S}}\right) &= \left(\frac{w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)^{\top}\pi}{\bar{\sigma}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \\ &= \mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \end{aligned}$$

- The σ-problem becomes:

$$\begin{aligned} (*) &= \max_{\bar{\sigma}}\left\{\max_{w}\left\{f\left(w;\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\text{ s.t. }w\in\Omega\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right)\right\}\right\} \\ &= \max_{\bar{\sigma}}\left\{\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)\bar{\sigma}-\frac{\bar{\gamma}}{2}\bar{\sigma}^{2}+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right\} \end{aligned}$$

- The first-order condition is $\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)-\bar{\gamma}\bar{\sigma}=0$ or $\bar{\sigma}=\bar{\gamma}^{-1}\,\mathrm{SR}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- We have:

$$\begin{aligned} f\left(w^{\star}\left(\bar{\sigma},\bar{\boldsymbol{\mathcal{S}}}\right);\pi,\Sigma,\boldsymbol{\mathcal{S}}\right) &= \bar{\gamma}^{-1}\,\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right) - \\ &\quad \frac{1}{2}\bar{\gamma}^{-1}\,\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right) \\ &= \frac{1}{2}\bar{\gamma}^{-1}\left(\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right) \end{aligned}$$

- We conclude that the $\boldsymbol{\mathcal{S}}$-problem becomes:

$$\boldsymbol{\mathcal{S}}^{\star}=\arg\max_{\bar{\boldsymbol{\mathcal{S}}}}\left\{\mathrm{SR}^{2}\left(\bar{\boldsymbol{\mathcal{S}}}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)+2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)\right\}$$

- The optimal portfolio is $w^{\star}=w^{\star}\left(\sigma^{\star},\boldsymbol{\mathcal{S}}^{\star}\right)$ where $\boldsymbol{\mathcal{S}}^{\star}$ is the solution of the $\boldsymbol{\mathcal{S}}$-problem and $\sigma^{\star}=\bar{\gamma}^{-1}\,\mathrm{SR}\left(\boldsymbol{\mathcal{S}}^{\star}\mid\pi,\Sigma,\boldsymbol{\mathcal{S}}\right)$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Pedersen *et al.* (2021) distinguished three groups of investors:

- Type-U or ESG-unware investors have no ESG preference and do not use the information of ESG scores
- Type-A or ESG-aware investors have no ESG preference, but they use the ESG scores to update their views on the risk premia
- Type-M or ESG-motivated investors have ESG preferences, implying that they would like to have a high ESG score

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- Type-U investors hold the same portfolio:

$$w_U^\star = \frac{\Sigma^{-1}\pi}{\mathbf{1}^\top \Sigma^{-1}\pi}$$

- Type-A investors choose the optimal portfolio with the highest Sharpe ratio ($\zeta(s) = 0$) $\Rightarrow \boldsymbol{\mathcal{S}}_A^\star$ is the optimal ESG score
- Type-M investors choose an optimal portfolio on the ESG-SR efficient frontier, with:

$$\boldsymbol{\mathcal{S}}_M^\star \geq \boldsymbol{\mathcal{S}}_A^\star$$

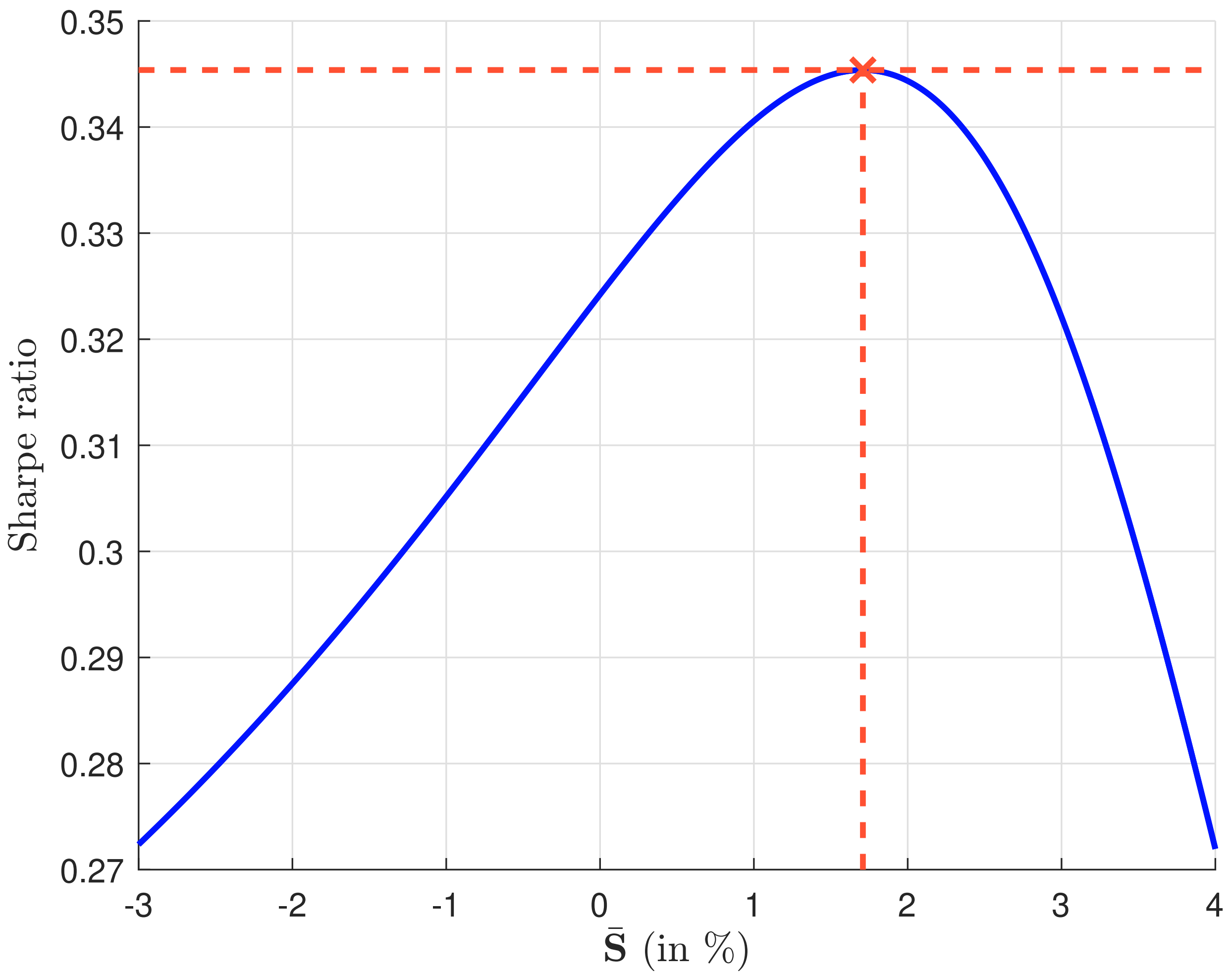
and:

$$\mathrm{SR}\left(\boldsymbol{\mathcal{S}}_M^\star \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) \leq \mathrm{SR}\left(\boldsymbol{\mathcal{S}}_A^\star \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right)$$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

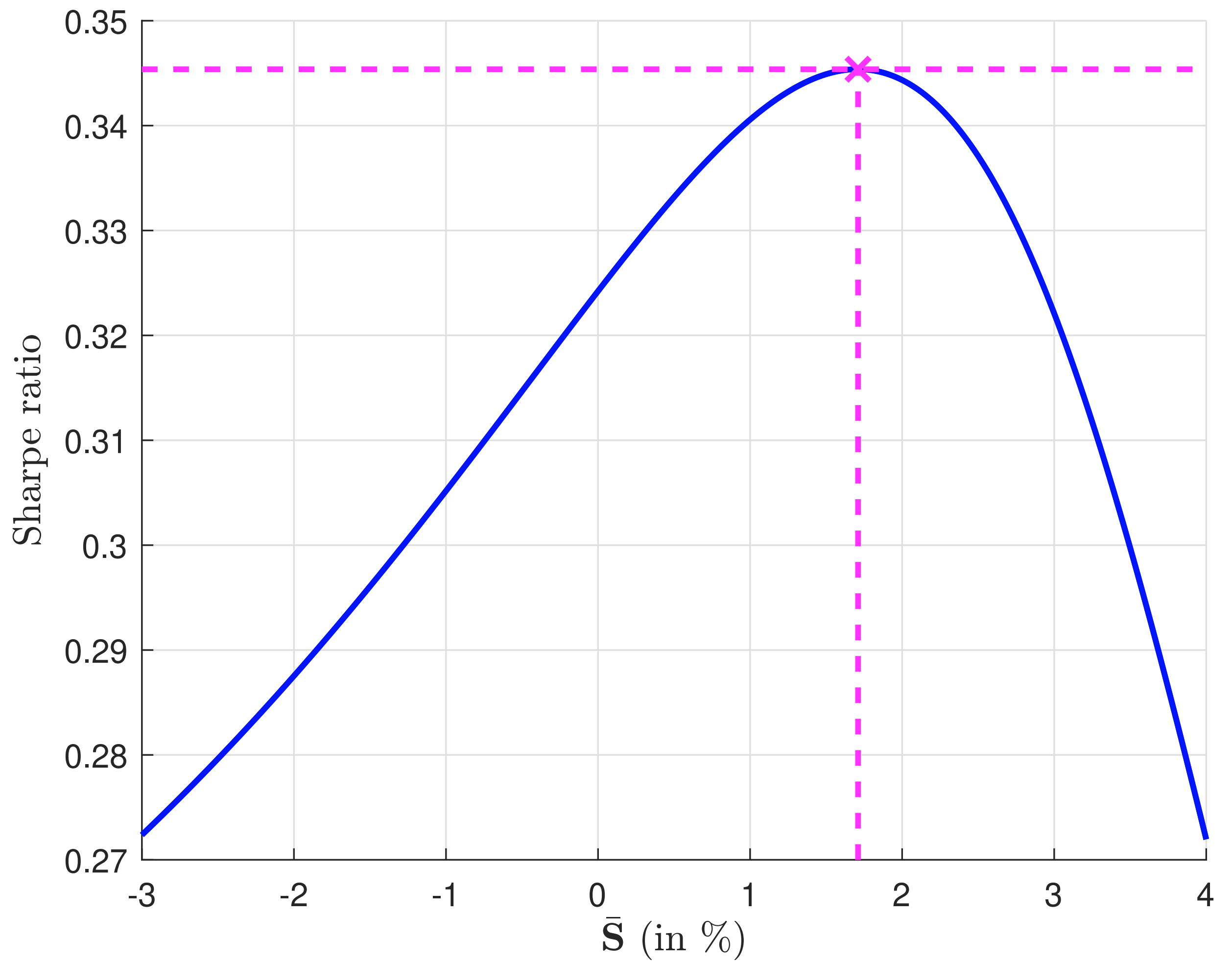
Figure 52: Optimal portfolio for type-U investors (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Figure 53: Optimal portfolio for type-A investors (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

- For type-M investors, we first compute the function $\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right)$:

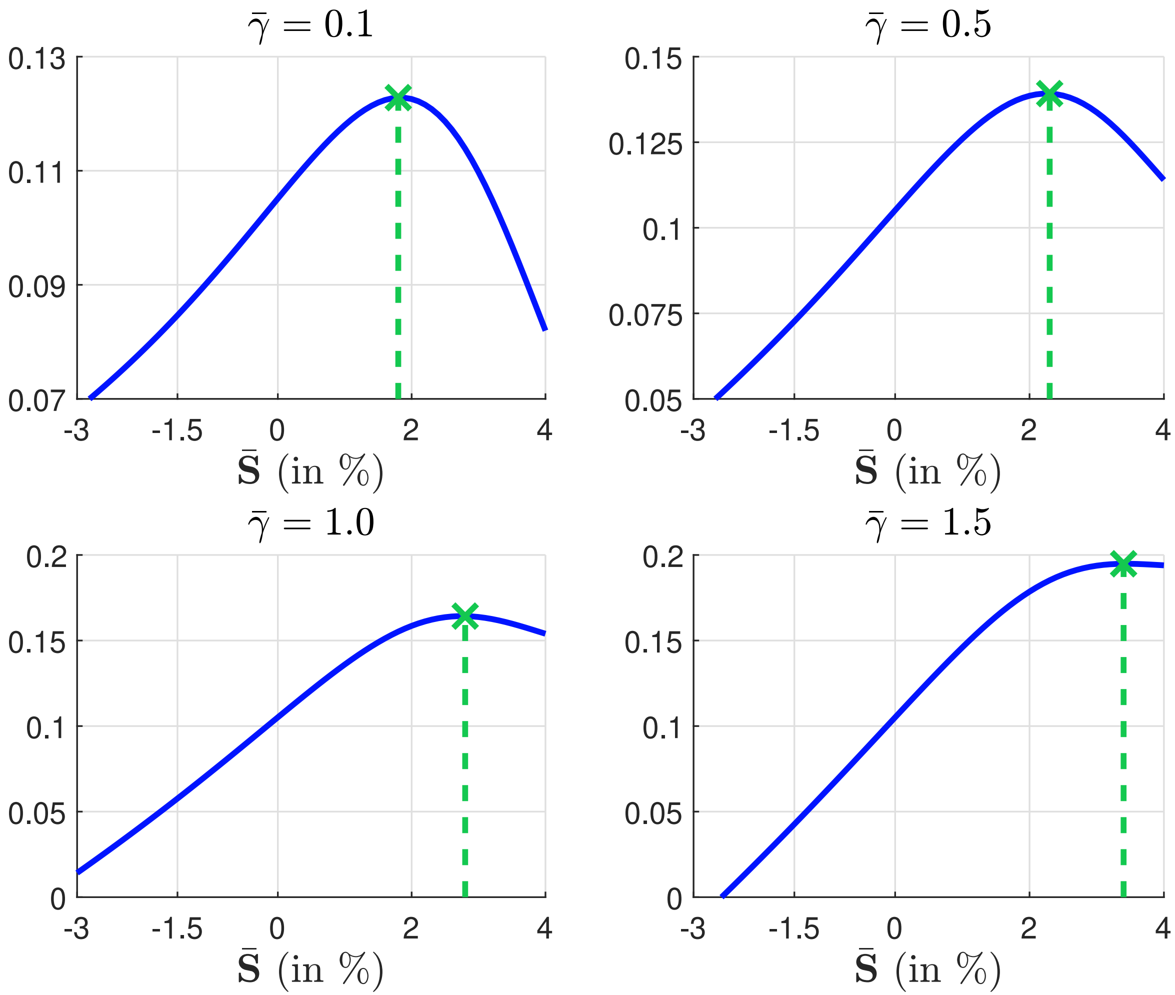
$$\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right) = \mathrm{SR}^2\left(\bar{\boldsymbol{\mathcal{S}}} \mid \pi, \Sigma, \boldsymbol{\mathcal{S}}\right) + 2\bar{\gamma}\zeta\left(\bar{\boldsymbol{\mathcal{S}}}\right)$$

- The optimal portfolio corresponds to the optimal ESG score that maximizes $\xi\left(\bar{\boldsymbol{\mathcal{S}}}\right)$

The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

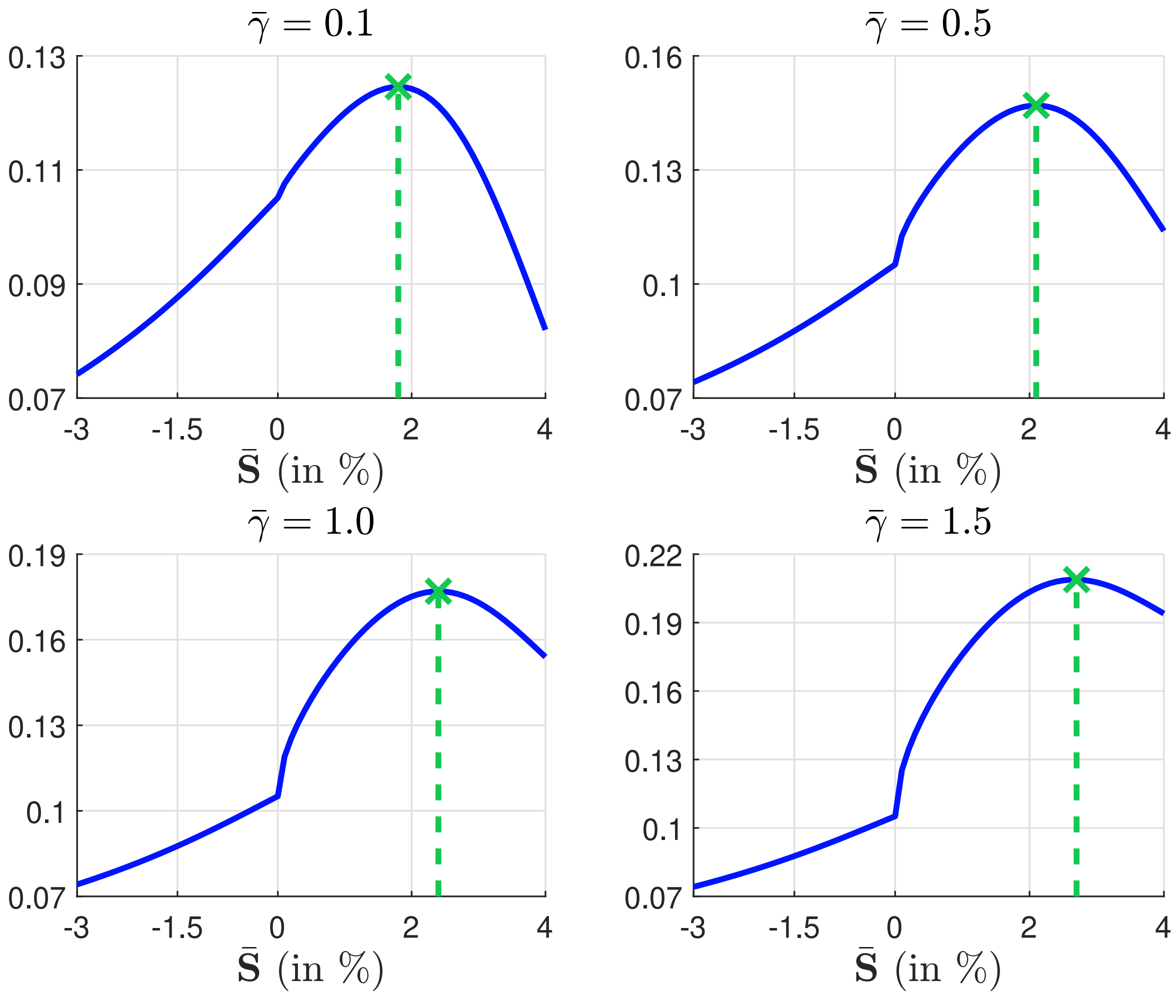
Figure 54: Optimal portfolio for type-M investors when $\zeta(s) = s$ (Example #5)



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Figure 55: Optimal portfolio for type-M investors when $\zeta(s) = 0.2\sqrt{\max(s, 0)}$



The Pedersen-Fitzgibbons-Pomorski model

The ESG-SR frontier

Table 41: Optimal portfolios (Example #5)

Statistics	Type-U	Type-A	Type-M					
			$\zeta(s) = s$			$\zeta(s) = 0.2\sqrt{\max(s,0)}$		
$\bar{\gamma}$			0.500	1.000	1.500	0.500	1.000	1.500
$\mathcal{S}(w^\star)$	0.017	0.017	0.023	0.028	0.034	0.021	0.024	0.027
$\sigma(w^\star)$	0.139	0.100	0.682	0.329	0.203	0.687	0.339	0.221
$\mathrm{SR}(w^\star \mid r)$	0.345	0.345	0.341	0.329	0.305	0.343	0.339	0.332
$w_1^\star$	0.524	0.378	3.028	1.623	1.090	2.900	1.542	1.072
$w_2^\star$	0.289	0.208	1.786	1.009	0.718	1.673	0.919	0.660
$w_3^\star$	0.120	0.086	0.383	0.073	−0.056	0.464	0.169	0.065
$w_4^\star$	0.067	0.048	−0.012	−0.144	−0.178	0.106	−0.035	−0.079
$w_r^\star$	0.000	0.280	−4.184	−1.562	−0.574	−4.143	−1.596	−0.718

The Pedersen-Fitzgibbons-Pomorski model

Impact on asset returns

- If $\omega^U = 1$ and $\omega^A = \omega^M = 0$, then unconditional expected returns are given by the CAPM:

$$\mathbb{E}[R_i] - r = \beta_i (\mathbb{E}[R_m] - r)$$

but conditional expected returns depend on the ESG scores:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \beta_i (\mathbb{E}[R_m] - r) + \theta \frac{\boldsymbol{\mathcal{S}}_i - \boldsymbol{\mathcal{S}}_m}{P_i}$$

where P_i is the asset price of asset i

- If $\omega^A = 1$ and $\omega^U = \omega^M = 0$, then the informational value of ESG scores is fully incorporated into asset prices, and we have:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \tilde{\beta}_i (\mathbb{E}[R_m \mid \boldsymbol{\mathcal{S}}] - r)$$

where $\tilde{\beta}_i$ is the ESG-adjusted beta coefficient

- If $\omega^M = 1$ and $\omega^U = \omega^A = 0$, then the conditional expected return is given by:

$$\mathbb{E}[R_i \mid \boldsymbol{\mathcal{S}}] - r = \tilde{\beta}_i (\mathbb{E}[R_m \mid \boldsymbol{\mathcal{S}}] - r) + \lambda_2 (\boldsymbol{\mathcal{S}}_i - \boldsymbol{\mathcal{S}}_m)$$

The Pedersen-Fitzgibbons-Pomorski model

Impact on asset returns

"If all types of investors exist, then several things can happen. If a security has a higher ESG score, then, everything else equal, its expected return can be higher or lower. A higher ESG score increases the demand for the stock from type-M investors, leading to a higher price and, therefore, a lower required return [...] Companies with poor ESG scores that are down-weighted by type-M investors will have lower prices and higher cost of capital. [...] Furthermore, the force that can increase the expected return is that the higher ESG could be a favorable signal of firm fundamentals, and if many type-U investors ignore this, the fundamental signal perhaps would not be fully reflected in the price [...] A future increase in ESG investing would lead to higher prices for high-ESG stocks [...]. If these flows are unexpected (or not fully captured in the price for other reasons), then high-ESG stocks would experience a return boost during the period of this repricing of ESG. If these flows are expected, then expected returns should not be affected."

(Pedersen et al., 2021)

What is the performance of ESG investing?

According to Coqueret (2022), we can classify the academic studies into four categories:

1. ESG improves performance
2. ESG does not impact performance
3. ESG is financially detrimental
4. The relationship between ESG and performance depends on many factors

What is the performance of ESG investing?

According to Friede *et al.* (2015), the first category dominates the other categories:

> *"[...] The results show that the business case for ESG investing is empirically very well founded. Roughly 90% of studies find a nonnegative ESG – CFP relation. More importantly, the large majority of studies reports positive findings. We highlight that the positive ESG impact on CFP appears stable over time. Promising results are obtained when differentiating for portfolio and non-portfolio studies, regions, and young asset classes for ESG investing such as emerging markets, corporate bonds, and green real estate."*

$\Rightarrow$ Many dimensions of CFP (cost of capital, G pillar, proxy variables, etc.)

Relationship between ESG and performance in equity markets

We can also find many studies, whose conclusion is more neutral or negative: Barnett and Salomon (2006), Fabozzi *et al.* (2008), Hong and Kacperczyk (2009), Johnson *et al.* (2009), Capelle-Blancard and Monjon (2014), Matos (2020), etc.

$\Rightarrow$ Sin stocks

Mixed results

What is the performance of ESG investing?

- Generally, academic studies that analyze the relationship between ESG and performance are based on long-term historical data, typically the last 20 years or the last 30 years.
- Two issues:
  1. ESG investing was marginal 15+ years ago
  2. ESG data are not robust or relevant before 2010
- The relationship between ESG and performance is dynamic
- Sometimes, ESG may create performance, but sometimes not

Simulated results

Sorted portfolios

Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date t, we rank the stocks according to their Amundi **ESG** z-score $s_{i,t}$
- We form the five quintile portfolios Q_i for $i = 1, \ldots, 5$
- The portfolio Q_i is invested during the period $]t, t+1]$:
  - Q_1 corresponds to the best-in-class portfolio (best scores)
  - Q_5 corresponds to the worst-in-class portfolio (worst scores)
- Quarterly rebalancing
- Universe: MSCI World Index
- Equally-weighted and sector-neutral portfolio (and region-neutral for the world universe)

Simulated results
Sorted portfolios

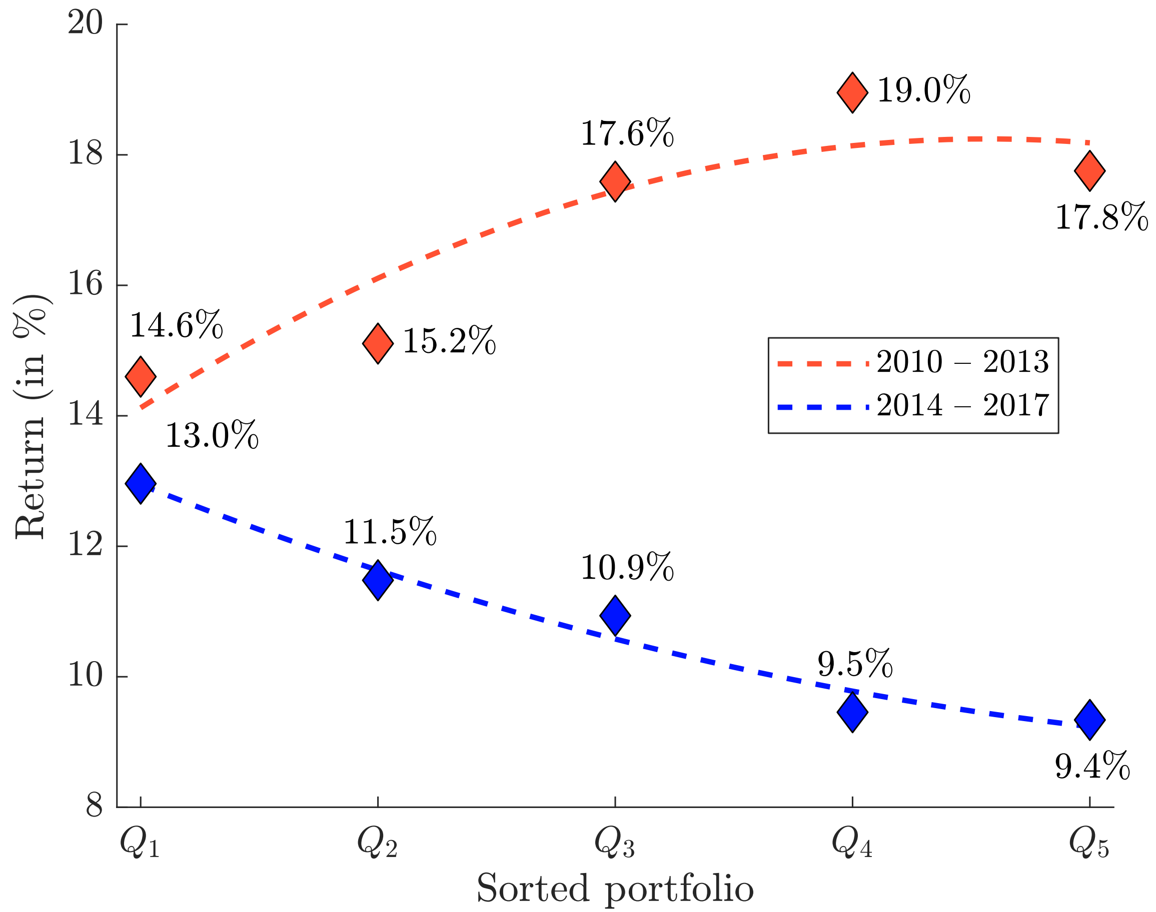
Table 42: An illustrative example

Asset	$\mathcal{S}_i$	Rank	Q_i	Weight
#1	−0.3	6	Q_3	+50%
#2	0.2	5	Q_3	+50%
#3	−1.0	7	Q_4	+50%
#4	1.5	3	Q_2	+50%
#5	−2.9	10	Q_5	+50%
#6	0.8	4	Q_2	+50%
#7	−1.4	8	Q_4	+50%
#8	2.3	2	Q_1	+50%
#9	2.8	1	Q_1	+50%
#10	−2.2	9	Q_5	+50%

Simulated results

Sorted portfolios

Figure 56: Annualized return of ESG-sorted portfolios (MSCI North America)

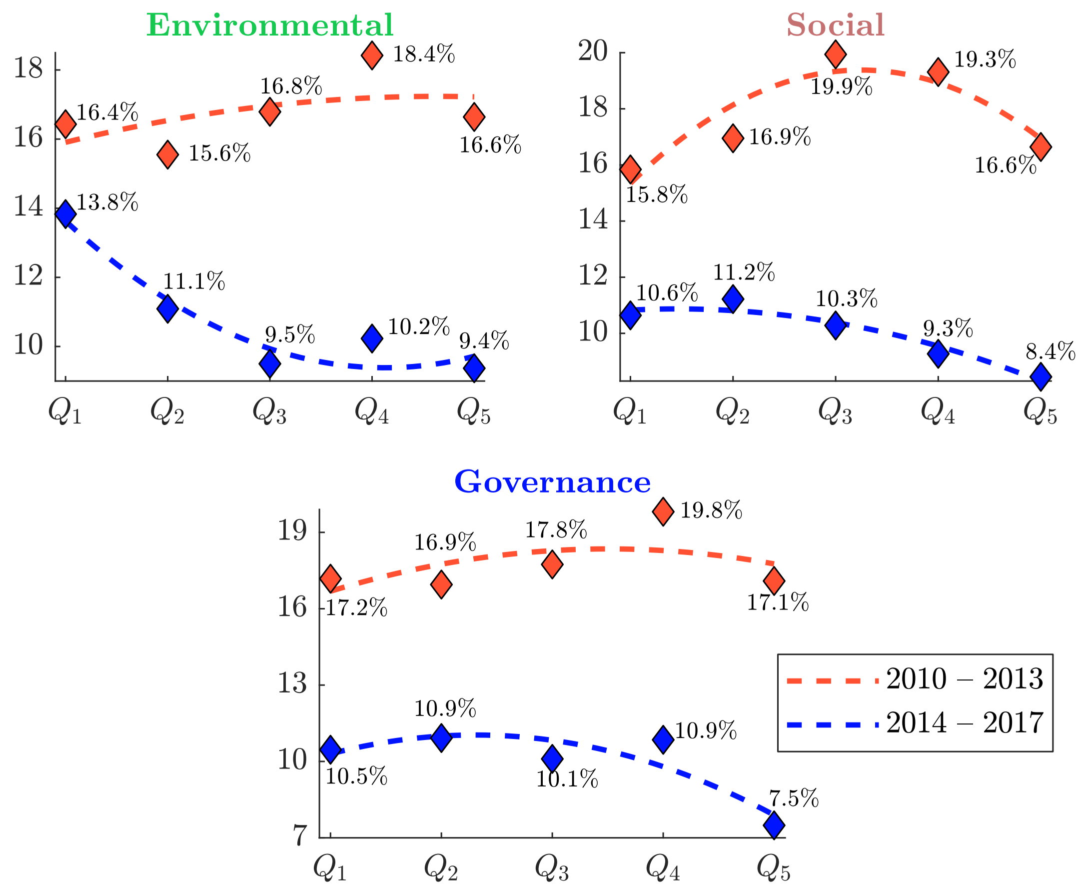


Source: Bennani *et al* (2018).

Simulated results

Sorted portfolios

Figure 57: Annualized return of ESG-sorted portfolios (MSCI North America)

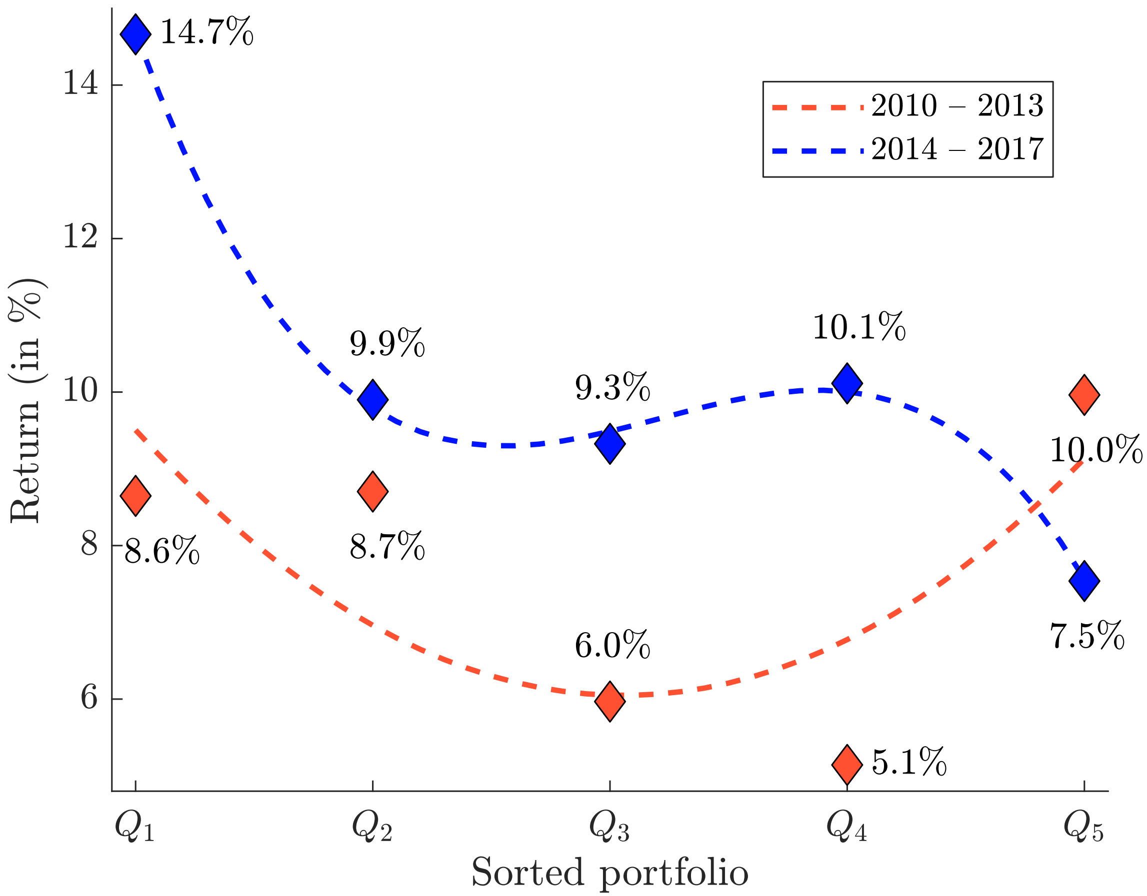


Source: Bennani *et al* (2018).

Simulated results

Sorted portfolios

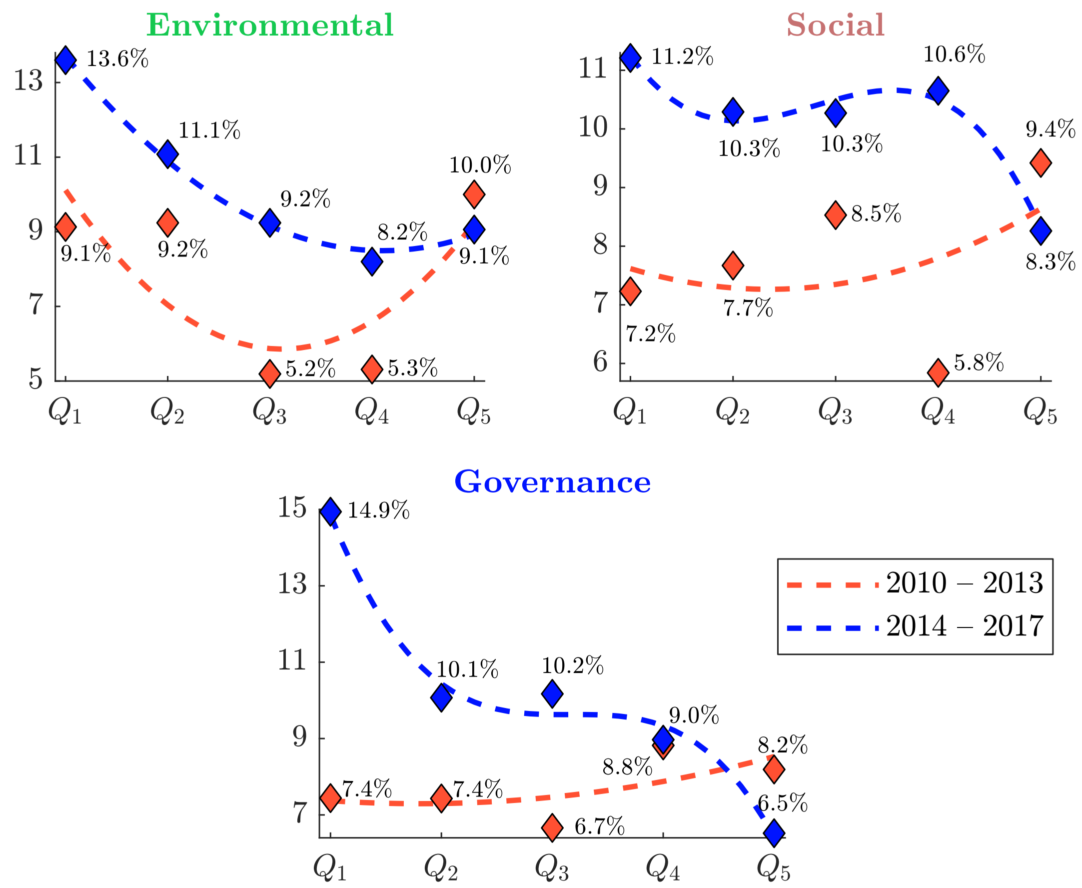
Figure 58: Annualized return of ESG-sorted portfolios (MSCI EMU)



Source: Bennani *et al* (2018).

Simulated results
Sorted portfolios

Figure 59: Annualized return of ESG-sorted portfolios (MSCI EMU)



Source: Bennani *et al* (2018).

Simulated results

Sorted portfolios

Table 43: Impact of ESG screening on sorted portfolio returns (2010 – 2017)

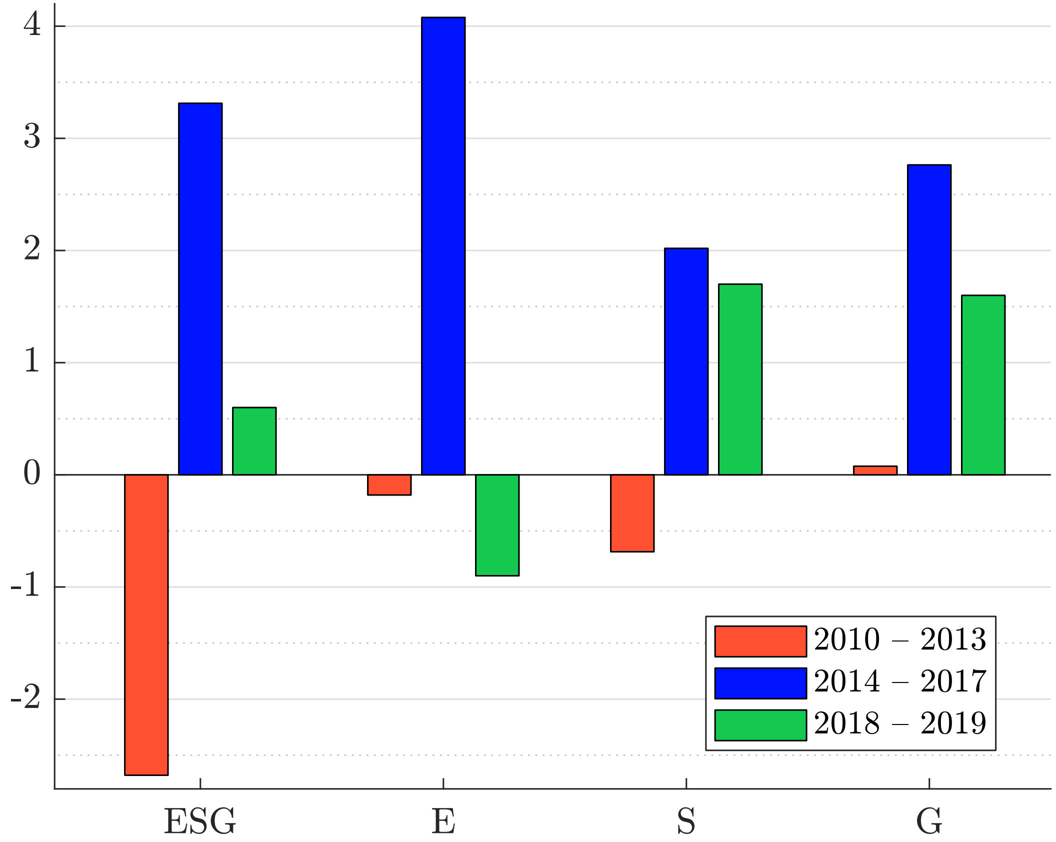
Period	Pillar	North America	EMU	Europe-ex-EMU	Japan	World
2010 – 2013	ESG	−−	−	**0**	+	**0**
	E	−	**0**	+	−	**0**
	S	−	−	**0**	−	−
	G	−	**0**	+	**0**	+
2014 – 2017	ESG	++	++	**0**	−	+
	E	++	++	−	+	++
	S	+	+	**0**	**0**	+
	G	+	++	**0**	+	++

Source: Bennani *et al* (2018).

Simulated results

Sorted portfolios

Figure 60: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI North America)

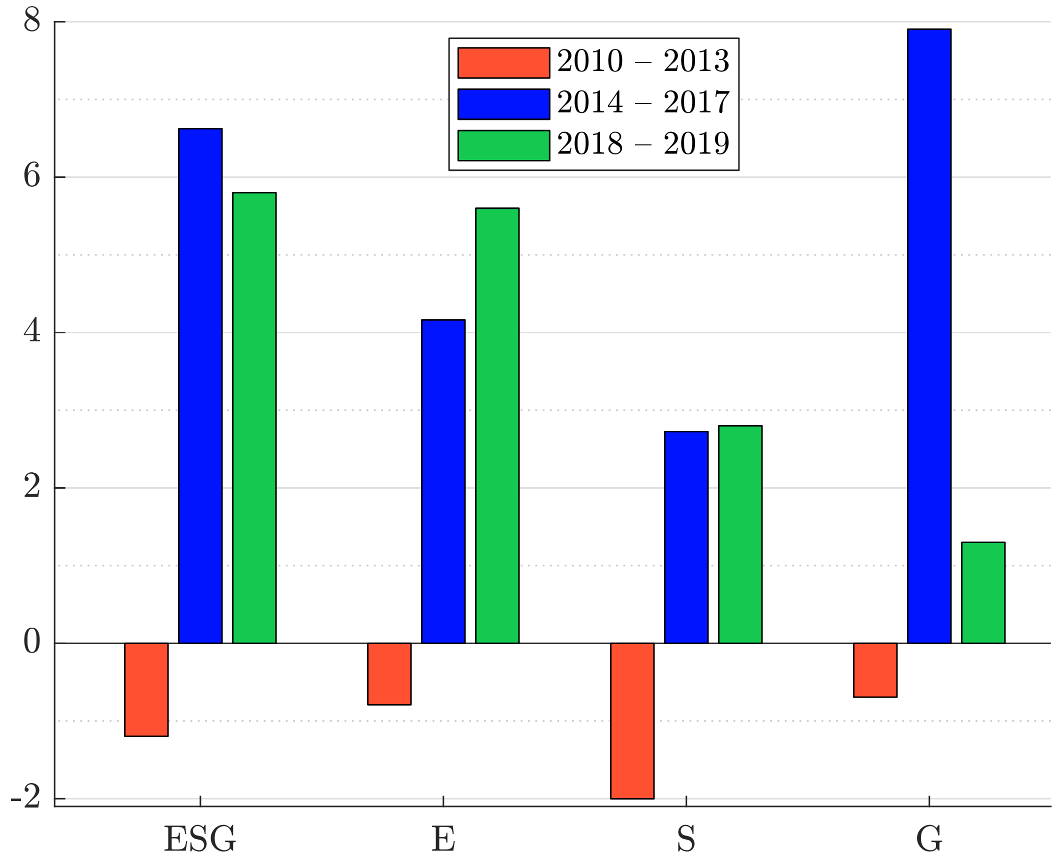


Source: Drei *et al* (2019).

Simulated results

Sorted portfolios

Figure 61: Annualized return of long/short $Q_1 - Q_5$ sorted portfolios (MSCI EMU)



Source: Drei *et al* (2019).

Simulated results

Sorted portfolios

The impact of investment flows

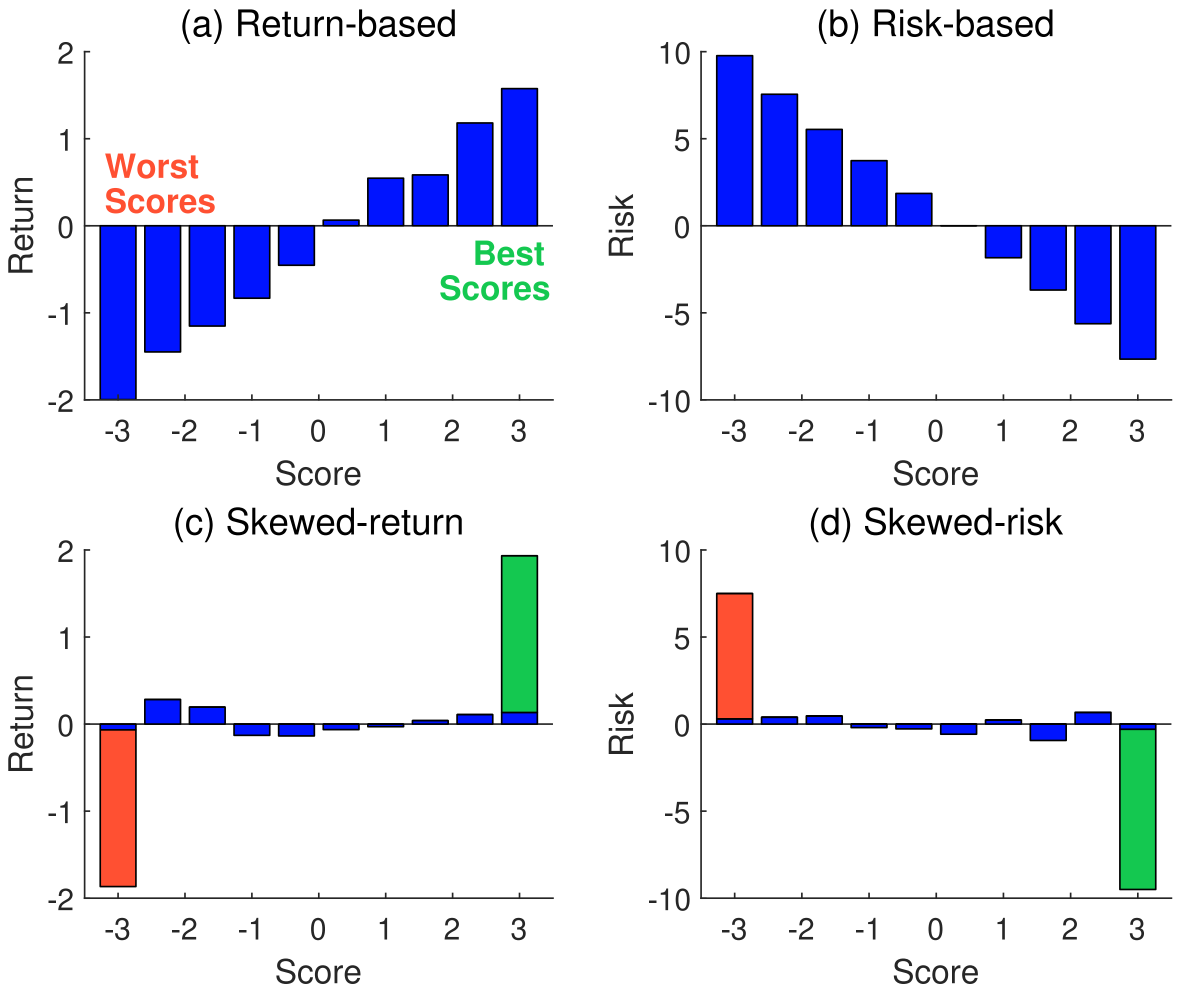
- The 2014 break
  - November 2013: Responsible Investment and the Norwegian Government Pension Fund Global (2013 Strategy Council)
  - Strong mobilization of the largest institutional European investors: NBIM, APG, PGGM, ERAFP, FRR, etc.
  - They are massively invested in European stocks and America stocks:

    NBIM $\succ$ CalPERS + CalSTRS + NYSCRF for U.S. stocks
- The 2018-2019 period
  - Implication of U.S. investors continues to be weak
  - Strong mobilization of medium (or tier two) institutional European investors, that have a low exposure on American stocks
  - Mobilization of European investors is not sufficient

Simulated results

Sorted portfolios

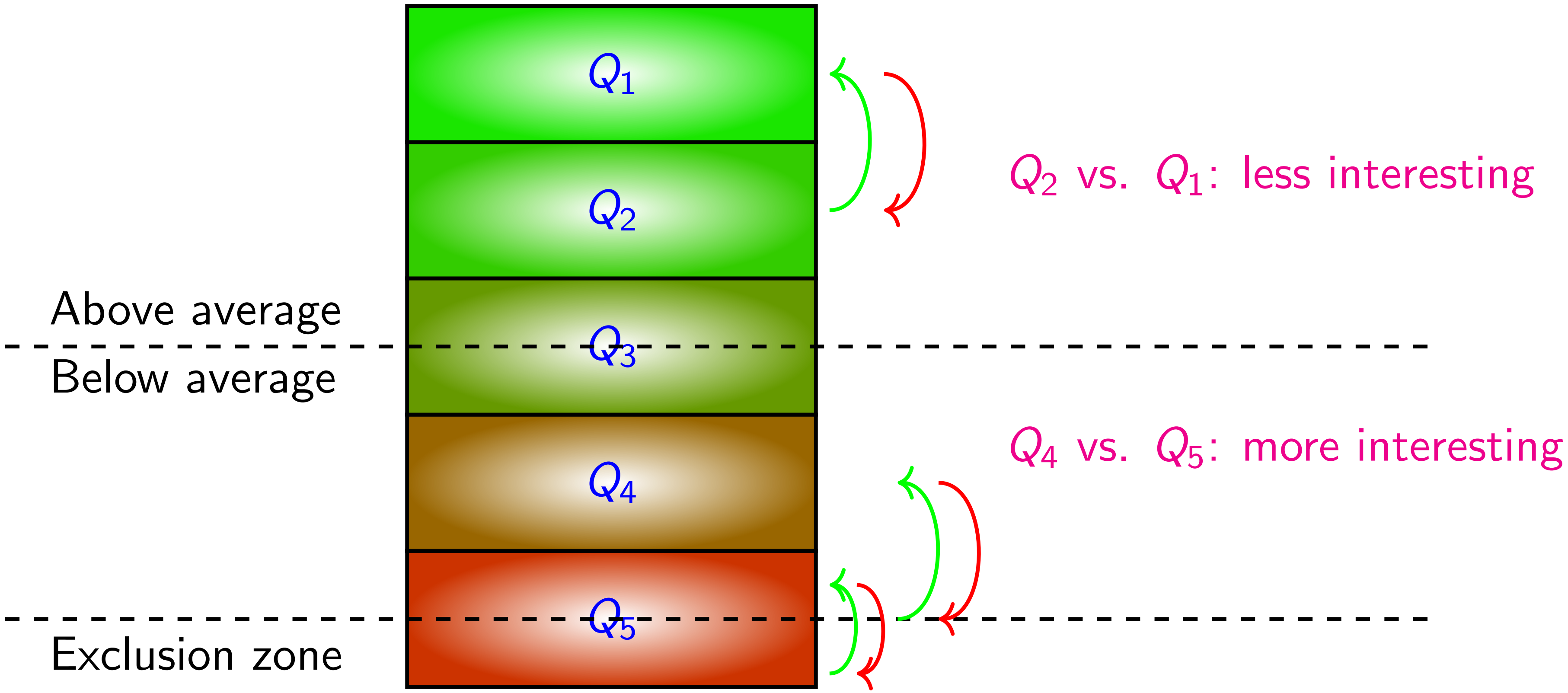
Figure 62: The monotonous assumption of the ESG-performance relationship



Simulated results

Sorted portfolios

Figure 63: How to play ESG momentum?



Simulated results

Optimized portfolios

- We note b the benchmark, $\boldsymbol{\mathcal{S}}$ the vector of ESG scores and Σ the covariance matrix
- We consider the following optimization problem:

$$w^{\star}(\gamma) = \arg\min \frac{1}{2}\sigma^{2}(w \mid b) - \gamma \boldsymbol{\mathcal{S}}(w \mid b)$$

where $\sigma^{2}(w \mid b) = (w - b)^{\top} \Sigma (w - b)$ and $\boldsymbol{\mathcal{S}}(w \mid b)$ are the ex-ante tracking error variance and the ESG excess score of portfolio w with respect to the benchmark b

- Since we have:

$$\boldsymbol{\mathcal{S}}(w \mid b) = (w - b)^{\top} \boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}}(w) - \boldsymbol{\mathcal{S}}(b)$$

we obtain the following optimization function:

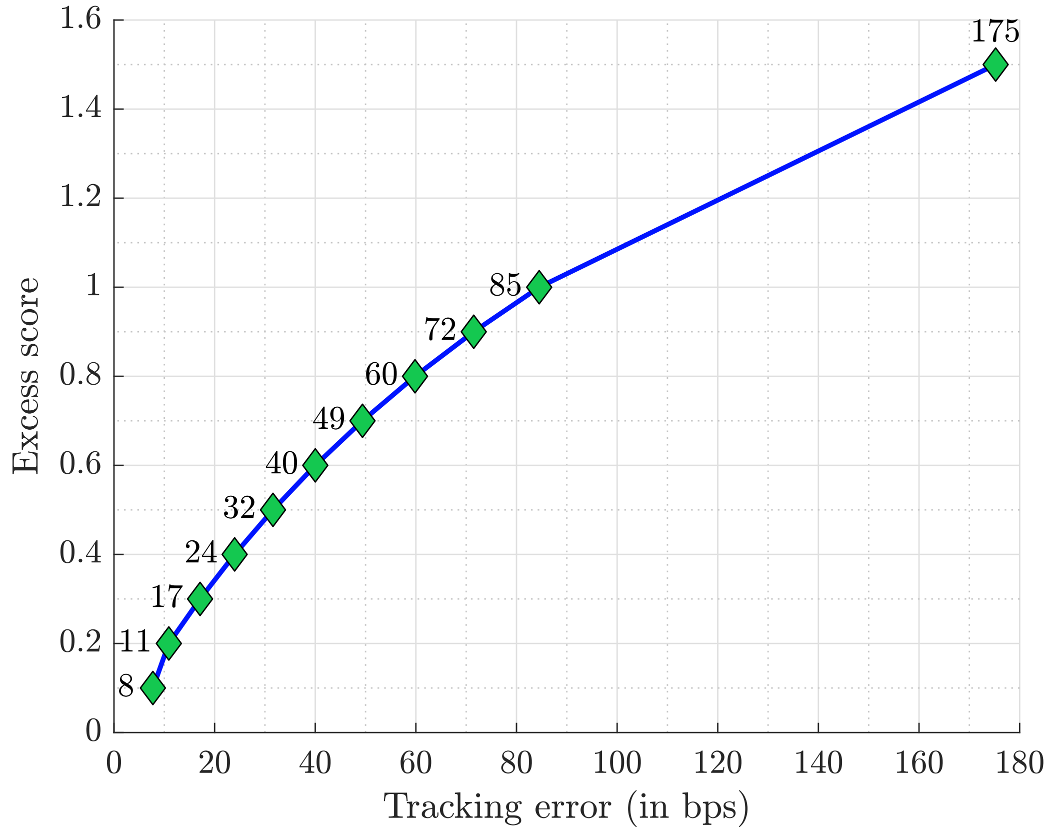
$$w^{\star}(\gamma) = \arg\min \frac{1}{2} w^{\top} \Sigma w - w^{\top}(\gamma \boldsymbol{\mathcal{S}} + \Sigma b)$$

- The QP form is given by $Q = \Sigma$ and $R = \gamma \boldsymbol{\mathcal{S}} + \Sigma b$

Simulated results

Optimized portfolios

Figure 64: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

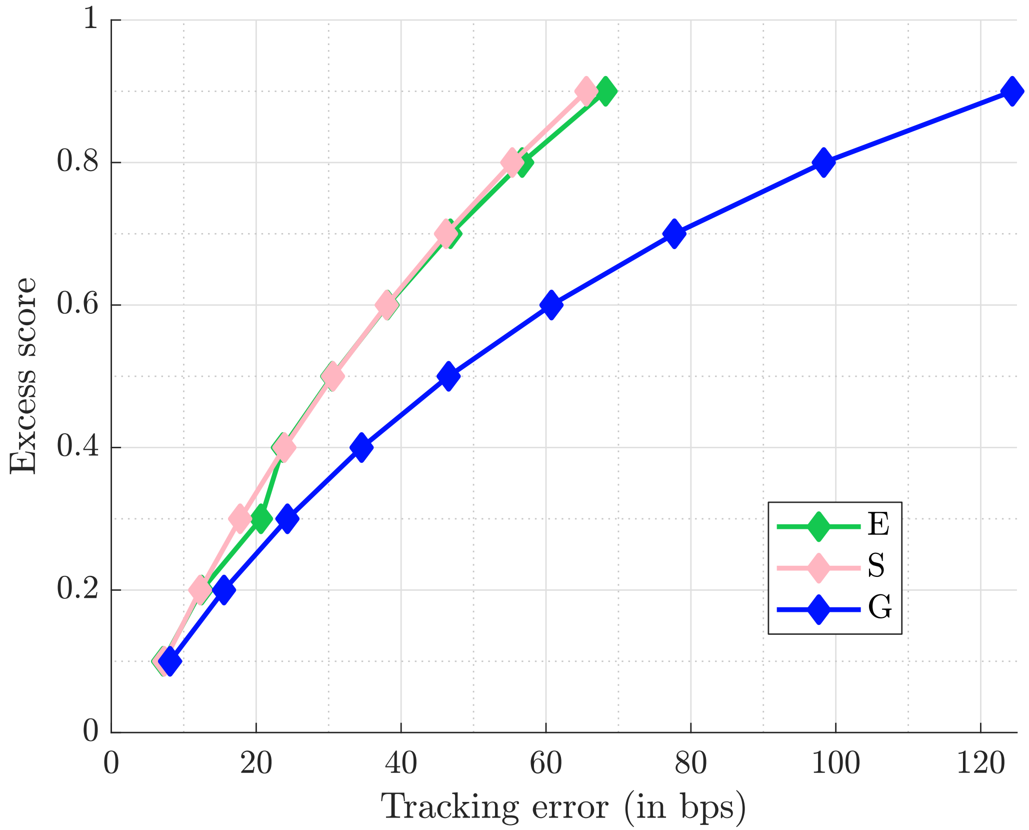


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

Figure 65: Efficient frontier of ESG-optimized portfolios (MSCI World, 2010-2017, individual pillars)

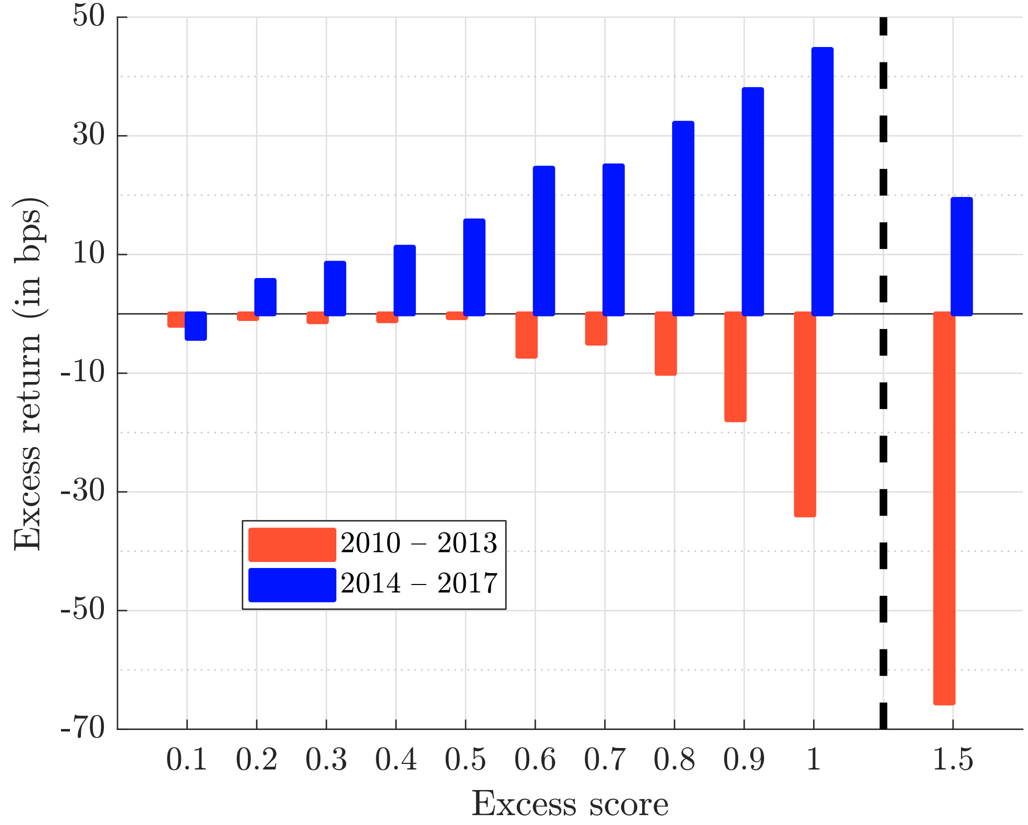


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

Figure 66: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2017, global score)

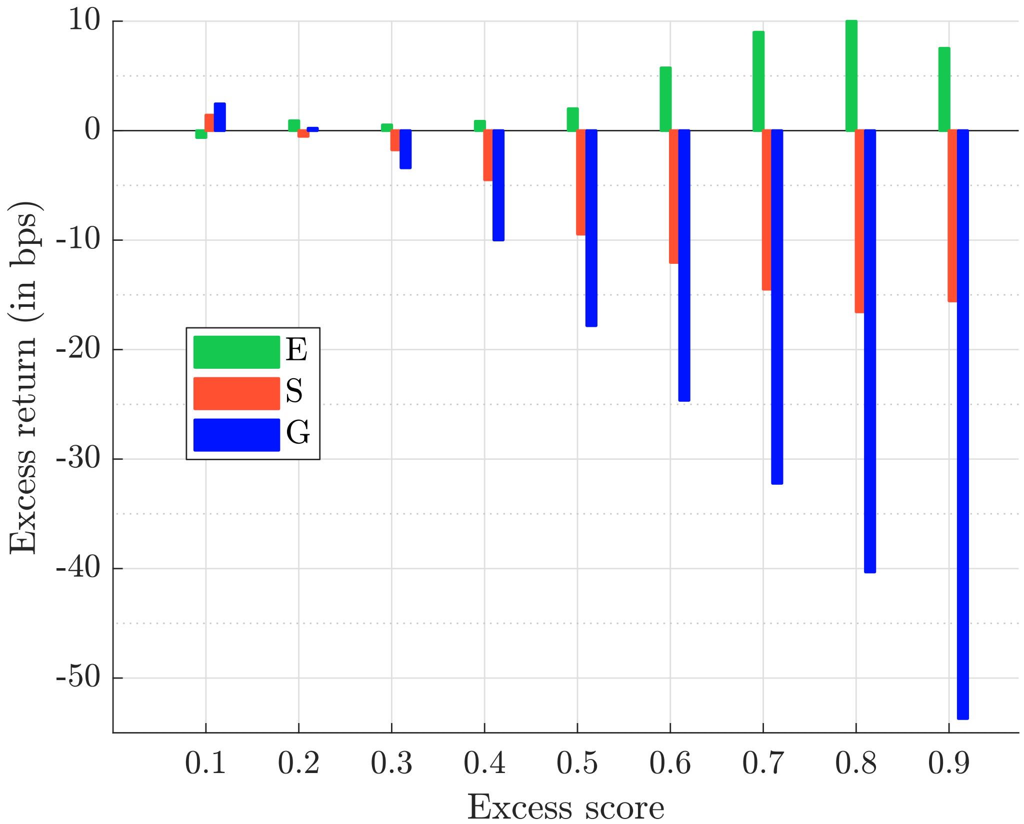


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

Figure 67: Annualized excess return of ESG-optimized portfolios (MSCI World, 2010-2013, individual pillars)

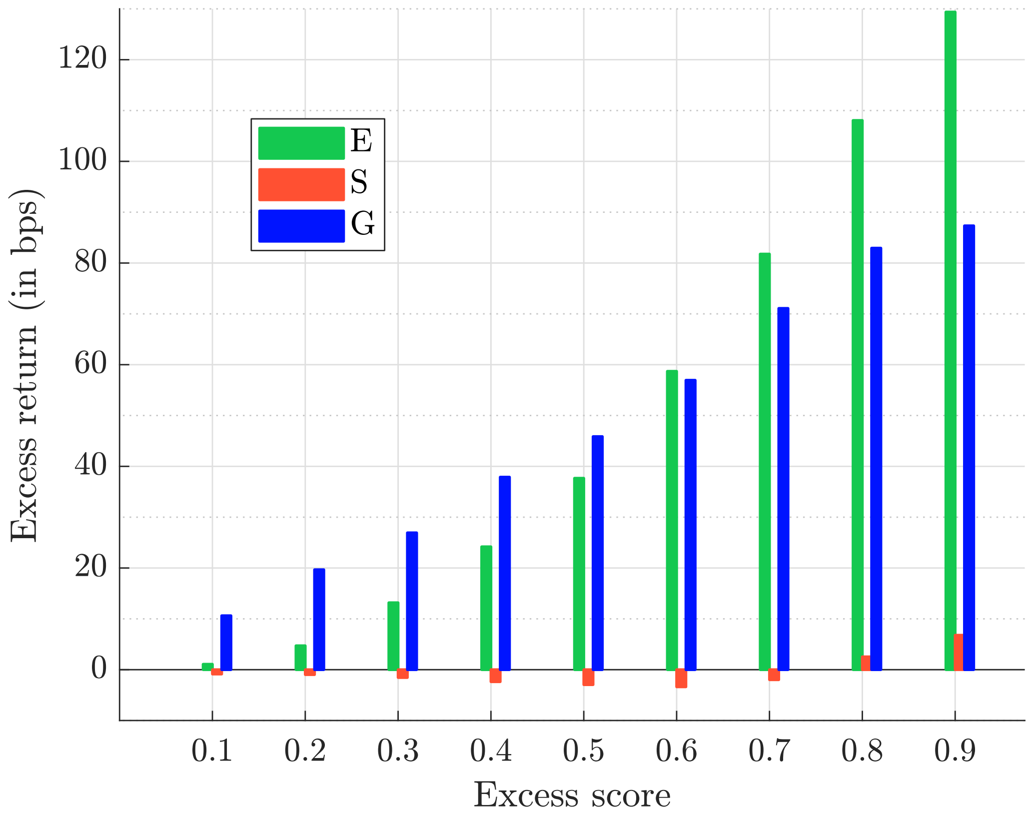


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

Figure 68: Annualized excess return of ESG-optimized portfolios (MSCI World, 2014-2017, individual pillars)

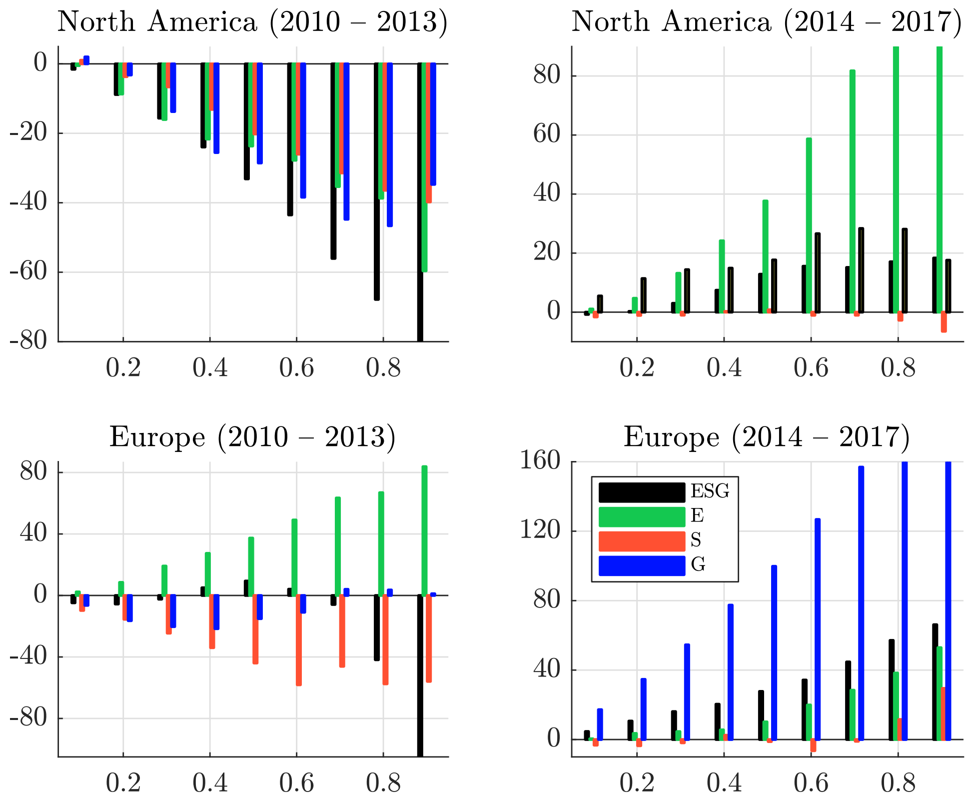


Source: Bennani *et al* (2018).

Simulated results

Optimized portfolios

Figure 69: Annualized excess return in bps of ESG-optimized portfolios (MSCI North America and EMU, 2010-2017



Source: Bennani *et al* (2018).

Single-factor model

Regression model

The single-factor model is:

$$R_{i,t} = \alpha_{i,j} + \beta_{i,j}\mathcal{F}_{j,t} + \varepsilon_{i,t}$$

where:

- $R_{i,t}$ is the return of stock i at time t
- $\mathcal{F}_{j,t}$ is the value of the j^{th} common risk factor at time t (market, size, value, momentum, low-volatility, quality or ESG)
- $\varepsilon_{i,t}$ is the idiosyncratic risk

The average proportion of the return variance explained by the common factor is given by:

$$\bar{\mathfrak{R}}_j^2 = \frac{1}{n}\sum_{i=1}^{n}\mathfrak{R}_{i,j}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \frac{\operatorname{var}\left(\varepsilon_{i,t}\right)}{\operatorname{var}\left(R_{i,t}\right)}\right)$$

Single-factor model

Table 44: Results of cross-section regression with long-only risk factors (single-factor linear regression model, average $\mathfrak{R}^2$)

Factor	North America		Eurozone	
	2010 – 2013	2014 – 2019	2010 – 2013	2014 – 2019
Market	40.8%	28.6%	42.8%	36.3%
Size	39.3%	26.1%	37.1%	23.3%
Value	38.9%	26.7%	41.6%	33.6%
Momentum	39.6%	26.3%	40.8%	34.1%
Low-volatility	35.8%	25.1%	38.7%	33.4%
Quality	39.1%	26.6%	42.4%	34.6%
ESG	40.1%	27.4%	42.6%	35.3%

Source: Roncalli (2020).

- Specific risk has increased during the period 2014 – 2019
- Since 2014, we find that:
  - ESG $\succ$ Value $\succ$ Quality $\succ$ Momentum $\succ$ ... (North America)
  - ESG $\succ$ Quality $\succ$ Momentum $\succ$ Value $\succ$ ... (Eurozone)

Multi-factor model

Regression model

We have:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{m} \beta_{i,j} \mathcal{F}_{j,t} + \varepsilon_{i,t}$$

where m is the number of risk factors

- 1F = market
- 5F = size + value + momentum + low-volatility + quality
- 6F = 5F + ESG

Multi-factor model

Table 45: Results of cross-section regression with long-only risk factors (multi-factor linear regression model, average $\mathfrak{R}^2$)

Model	North America		Eurozone	
	2010 – 2013	2014 – 2019	2010 – 2013	2014 – 2019
CAPM	40.8%	28.6%	42.8%	36.3%
5F model	46.1%	38.4%	49.5%	45.0%
6F model (5F + ESG)	46.7%	39.7%	50.1%	45.8%

Source: Roncalli (2020).

***p-value statistic for the MSCI Index (time-series, 2014 – 2019):

- 6F = ~~Size~~, Value, Momentum, Low-volatility, Quality, ~~ESG~~ (North America)
- 6F = Size, Value, Momentum, ~~Low-volatility~~, Quality, ESG (Eurozone)

Factor selection

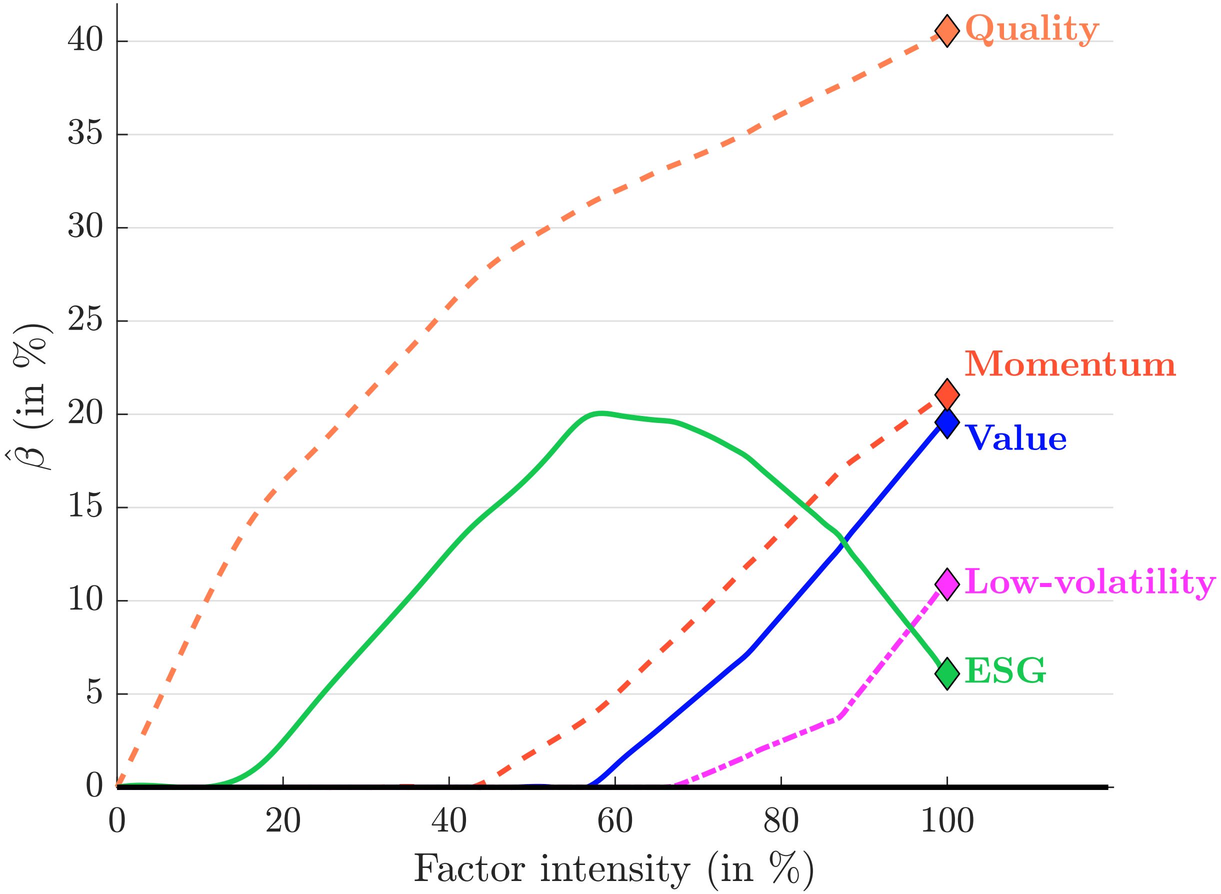
- We use a lasso penalized regression is used in place of the traditional least squares regression:

$$\left\{\hat{\alpha}_i, \hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,m}\right\} = \arg\min \left\{\frac{1}{2}\operatorname{var}\left(\varepsilon_{i,t}\right) + \lambda \left\|\beta_i\right\|_1\right\}$$

- Low-factor intensity ($\lambda \approx \infty$) $\Rightarrow$ we determine which risk factor is the most important
- When the factor intensity reaches 100% ($\lambda = 0$), we obtain the same results calculated previously with the linear regression

Factor selection

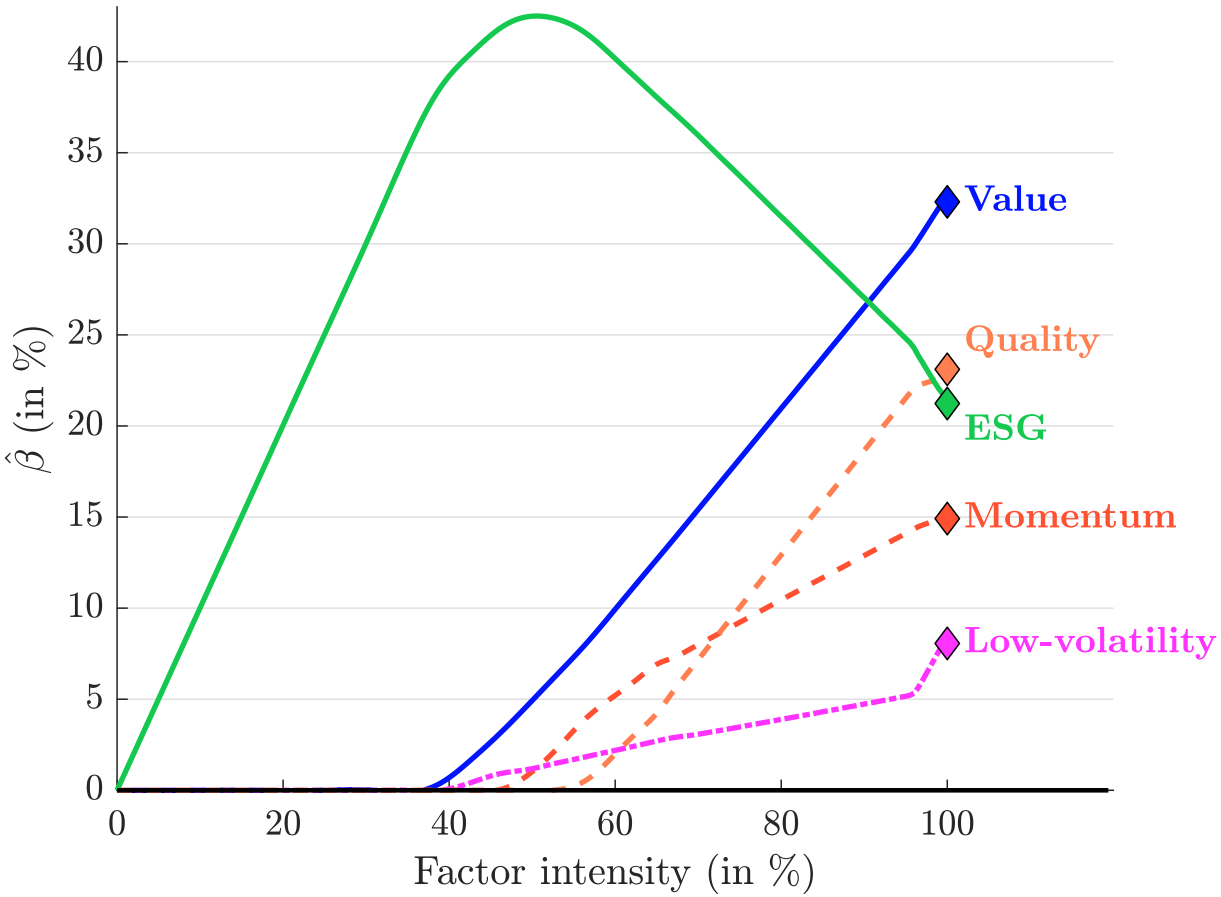
Figure 70: Factor picking (MSCI North America, 2014-2019, global score)



Source: Roncalli (2020).

Factor selection

Figure 71: Factor picking (MSCI EMU, 2014-2019, global score)



Source: Roncalli (2020).

What is the difference between alpha and beta?

α or β?

"[...] When an alpha strategy is massively invested, it has an enough impact on the structure of asset prices to become a risk factor.
[...] Indeed, an alpha strategy becomes a common market risk factor once it represents a significant part of investment portfolios and explains the cross-section dispersion of asset returns" (Roncalli, 2020)

- ESG remains an alpha strategy in North America
- ESG becomes a beta strategy (or a risk factor) in Europe
- Forward looking, ESG will be a beta strategy in North America

Equity indices

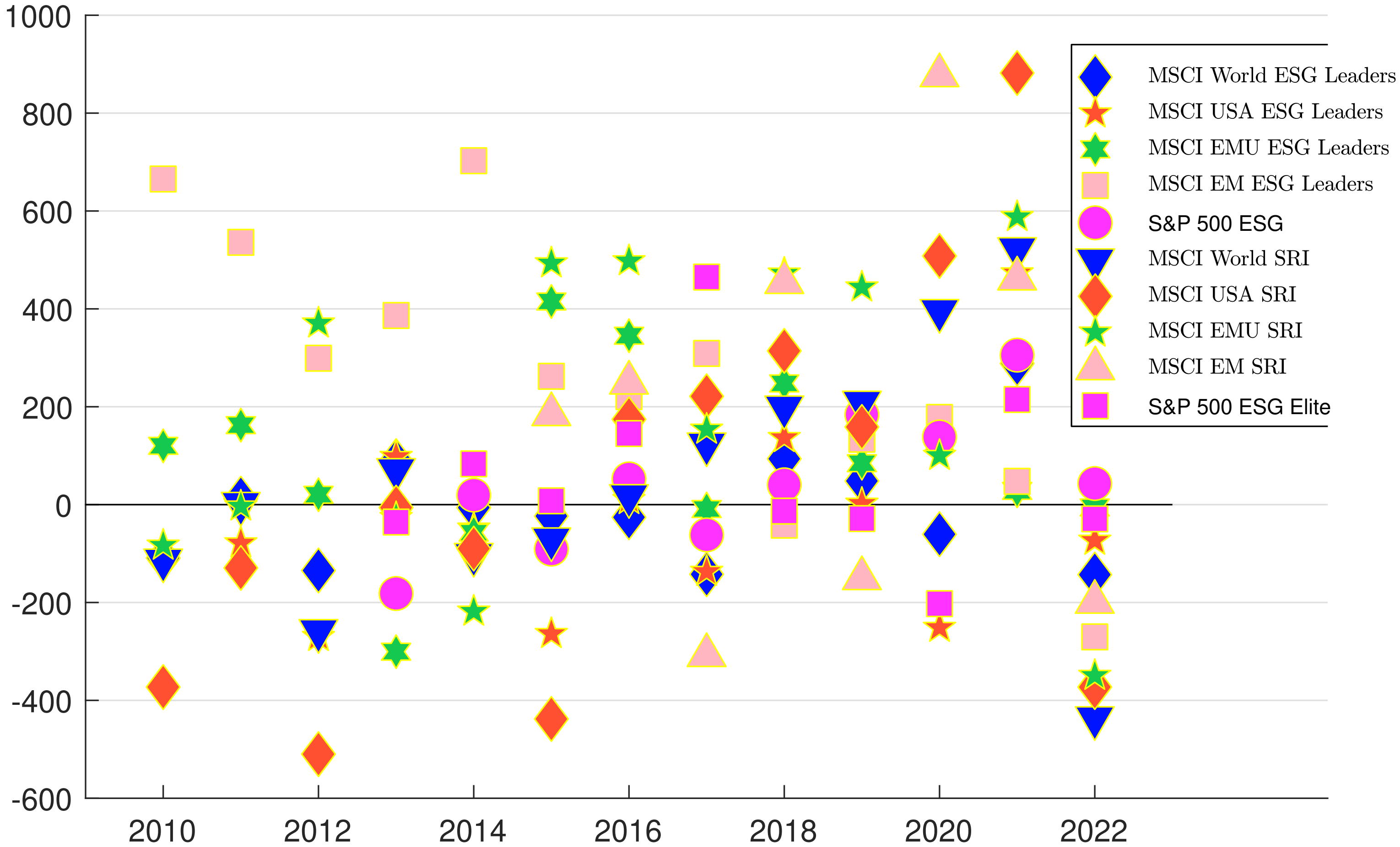
Table 46: Performance of ESG indexes (MSCI World, 2010 – 2022)

Year	Return (in %)			Alpha (in bps)	
	CW	ESG	SRI	ESG	SRI
2010	11.8	10.7	10.6	−109	−114
2011	−5.5	−5.4	−5.5	12	2
2012	15.8	14.5	13.2	−135	−258
2013	26.7	27.6	27.4	89	71
2014	4.9	4.9	3.9	−6	−102
2015	−0.9	−1.1	−1.6	−23	−71
2016	7.5	7.3	7.7	−26	18
2017	22.4	21.0	23.6	−142	124
2018	−8.7	−7.8	−6.7	94	199
2019	27.7	28.2	29.8	48	209
2020	15.9	15.3	19.9	−61	396
2021	21.8	24.7	27.0	288	523
2022	−18.1	−19.6	−22.5	−143	−436
3Y	4.9	5.0	5.7	2	73
5Y	6.1	6.4	7.4	31	125
7Y	8.5	8.5	9.6	1	110
10Y	8.9	8.9	9.5	5	64

Source: MSCI, Factset & Author's calculation.

Equity indices

Figure 72: Alpha return of several ESG equity indexes (in bps)



Source: MSCI, Factset & Author's calculation.

Bond markets $\neq$ stock markets

Stocks

- ESG scoring is incorporated in portfolio management
- ESG = long-term business risk
  $\Rightarrow$ strongly impacts the equity
- Portfolio integration
- Managing the business risk

Bonds

- ESG integration is generally limited to exclusions
- ESG lowly impacts the debt
- Portfolio completion
- Fixed income = impact investing
- Development of pure play ESG securities (green and social bonds)

$\Rightarrow$ Stock holders are more ESG sensitive than bond holders because of the capital structure

Bond markets $\neq$ stock markets

ESG investment flows affect asset pricing differently

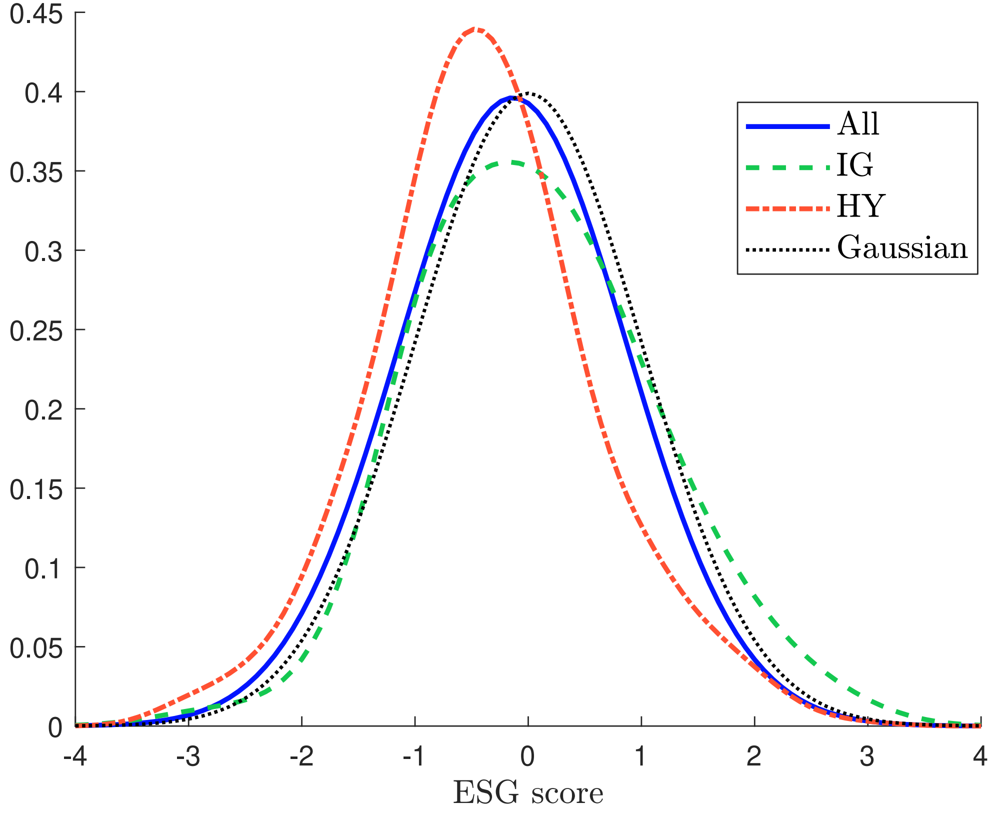
- Impact on carry (coupon effect)?
- Impact on price dynamics (credit spread/mark-to-market effect)?
- Buy-and-hold portfolios $\neq$ managed portfolios

The distinction between IG and HY bonds

- ESG and credit ratings are correlated
- There are more worst-in-class issuers in the HY universe, and best-in-class issuers in the IG universe
- Non-neutrality of the bond universe (bonds $\neq$ stocks)

Bond markets ≠ stock markets

Figure 73: Probability density function of ESG scores



Source: Ben Slimane *et al.* (2019).

- The average z-score for IG bonds is positive
- The average z-score for HY bonds is negative

Simulated results

Sorted portfolios

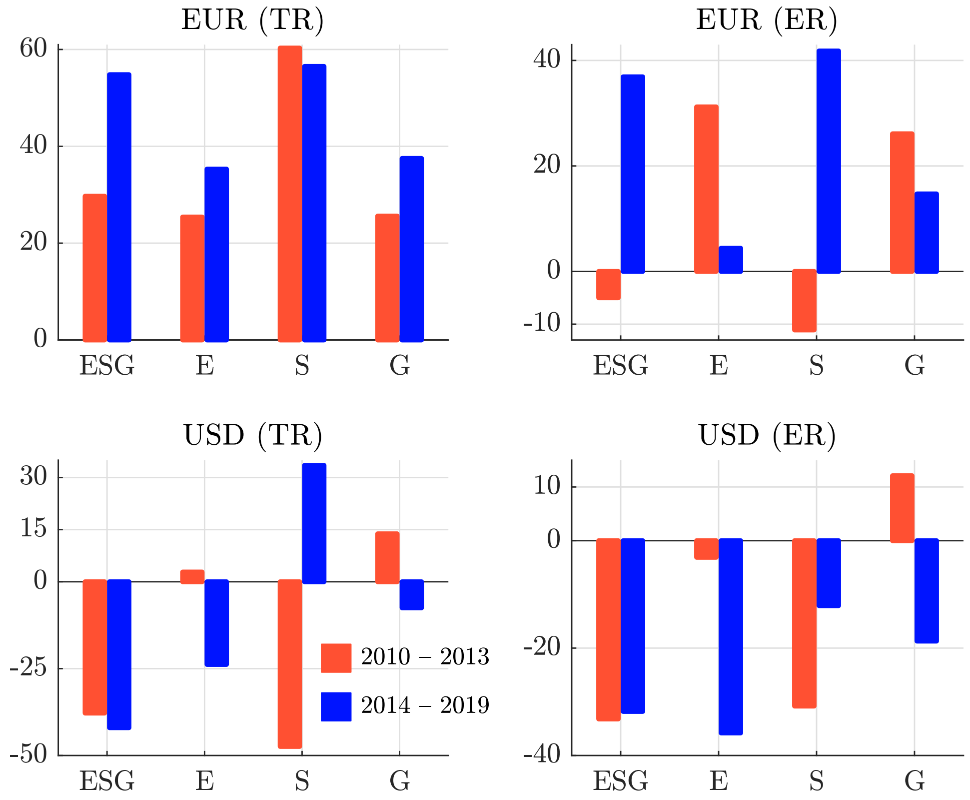
Sorted-portfolio approach

- Sorted-based approach of Fama-French (1992)
- At each rebalancing date t, we rank the bonds according to their Amundi **ESG** z-score
- We form the five quintile portfolios Q_i for $i = 1, \ldots, 5$
- The portfolio Q_i is invested during the period $]t, t+1]$:
  - Q_1 corresponds to the best-in-class portfolio (best scores)
  - Q_5 corresponds to the worst-in-class portfolio (worst scores)
- Monthly rebalancing
- Universe: ICE (BofAML) Large Cap IG EUR Corporate Bond
- Sector-weighted and sector-neutral portfolio
- Within a sector, bonds are equally-weighted

Simulated results

Sorted portfolios

Figure 74: Annualized return in bps of the long short $Q_1 - Q_5$ strategy (IG, 2010 – 2019)



Source: Ben Slimane *et al.* (2019).

Simulated results

Sorted portfolios

Table 47: Carry statistics (in bps)

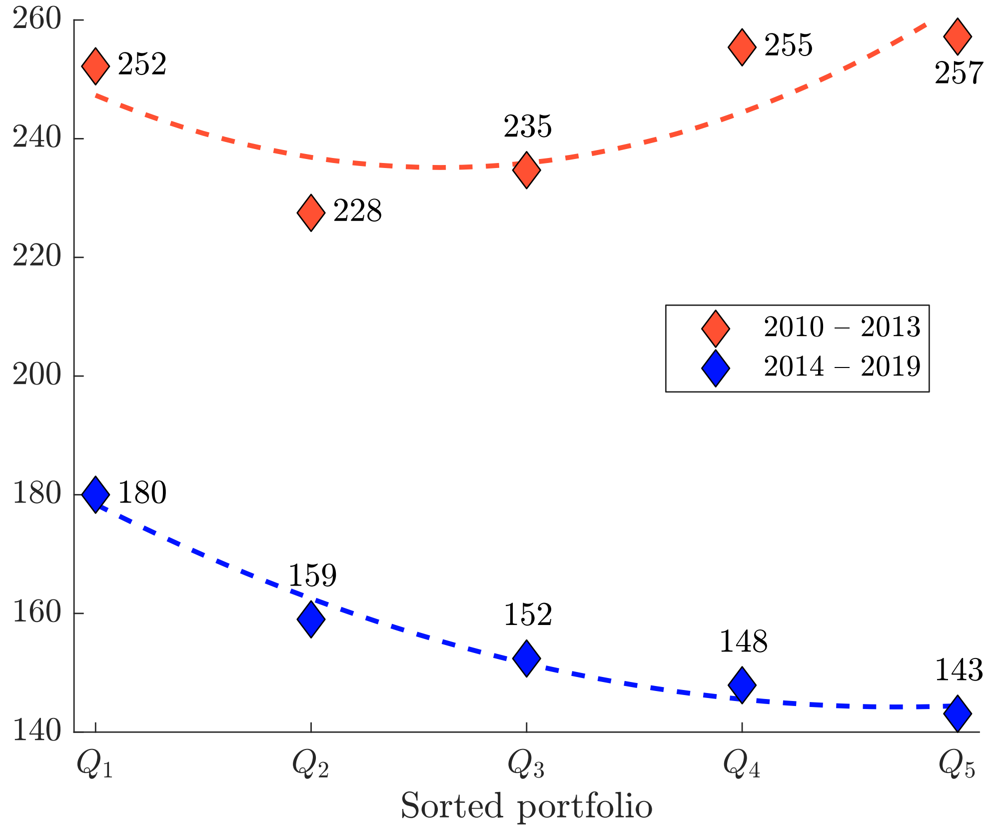
Period	Q_1	Q_5	$Q_1 - Q_5$
2010 – 2013	175	192	-17
2014 – 2019	113	128	-15

Source: Ben Slimane *et al.* (2019).

Simulated results

Sorted portfolios

Figure 75: Annualized credit return in bps of ESG sorted portfolios (EUR IG, 2010 – 2019)



Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

- Portfolio $w = (w_1, \ldots, w_n)$ and benchmark $b = (b_1, \ldots, b_n)$
- ESG score of the portfolio:

$$\mathcal{S}(w) = \sum_{i=1}^{n} w_i \mathcal{S}_i$$

- ESG excess score of portfolio w with respect to benchmark b:

$$\begin{aligned} \mathcal{S}(w \mid b) &= \sum_{i=1}^{n} (w_i - b_i) \mathcal{S}_i \\ &= \mathcal{S}(w) - \mathcal{S}(b) \end{aligned}$$

- z-scores $\Rightarrow \mathcal{S}(w \mid b) > 0$
- Active or tracking risk $\mathcal{R}(w \mid b)$
- The optimization problem becomes:

$$w^{\star}(\gamma) = \arg\min \mathcal{R}(w \mid b) - \gamma \mathcal{S}(w \mid b)$$

Simulated results

Optimized portfolios

- The modified duration risk of portfolio w with respect to benchmark b is:

$$\mathcal{R}_{\mathrm{MD}}\left(x \mid b\right) = \sum_{j=1}^{n_{\mathcal{S}}} \left(\left(\sum_{i \in \mathcal{S}ector(j)} w_i \, \mathrm{MD}_i \right) - \left(\sum_{i \in \mathcal{S}ector(j)} b_i \, \mathrm{MD}_i \right) \right)^2$$

  where $n_{\mathcal{S}}$ is the number of sectors and MD_i is the modified duration of bond i

- An alternative is to use the DTS risk measure:

$$\mathcal{R}_{\mathrm{DTS}}\left(x \mid b\right) = \sum_{j=1}^{n_{\mathcal{S}}} \left(\left(\sum_{i \in \mathcal{S}ector(j)} w_i \, \mathrm{DTS}_i \right) - \left(\sum_{i \in \mathcal{S}ector(j)} b_i \, \mathrm{DTS}_i \right) \right)^2$$

  where DTS_i is the DTS of bond i

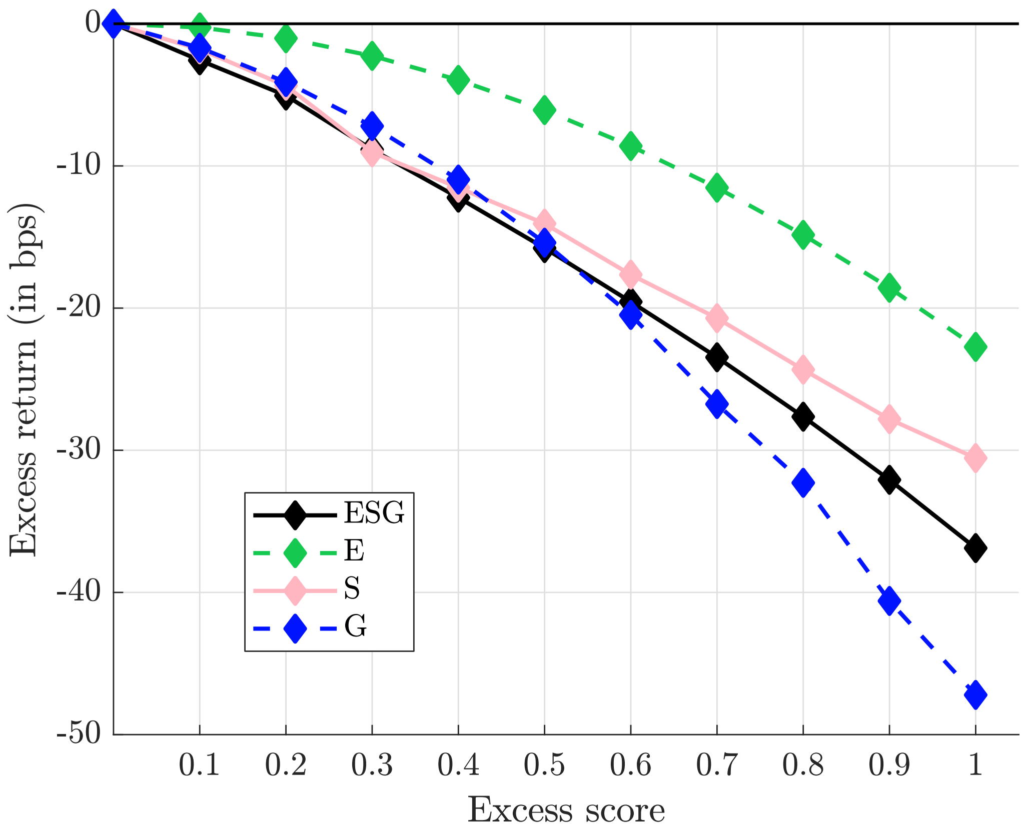
- Hybrid approach:

$$\mathcal{R}\left(w \mid b\right) = \frac{1}{2} \mathcal{R}_{\mathrm{MD}}\left(w \mid b\right) + \frac{1}{2} \mathcal{R}_{\mathrm{DTS}}\left(w \mid b\right)$$

Simulated results

Optimized portfolios

Figure 76: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2010 – 2013)

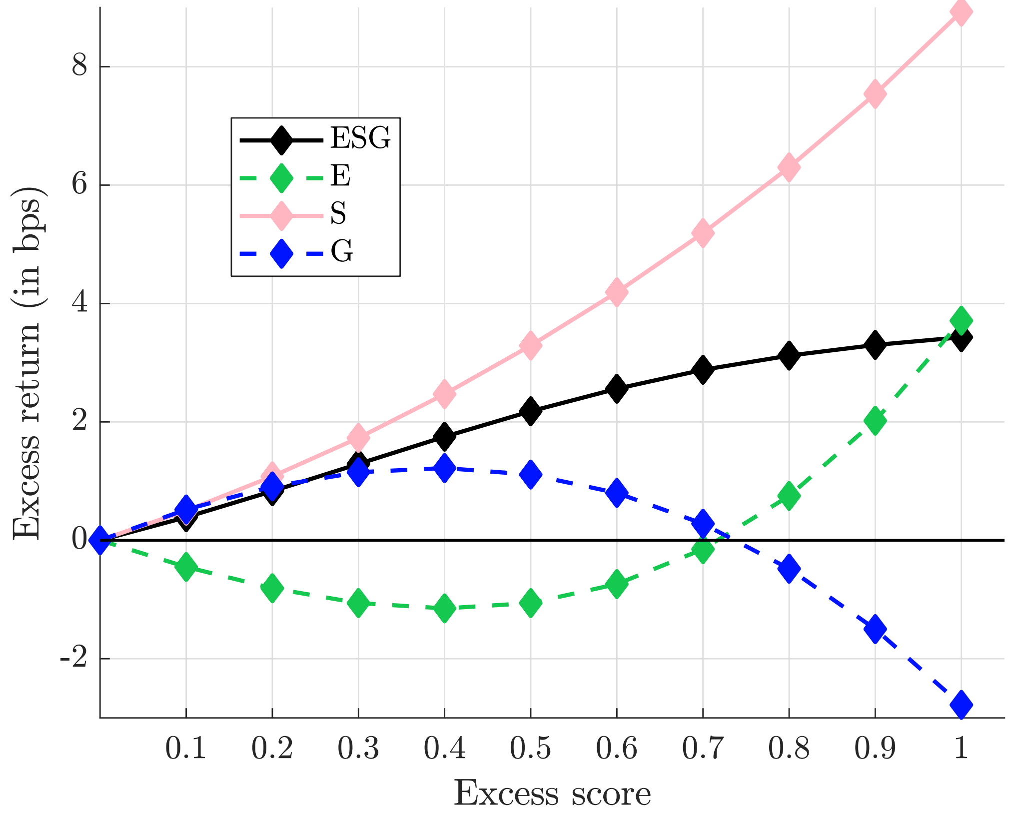


Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

Figure 77: Annualized excess return in bps of ESG optimized portfolios (EUR IG, 2014 – 2016)

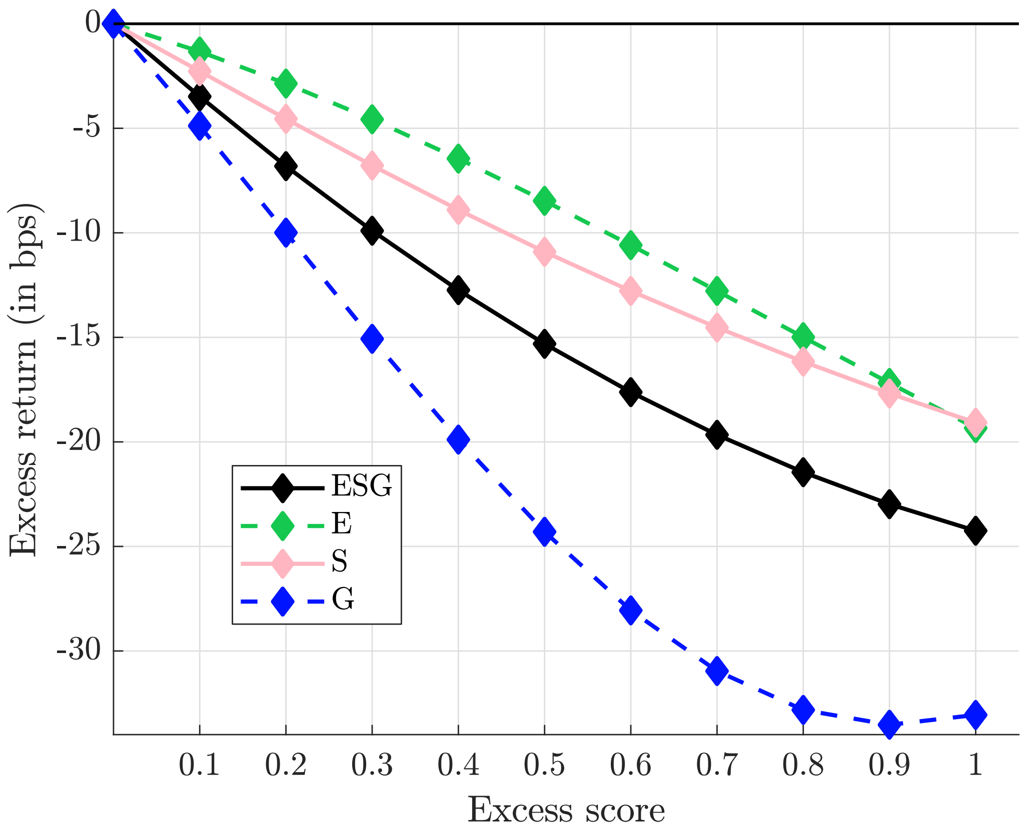


Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

Figure 78: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2010 – 2013)

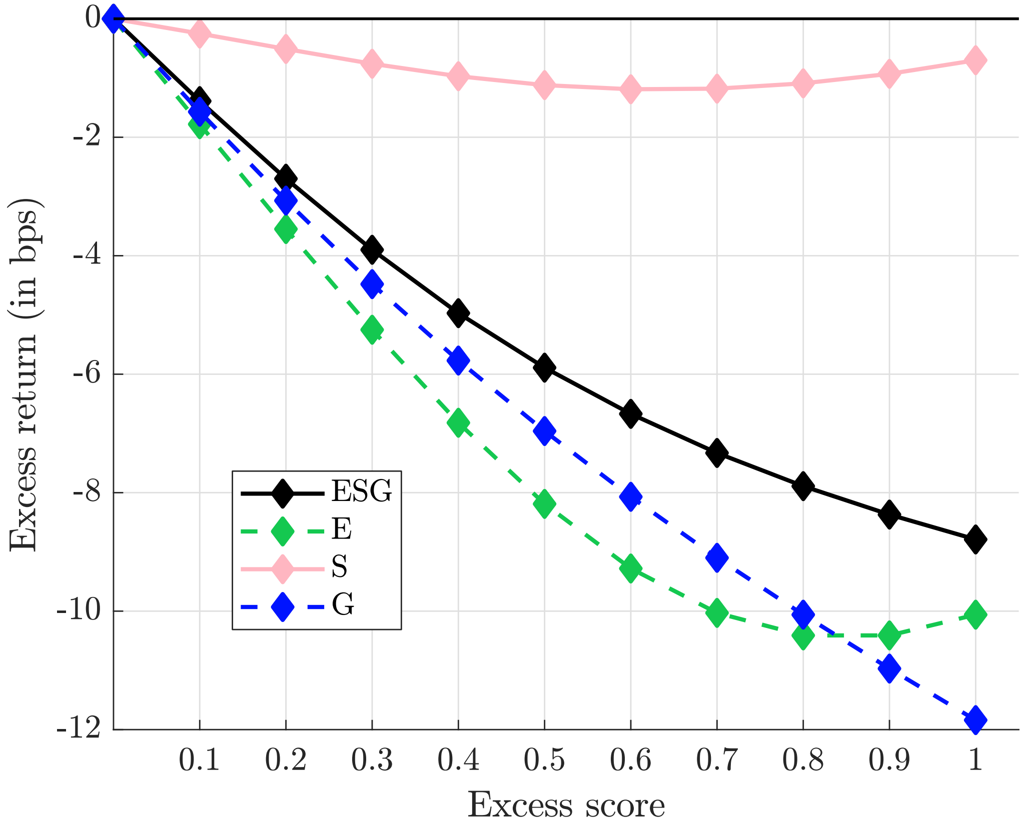


Source: Ben Slimane *et al.* (2019).

Simulated results

Optimized portfolios

Figure 79: Annualized excess return in bps of ESG optimized portfolios (USD IG, 2014 – 2016)



Source: Ben Slimane *et al.* (2019).

Bond indices

Table 48: Performance of ESG bond indexes (sovereign)

Year	FTSE WGBI Return BM	FTSE WGBI Return ESG	FTSE WGBI Alpha ESG	FTSE EGBI Return BM	FTSE EGBI Return ESG	FTSE EGBI Alpha ESG
2010	4.61	4.31	−30	0.61	4.14	353
2011	6.35	7.05	69	3.41	7.31	391
2012	1.65	3.06	141	10.65	7.39	−326
2013	−4.00	−2.95	105	2.21	−1.40	−362
2014	−0.48	−0.22	26	13.19	11.44	−175
2015	−3.57	−4.85	−128	1.65	0.39	−126
2016	1.60	1.02	−59	3.20	4.00	81
2017	7.49	8.16	67	0.15	−0.47	−62
2018	−0.84	−1.41	−57	0.88	1.65	78
2019	5.90	5.56	−34	6.72	4.45	−227
2020	10.11	10.90	79	5.03	4.11	−92
2021	−6.97	−7.15	−17	−3.54	−3.76	−21
2022	−18.26	−20.00	−173	−18.52	−19.06	−54
3Y	−5.75	−6.26	−51	−6.19	−6.74	−55
5Y	−2.54	−3.03	−49	−2.33	−2.95	−61
7Y	−0.58	−0.93	−35	−1.21	−1.63	−42
10Y	−1.22	−1.46	−24	0.77	−0.17	−94

Bond indices

Table 49: Performance of ESG bond indexes (corporates)

Year	Bloomberg Euro Aggregate Corporate						
	Return				Alpha		
	BM	SRI	S-SRI	ESG-S	SRI	S-SRI	ESG-S
2010	3.07	2.93	2.96		−13	−10	
2011	1.49	1.17	1.43		−32	−5	
2012	13.59	13.99	12.96		40	−63	
2013	2.37	2.49	2.36		12	−1	
2014	8.40	8.31	8.49		−8	10	
2015	−0.56	−0.59	−0.50	−0.59	−3	6	−3
2016	4.73	4.60	4.44	4.60	−13	−29	−13
2017	2.41	2.47	2.48	2.47	6	6	6
2018	−1.25	−1.12	−1.11	−1.12	13	14	13
2019	6.24	6.01	5.92	6.01	−24	−32	−24
2020	2.77	2.69	2.70	2.52	−8	−7	−25
2021	−0.97	−0.96	−0.99	−0.99	1	−2	−2
2022	−13.65	−13.62	−13.48	−13.48	3	16	17
3Y	−4.21	−4.22	−4.18	−4.23	−1	3	−2
5Y	−1.61	−1.63	−1.62	−1.64	−2	−1	−3
7Y	−0.16	−0.19	−0.20	−0.19	−3	−4	−3
10Y	0.88	0.86	0.86		−2	−1	

Bond indices

Table 50: Performance of ESG bond indexes (corporates)

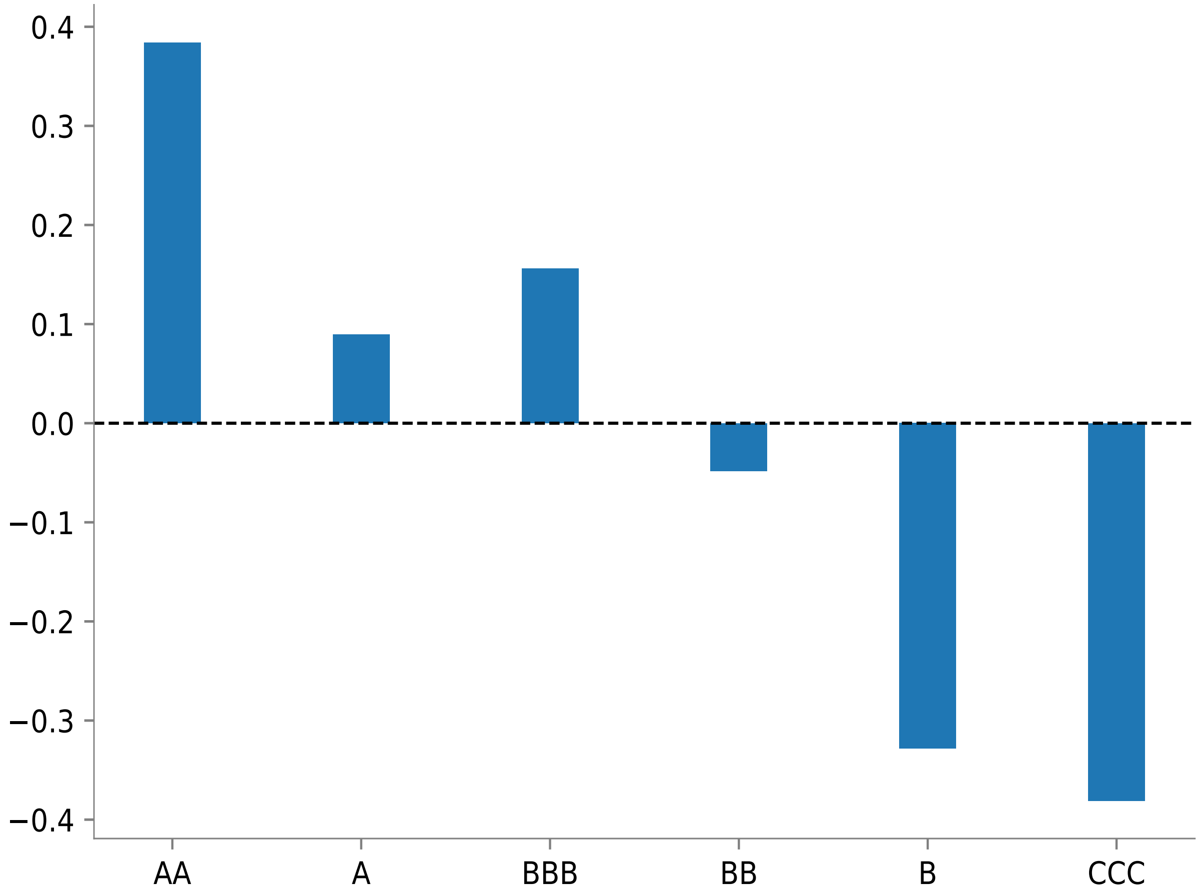
Year	Bloomberg US Corporate							Bloomberg Global High Yield				
	Return				Alpha			Return			Alpha	
	BM	SRI	S-SRI	ESG-S	SRI	S-SRI	ESG-S	BM	SRI	SUS	SRI	SUS
2019								1.00	0.96		−4	
2020								2.47	2.80	2.87	32	40
2021	−1.04	−1.55	9.56	2.34	−51	1 060	338	1.10	0.40	0.21	−70	−89
2022	−15.76	−15.12	−1.10	−13.86	64	1 467	190	−5.00	−5.95	−5.73	−95	−72

Definition

Equities

Correlation between Credit ratings and ESG ratings

Figure 80: Average **ESG** z-score with respect to the credit rating (2010 – 2019)



An integrated Credit-ESG model

We consider the following regression model:

$$\begin{aligned}\ln \mathrm{OAS}_{i,t} &= \alpha_t + \beta_{esg} \cdot \mathcal{S}_{i,t} + \beta_{md} \cdot \mathrm{MD}_{i,t} + \sum_{j=1}^{N_{\mathcal{S}ector}} \beta_{\mathcal{S}ector}(j) \cdot \mathcal{S}ector_{i,t}(j) + \\ & \beta_{sub} \cdot \mathrm{SUB}_{i,t} + \sum_{k=1}^{N_{\mathcal{R}ating}} \beta_{\mathcal{R}ating}(k) \cdot \mathcal{R}ating_{i,t}(k) + \varepsilon_{i,t}\end{aligned}$$

where:

- $\mathcal{S}_{i,t}$ is the **ESG** z-score of Bond i at time t
- $\mathrm{SUB}_{i,t}$ is a dummy variable accounting for subordination of the bond
- $\mathrm{MD}_{i,t}$ is the modified duration
- $\mathcal{S}ector_{i,t}(j)$ is a dummy variable for the j^{th} sector
- $\mathcal{R}ating_{i,t}(k)$ is a dummy variable for the k^{th} rating

An integrated Credit-ESG model

Table 51: Results of the panel data regression model (EUR IG, 2010 – 2019)

	2010–2013				2014–2019			
	ESG	**E**	**S**	**G**	**ESG**	**E**	**S**	**G**
R^2	60.0%	59.4%	59.5%	60.3%	66.3%	65.0%	65.2%	64.6%
Excess R^2 of ESG	0.6%	0.0%	0.2%	1.0%	4.0%	2.6%	2.9%	2.3%
$\hat{\beta}_{esg}$	-0.05	-0.01	-0.02	-0.07	-0.09	-0.08	-0.08	-0.08
t-statistic	-32	-7	-16	-39	-124	-98	-104	-92

Source: Ben Slimane *et al.* (2020)

The assumption $\mathcal{H}_0 : \beta_{esg} < 0$ is not rejected

ESG cost of capital with min/max score bounds

We calculate the difference between:

(1) the funding cost of **the worst-in-class issuer** and

(2) the funding cost of **the best-in-class issuer**

by assuming that:

- the two issuers have the same credit rating;
- the two issuers belong to the same sector;
- the two issuers have the same capital structure;
- the two issuers have the same debt maturity.

⇒ Two approaches:

1. Theoretical approach: ESG scores are set to -3 and $+3$ (not realistic)
2. Empirical approach: ESG scores are set to observed min/max score bounds (e.g. min/max $= -2.0/+1.9$ for Consumer Cyclical A-rated EUR, $-2.1/+3.2$ for Banking A-rated EUR, etc.)

ESG cost of capital with min/max score bounds

Table 52: **ESG** cost of capital (IG, 2014 – 2019)

	EUR				USD			
	AA	A	BBB	Average	AA	A	BBB	Average
Banking	23	45	67	45	11	19	33	21
Basic	9	25	44	26	5	15	34	18
Capital Goods	8	32	42	27	6	15	26	16
Communication		26	48	37	5	11	23	13
Consumer Cyclical	3	26	43	28	2	8	17	10
Consumer Non-Cyclical	15	29	31	25	6	12	19	12
Utility & Energy	12	32	56	33	9	14	31	18
Average	12	31	48	**31**	7	13	26	**15**

Source: Ben Slimane *et al.* (2020)

ESG and sovereign risk

Motivation

- Financial analysis **versus/and** extra-financial analysis
- Sovereign risk $\neq$ Corporate risk
- Which ESG metrics are priced and not priced in by the market?
- What is the nexus between ESG analysis and credit analysis?

The economics of sovereign risk

A Tale of Two Countries

- Henry, P.B., and Miller, C. (2009), Institutions versus Policies: A Tale of Two Islands, *American Economic Review*, 99(2), pp. 261-267.
- The example of Barbados and Jamaica
- Why the economic growth of two countries with the same economic development at time t is different 10, 20 or 30 years later?

Sovereign ESG themes

Environmental

- Biodiversity
- Climate change
- Commitment to environmental standards
- Energy mix
- Natural hazard
- Natural hazard outcome
- Non-renewable energy resources
- **Temperature**
- Water management

Social

- Civil unrest
- Demographics
- **Education**
- Gender
- Health
- Human rights
- **Income**
- Labour market standards
- Migration
- Water and electricity access

Governance

- Business environment and R&D
- **Governance effectiveness**
- Infrastructure and mobility
- International relations
- Justice
- **National security**
- **Political stability**

The economics of sovereign risk

Assessment of a country's creditworthiness

- Confidence in the country? Only financial reasons?
- Mellios, C., and Paget-Blanc, E. (2006), Which Factors Determine Sovereign Credit Ratings?, *European Journal of Finance*, 12(4), pp. 361-377 $\Rightarrow$ credit ratings are correlated to the corruption perception index
- Country default risk cannot be summarized by only financial figures!
- Why some rich countries have to pay a credit risk premium?
- How to explain the large differences in Asia?

Single-factor analysis

Data

Endogenous variable

10Y sovereign bond yield

Explanatory variables

- 269 ESG variables grouped into 26 ESG thematics
- 183 indicators come from Verisk Maplecrof database, the 86 remaining metrics were retrieved from the World Bank, ILO, WHO, FAO, UN...
- 6 control variables: GDP Growth, Net Debt, Reserves, Account Balance, Inflation and **Credit Rating**

Panel dimensions

- 67 countries
- 2015–2020

Single-factor analysis
Regression model

Let $s_{i,t}$ be the bond yield spread of the country i at time t. We consider the following regression model estimated by OLS:

$$s_{i,t} = \alpha + \underbrace{\beta x_{i,t}}_{\text{ESG metric}} + \underbrace{\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)}}_{\substack{\text{Control variables/} \\ \textbf{Fundamental model}}} + \varepsilon_{i,t}$$

and:

$$\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)} = \gamma_1 g_{i,t} + \gamma_2 \pi_{i,t} + \gamma_3 d_{i,t} + \gamma_4 ca_{i,t} + \gamma_5 r_{i,t} + \gamma_6 \mathcal{R}_{i,t}$$

where $g_{i,t}$ is the economic growth, $\pi_{i,t}$ is the inflation, $d_{i,t}$ is the debt ratio, $ca_{i,t}$ is the current account balance, $r_{i,t}$ is the reserve adequacy and $\mathcal{R}_{i,t}$ is the credit rating

Single-factor analysis
Results

Table 53: 7 most relevant indicators of the single-factor analysis per pillar

Pillar	Thematic	Indicator	$\Delta\mathfrak{R}_c^2$	F-test	Rank
	Climate change	Climate change vulnerability (acute)	5.51%	57.19	1
	Climate change	Climate change exposure (extreme)	4.80%	48.60	2
	Water management	Agricultural water withdrawal	4.02%	47.10	3
E	Climate change	Climate change sensitivity (acute)	3.95%	38.79	4
	Biodiversity	Biodiversity threatening score	3.53%	35.32	5
	Climate change	Climate change exposure (acute)	3.39%	32.95	6
	Climate change	Climate change vulnerability (average)	3.11%	31.16	7
	Human rights	Freedom of assembly	8.74%	89.58	1
	Human rights	Extent of arbitrary unrest	8.04%	80.10	2
	Human rights	Extent of torture and ill treatment	7.63%	75.48	3
S	Labour market standards	Severity of working time violations	7.21%	70.46	4
	Labour market standards	Forced labour violations (extent)	6.10%	54.40	5
	Labour market standards	Child labour (extent)	5.83%	54.68	6
	Migration	Vulnerability of migrant workers	5.83%	53.76	7
	National security	Severity of kidnappings	6.80%	64.49	1
	Business environment and R&D	Ease of access to loans	6.77%	73.57	2
	Infrastructure and mobility	Roads km	6.45%	63.66	3
G	Business environment and R&D	Capacity for innovation	5.65%	58.58	4
	Business environment and R&D	Ethical behaviour of firms	5.37%	55.14	5
	National security	Frequency of kidnappings	5.27%	48.49	6
	Infrastructure and mobility	Physical connectivity	4.94%	50.76	7

Source: Semet *et al.* (2021)

Single-factor analysis
Results

Table 54: Summary of the results

	E	S	G
Relevant	**Temperature** Climate change Natural hazard outcome	Labour market standards Human rights Migration	Infrastructure and mobility **National security** Justice
Less relevant	Water management Energy mix	**Income** **Education** Water and electricity access	**Political stability**

Multi-factor analysis

Regression model

We consider the following multi-factor regression model:

$$s_{i,t} = \alpha + \underbrace{\sum_{j=1}^{m} \beta_j x_{i,t}^{(j)}}_{\substack{\text{ESG variables/} \\ \textbf{Extra-financial model}}} + \underbrace{\sum_{k=1}^{6} \gamma_k z_{i,t}^{(k)}}_{\substack{\text{Control variables/} \\ \textbf{Fundamental model}}} + \varepsilon_{i,t}$$

A 4-step process

1. We consider the significant variables of the single-factor analysis at the 1% level
2. We filter the variables selected at Step 1 in order to eliminate redundant variables in each ESG theme
3. We perform a lasso regression to retain the seven most relevant variables within each ESG pillar
4. We perform a multi-factor analysis ($m = 21 \Rightarrow m = 7$)

Multi-factor analysis

The collinearity issue

Table 55: Example of variables exhibiting high correlations

Variable	$\Delta\mathfrak{R}_c^2$	Correlation$_{i,j}$					
Climate change exposure (average)	2.12%	1.00	0.74	0.80	0.48	**0.92**	0.77
Climate change exposure (acute)	3.89%	0.74	1.00	0.65	0.51	0.73	**0.89**
Climate change exposure (extreme)	4.80%	**0.80**	0.65	1.00	0.54	0.79	0.71
Climate change sensitivty (average)	3.95%	0.48	0.51	0.54	1.00	0.76	**0.81**
Climate change vulnerability (average)	3.11%	**0.92**	0.73	0.79	0.76	1.00	0.89
Climate change vulnerability (acute)	5.51%	0.77	0.89	0.71	0.81	**0.89**	1.00

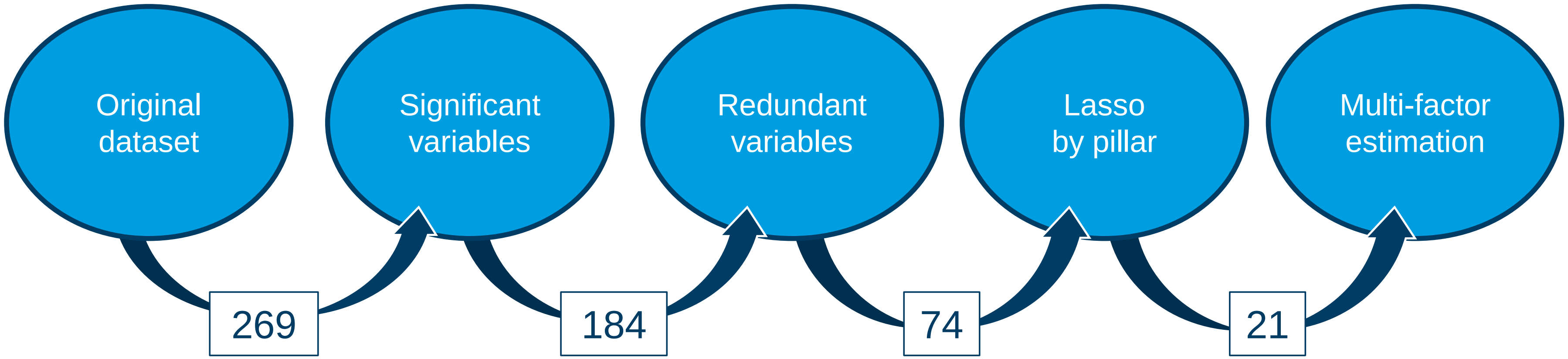
Source: Semet *et al.* (2021)

Selecting the variables

1. For each variable, we identify the highest pairwise correlation
2. Among each couple, we retain the variable showing the highest $\Delta\mathfrak{R}_c^2$
3. Among these variables, we select the variable with the lowest correlation

Multi-factor analysis
The collinearity issue

Figure 81: Filtering process



Source: Semet *et al.* (2021)

Multi-factor analysis
Results

Table 56: Results after Step 3 : Lasso regression pillar by pillar

Rank	Pillar	Thematic	Variable	Sign
1	E	Non-renewable energy resources	Total GHG emissions	−
2	E	Biodiversity	Biodiversity threatening score	−
3	E	Natural hazard	Severe storm hazard (absolute high extreme)	−
4	E	Temperature	Temperature change	+
5	E	Non-renewable energy resources	Fossil fuel intensity of the economy	−
6	E	Natural hazard	Drought hazard (absolute high extreme)	−
7	E	Commitment to environmental standards	Paris Agreement	−
1	S	Migration	Vulnerability of migrant workers	−
2	S	Demographics	Projected population change (5 years)	+
3	S	Civil unrest	Frequency of civil unrest incidents	−
4	S	Labor market standards	Index of labor standards	−
5	S	Labor market standards	Right to join trade unions (protection)	−
6	S	Human rights	Food import security	−
7	S	Income	Average monthly wage	−
1	G	International relationships	Exporting across borders (cost)	+
2	G	Business environment and R&D	Ethical behaviour of firms	−
3	G	National security	Severity of kidnappings	−
4	G	Business environment and R&D	Capacity for innovation	−
5	G	Infrastructure and mobility	Physical connectivity	−
6	G	Infrastructure and mobility	Air transport departures	−
7	G	Infrastructure and mobility	Rail lines km	−

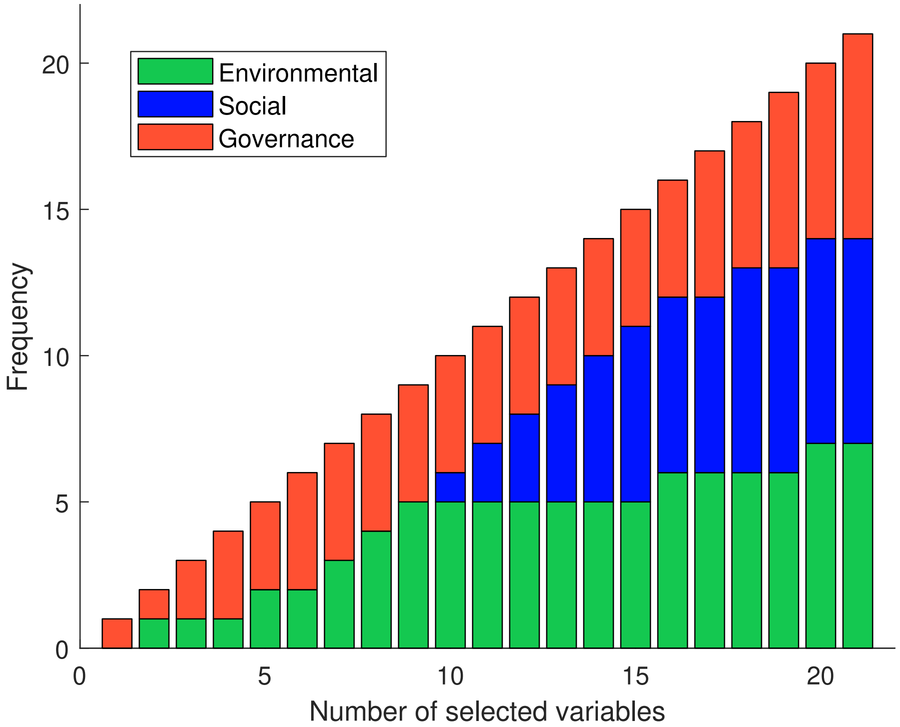
Source: Semet *et al.* (2021)

Multi-factor analysis

Global analysis - Lasso regression on the three pillars

Pillar	Indicator	Rank
G	Exporting across borders (cost)	1
E	Severe storm hazard	2
G	Capacity for innovation	3
G	Ethical behaviour of firms	4
E	Temperature change	5
G	Severity of kidnappings	6
E	Drought hazard	7
E	Fossil fuel intensity of the economy	8
E	Biodiversity threatening score	9
S	Index of labor standards	10

ESG pillar importance



Source: Semet *et al.* (2021)

Multi-factor analysis

Global analysis

Table 57: Final multi-factor model

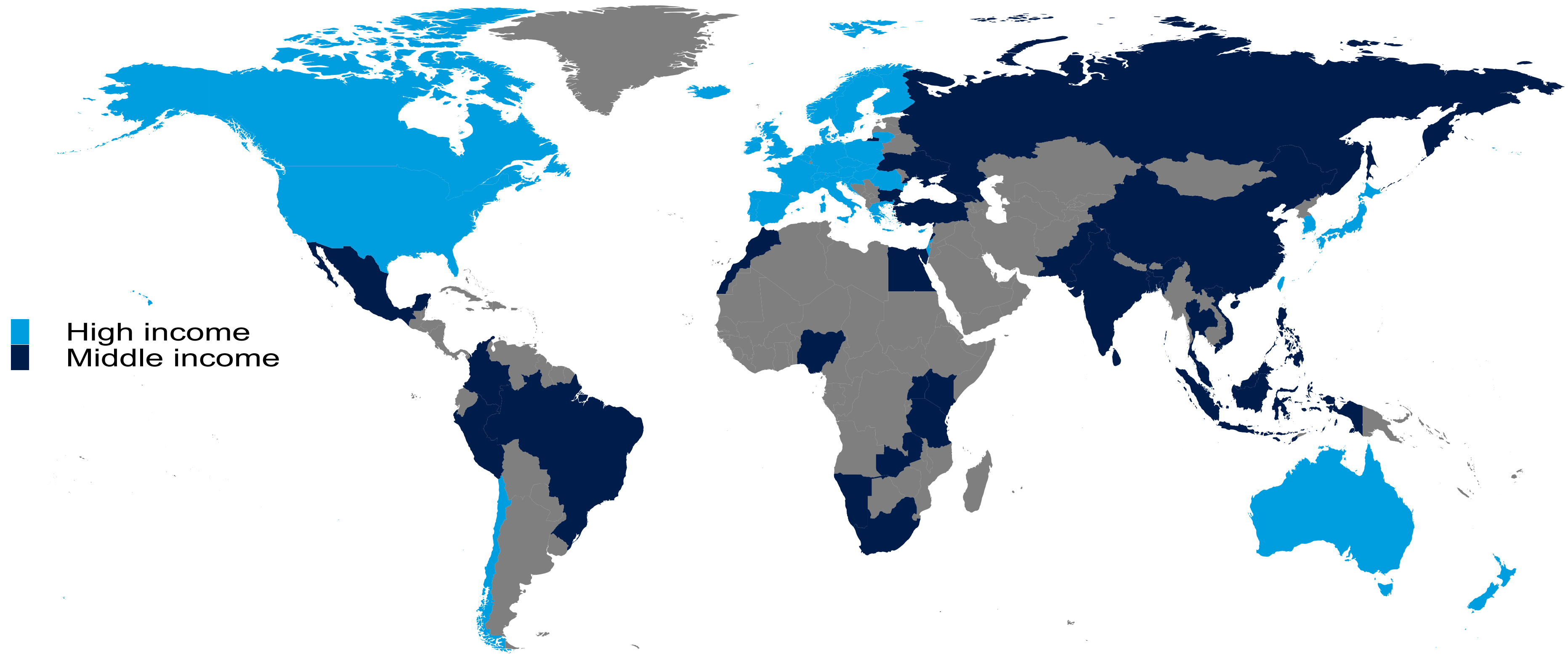
	Variable	$\hat{\beta}$	$\hat{\sigma}\left(\hat{\beta}\right)$	t-student	p-value
Financial	Intercept α	2.834	0.180	15.72***	0.00
	GDP growth $g_{i,t}$	0.017	0.012	1.37	0.17
	Inflation $\pi_{i,t}$	0.048	0.007	6.64***	0.00
	Debt ratio $d_{i,t}$	−0.001	0.001	−1.71*	0.08
	Current account balance $ca_{i,t}$	−0.012	0.005	−2.45**	0.01
	Reserve adequacy $r_{i,t}$	0.005	0.007	0.74	0.45
	Rating score $\mathcal{R}_{i,t}$	−0.013	0.001	−9.08***	0.00
Extra-financial	Exporting across borders (cost)	$4.05e^{-04}$	$9.83e^{-05}$	4.11***	0.00
	Severe storm hazard (absolute high extreme)	−0.015	0.009	−1.66*	0.09
	Capacity for innovation	−0.004	0.001	−4.99***	0.00
	Ethical behavior of firms	−0.061	0.021	−2.79***	0.00
	Temperature change	−0.149	0.042	−3.50***	0.00
	Severity of kidnappings	−0.032	0.007	−4.25***	0.00
	Drought hazard (absolute high extreme)	$3.33e^{-08}$	$1.27e^{-08}$	2.60***	0.00

$\Delta\Re_c^2 = 13.51\%$, F-test $= 29.28^{***}$

Source: Semet *et al.* (2021)

Multi-factor analysis

High income vs middle income countries

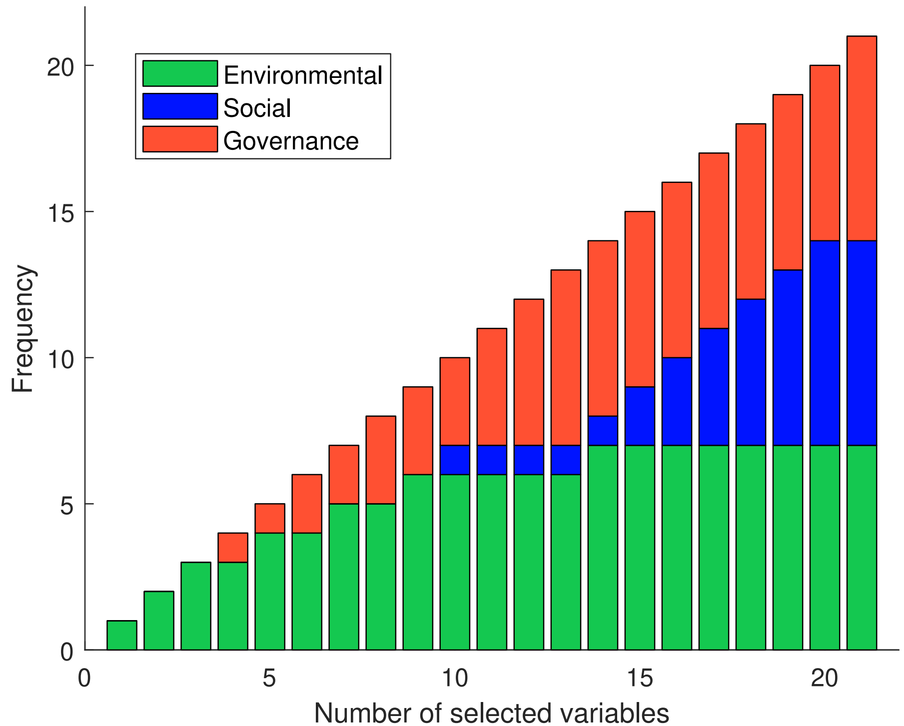


Multi-factor analysis

High income countries

Pillar	Indicator	Rank
E	Fossil fuel intensity of the economy	1
E	Temperature change	2
E	Cooling degree days annual average	3
G	Capacity for innovation	4
E	Heat stress (future)	5
G	Severity of kidnappings	6
E	Biodiversity threatening score	7
G	Efficacy of corporate boards	8
E	Total GHG emissions	9
S	Significant marginalized group	10

ESG pillar importance



Source: Semet *et al.* (2021)

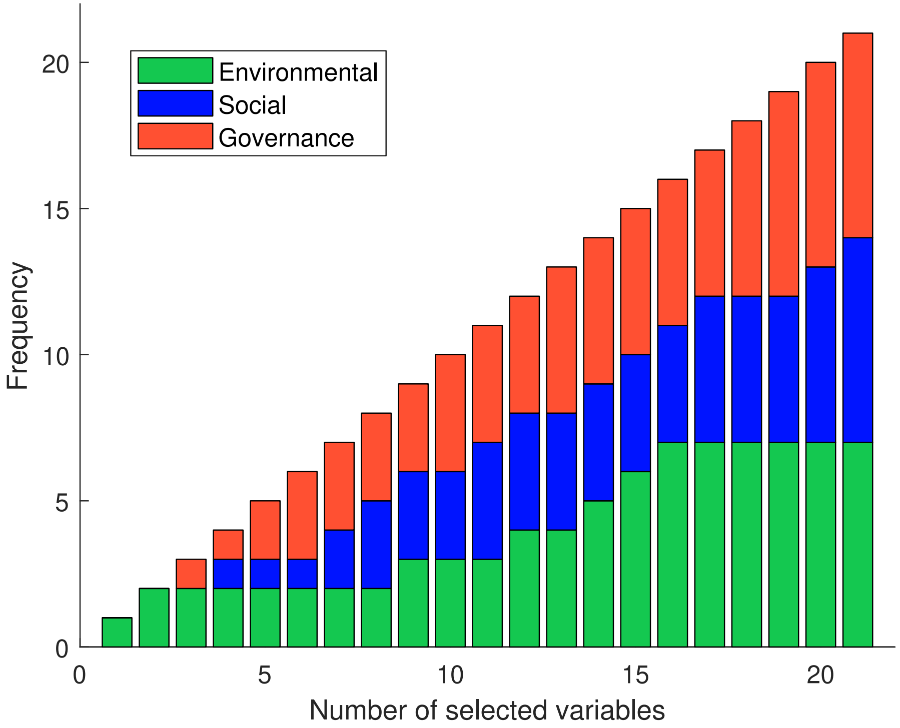
- Transition risk
- S is lagging

Multi-factor analysis

Middle income countries

Pillar	Indicator	Rank
E	Tsunami hazard	1
E	Transport infrastructure exposed to natural hazards	2
G	Severity of kidnappings	3
S	Discrimination based on LGBT status	4
G	Air transport departures	5
G	Exporting across borders (cost)	6
S	Index of labour standards	7
S	Vulnerability of migrant workers	8
E	Paris Agreement	9
G	Military expenditure (% of GDP)	10

ESG pillar importance



Source: Semet *et al.* (2021)

- Physical risk
- Social issues are priced

Explaining credit ratings with ESG metrics

Statistical framework

We consider the logit model:

$$\Pr\left\{\mathcal{G}_{i,t}=1\right\}=\mathbf{F}\left(\beta_0+\underbrace{\sum_{j=1}^{m}\beta_j x_{i,t}^{(j)}}_{\text{ESG variables}}\right)$$

where:

- $\mathcal{G}_{i,t}=1$ indicates if the country i is rated upper grade at time t
  - If the rating $\succeq$ A then $\mathcal{G}_{i,t}=1$
  - if the rating $\preceq$ BBB then $\mathcal{G}_{i,t}=0$
- $\mathbf{F}(z)$ is the logistic cumulative density function
- $x_{i,t}^{(j)}$ is the j^{th} selected indicator

We note $\theta_j=e^{\beta_j}$ is the odds-ratio coefficient

Lasso-penalized logit regression

Again, we perform a lasso regression to retain the seven most relevant variables for each ESG pillar and then we perform a multi-factor analysis

Explaining credit ratings with ESG metrics

Lasso selection process

Table 58: List of selected ESG variables for the logistic regression

Theme	Variable	Rank
Commitment to environmental standards	Domestic regulatory framework	1
Climate change	Climate change vulnerability (average)	2
Water management	Water import security (average)	3
Energy mix	Energy self sufficiency	4
Water management	Wastewater treatment index	5
Water management	Water intensity of the economy	6
Biodiversity	Biodiversity threatening score	7
Health	Health expenditure per capita	1
Water and electricity access	Public dissatisfaction with water quality	2
Education	Mean years of schooling of adults	3
Income	Base pay / value added per worker	4
Demographics	Urban population change (5 years)	5
Human rights	Basic food stuffs net imports per person	6
Human rights	Food import security	7
Government effectiveness	Government effectiveness index	1
Business environment and R&D	Venture capital availability	2
Business environment and R&D	R&D expenditure (% of GDP)	3
Infrastructure and mobility	Customs efficiency	4
Business environment and R&D	Enforcing a contract (time)	5
Business environment and R&D	Paying tax (process)	6
Business environment and R&D	Getting electricity (time)	7

Source: Semet *et al.* (2021)

Explaining credit ratings with ESG metrics

E pillar

Table 59: Logit model with environmental variables

Variable	$\hat{\theta}_j$	$\hat{\sigma}\left(\hat{\theta}_j\right)$	t-student	p-value
Domestic regulatory framework	1.415	0.156	3.16^{***}	0.00
Climate change vulnerability (average)	2.929	0.572	5.51^{***}	0.00
Water import security (average)	1.385	0.147	3.07^{***}	0.00
Energy self sufficiency	0.960	0.033	-1.16	0.24
Wastewater treatment index	1.011	0.008	1.36	0.17
Water intensity of the economy	1.000	0.000	-1.02	0.30
Biodiversity threatening score	0.887	0.026	-4.02^{***}	0.00

$\ell\left(\hat{\beta}\right) = -107.60$, $\mathrm{AIC} = 231.19$, $\mathfrak{R}^2 = 49.1\%$, $\mathrm{ACC} = 83.6\%$

Source: Semet *et al.* (2021)

Explaining credit ratings with ESG metrics

S pillar

Table 60: Logit model with social variables

Variable	$\hat{\theta}_j$	$\hat{\sigma}\left(\hat{\theta}_j\right)$	t-student	p-value
Health expenditure per capita	1.001	0.000	3.47***	0.00
Public dissatisfaction with water quality	0.889	0.024	−4.27***	0.00
Mean years of schooling of adults	2.710	0.583	4.64***	0.00
Base pay / value added per worker	0.000	0.000	−5.13***	0.00
Urban population change (5 years)	1.653	0.131	6.36***	0.00
Basic food stuffs net imports per person	0.996	0.001	−3.58***	0.00
Food import security	0.973	0.006	−4.33***	0.00

$\ell\left(\hat{\beta}\right) = -72.41$, $\mathrm{AIC} = 160.83$, $\mathfrak{R}^2 = 65.6\%$, $\mathrm{ACC} = 87.9\%$

Source: Semet *et al.* (2021)

Explaining credit ratings with ESG metrics

G pillar

Table 61: Logit model with governance variables

Variable	$\hat{\theta}_j$	$\hat{\sigma}\left(\hat{\theta}_j\right)$	t-student	p-value
Government effectiveness index	1.096	0.035	2.81^{***}	0.00
Venture capital availability	1.020	0.005	4.16^{***}	0.00
R&D expenditure (% of GDP)	2.259	1.006	1.83^{*}	0.06
Customs efficiency	2.193	1.657	1.04	0.29
Enforcing a contract (time)	0.997	0.001	-3.69^{***}	0.00
Paying tax (process)	0.914	0.031	-2.63^{***}	0.00
Getting electricity (time)	0.989	0.004	-2.73^{***}	0.00

$\ell\left(\hat{\beta}\right) = -67.78$, $\mathrm{AIC} = 151.57$, $\mathfrak{R}^2 = 67.9\%$, $\mathrm{ACC} = 90.1\%$

Source: Semet *et al.* (2021)

Explaining credit ratings with ESG metrics

E, S and G pillars

Table 62: Logit model with the ESG selected variables

Pillar	Variable	$\hat{\theta}_j$	$\hat{\sigma}\left(\hat{\theta}_j\right)$	t-student	p-value
E	Domestic regulatory framework	2.881	2.108	1.44	0.14
	Climate change vulnerability (average)	0.275	0.302	−1.17	0.24
	Water import security (average)	0.717	0.467	−0.50	0.61
	Biodiversity threatening score	1.029	0.199	0.14	0.88
S	Health expenditure per capita	0.998	0.002	−1.10	0.26
	Public dissatisfaction with water quality	1.332	0.269	1.41	0.15
	Mean years of schooling of adults	68.298	85.559	3.37***	0.00
	Base pay / value added per worker	0.000	0.000	−1.07	0.28
	Urban population change (5 years)	3.976	1.857	2.95***	0.00
	Basic food stuffs net imports per person	0.990	0.004	−2.07**	0.03
	Food import security	0.803	0.067	−2.59***	0.00
G	Government effectiveness index	1.751	0.412	2.37**	0.01
	Venture capital availability	1.099	0.035	2.93***	0.00
	Enforcing a contract (time)	0.999	0.004	−0.31	0.75
	Paying tax (process)	0.846	0.096	−1.47	0.14
	Getting electricity (time)	0.882	0.037	−2.95***	0.00

$\ell\left(\hat{\beta}\right) = -18.91$, $\text{AIC} = 71.83$, $\mathfrak{R}^2 = 91.1\%$, $\text{ACC} = 96.7\%$

Source: Semet *et al.* (2021)

Explaining credit ratings with ESG metrics

Prediction accuracy of credit ratings

Table 63: Summary of the results

	***	$\mathfrak{R}^2$	Accuracy	Sensitivity	Specificity	AIC
E*	4	48.02%	84.97%	86.90%	83.23%	230.04
S*	7	65.60%	87.90%	88.80%	86.90%	160.83
G*	4	67.70%	89.54%	91.72%	87.58%	150.65
ESG*	7	79.02%	92.50%	93.80%	91.30%	104.80

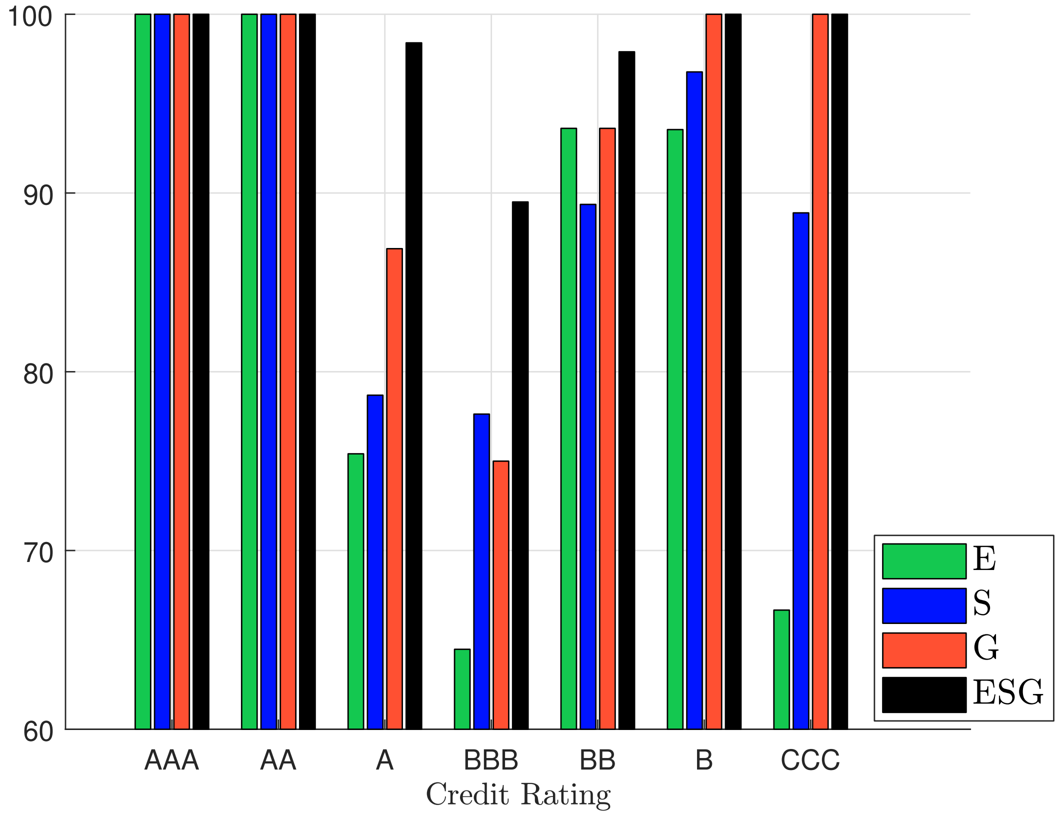
Source: Semet *et al.* (2021)

⇒ Final model: Education, Demographics, Human rights, Government effectiveness, Business environment and R&D

Explaining credit ratings with ESG metrics

Prediction accuracy of credit ratings

Figure 82: Prediction accuracy (in %) of credit ratings



	Rating	Probabilty range
Upper-grade	AAA	83% – 100%
	AA	67% – 82%
	A	50% – 66%
Lower-grade	BBB	39% – 49%
	BB	29% – 38%
	B	11% – 28%
	C	0% – 10%

Source: Semet *et al.* (2021)

ESG and sovereign risk

Summary of the results

What is directly priced by the bond market?		What is indirectly priced by credit rating agencies?
E $\succ$ G $\succ$ S Significant market-based ESG indicators	$\neq$	G $\succ$ S $\succ$ E Relevant CRA-based ESG indicators
• High-income countries Transition risk $\succ$ Physical risk • Middle-income countries Physical risk $\succ$ Transition risk		E metrics are second-order variables: • Environmental stantards • Water management • Biodiversity • Climate change
S matters for middle-income countries, especially for Gender inequality, Working conditions and Migration		Education, Demographic and Human rights are prominent indicators for the S pillar
National security, Infrastructure and mobility and International relationships are the relevant G metrics		Government effectiveness, Business environment and R&D dominate the G pillar
Fundamental analysis: $\mathfrak{R}_c^2 \approx 70\%$ Extra-financial analysis: $\Delta\mathfrak{R}_c^2 \approx 13.5\%$		Accuracy $> 95\%$ AAA, AA, B, CCC $\succ$ A $\succ$ BB $\succ$ BBB

Course 2022-2023 in Sustainable Finance
Lecture 4. Exercise
Equity Portfolio Optimization with ESG Scores

Thierry Roncalli*

*Amundi Asset Management[7]

*University of Paris-Saclay

March 2023

[7] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

We consider the CAPM model:

$$R_i - r = \beta_i (R_m - r) + \varepsilon_i$$

where R_i is the return of asset i, R_m is the return of the market portfolio w_m, r is the risk free asset, β_i is the beta of asset i with respect to the market portfolio and ε_i is the idiosyncratic risk of asset i. We have $R_m \perp \varepsilon_i$ and $\varepsilon_i \perp \varepsilon_j$. We note σ_m the volatility of the market portfolio. Let $\tilde{\sigma}_i$, μ_i and $\boldsymbol{\mathcal{S}}_i$ be the idiosyncratic volatility, the expected return and the ESG score of asset i. We use a universe of 6 assets with the following parameter values:

Asset i	1	2	3	4	5	6
β_i	0.10	0.30	0.50	0.90	1.30	2.00
$\tilde{\sigma}_i$ (in %)	17.00	17.00	16.00	10.00	11.00	12.00
μ_i (in %)	1.50	2.50	3.50	5.50	7.50	11.00
$\boldsymbol{\mathcal{S}}_i$	1.10	1.50	2.50	-1.82	-2.35	-2.91

and $\sigma_m = 20\%$. The risk-free return r is set to 1% and the expected return of the market portfolio w_m is equal to $\mu_m = 6\%$.

Question 1

We assume that the CAPM is valid.

Question (a)

Calculate the vector μ of expected returns.

- Using the CAPM, we have:

$$\mu_i = r + \beta_i \left(\mu_m - r\right)$$

- For instance, we have:

$$\mu_1 = 1\% + 0.10 \times (6\% - 1\%) = 1.5\%$$

  and:

$$\mu_2 = 1\% + 0.30 \times 5\% = 2.5\%$$

- Finally, we obtain $\mu = (1.5\%, 2.5\%, 3.5\%, 5.5\%, 7.5\%, 11\%)$

Question (b)

Compute the covariance matrix Σ. Deduce the volatility σ_i of the asset i and find the correlation matrix $\mathbb{C} = (\rho_{i,j})$ between asset returns.

- We have:

$$\Sigma = \sigma_m^2 \beta \beta^\top + D$$

where:

$$D = \operatorname{diag}\left(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_6^2\right)$$

- The numerical value of Σ is:

$$\Sigma = \begin{pmatrix} 293 & & & & & \\ 12 & 325 & & & & \\ 20 & 60 & 356 & & & \\ 36 & 108 & 180 & 424 & & \\ 52 & 156 & 260 & 468 & 797 & \\ 80 & 240 & 400 & 720 & 1\,040 & 1\,744 \end{pmatrix} \times 10^{-4}$$

- We have:

$$\sigma_i = \sqrt{\Sigma_{i,i}}$$

- We obtain:

$$\sigma = (17.12\%, 18.03\%, 18.87\%, 20.59\%, 28.23\%, 41.76\%)$$

- We have:

$$\rho_{i,j} = \frac{\Sigma_{i,j}}{\sigma_i \sigma_j}$$

- We obtain the following correlation matrix expressed in %:

$$\mathbb{C} = \begin{pmatrix} 100.00 & & & & & \\ 3.89 & 100.00 & & & & \\ 6.19 & 17.64 & 100.00 & & & \\ 10.21 & 29.09 & 46.33 & 100.00 & & \\ 10.76 & 30.65 & 48.81 & 80.51 & 100.00 & \\ 11.19 & 31.88 & 50.76 & 83.73 & 88.21 & 100.00 \end{pmatrix}$$

Question (c)

Compute the tangency portfolio w^*. Calculate $\mu(w^*)$ and $\sigma(w^*)$. Deduce the Sharpe ratio and the ESG score of the tangency portfolio.

- We have:

$$w^* = \frac{\Sigma^{-1}(\mu - r\mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\mu - r\mathbf{1})} = \begin{pmatrix} 0.94\% \\ 2.81\% \\ 5.28\% \\ 24.34\% \\ 29.06\% \\ 37.57\% \end{pmatrix}$$

- We deduce:

$$\begin{aligned}
\mu(w^*) &= w^{*\top}\mu = 7.9201\% \\
\sigma(w^*) &= \sqrt{w^{*\top}\Sigma w^*} = 28.3487\% \\
\mathrm{SR}(w^* \mid r) &= \frac{7.9201\% - 1\%}{28.3487\%} = 0.2441 \\
\boldsymbol{\mathcal{S}}(w^*) &= \sum_{i=1}^{6} w_i^* \boldsymbol{\mathcal{S}}_i = -2.0347
\end{aligned}$$

Question (d)

Compute the beta coefficient $\beta_i(w^*)$ of the six assets with respect to the tangency portfolio w^*, and the implied expected return $\tilde{\mu}_i$:

$$\tilde{\mu}_i = r + \beta_i(w^*)(\mu(w^*) - r)$$

- We have:

$$\beta_i\left(w^*\right)=\frac{\mathbf{e}_i^\top \Sigma w^*}{\sigma^2\left(w^*\right)}$$

- We obtain:

$$\beta\left(w^*\right)=\left(\begin{array}{c}0.0723\\0.2168\\0.3613\\0.6503\\0.9393\\1.4451\end{array}\right)$$

- The computation of $\tilde{\mu}_i=r+\beta_i\left(w^*\right)\left(\mu\left(w^*\right)-r\right)$ gives:

$$\tilde{\mu}=\left(\begin{array}{r}1.50\%\\2.50\%\\3.50\%\\5.50\%\\7.50\%\\11.00\%\end{array}\right)$$

Question (e)

Deduce the market portfolio w_m. Comment on these results.

- $\beta_i(w^*) \neq \beta_i(w_m)$ but risk premia are exact
- Let us assume that the allocation of w_m is equal to α of the tangency portfolio w^* and $1-\alpha$ of the risk-free asset. We deduce that:

$$\beta(w_m) = \frac{\Sigma w_m}{\sigma^2(w_m)} = \frac{\alpha \Sigma w^*}{\alpha^2 \sigma^2(w^*)} = \frac{1}{\alpha}\beta(w^*)$$

- We have:

$$\alpha = \frac{\beta_i(w^*)}{\beta_i(w_m)} = 72.25\%$$

- The market portfolio w_m is equal to 72.25% of the tangency portfolio w^* and 27.75% of the risk-free asset

- We have:

$$\mu(w_m) = r + \alpha(\mu(w^*) - r) = 1\% + 72.25\% \times (7.9201\% - 1\%) = 6\%$$

and:

$$\sigma(w_m) = \alpha\,\sigma(w^*) = 72.25\% \times 28.3487\% = 20.48\%$$

- We deduce that:

$$\mathrm{SR}(w_m \mid r) = \frac{6\% - 1\%}{20.48\%} = 0.2441$$

- We do not obtain the true value of the Sharpe ratio:

$$\mathrm{SR}(w_m \mid r) = \frac{6\% - 1\%}{20\%} = 0.25$$

- The tangency portfolio has an idiosyncratic risk:

$$\sqrt{w_m^\top\left(\sigma_m^2 \beta\beta^\top\right) w^\top} = 20\% < \sigma(w_m) = 20.48\%$$

Question 2

We consider long-only portfolios and we also impose a minimum threshold $\mathcal{S}^{\star}$ for the portfolio ESG score:

$$\mathcal{S}(w) = w^{\top}\mathcal{S} \geq \mathcal{S}^{\star}$$

Question (a)

Let γ be the risk tolerance. Write the mean-variance optimization problem.

- We have:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2} w^{\top} \Sigma w - \gamma w^{\top} \mu \\
\text{s.t.} & \left\{ \begin{array}{l} \mathbf{1}_6^{\top} w = 1 \\ w^{\top} \boldsymbol{\mathcal{S}} \geq \boldsymbol{\mathcal{S}}^{\star} \\ \mathbf{0}_6 \leq w \leq \mathbf{1}_6 \end{array} \right.
\end{aligned}
$$

Question (b)

Find the QP form of the MVO problem.

- The matrix form of the QP problem is:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2} w^{\top} Q w - w^{\top} R \\
& \text{s.t.} \quad \left\{ \begin{array}{l} A w = B \\ C w \leq D \\ w^{-} \leq w \leq w^{+} \end{array} \right.
\end{aligned}
$$

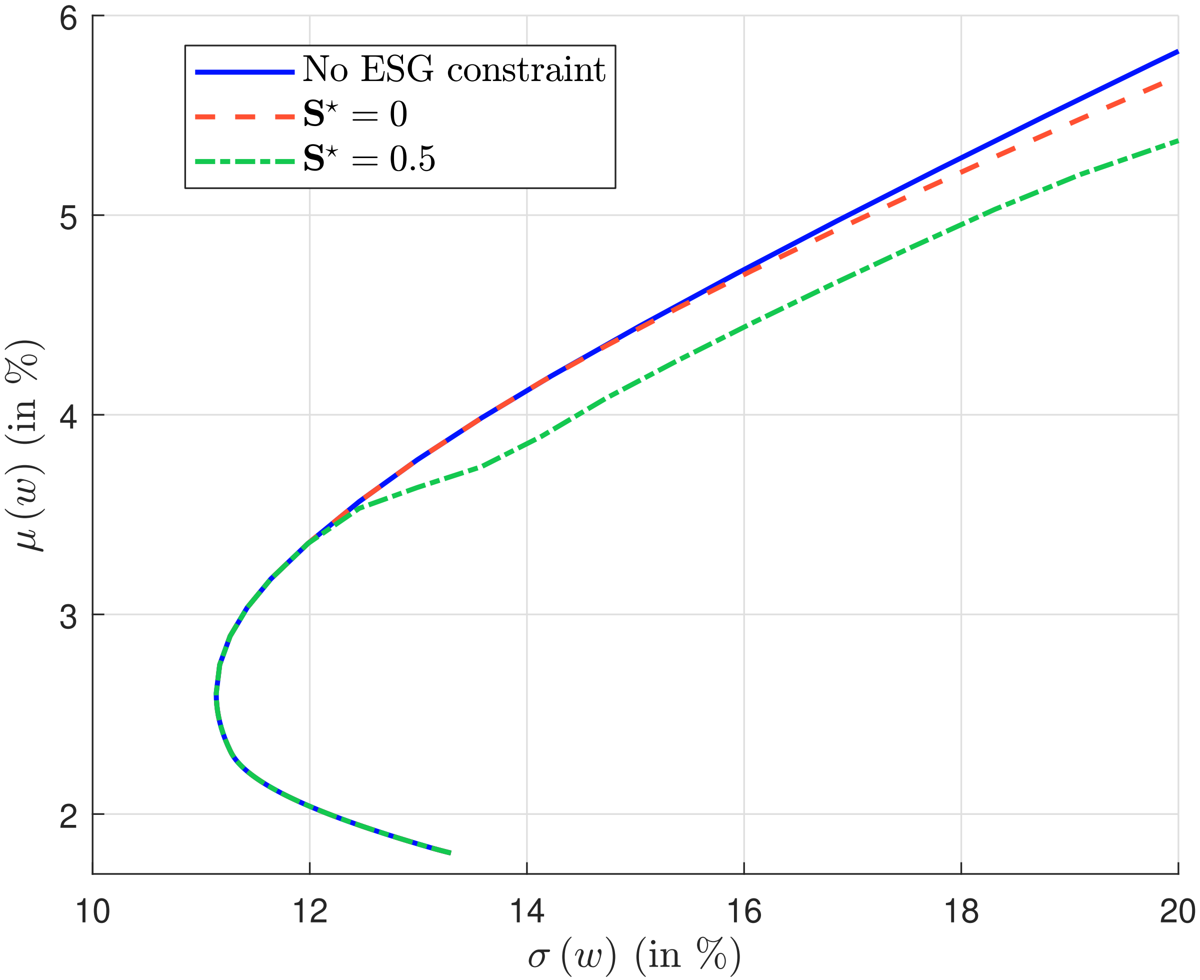
- We deduce that $Q = \Sigma$, $R = \gamma \mu$, $A = \mathbf{1}_6^{\top}$, $B = 1$, $C = -\boldsymbol{\mathcal{S}}^{\top}$, $D = -\boldsymbol{\mathcal{S}}^{\star}$, $w^{-} = \mathbf{0}_6$ and $w^{+} = \mathbf{1}_6$

Question (c)

Compare the efficient frontier when (1) there is no ESG constraint ($\mathcal{S}^{\star} = -\infty$), (2) we impose a positive ESG score ($\mathcal{S}^{\star} = 0$) and (3) the minimum threshold is set to 0.5 ($\mathcal{S}^{\star} = 0.5$). Comment on these results.

- To compute the efficient frontier, we consider several value of $\gamma \in [-1, 2]$
- For each value of γ, we compute the optimal portfolio $w^\star$ and deduce its expected return $\mu(w^\star)$ and its volatility $\sigma(w^\star)$

Figure 83: Impact of the minimum ESG score on the efficient frontier



Question (d)

For each previous cases, find the tangency portfolio w^* and the corresponding risk tolerance γ^*. Compute then $\mu(w^*)$, $\sigma(w^*)$, $\mathrm{SR}(w^* \mid r)$ and $\boldsymbol{\mathcal{S}}(w^*)$. Comment on these results.

- Let $w^{\star}(\gamma)$ be the MVO portfolio when the risk tolerance is equal to γ
- By using a fine grid of γ values, we can find the optimal value γ^{*} by solving numerically the following optimization problem with the brute force algorithm:

$$\gamma^{*} = \arg\max \frac{\mu\left(w^{\star}(\gamma)\right) - r}{\sigma\left(w^{\star}(\gamma)\right)} \qquad \text{for } \gamma \in [0, 2]$$

- We deduce the tangency portfolio $w^{*} = w^{\star}(\gamma^{*})$

Table 64: Impact of the minimum ESG score on the efficient frontier

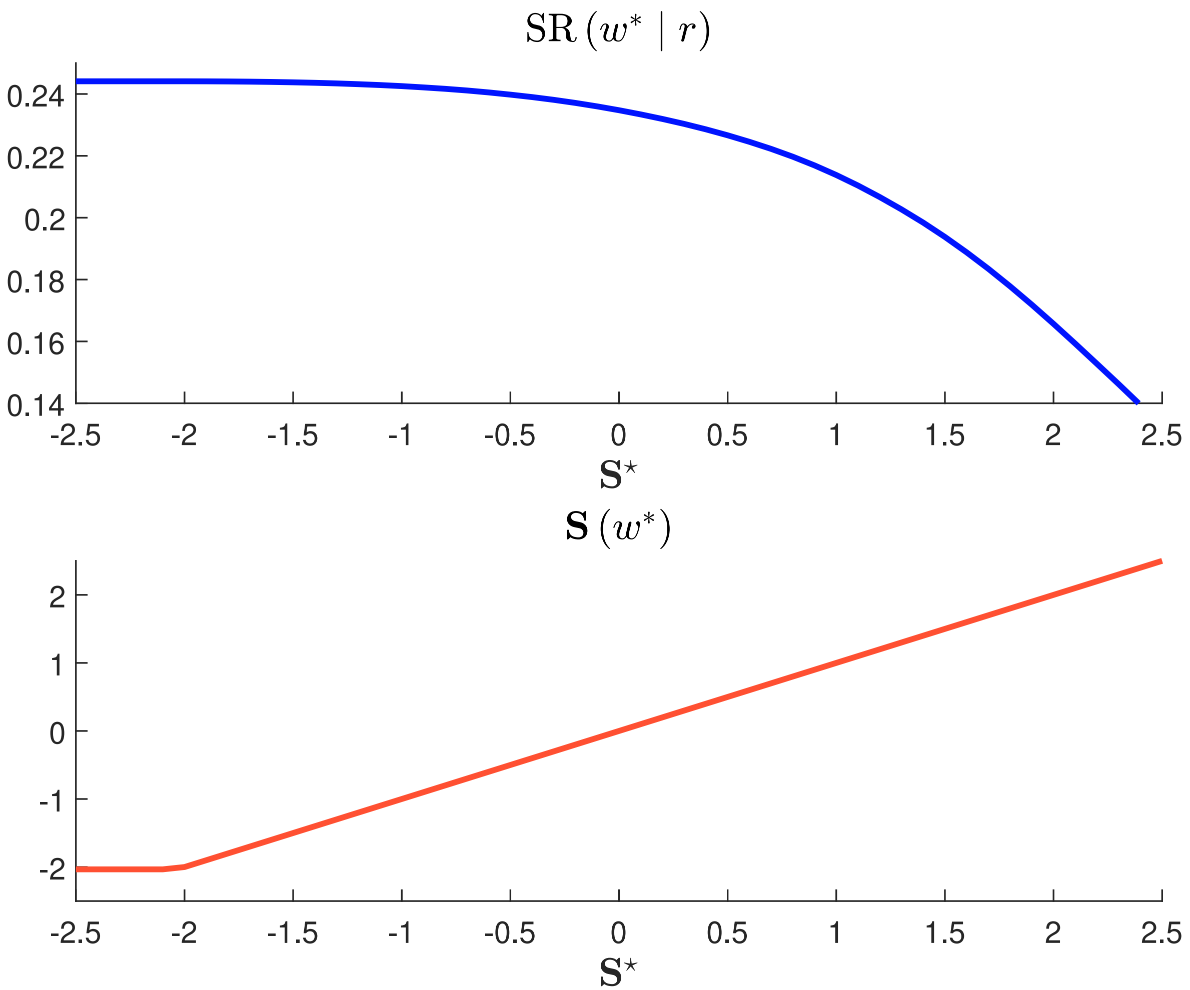
$\mathcal{S}^{\star}$	$-\infty$	0	0.5
γ^*	1.1613	0.8500	0.8500
w^* (in %)	0.9360	9.7432	9.1481
	2.8079	16.3317	19.0206
	5.2830	31.0176	40.3500
	24.3441	5.1414	0.0000
	29.0609	11.6028	3.8248
	37.5681	26.1633	27.6565
$\mu(w^*)$ (in %)	7.9201	5.6710	5.3541
$\sigma(w^*)$ (in%)	28.3487	19.8979	19.2112
$\mathrm{SR}(w^* \mid r)$	0.2441	0.2347	0.2266
$\mathcal{S}(w^*)$	-2.0347	0.0000	0.5000

Question (e)

Draw the relationship between the minimum ESG score $\boldsymbol{\mathcal{S}}^{\star}$ and the Sharpe ratio $\mathrm{SR}\left(w^{*} \mid r\right)$ of the tangency portfolio.

- We perform the same analysis as previously for several values $\boldsymbol{\mathcal{S}}^{\star} \in [-2.5, 2.5]$
- We verify that the Sharpe ratio is a decreasing function of $\boldsymbol{\mathcal{S}}^{\star}$

Figure 84: Relationship between the minimum ESG score $\mathcal{S}^{\star}$ and the Sharpe ratio $\mathrm{SR}\left(w^{*} \mid r\right)$ of the tangency portfolio



Question (f)

We assume that the market portfolio w_m corresponds to the tangency portfolio when $\mathcal{S}^\star = 0.5$.

- The market portfolio w_m is then equal to:

$$w_m = \begin{pmatrix} 9.15\% \\ 19.02\% \\ 40.35\% \\ 0.00\% \\ 3.82\% \\ 27.66\% \end{pmatrix}$$

- We deduce that:

$$\begin{array}{rcl} \mu(w_m) & = & 5.3541\% \\ \sigma(w_m) & = & 19.2112\% \\ \mathrm{SR}(w_m \mid r) & = & 0.2266 \\ \mathcal{S}(w_m) & = & 0.5 \end{array}$$

Question (f).i

Compute the beta coefficient $\beta_i(w_m)$ and the implied expected return $\tilde{\mu}_i(w_m)$ for each asset. Deduce then the alpha return α_i of asset i. Comment on these results.

- We have:

$$\beta_i(w_m) = \frac{\mathbf{e}_i^\top \Sigma w_m}{\sigma^2(w_m)}$$

and:

$$\tilde{\mu}_i(w_m) = r + \beta_i(w_m)(\mu(w_m) - r)$$

- We deduce that the alpha return is equal to:

$$\begin{aligned} \alpha_i &= \mu_i - \tilde{\mu}_i(w_m) \\ &= (\mu_i - r) - \beta_i(w_m)(\mu(w_m) - r) \end{aligned}$$

- We notice that $\alpha_i < 0$ for the first three assets and $\alpha_i > 0$ for the last three assets, implying that:

$$\begin{cases} \boldsymbol{\mathcal{S}}_i > 0 \Rightarrow \alpha_i < 0 \\ \boldsymbol{\mathcal{S}}_i < 0 \Rightarrow \alpha_i > 0 \end{cases}$$

Table 65: Computation of the alpha return due to the ESG constraint

Asset	$\beta_i(w_m)$	$\tilde{\mu}_i(w_m)$ (in %)	$\tilde{\mu}_i(w_m) - r$ (in %)	α_i (in bps)
1	0.1660	1.7228	0.7228	−22.28
2	0.4321	2.8813	1.8813	−38.13
3	0.7518	4.2733	3.2733	−77.33
4	0.8494	4.6984	3.6984	80.16
5	1.2395	6.3967	5.3967	110.33
6	1.9955	9.6885	8.6885	131.15

Question (f).ii

We consider the equally-weighted portfolio w_{ew}. Compute its beta coefficient $\beta\left(w_{\mathrm{ew}} \mid w_m\right)$, its implied expected return $\tilde{\mu}\left(w_{\mathrm{ew}}\right)$ and its alpha return $\alpha\left(w_{\mathrm{ew}}\right)$. Comment on these results.

- We have:

$$\beta\left(w_{\mathrm{ew}} \mid w_m\right) = \frac{w_{\mathrm{ew}}^\top \Sigma w_m}{\sigma^2\left(w_m\right)} = 0.9057$$

and:

$$\tilde{\mu}\left(w_{\mathrm{ew}}\right) = 1\% + 0.9057 \times \left(5.3541\% - 1\%\right) = 4.9435\%$$

- We deduce that:

$$\alpha\left(w_{\mathrm{ew}}\right) = \mu\left(w_{\mathrm{ew}}\right) - \tilde{\mu}\left(w_{\mathrm{ew}}\right) = 5.25\% - 4.9435\% = 30.65 \text{ bps}$$

- We verify that:

$$\alpha\left(w_{\mathrm{ew}}\right) = \sum_{i=1}^{6} w_{\mathrm{ew},i}\alpha_i = \frac{\sum_{i=1}^{6}\alpha_i}{6} = 30.65 \text{ bps}$$

- The equally-weighted portfolio has a positive alpha because:

$$\boldsymbol{\mathcal{S}}\left(w_{\mathrm{ew}}\right) = -0.33 \ll \boldsymbol{\mathcal{S}}\left(w_m\right) = 0.50$$

Question 3

The objective of the investor is twice. He would like to manage the tracking error risk of his portfolio with respect to the benchmark $b = (15\%, 20\%, 19\%, 14\%, 15\%, 17\%)$ and have a better ESG score than the benchmark. Nevertheless, this investor faces a long-only constraint because he cannot leverage his portfolio and he cannot also be short on the assets.

Question (a)

What is the ESG score of the benchmark?

- We have:

$$\mathcal{S}(b) = \sum_{i=1}^{6} b_i \mathcal{S}_i = -0.1620$$

Question (b)

We assume that the investor's portfolio is $w = (10\%, 10\%, 30\%, 20\%, 20\%, 10\%)$. Compute the excess score $\mathcal{S}(w \mid b)$, the expected excess return $\mu(w \mid b)$, the tracking error volatility $\sigma(w \mid b)$ and the information ratio $\mathrm{IR}(w \mid b)$. Comment on these results.

- We have:

$$\left\{\begin{array}{l} \mathcal{S}\left(w \mid b\right) = \left(w - b\right)^\top \mathcal{S} = 0.0470 \\ \mu\left(w \mid b\right) = \left(w - b\right)^\top \mu = -0.5 \text{ bps} \\ \sigma\left(w \mid b\right) = \sqrt{\left(w - b\right)^\top \Sigma \left(w - b\right)} = 2.8423\% \\ \mathrm{IR}\left(w \mid b\right) = \dfrac{\mu\left(w \mid b\right)}{\sigma\left(w \mid b\right)} = -0.0018 \end{array}\right.$$

- The portfolio w is not optimal since it improves the ESG score of the benchmark, but its information ratio is negative. Nevertheless, the expected excess return is close to zero (less than -1 bps).

Question (c)

Same question with the portfolio $w = (10\%, 15\%, 30\%, 10\%, 15\%, 20\%)$.

- We have: We have:

$$\left\{\begin{array}{l}\boldsymbol{\mathcal{S}}\left(w \mid b\right) = \left(w - b\right)^{\top}\boldsymbol{\mathcal{S}} = 0.1305 \\ \mu\left(w \mid b\right) = \left(w - b\right)^{\top}\mu = 29.5 \text{ bps} \\ \sigma\left(w \mid b\right) = \sqrt{\left(w - b\right)^{\top}\Sigma\left(w - b\right)} = 2.4949\% \\ \mathrm{IR}\left(w \mid b\right) = \dfrac{\mu\left(w \mid b\right)}{\sigma\left(w \mid b\right)} = 0.1182\end{array}\right.$$

Question (d)

In the sequel, we assume that the investor has no return target. In fact, the objective of the investor is to improve the ESG score of the benchmark and control the tracking error volatility. We note γ the risk tolerance. Give the corresponding esg-variance optimization problem.

- The optimization problem is:

$$
\begin{aligned}
w^\star & = & \arg\min \frac{1}{2}\sigma^2\left(w \mid b\right) - \gamma \boldsymbol{\mathcal{S}}\left(w \mid b\right) \\
& \text{s.t.} & \left\{
\begin{array}{l}
\mathbf{1}_6^\top w = 1 \\
\mathbf{0}_6 \leq w \leq \mathbf{1}_6
\end{array}
\right.
\end{aligned}
$$

Question (e)

Find the matrix form of the corresponding QP problem.

- The objective function is equal to:

$$\begin{aligned}
(*) &= \frac{1}{2}\sigma^2(w \mid b) - \gamma \boldsymbol{\mathcal{S}}(w \mid b) \\
&= \frac{1}{2}(w-b)^\top \Sigma (w-b) - \gamma (w-b)^\top \boldsymbol{\mathcal{S}} \\
&= \frac{1}{2} w^\top \Sigma w - w^\top (\Sigma b + \gamma \boldsymbol{\mathcal{S}}) + \underbrace{\left(\gamma b^\top \boldsymbol{\mathcal{S}} + \frac{1}{2} b^\top \Sigma b\right)}_{\text{does not depend on } w}
\end{aligned}$$

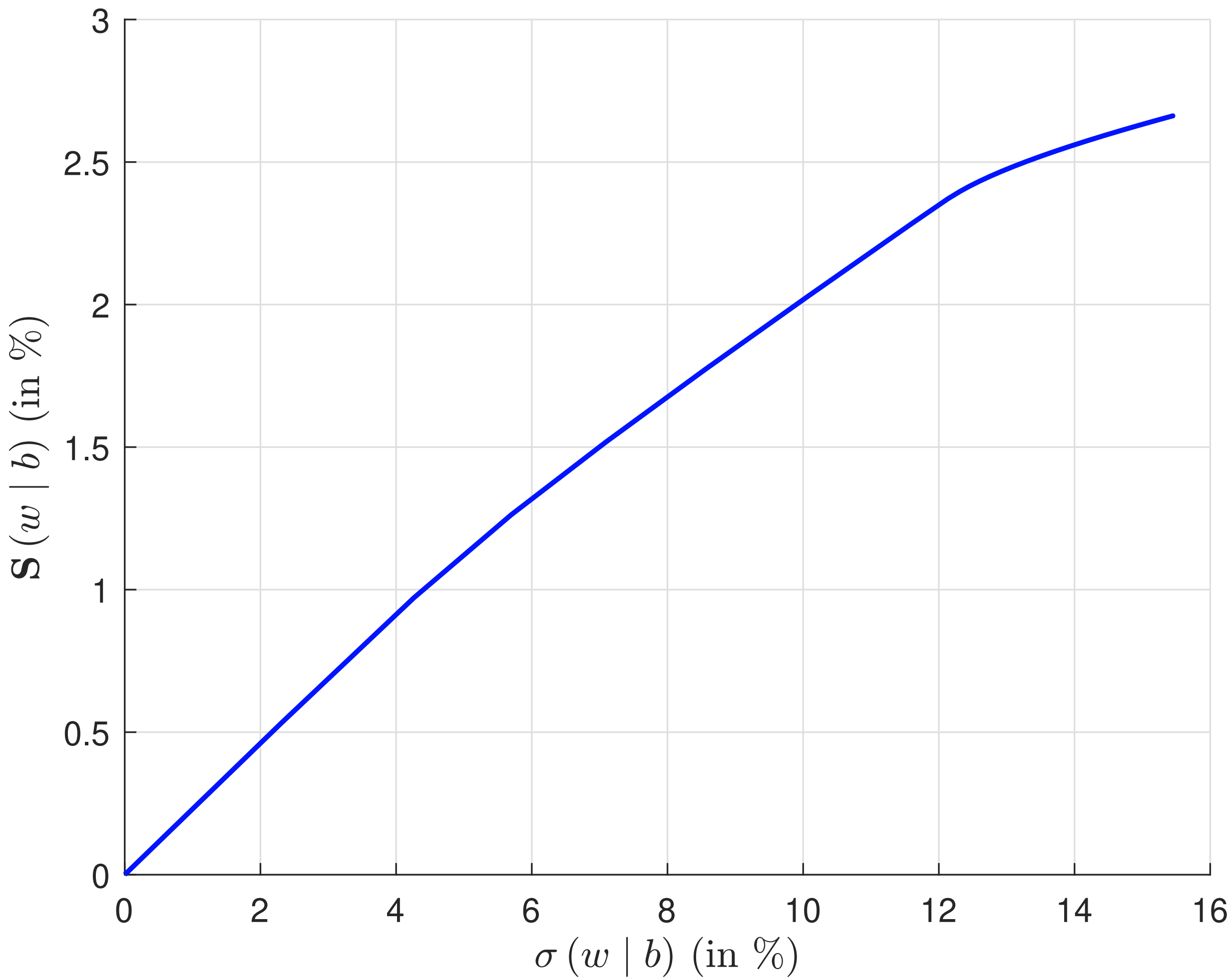
- We deduce that $Q = \Sigma$, $R = \Sigma b + \gamma \boldsymbol{\mathcal{S}}$, $A = \mathbf{1}_6^\top$, $B = 1$, $w^- = \mathbf{0}_6$ and $w^+ = \mathbf{1}_6$

Question (f)

Draw the esg-variance efficient frontier $\left(\sigma\left(w^{\star} \mid b\right), \boldsymbol{\mathcal{S}}\left(w^{\star} \mid b\right)\right)$ where $w^{\star}$ is an optimal portfolio.

- We solve the QP problem for several values of $\gamma \in [0, 5\%]$ and obtain Figure 85

Figure 85: Efficient frontier of tracking a benchmark with an ESG score objective



Question (g)

Find the optimal portfolio $w^{\star}$ when we target a given tracking error volatility $\sigma^{\star}$. The values of $\sigma^{\star}$ are 0%, 1%, 2%, 3% and 4%.

- Using the QP numerical algorithm, we compte the optimal value $\sigma(w \mid b)$ for $\gamma = 0$ and $\gamma = 5\%$
- Then, we apply the bisection algorithm to find the optimal value $\gamma^\star$ such that:

$$\sigma(w \mid b) = \sigma^\star$$

Table 66: Solution of the σ-problem

Target $\sigma^\star$	0	1%	2%	3%	4%
$\gamma^\star$ (in bps)	0.000	4.338	8.677	13.015	18.524
$w^\star$ (in %)	15.000	15.175	15.350	15.525	14.921
	20.000	21.446	22.892	24.338	25.385
	19.000	23.084	27.167	31.251	35.589
	14.000	9.588	5.176	0.763	0.000
	15.000	12.656	10.311	7.967	3.555
	17.000	18.052	19.104	20.156	20.550
$\mathcal{S}(w^\star \mid b)$	0.000	0.230	0.461	0.691	0.915

Question (h)

Find the optimal portfolio $w^{\star}$ when we target a given excess score $\boldsymbol{\mathcal{S}}^{\star}$. The values of $\boldsymbol{\mathcal{S}}^{\star}$ are 0, 0.1, 0.2, 0.3 and 0.4.

- Same method as previously with the following equation:

$$\mathcal{S}(w \mid b) = \mathcal{S}^{\star}$$

- An alternative approach consists in solving the following optimization problem:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2}\sigma^{2}(w \mid b) \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_6^{\top} w = 1 \\ \mathcal{S}(w \mid b) = \mathcal{S}^{\star} \\ \mathbf{0}_6 \leq w \leq \mathbf{1}_6 \end{array} \right. \end{aligned}$$

- We have: $Q = \Sigma$, $R = \Sigma b$, $A = \left(\begin{array}{c} \mathbf{1}_6^{\top} \\ \mathcal{S}^{\top} \end{array} \right)$, $B = \left(\begin{array}{c} 1 \\ \mathcal{S}^{\star} + \mathcal{S}^{\top} b \end{array} \right)$, $w^{-} = \mathbf{0}_6$ and $w^{+} = \mathbf{1}_6$

Table 67: Solution of the $\mathcal{S}$-problem

Target $\mathcal{S}^\star$	0	0.1	0.2	0.3	0.4
$\gamma^\star$ (in bps)	0.000	1.882	3.764	5.646	7.528
$w^\star$ (in %)	15.000	15.076	15.152	15.228	15.304
	20.000	20.627	21.255	21.882	22.509
	19.000	20.772	22.544	24.315	26.087
	14.000	12.086	10.171	8.257	6.343
	15.000	13.983	12.966	11.949	10.932
	17.000	17.456	17.913	18.369	18.825
$\sigma(w^\star \mid b)$ (in %)	0.000	0.434	0.868	1.301	1.735

Question (i)

We would like to compare the efficient frontier obtained in Question 3(f) with the efficient frontier when we implement a best-in-class selection or a worst-in-class exclusion. The selection strategy consists in investing only in the best three ESG assets, while the exclusion strategy implies no exposure on the worst ESG asset. Draw the three efficient frontiers. Comment on these results.

- For the best-in-class strategy, the optimization problem becomes:

$$
\begin{aligned}
w^{\star} & = \arg\min \frac{1}{2}\sigma^2 \left(w \mid b\right) - \gamma \boldsymbol{\mathcal{S}} \left(w \mid b\right) \\
\text{s.t.} & \left\{
\begin{array}{l}
\mathbf{1}_6^{\top} w = 1 \\
w_4 = w_5 = w_6 = 0 \\
\mathbf{0}_6 \leq w \leq \mathbf{1}_6
\end{array}
\right.
\end{aligned}
$$

- The QP form is defined by $Q = \Sigma$, $R = \Sigma b + \gamma \boldsymbol{\mathcal{S}}$, $A = \mathbf{1}_6^{\top}$, $B = 1$, $w^{-} = \mathbf{0}_6$ and $w^{+} = \left(\begin{array}{c} \mathbf{1}_3 \\ \mathbf{0}_3 \end{array}\right)$

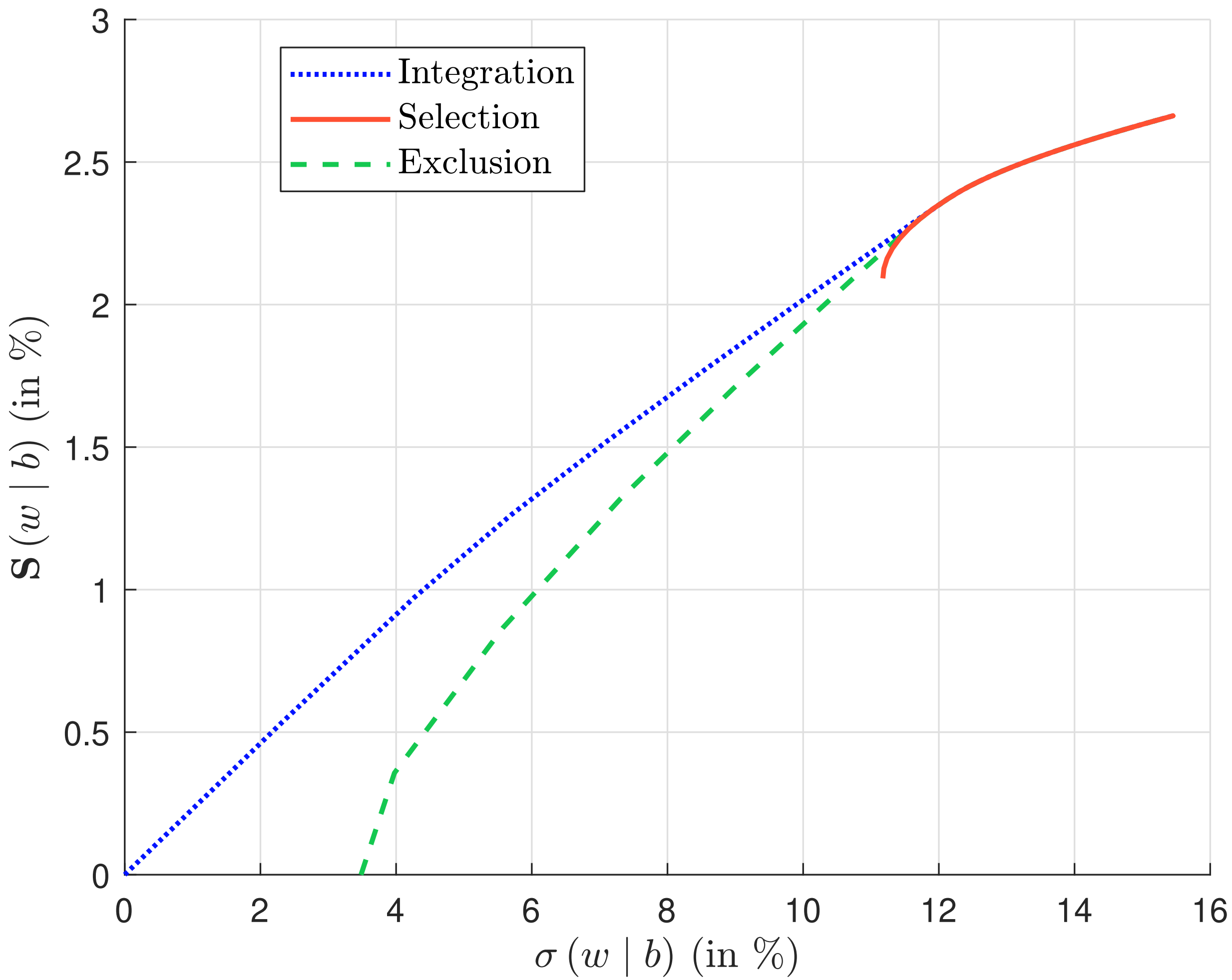
- For the worst-in-class strategy, the optimization problem becomes:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2}\sigma^2\left(w \mid b\right) - \gamma \boldsymbol{\mathcal{S}}\left(w \mid b\right) \\ \text{s.t.} &\quad \left\{ \begin{array}{l} \mathbf{1}_6^\top w = 1 \\ w_6 = 0 \\ \mathbf{0}_6 \leq w \leq \mathbf{1}_6 \end{array} \right. \end{aligned}$$

- The QP form is defined by $Q = \Sigma$, $R = \Sigma b + \gamma \boldsymbol{\mathcal{S}}$, $A = \mathbf{1}_6^\top$, $B = 1$, $w^- = \mathbf{0}_6$ and $w^+ = \begin{pmatrix} \mathbf{1}_5 \\ 0 \end{pmatrix}$

- The efficient frontiers are reported in Figure 86
- The exclusion strategy has less impact than the selection strategy
- The selection strategy implies a high tracking error risk

Figure 86: Comparison of the efficient frontiers (ESG integration, best-in-class selection and worst-in-class exclusion)



Question (j)

Which minimum tracking error volatility must the investor accept to implement the best-in-class selection strategy? Give the corresponding optimal portfolio.

- We solve the first problem of Question 3(i) with $\gamma = 0$
- We obtain:

$$\sigma(w \mid b) \geq 11.17\%$$

- The lower bound $\sigma(w^{\star} \mid b) = 11.17\%$ corresponds to the following optimal portfolio:

$$w^{\star} = \begin{pmatrix} 16.31\% \\ 34.17\% \\ 49.52\% \\ 0\% \\ 0\% \\ 0\% \end{pmatrix}$$

Remark

The impact of ESG scores on optimized portfolios depends on their relationship with expected returns, volatilities, correlations, beta coefficients, etc. In the previous exercise, the results are explained because the best-in-class assets are those with the lowest expected returns and beta coefficients while the worst-in-class assets are those with the highest expected returns and beta coefficients. For instance, we obtain a high tracking error risk for the best-in-class selection strategy, because the best-in-class assets have low volatilities and correlations with respect to worst-in-class assets, implying that it is difficult to replicate these last assets with the other assets.

Course 2022-2023 in Sustainable Finance
Lecture 5. Sustainable Financial Products, Impact Investing & Engagement

Thierry Roncalli*

*Amundi Asset Management[8]

*University of Paris-Saclay

March 2023

[8] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Greenwashing

The big issue for an investor is:

How to avoid Greenwashing (& ESG washing)?

Greenwash (also greenwashing)

- Activities by a company or an organization that are intended to make people think that it is concerned about the environment, even if its real business actually harms the environment
- A common form of greenwash is to publicly claim a commitment to the environment while quietly lobbying to avoid regulation

Source: Oxford English Dictionary (2020), `https://www.oed.com`

In finance, greenwashing is understood as making misleading claims about environmental practices, performance or products

Greenwashing

We must distinguish two types of risk:

- Explicit & deliberate greenwashing

Deliberate greenwashing = mis-selling risk

- Unintentional greenwashing

Unintentional greenwashing = misinterpretation risk

SRI Investment funds
Market

- Investment vehicles
  - Mutual funds
  - ETFs
  - Mandates & dedicated funds
- Investment strategies
  - Thematic strategies (e.g. water, social, wind energy, climate, plastic, etc.)
  - ESG-tilted strategies (e.g. exclusion, negative screening, best-in-class, enhanced ESG score, controlled tracking error, etc.)
  - Climate strategies (e.g. low carbon, $2^\circ\mathrm{C}$ alignment, activity exclusions[9], etc.)
  - Sustainability-linked securities (e.g. green bonds, social bonds, etc.)

Both α and β management

[9] e.g. coal exploration, oil exploration, electricity generation with a high GHG intensity

SRI Investment funds

Market

Mutual funds

- Amundi Climate Transition
- Amundi ARI European Credit SRI
- AXA World Funds – Euro Bonds SRI
- CPR Invest Social Impact
- Fidelity U.S. Sustainability Index
- Fidelity Sustainable Water & Waste
- Natixis ESG Dynamic Fund
- Vanguard FTSE Social Index
- Etc.

ETFs

- Amundi Index MSCI Europe SRI UCITS ETF
- Amundi MSCI Emerging ESG Leaders UCITS ETF
- Amundi EURO ISTOXX Climate Paris Aligned PAB UCITS ETF
- Lyxor New Energy UCITS ETF
- Lyxor World Water UCITS ETF
- SPDR S&P 500 ESG
- First Trust Global Wind Energy ETF
- Invesco S&P 500 ESG UCITS ETF
- Etc.

SRI Investment funds

Market

- ESG represents **58% of the net new assets** (NNA) in the European ETF market
- ESG fund assets reach $1 652 bn
  - Europe: $1 343 bn (or 81.3%)
  - US: $236.4 bn (or 14.3%)
  - Asia: $43.1 bn (or 2.6%)
- Net flows into sustainable mutual funds and ETFs in Q4 2020: $370 bn (or **+29% of assets**)
- Net flows into sustainable mutual funds and ETFs in 2020
  - Europe: $273 bn, almost double the total for 2019, almost 5 times more than in 2017
  - US: $51.2 bn, more than double the total for 2019, almost 10 times more than in 2018

Source: Morningstar, Global Sustainable Fund Flows: Q4 2020 in Review (January 2021)

SRI Investment funds

Labels

European sustainable finance labels

- Novethic label (pioneer label in 2009, suspended in 2016)
- French SRI label — `https://www.lelabelisr.fr`
- FNG label (Germany) — `https://fng-siegel.org`
- Towards Sustainability label (Belgium) — `https://www.towardssustainability.be`
- LuxFLAG label (Luxembourg) — `https://www.luxflag.org`
- Nordic Swan Ecolabel (Nordic countries) — `https://www.nordic-ecolabel.org`
- Umweltzeichen Ecolabel (Austria) — `https://www.umweltzeichen.at/en`
- French Greenfin label — `https://www.ecologie.gouv.fr/label-greenfin`

SRI Investment funds

Labels

Remark

According to Novethic (2020), 806 funds had a label at the end of December 2019. Nine months later, this number has increased by 392 and the AUM has be multiplied by 3.2!

SRI Investment funds

Regulation

"Today it is difficult for consumers, companies and other market actors to make sense of the many environmental labels and initiatives on the environmental performance of products and companies. There are more than 200 environmental labels active in the EU, and more than 450 active worldwide; there are more than 80 widely used reporting initiatives and methods for carbon emissions only. Some of these methods and initiatives are reliable, some not; they are variable in the issues they cover" (European Commission, 2020).

Source: `https://ec.europa.eu/environment/eussd/index.htm`

SRI Investment funds

Regulation

1. EU taxonomy regulation
2. Sustainable Finance disclosure regulation (SFDR)
3. Climate benchmarks
4. Sustainability preferences (MiFID II & IDD)

SRI Investment funds

Regulation

SFDR

- Article 6: Non-ESG funds (standard funds)
- Article 8: ESG funds (funds that promote **E** or **S** characteristics)
- Article 9: Sustainable funds (funds that have a sustainable investment objective: impact investing or reduction of carbon emissions)

SRI Investment funds

Regulation

New benchmark rules

- Climate transition benchmarks (CTB): high level of decarbonization (-30%), no controversial weapons and tobacco, high positive impact on climate change, etc.
- Paris-aligned benchmarks (PAB): high level of decarbonization (-50%), no controversial weapons and tobacco, no activities in coal, oil and natural gas, global warming below 2°, etc.

- MSCI Climate Paris Aligned Indexes — `www.msci.com/esg/climate-paris-aligned-indexes`
- FTSE TPI Climate Transition Index Series — `www.ftserussell.com/products/indices/tpi-climate-transition`
- STOXX Climate Transition Benchmark (CTB) and STOXX Paris-Aligned Benchmark (PAB) Indices — `qontigo.com/solutions/climate-indices`

Sustainable fixed-income products

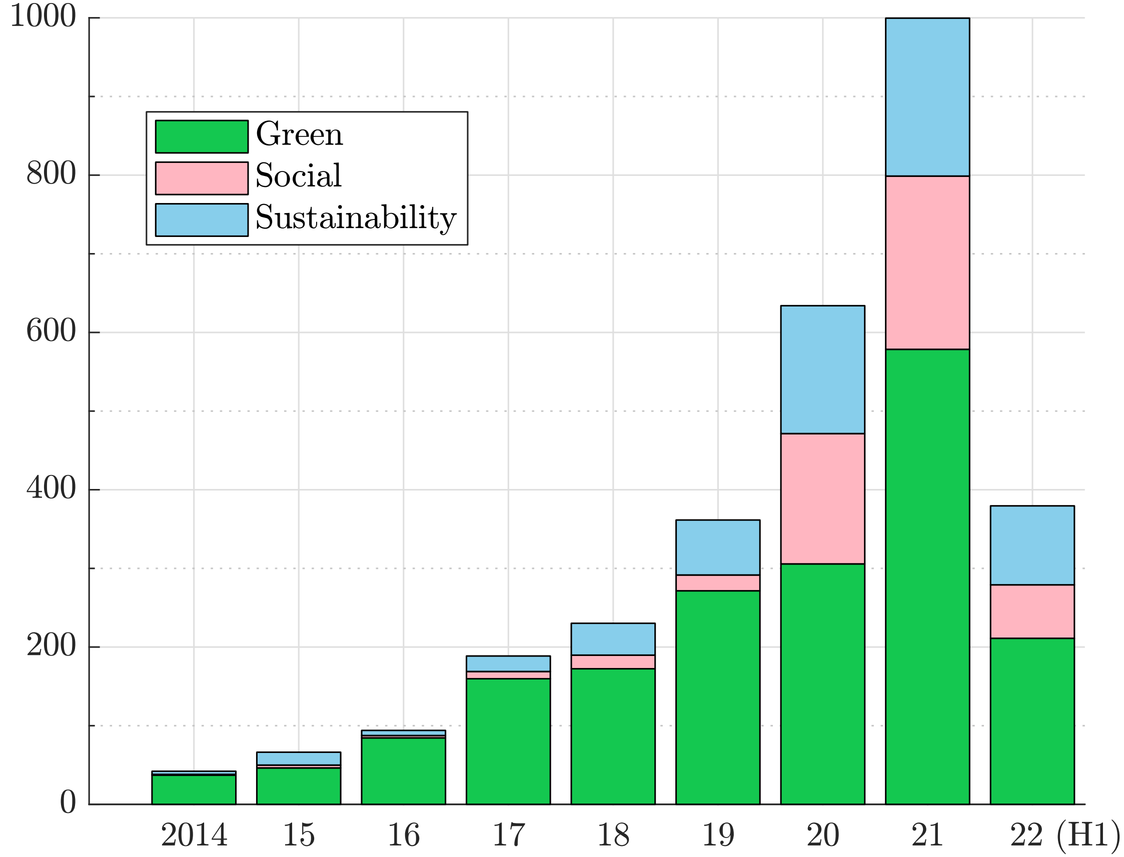
Table 68: Sustainable fixed-income market

Theme		Label	Format
GSS+	GSS	Green	Use of proceeds
		Social	Use of proceeds
		Sustainability	Use of proceeds
	Transition	Sustainability-Linked	Entity KPI-linked
		Transition	Use of proceeds

Source: CBI (2022).

Sustainable fixed-income products

Figure 87: Issuance of GSS securities (in $ bn)



Source: `https://www.climatebonds.net/market/data`.

Green bonds

Definition

Definition

Green bonds are any type of bond instrument where the proceeds or an equivalent amount will be exclusively applied to finance or re-finance, in part or in full, new and/or existing eligible **green projects** and which are aligned with the four core components of the Green Bond Principles (GBP).

Source: ICMA (2021).

⇒ Green bonds are "*regular*" bonds[10] aiming at funding projects with positive environmental and/or climate benefits

[10] A regular bond pays regular interest to bondholders

Green bonds

Green Bonds Principles

Green Bonds Principles (GBP)

The 4 core components of the GBP are:

1. Use of proceeds
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

https://www.icmagroup.org/sustainable-finance/the-principles-guidelines-and-handbooks

Green bonds

Green Bonds Principles

The use of proceeds includes:

- Renewable energy
- Energy efficiency
- Pollution prevention (e.g. GHG control, soil remediation, waste recycling)
- Sustainable management of living natural resources (e.g. sustainable agriculture, sustainable forestry, restoration of natural landscapes)
- Terrestrial and aquatic biodiversity conservation (e.g. protection of coastal, marine and watershed environments)
- Clean transportation
- Sustainable water management
- Climate change adaptation
- Eco-efficient products
- Green buildings

Green bonds

Green Bonds Principles

With respect to the **process for project evaluation and selection** (component 2), the issuer of a green bond should clearly communicate:

- the environmental sustainability objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the "*balance sheet*" and the allocation of funds[11]
- An external review (not mandatory but highly recommended)

[11] The proceeds should be credited to a sub-account

Green bonds

Green Bonds Principles

The **reporting** (component 4) must be based on the following pillars:

- Transparency
- Description of the projects, allocated amounts and expected impacts
- Qualitative performance indicators
- Quantitative performance measures (e.g. energy capacity, electricity generation, GHG emissions reduced/avoided, number of people provided with access to clean power, decrease in water use, reduction in the number of cars required)

Green bonds

Several standards

Standardization is strongly required by investors and regulators

- Green Bond Principles[12] (ICMA, 2021)
- Climate Bonds Standard[13] (CBI, 2019)
- EU Green Bond Standard (2021)
- China Green Bond Principles (PBOC, CBIRC, July 2022)
- Asean Green Bond Standards (ACMF, 2018)

[12] The first version is published in January 2014

[13] The first version is published in November 2011

Green bonds

Types of debt instruments

Asset-linked bond structures

- Regular bond
- Revenue bond
- Project bond
- Green loans

Asset-backed bond structures

- Securitized bond
- Project bond
- ABS/MBS/CLO/CDO
- Covered bond

Green bonds

Certification

- Second party opinion provided by ESG rating agencies (ISS, Sustainalytics, Vigeo-Eiris);
- Certification by specialized green bond entities (CBI, CICERO, DNV);
- Green bond assessment by statistical rating organizations (Moody's, S&P).

Green bonds

Examples

- Solar bond by the City of San Francisco in 2001
- Equity-linked climate awareness bond by the European Investment Bank (EIB) in 2007
- First green bond issued by the World Bank (in collaboration with Skandinaviska Enskilda Banken) in November 2008
- First corporate green bonds: French utility company EDF ($1.8 bn) and Swedish real estate company Vasakronan ($120 bn)
- Toyota introduced the auto industry's first-ever asset-backed green bond in 2014 ($1.75 bn)
- The Commonwealth of Massachusetts issued the first municipal green bond in 2013 ($100 mn)
- The first sovereign green are: Poland in December 2016 ($1 bn) and France[14] in January 2017 ($10 bn)

[14] Green OAT 1.75% 25 June 2039.

Green bonds
The green bond market

Green bond issuers

- Sovereigns (agencies, municipals, governments)
- Multilateral development banks (MDB)
- Energy and utility companies
- Banks
- Other corporates

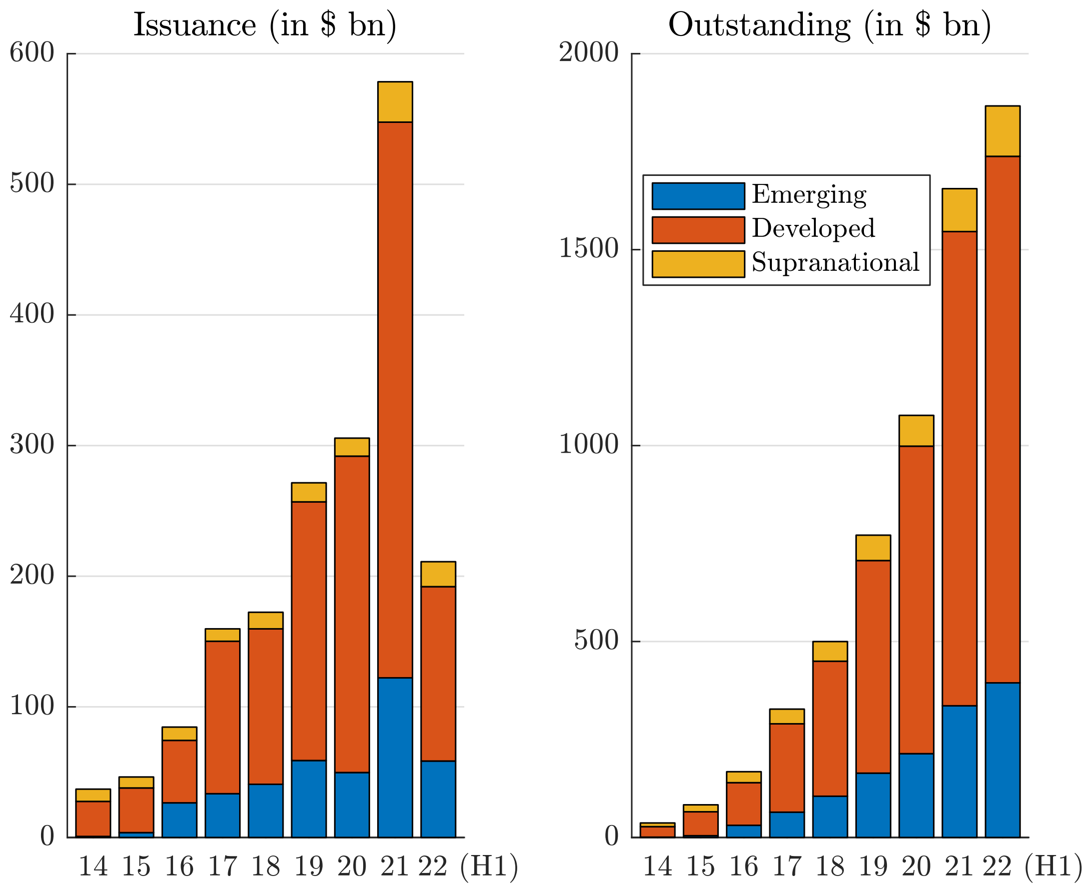
Green bond investors

- Pension funds
- Sovereign wealth funds
- Insurance companies
- Asset managers
- Retail investors (e.g. employee savings plans)

Strong imbalance between supply and demand

Green bonds
The green bond market

Figure 88: Issuance and notional outstanding of green debt by market type

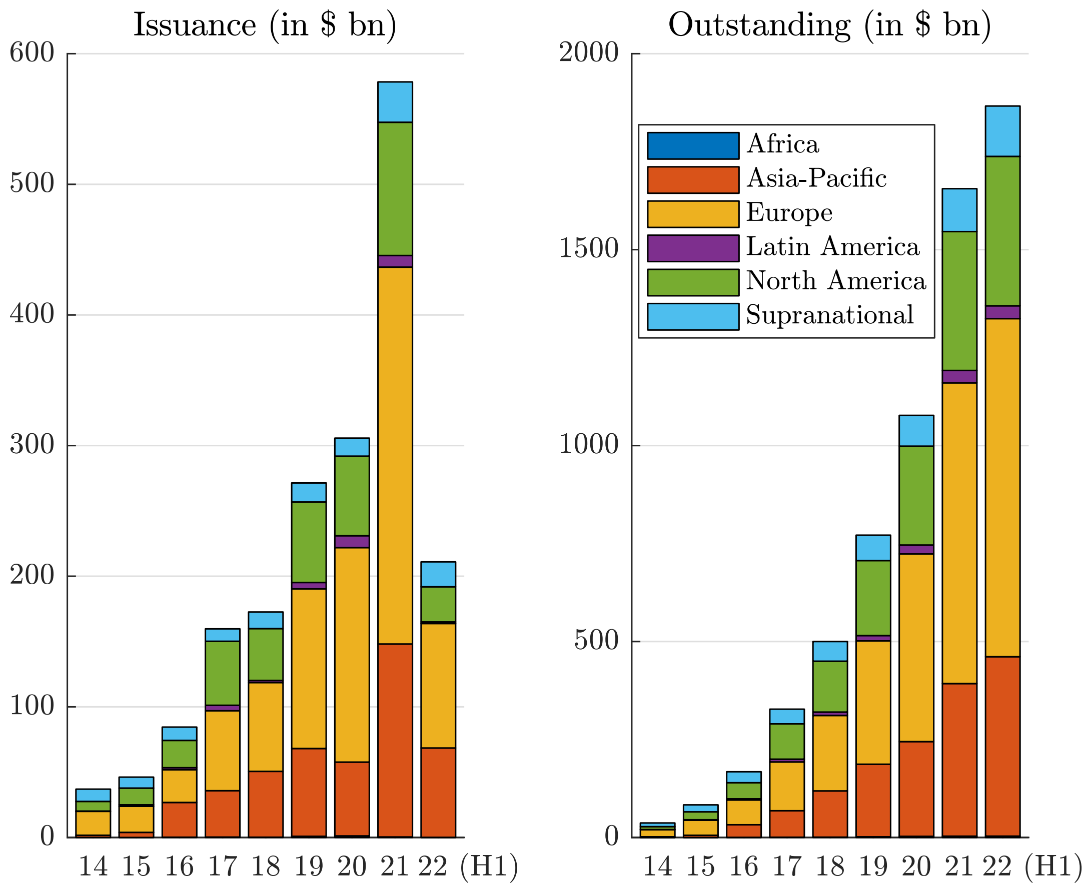


Source: `https://www.climatebonds.net/market/data`.

Green bonds

The green bond market

Figure 89: Issuance and notional outstanding of green debt by region

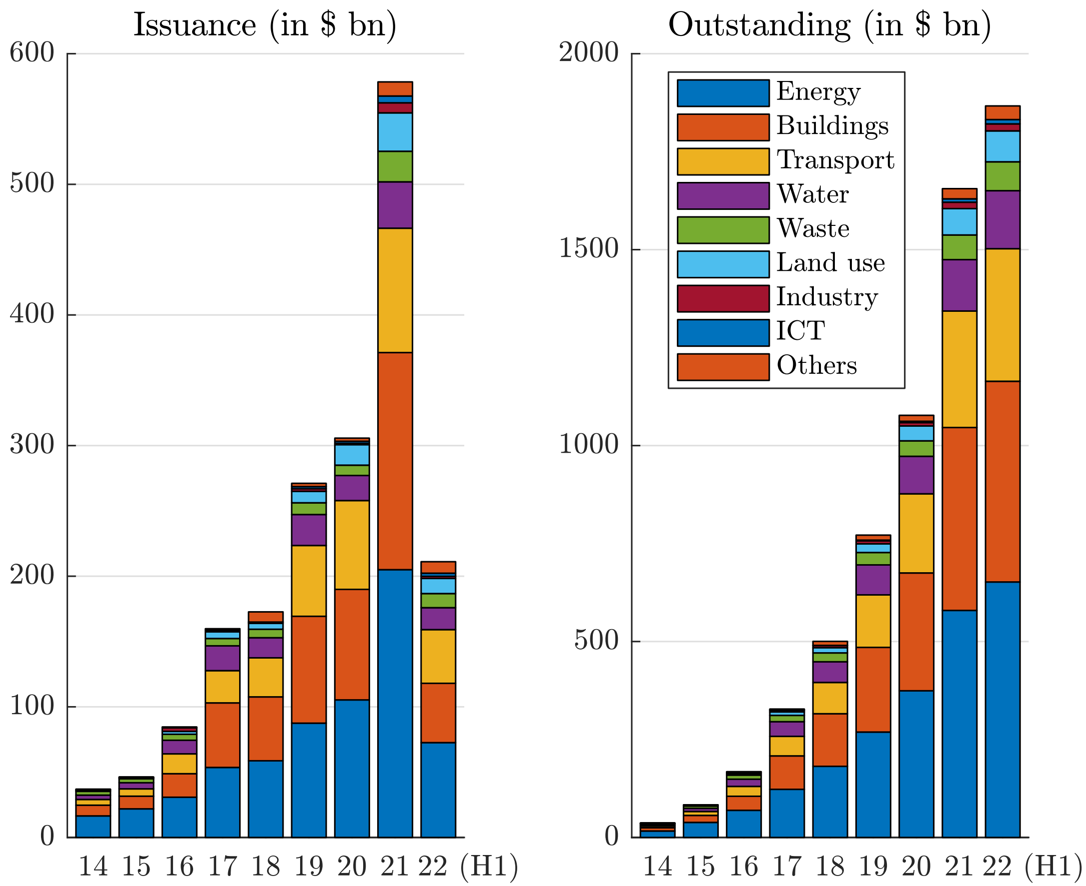


Source: `https://www.climatebonds.net/market/data`.

Green bonds

The green bond market

Figure 90: Issuance and notional outstanding of green debt by use of proceeds

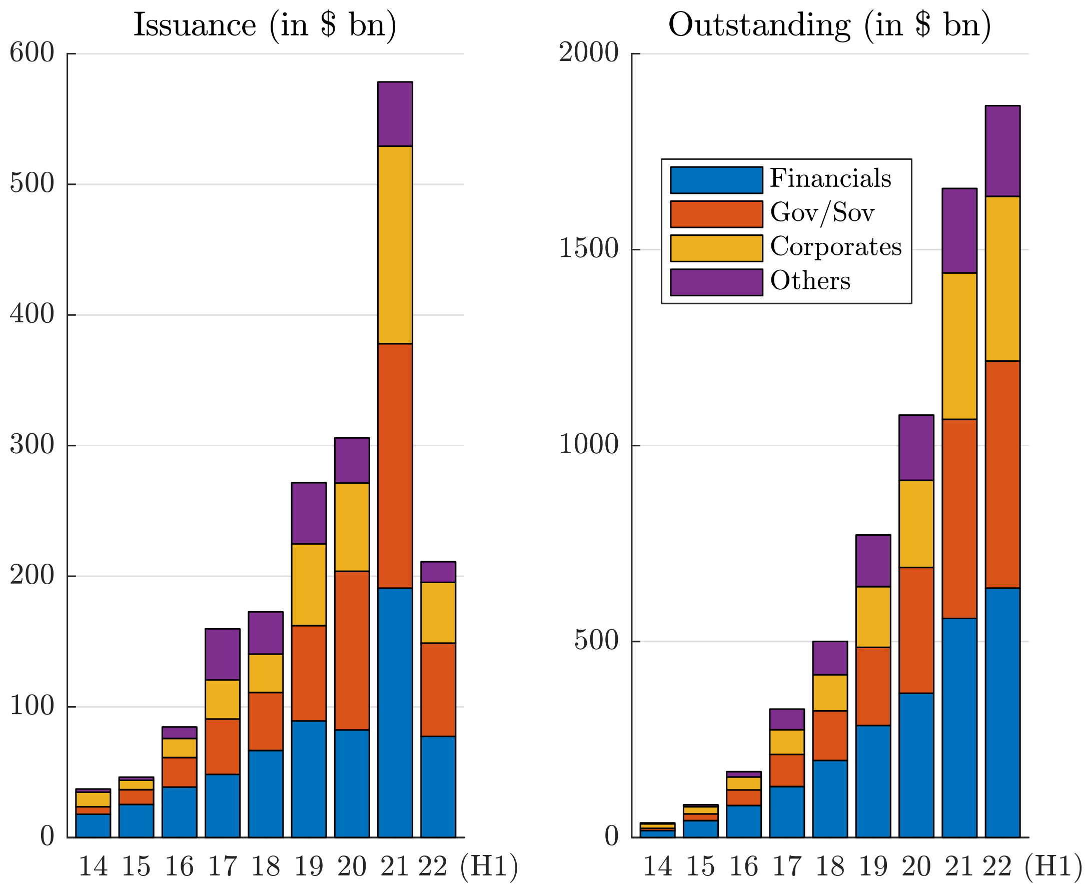


Source: `https://www.climatebonds.net/market/data`.

Green bonds

The green bond market

Figure 91: Issuance and notional outstanding of green debt by issuer type



Source: `https://www.climatebonds.net/market/data`.

Green bonds

How to investing in green bonds

Example of green bond funds:

- Allianz IG green bond fund
- Amundi RI impact green bonds
- AXA WF ACT green bonds
- BNP Paribas green bond
- Calvert green bond fund
- Mirova global green bond fund
- TIAA-CREF green bond fund
- Etc.

Green bonds

How to investing in green bonds

List of green bond indices:

- Bloomberg Barclays MSCI Global Green Bond Index
- S&P Green Bond Index
- Solactive Green Bond Index
- ChinaBond China Climate-Aligned Bond Index:
- ICE BofA Green Index

$\Rightarrow$ ETF and index funds

Green bonds

The economics of green bonds

[...] "I show that investors respond positively to the issuance announcement, a response that is stronger for first-time issuers and bonds certified by third parties. The issuers improve their environmental performance post-issuance (i.e., higher environmental ratings and lower CO_2 emissions) and experience an increase in ownership by long-term and green investors. Overall, the findings are consistent with a signaling argument – by issuing green bonds, companies credibly signal their commitment toward the environment." (Flammer, 2021, page 499).

Green bonds
The economics of green bonds

Green bonds = second-best instrument

Green bonds

The green bond premium

Definition

- The green bond premium (or greenium) is the difference in pricing between green bonds and regular bonds
- The greenium is defined as:

$$\boldsymbol{g} = y\left(\mathrm{GB}\right) - y\left(\mathrm{CB}\right)$$

where $y\left(\mathrm{GB}\right)$ is the yield (or return) of the green bond and $y\left(\mathrm{CB}\right)$ is the yield (or return) of the conventional twin bond

Green bonds

The green bond premium

- From the issuer's point of view, a green bond issuance is more expensive than a conventional issuance due to the need for external review, regular reporting and impact assessments
- From the investor's point of view, there is no fundamental difference between a green bond and a conventional bond, meaning that one should consider a negative green bond premium as a market anomaly

Green bonds

The green bond premium

Green twin bonds

- Introduced in 2020 by Germany
- Issuance of a green and conventional bond at the same time with the same characteristics
- Investors may swap the green bond with the conventional bond any time, but not vice-versa
- Liquidity of the green bond market $\nearrow$

Green bonds
The green bond premium

Examples of twin bonds:

- On 3 September 2020, the 10-year German green bond with a coupon of 0.00% was priced 1 basis point below the 10-year conventional German bond
- On 19 January 2022, Denmark issued a 10-year green bond with the same maturity, interest payment dates and coupon rate as the conventional 2031 Danish bond. The effective yield of the green bond was 5 basis points below the twin regular bond

Green bonds

The green bond premium

Example #1

We consider a 10-year green bond GB_1 whose current price is equal to 91.35. The corresponding conventional twin bond is a 20-year regular bond, whose remaining maturity is exactly equal to ten years and its price is equal to 90.07%. We assume that the two bonds have the same coupon level, which is equal to 4%.

Green bonds

The green bond premium

Computation of the greenium with the current yield:

- We have:

$$y\,(\mathrm{GB}) = \frac{4}{91.35} = 4.379\%$$

  and:

$$y\,(\mathrm{CB}) = \frac{4}{90.07} = 4.441\%$$

- We deduce that the greenium is equal to:

$$\boldsymbol{g} = 4.441\% - 4.379\% = -6.2 \text{ bps}$$

Green bonds

The green bond premium

Computation of the greenium with the yield to maturity:

- We solve the equation:

$$\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 91.35$$

and find:

$$y\left(\mathrm{GB}\right) = 5\%$$

- We solve the equation:

$$\sum_{t=1}^{10} 4e^{-ty} + 100e^{-10y} = 90.07$$

and find:

$$y\left(\mathrm{CB}\right) = 5.169\%$$

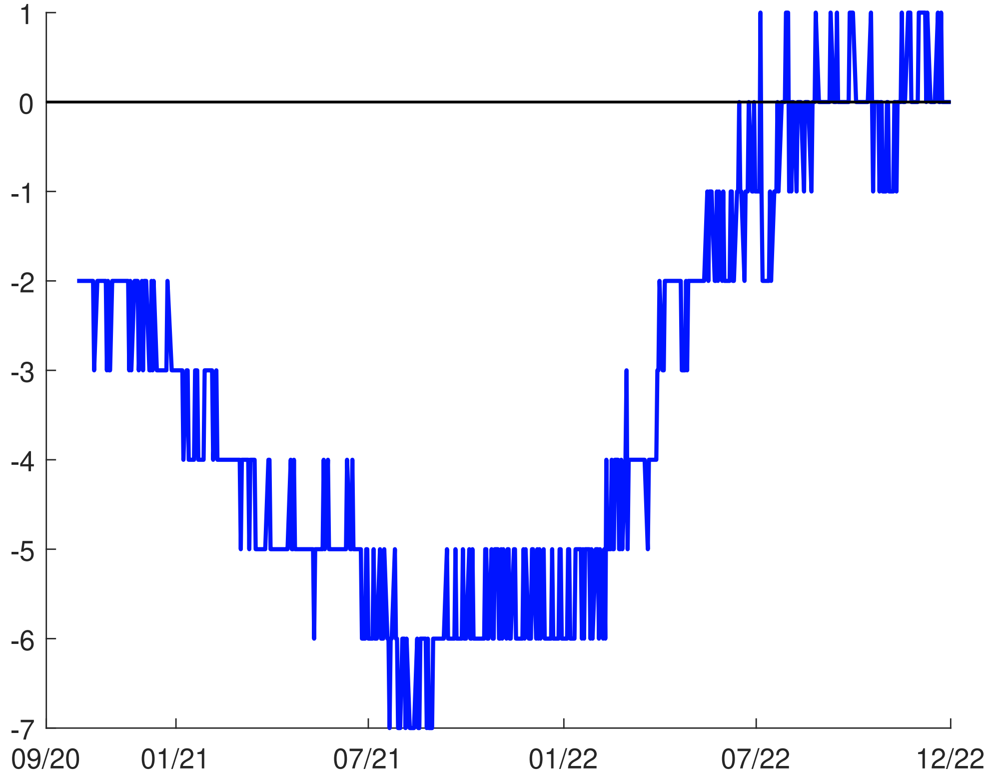
- We deduce that the greenium is equal to:

$$\boldsymbol{g} = 5\% - 5.169\% = -16.9 \text{ bps}$$

Green bonds

The green bond premium

Figure 92: Greenium in bps of the German green (twin) bond (DBR 0% 15/08/2030)



Source: ICE (2022).

Green bonds

The green bond premium

What about the greenium when the green bond is not a twin bond?

$\Rightarrow$ We must distinguish primary and secondary markets

Green bonds

The green bond premium

- In the primary market, the greenium is negative ($\approx 5 - 10$ bps on average)
- How to measure the persistence of the greenium in the secondary market?

Green bonds

The green bond premium

There are two approaches:

1. Bottom-up approach
   - Compares the green bond of an issuer with a synthetic conventional bond of the same issuer
   - Same characteristics in terms of currency, seniority and duration
2. Top-down approach
   - Compare a green bond index portfolio to a conventional bond index portfolio
   - Same characteristics in terms of currency, sector, credit quality and maturity

Green bonds

The green bond premium

Bottom-up approach

1. We filter all the conventional bonds, which has the same issuer, the same currency, and the same seniority of the green bond GB
2. We select the two conventional bonds CB_1 and CB_2 which are the nearest in terms of modified duration:

$$\left|\mathrm{MD}\left(\mathrm{GB}\right)-\mathrm{MD}\left(\mathrm{CB}_j\right)\right|_{j\neq 1,2}\geq \sup_{j=1,2}\left|\mathrm{MD}\left(\mathrm{GB}\right)-\mathrm{MD}\left(\mathrm{CB}_j\right)\right|$$

3. We perform the linear interpolation/extrapolation of the two yields $y\left(\mathrm{CB}_1\right)$ and $y\left(\mathrm{CB}_2\right)$:

$$y\left(\mathrm{CB}\right)=y\left(\mathrm{CB}_1\right)+\frac{\mathrm{MD}\left(\mathrm{GB}\right)-\mathrm{MD}\left(\mathrm{CB}_1\right)}{\mathrm{MD}\left(\mathrm{CB}_2\right)-\mathrm{MD}\left(\mathrm{CB}_1\right)}\left(y\left(\mathrm{CB}_2\right)-y\left(\mathrm{CB}_1\right)\right)$$

Green bonds

The green bond premium

Example #2

- We consider a green bond, whose modified duration is 8 years. Its yield return is equal to 132 bps
- We can surround the green bond by two conventional bonds with modified duration 7 and 9.5 years. The yield is respectively equal to 125 and 148 bps
- The interpolated yield is equal to:

$$\begin{aligned} y(\mathrm{CB}) &= 125 + \frac{8-7}{9.5-7}(148-125) \\ &= 134.2 \text{ bps} \end{aligned}$$

- It follows that the greenium is equal to -2.2 bps:

$$\boldsymbol{g} = 132 - 134.2 = -2.2 \text{ bps}$$

Green bonds

The green bond premium

Top-down approach

1. We consider a portfolio $w = (w_1, \ldots, w_n)$ of green bonds.
2. We perform a clustering analysis by considering the 4-uplets (Currency $\times$ Sector $\times$ Credit quality $\times$ Maturity)
3. Let (C_h, S_j, R_k, M_l) be an observation for the 4-uplet (e.g. EUR, Financials, AAA, 1Y-3Y). We compute its weight:

$$\omega_{h,j,k,l} = \sum_{i \in (C_h, S_j, R_k, M_l)} w_i$$

4. The greenium is then defined as the weighted excess yield:

$$\boldsymbol{g} = \sum_{h,j,k,l} \omega_{h,j,k,l} \left(y_{h,j,k,l}(\mathrm{GB}) - y_{h,j,k,l}(\mathrm{CB}) \right)$$

Green bonds

The green bond premium

Main result (Ben Slimane *et al.*, 2020)

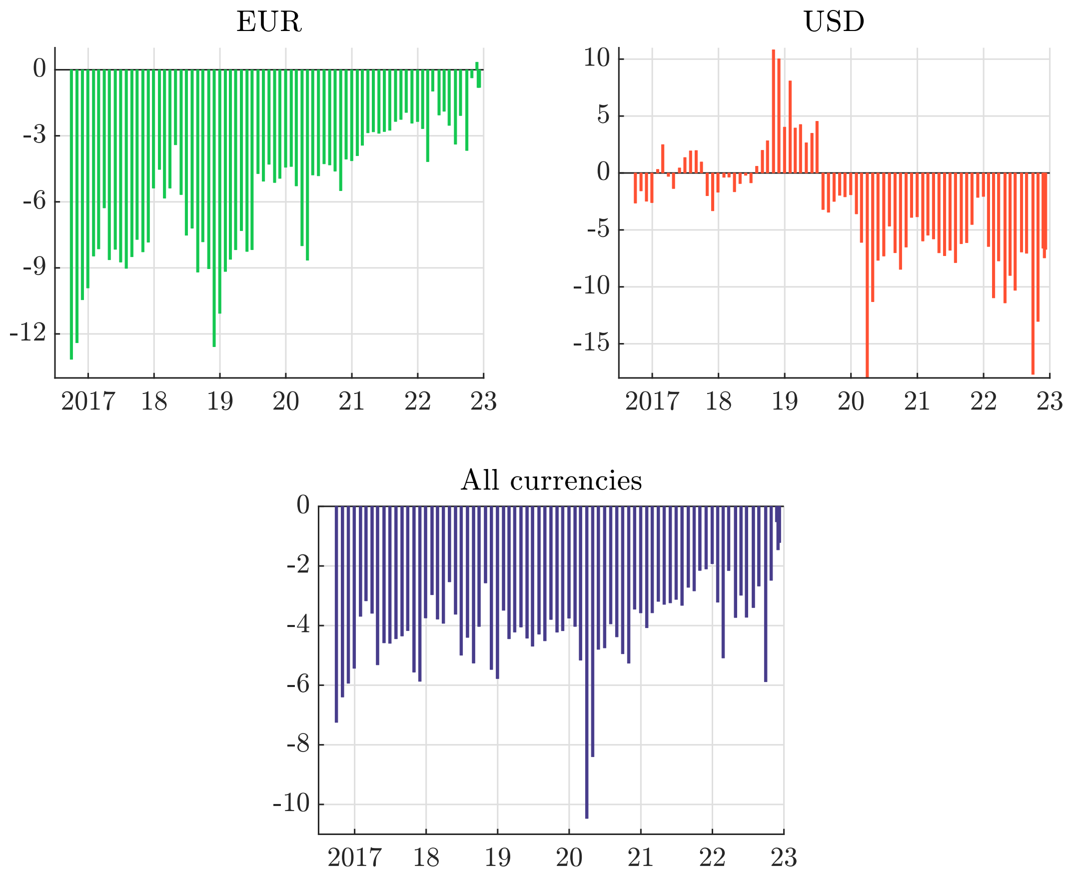
The greenium is negative between -5 and -2 bps on average

Other results:

- Differences between sectors, currencies, maturities, regions and ratings
- Transatlantic divided between US and Europe
- The volatility of green bond portfolios are lower than the volatility of conventional bond portfolios $\Rightarrow$ identical Sharpe ratio since the last four years
- Time-varying property of the greenium

Green bonds
The green bond premium

Figure 93: Evolution of the greenium (in bps)



Source: Ben Slimane *et al.* (2020)

Green bonds

The green bond premium

Green financing $\Leftrightarrow$ green investing

1. Bond issuers have a competitive advantage to finance their environmental projects using green bonds instead of conventional bonds
2. Another premium? the “green bond issuer premium”

Social bonds

Definition

Social Bonds are any type of bond instrument where the proceeds, or an equivalent amount, will be exclusively applied to finance or re-finance in part or in full new and/or existing eligible **social projects** and which are aligned with the four core components of the Social Bond Principles (SBP).

Source: ICMA (2021), `https://www.icmagroup.org/sustainable-finance`

Social bonds

Social Bonds Principles

Social Bonds Principles (SBP)

The 4 core components of the SBP are:

1. Use of proceeds
    1. Eligible social project categories
    2. **Target populations**
2. Process for project evaluation and selection
3. Management of proceeds
4. Reporting

https://www.icmagroup.org/sustainable-finance/the-principles-guidelines-and-handbooks

Social bonds

Social Bonds Principles

The **eligible social projects categories** (component 1) are:

- Affordable basic infrastructure (e.g. clean drinking water, sanitation, clean energy)
- Access to essential services (e.g. health, education)
- Affordable housing (e.g. sustainable cities)
- Employment generation (e.g. pandemic crisis)
- Food security and sustainable food systems (e.g. nutritious and sufficient food, resilient agriculture)
- Socioeconomic advancement and empowerment (e.g. income inequality, gender inequality)
- Etc.

Social bonds

Social Bonds Principles

The **target populations** (component 1) are:

- Living below the poverty line
- Excluded and/or marginalised populations/communities
- People with disabilities
- Migrants and /or displaced persons
- Undereducated
- Unemployed
- Women and/or sexual and gender minorities
- Aging populations and vulnerable youth
- Etc.

Social bonds
Social Bonds Principles

With respect to the **process for project evaluation and selection** (component 2), the issuer of a social bond should clearly communicate:

- the social objectives
- the eligible projects
- the related eligibility criteria

The **management of proceeds** (component 3) includes:

- The tracking of the "*balance sheet*" and the allocation of funds[15]
- An external review (not mandatory but highly recommended)

[15] The proceeds should be credited to a sub-account

Social bonds

Social Bonds Principles

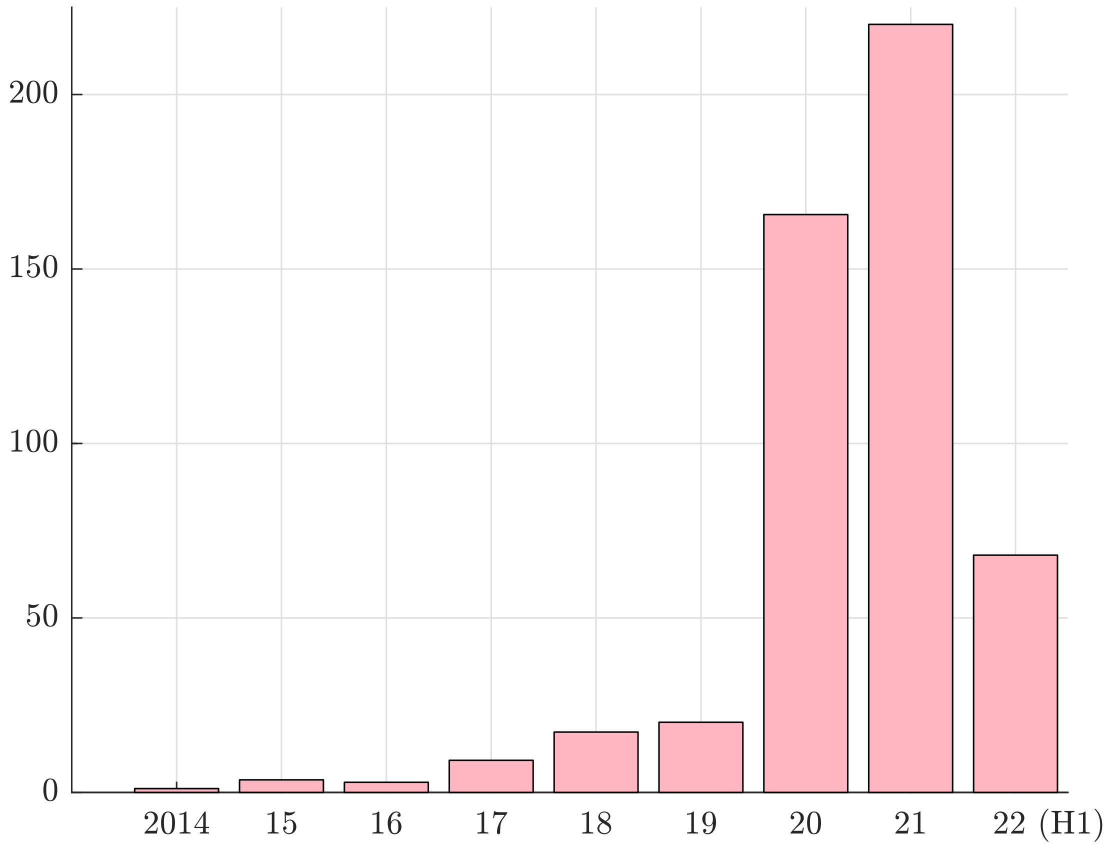
The **reporting** (component 4) must be based on the following pillars:

- Transparency
- Description of the projects, allocated amounts and expected impacts
- Qualitative performance indicators
- Quantitative performance measures (e.g. number of beneficiaries)

Social bonds

Market

Figure 94: Issuance of social bonds



Source: `https://www.climatebonds.net/market/data`.

Social bonds

Examples

- Instituto de Crédito Oficial (Spanish state-owned bank, March 2020)
  "*The Social Bond proceeds under ICO's Second – Floor facilities will be allocated to loans to finance small, medium and micro enterprises with an emphasis on employment creation or employment retention in: (1) specific economically underperforming regions of Spain; (2) specific municipalities of Spain facing depopulation; (3) regions affected by a natural disaster. [...] The target populations are SMEs in line with European Union's standards.*"

Social bonds

Examples

- Pepper Money (non-bank lender in Australia and New Zealand, April 2022)
  "*The positive social impact of a Pepper Money eligible social project derives from its direct contribution to improving access to financial services and socio-economic empowerment, by using proprietary systems to make flexible loan solutions available to applicants who are not served by traditional banks. [...] Pepper Money is seeking to achieve positive social outcomes for a target population of Australians that lack access to essential financial services and experience inequitable access to and lack of control over assets. Pepper Money directly aims to address the positive social outcome of home ownership for borrowers who may have complexity in their income streams, gaps in their loan documentation or have adverse credit history. Traditionally, this cohort has been underserved by banks that rely on inflexible algorithmic loan application processing.*"

Social bonds
Examples

- Danone (French multinational food-products corporation, March 2018)
  "*The eligible project categories are: (1) research & innovation for advanced medical nutrition (target populations: infants, pregnant women, patients and elderly people with specific nutritional needs), (2) social inclusiveness (target poputions: farmers, excluded and/or marginalised populations and/or communities, people living under the poverty line, rural communities in developing countries), (3) responsible farming and agriculture (target populations: milk producers, farmers), etc.*"

Social bonds
Examples

- Korian (European care group, October 2021)
  "The proceeds of any instrument issued under the framework will be used [...] to provide services, solutions, and technologies that will enable Korian to meet at least one of its social objectives: (1) to increase and improve long-term care nursing home capacity for dependent older adults; (2) to increase and improve medical capacity for people in need of medical support; (3) to increase and improve access to alternative, nonmedical services, technologies, and housing solutions that facilitate the retention of older adults' autonomy; and (4) to improve the daily provision of care to and foster a safer living environment for its patients. [...] Furthermore, Korian's target populations are older adults, which Korian defines as being over 65 years of age, and those who are dependent on others for some degree of care, which is defined by the health authorities or insurance system of the respective country."

Social bonds

Examples

- JASSO (Japan Student Services Organization, July 2022)
  "*The social project categories concern the financing of the 'Category 2 Scholarship Loans' (interest-bearing scholarship loans that have to be repaid) while the target population is made up of students with financial difficulties.*"

Other sustainability-related instruments

Sustainability bonds

$$\text{Sustainability bond} = \text{GBP} + \text{SBP}$$

Remark

According to CBI, the cumulative issuance of sustainability bonds reaches $620 bn at the end of June 2022

Other sustainability-related instruments

Sustainability-linked bonds (SLB)

Sustainability-linked bond (SLB)

- Two principles:
  - = a sustainability bond (green/social)
  - + a step up coupon if the KPI is not satisfied

  ⇒ forward-looking performance-based instrument
- The financial characteristics of the bond depends on whether the issuer achieves predefined ESG objectives
- Those objectives are:
  1. measured through predefined Key Performance Indicators (KPI)
  2. assessed against predefined Sustainability Performance Targets (SPT)

Other sustainability-related instruments

Sustainability-linked bonds (SLB)

ENEL General Purpose SDG Linked Bond

- SDG: 7 (affordable and clean energy), 13 (climate action), 9 (industry, innovation and infrastructure) and 11 (sustainable cities and communities)
- SDG 7 target: renewables installed capacity as of December 31, 2021 $\geq$ 55% (confirmed by external verifier)
- One time step up coupon of 25 bps if SDG 7 is not achieved
- On April 2022, the independent report produced by KPMG certifies that "*the renewables installed capacity percentage as of December 31, 2021 is equal to* 57.5%".

Other sustainability-related instruments

Sustainability-linked bonds (SLB)

H&M sustainability-linked bond

- 18 February 2021
- €500 mn
- Maturity of 8.5 years
- The annual coupon rate is 25 bps
- The objectives to achieve by 2025 are:
  - KPI_1 Increase the share of recycled materials used to 30% (SPT_1)
  - KPI_2 Reduce emissions from the Group's own operations (scopes 1+2) by 20% (SPT_2)
  - KPI_3 Reduce scope 3 emissions from fabric production, garment manufacturing, raw materials and upstream transport by 10% (SPT_3)
- The global KPI is equal to $40\% \times \mathrm{KPI}_1 + 20\% \times \mathrm{KPI}_2 + 40\% \times \mathrm{KPI}_3$
- The step-up of the coupons can consequently be 0%, 20%, 40%, 60%, 80% or 100% of the total step-up rate

Other sustainability-related instruments

Sustainability-linked bonds (SLB)

According to Berrada *et al.* (2022), "*the SLB market has grown strongly since its inception. [...] Bloomberg identifies a total of* 434 *outstanding bonds flagged as 'sustainability-linked' as of February 2022. In contrast, in 2018, there was only a single SLB. The amount raised through the single 2018 SLB issue was $0.22 bn, whereas the total amount raised through all SLBs issued in 2021 was approximately $160 bn*".

- The large majority of SLB issues address exclusively **E** issues (65%) or a combination of **E**, **S** and **G** issues (17%) or **E** and **G** issues (3%)
- The most frequent KPI concerns GHG emissions (40 %), followed by the issuer's global ESG score (14 %)

Other sustainability-related instruments

Transition bonds

- Financial instruments to support the transition of an issuer, which has significant current carbon emissions
- Fund projects such as renewable energy developments, energy efficiency upgrades, etc.
- The final objective of the bond issuer is always to reduce their carbon emissions
- For example, transition bonds can be used to switch diesel powered ships to natural gas or to implement carbon capture and storage.

Sustainable real assets

Definition

Principle

- Financial risks ⇒ financial performance (return, volatility, Sharpe ratio, etc.)
- Extra-financial risks ⇒ financial performance (return, volatility, Sharpe ratio, etc.)
- **Extra-financial risks ⇒ extra-financial performance** (ESG KPIs)

What is the final motivation of the ESG investor?

Financial performance or/and extra-financial performance?

Definition

Definition

The key elements of impact investing are:

1. Intentionality
   The intention of an investor to generate a positive and measurable social and environmental impact
2. Additionality
   Fulfilling a positive impact beyond the provision of private capital
3. **Measurement**
   Being able to account for in a transparent way on the financial, social and environmental performance of investments

Source: Eurosif (2019)

The investor must be able to measure its impact from a quantitative point of view

GIIN

Figure 95: Global Impact Investing Network (GIIN)

GIIN

GLOBAL IMPACT INVESTING NETWORK

https://thegiin.org

The example of social impact bonds

Social impact bond (SIB) = **pay-for-success bond** ($\approx$ call option)

The Peterborough SIB

- On 18 March 2010, the UK Secretary of State for Justice announced a six-year SIB pilot scheme that will see around 3 000 short term prisoners from Peterborough prison, serving less than 12 months, receiving intensive interventions both in prison and in the community
- Funding from investors will be initially used to pay for the services
- If reoffending is not reduced by at least 7.5%, the investors will receive no recompense

The example of sustainability-linked bonds

Sustainability-linked[16] (SLB) = **pay-for-failure bond** ($\approx$ cap option)

Risk taker

SIB: investor viewpoint $\neq$ SLB: issuer viewpoint

[16] See the examples of ENEL and H&M previouly

Measurement tools

Impact assessment and metrics

- Avoided CO2 emissions in tons per $M invested
- Amount of clean water produced by the project
- Number of children who are less obese
- Land management
- Affordable housing
- Job creation
- Construction of student housing

Sustainable development goals (SDG)

The sustainable development goals are a collection of 17 interlinked global goals designed to be a "*blueprint to achieve a better and more sustainable future for all*"

https://sdgs.un.org

Sustainable development goals (SDG)

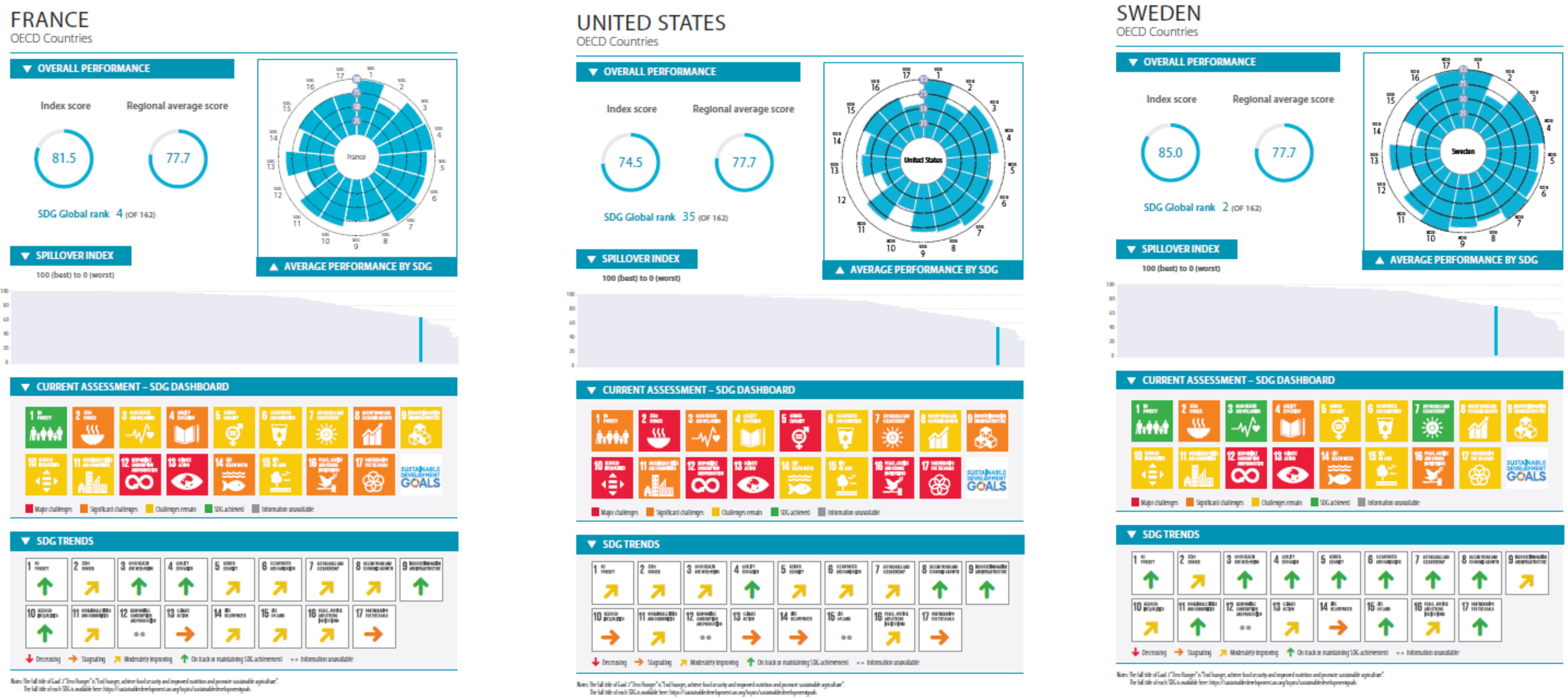
Figure 96: The map of sustainable development goals

Sustainable development goals (SDG)

Figure 97: Mapping the SDGs across E, S and G

Sustainable development goals (SDG)

Figure 98: Examples of sovereign SDG reports



Source: Sustainable Development Report 2019, `https://dashboards.sdgindex.org`

The challenge of reporting

- Impact reporting and investment standards (IRIS) proposed by GIIN
- EU taxonomy on sustainable finance
- Non-financial reporting directive 2014/95/EU (NFRD)
- Carbon accounting

The challenge of reporting

Table 69: Impact reporting of the CPR Invest — Social Impact fund

	Social indicator		Coverage ratio	
	Global Index	CPR Fund	Global Index	CPR Fund
CEO pay ratio	333	114	82%	84%
% of women in the board direction	18%	19%	79%	75%
Hours of training	33 hours	39 hours	33%	45%
Trade union rate	36%	45%	25%	36%

Source: CPR Asset Management (2021)

The challenge of reporting

- Amundi ARI – Impact Green Bonds (Annual impact record 2020)
  - GHG avoided emissions per €1 mn invested per year : 586.5 $\mathrm{tCO_2e}$
  - GHG avoided emissions rebased per €1 mn invested per year 882.7 $\mathrm{tCO_2e}$
- CPR Invest – Climate Action
  - -69% of $\mathrm{tCO_2e}$ wrt MSCI ACWI
- CPR Invest – Food For Generations
  - **Water consumption**: 6 765 m3/meur for the fund vs 13 258 for the benchmark and 18 869 for the universe
  - **Waste recycling ratio**: 71.14% for the fund vs 66.45% for the benchmark and 67.22% for the universe

Source: Amundi (2021) and CPR Asset Management (2021)

The challenge of reporting

Table 70: Impact investing reporting of the Amundi Finance & Solidarité fund

	2020	Since inception (2012)
People housed	2 364	10 336
Job created/preserved	9 439	43 655
Care recipients	83 240	250 314
Trained people	18 702	59 686
Preserved agricultural farmland (hectare)	438	987
Waste recycling (ton)	82 590	219 287
Microcredit beneficiaries	60 171	276 514

Source: Amundi (2021)

The challenge of reporting

Figure 99: Companies' portfolio contribution of the Finance & Solidarité fund

Source: Amundi (2021)

Stewardship vs. engagement

$$\text{Voting} \subset \text{Engagement} \subset \text{Stewardship}$$

Stewardship vs. engagement

Figure 100: Difference between stewardship and engagement reports

Amundi Stewardship Report (2021)

Amundi Engagement Report (2021)

Source: Amundi corporate website, `https://about.amundi.com/esg-documentation`.

Stewardship

"It guides investors on how to implement the PRI's Principle 2, which sets out signatories' commitment to stewardship, stating: we will be active owners and incorporate ESG issues into our ownership policies and practices. [...] The PRI defines stewardship as the use of influence by institutional investors to maximise overall long-term value including the value of common economic, social and environmental assets, on which returns and clients' and beneficiaries' interests depend." (PRI, 2021).

Definition

Active ownership $\approx$ Engagement $\approx$ Shareholder activism

"investors who, dissatisfied with some aspect of a company's management or operations, try to bring about change within the company without a change in control" Gillan and Starks (2000).

Definition

- Conflicting interests between shareholders and management (separation between ownership and control)
- Stakeholder theory (Freeman, 2004)

Milton Friedman (1970)

"*the social responsibility of business is to increase its profits*"

Peter Drucker (1954)

"*leaders in every single institution and in every single sector . . . have two responsibilities. They are responsible and accountable for the performance of their institutions, and that requires them and their institutions to be concentrated, focused, limited. They are responsible also, however, for the community as a whole*"

Shareholder activism

Shareholder activism can take various forms

1. Engage behind the scene with management and the board
2. Propose resolutions (shareholder proposals)
3. Vote (form coalition/express dissent/call back lent shares)
4. Voice displeasure publicly (in the media)
5. Initiate a takeover (acquire a sizable equity share)
6. Exit (sell shares, take an offsetting bet)

Source: Bekjarovski and Brière (2018)

Shareholder activism

Engage behind the scenes

"Behind the curtain engagement involves private communication between activist shareholders and the firm's board or management, that tends to precede public measures such as vote, shareholder proposals and voice. In a sense, the existence of other forms of public activism can be taken as a signal that behind the scene engagements were unsuccessful. When it comes to environmental and social issues, writing to the board or management is a common method though which shareholders can express concern and attempt to influence corporate policy behind the curtain; alternatively, face to face meetings with management or non-executive directors are a more common behind the scene engagement method when it comes to governance." Bekjarovski and Brière (2018).

Shareholder activism

Engage behind the scenes

Three families of engagement:

1. on-going engagement, where the goal for investors is to explain their ESG policy and collect information from the company. For instance, they can encourage companies to adopt best ESG practices, alert companies on ESG risks or better understand sectorial ESG challenges;
2. engagement for influence (or protest), where the goal is to express dissatisfaction with respect to some ESG issues, make recommendations to the firm and measure/control ESG progress of companies;
3. pre-AGM engagement, where the goal is to discuss with companies any resolution items that the investor may vote against.

Shareholder activism

Engage behind the scenes

The three steps of identification are:

1. List of engagement issues
2. Screening of companies
3. List of targeted companies

The different stages of engagement tracking are:

- Issues are raised to the company;
- Issues are acknowledged by the company;
- The company develops a strategy to address the issues;
- The company implements changes and the issues are resolved;
- The company did not solve the issues and the engagement failed.

Shareholder activism

Propose resolutions

According to the SEC (Securities Exchange Act Rule 14a-8, §240):

> *"a shareholder proposal or resolution is a recommendation or requirement that the company and/or its board of directors take action, which the shareholder intend to present at a meeting of the company's shareholders. The proposal should state as clearly as possible the course of action that the shareholder believes the company should follow. If the proposal is placed on the company's proxy card, the company must also provide in the form of proxy means for shareholders to specify by boxes a choice between approval or disapproval, or abstention."*

Shareholder activism

Propose resolutions

Threshold criteria:

- US: $2 000 + No-action letter
- France, Germany and UK: 5% of the capital
- Italy: 2.5% of the capital
- Netherlands: 0.33%
- Spain: 3% of the capital

⇒ Collective shareholder proposals

Shareholder resolution = Escalation

Shareholder activism

Propose resolutions

Some figures (Russell 300 & 2022 proxy season)

- 98% of proposals are filed by the management, while less than 2% corresponds to shareholder resolutions;
- Only 60% of shareholder resolutions are voted; The other 40% are omitted, not presented, withdrawn or pending;
- The average number of proposals per company is around two;
- The proponents of shareholder resolutions are concentrated on a small number of investors or organisations (15 proponents were responsible of 75% of shareholder proposals);
- The repartition of shareholder proposals voted in 2022 was the following: 11% related to **E** issues, 41% related to **S** issues and 48% related to **G** issues

Shareholder activism

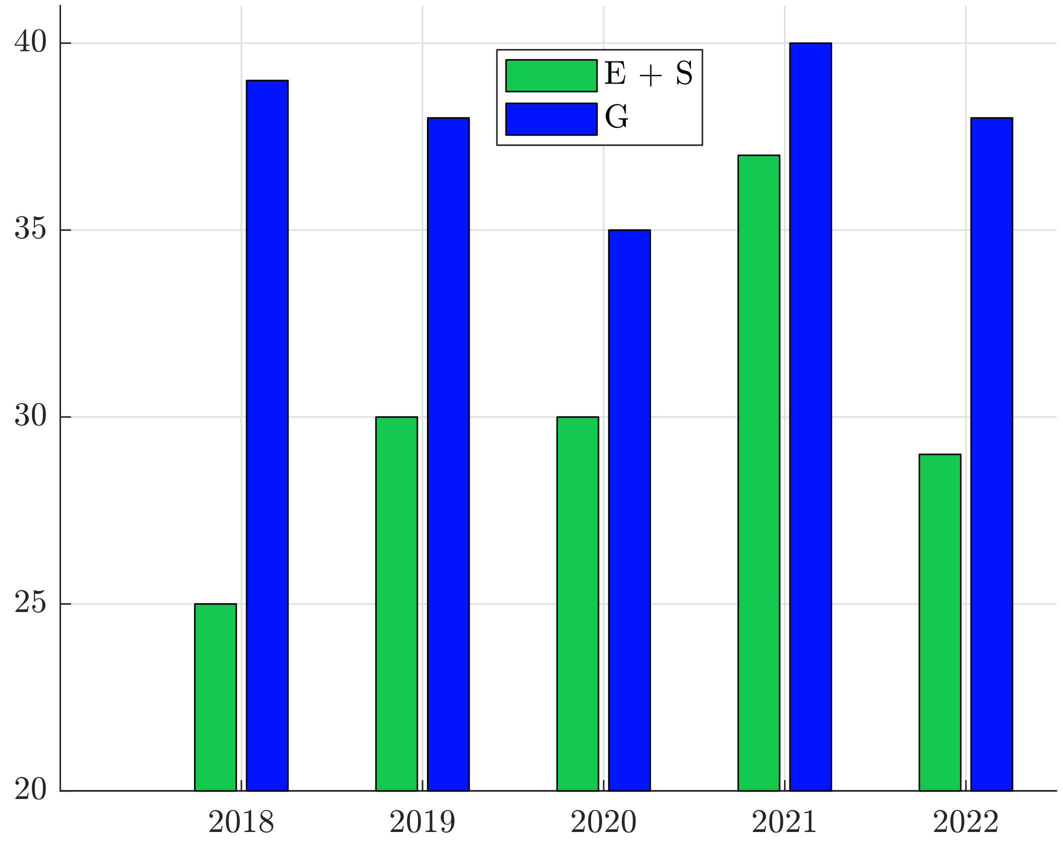
Vote

- Historical perspectives
- Importance of voting associations and NGOs
- US $\succ$ Europe
- The concept of proxy voting
  - Institutional Shareholder Services (ISS)
  - Glass Lewis
- Say on Pay (2002)
  - Support rate for Russell 3000 companies: 87% in 2022 (from 15.4% to 99%)
  - Results for Germany, France and Spain
- Say on Climate (2020)

Shareholder activism

Vote

Figure 101: Average support rate of shareholder proposals (Russell 3000 companies)



Source: PwC's Governance Insights Center (2022).

Shareholder activism

Vote

Some figures with Russell 3000 companies

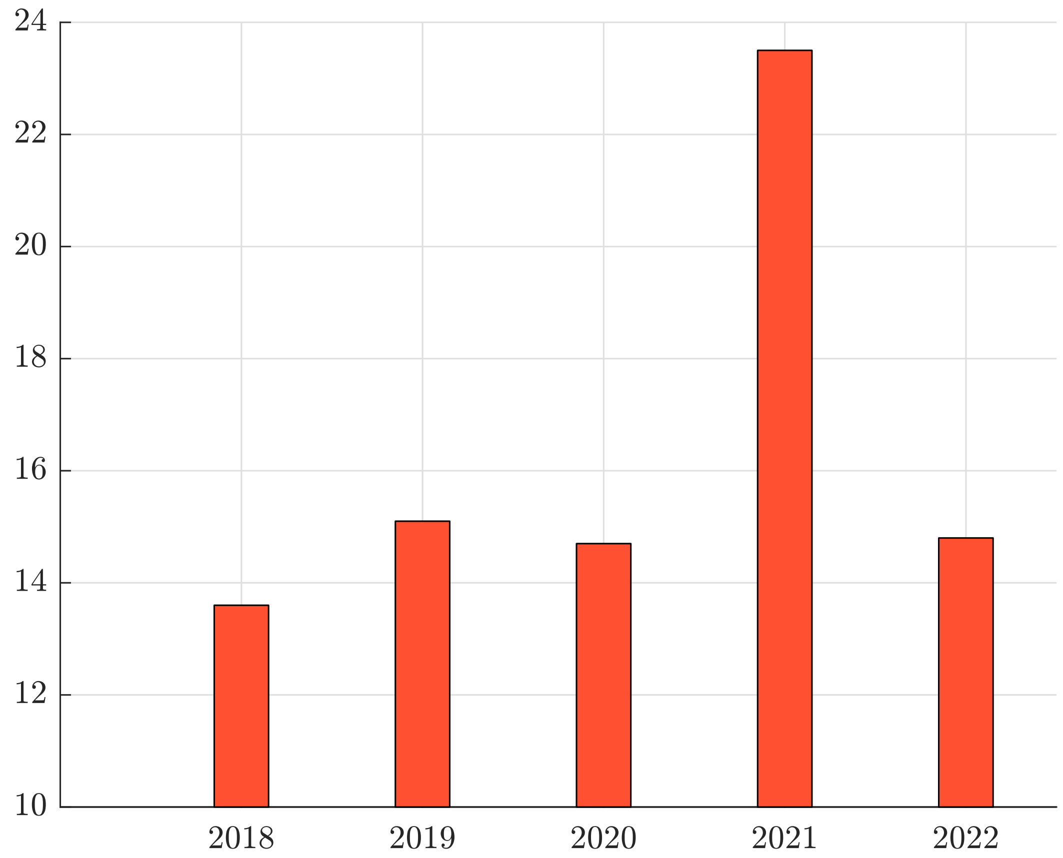
- 555 shareholder resolutions have been voted
- Only 82 have received majority support
- This means that one shareholder resolution was adopted for 37 companies!

What is the efficiency of vote? $\neq$ What is the impact of vote?

Shareholder activism

Vote

Figure 102: Pass rate of shareholder proposals (Russell 3000 companies)



Source: Tonello (2022).

Shareholder activism
Voice

- 1970: Publication of the book *Exit, Voice, and Loyalty: Responses to Decline in Firms, Organizations, and States* by the economist Albert Hirschman
- Exist-voice model: exist **versus** voice or exit **and** voice
- Voice as a form of escalation
- Impact of collaborative engagement (e.g., Climate Action 100+)
- Increasing involvement of NGOs in the debate on engagement and greenwashing

Shareholder activism

Initiate a takeover

$\Rightarrow$ Hedge funds

Shareholder activism

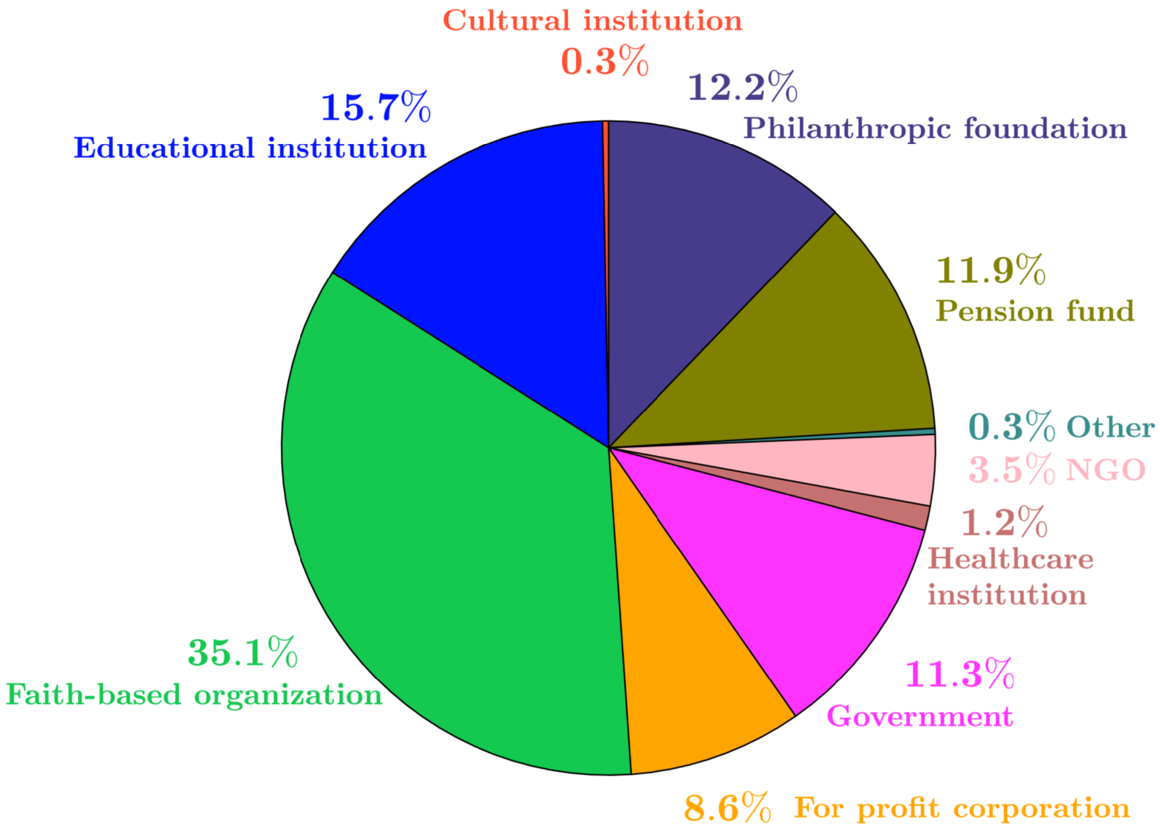
Exit

- **Exit** refers to the process of selling off investments in a particular company or industry
- **Divestment** is a more general term that implies a significant exposure reduction
- Divestment: Final step in an escalation strategy?

Shareholder activism

Exit

Figure 103: What kinds of institutions are divesting from fossil fuel?



Source: `https://divestmentdatabase.org`.

Shareholder activism

Exit

Case study: the Cambridge University endowment fund

"A dilemma faced by an increasing number of investors is whether to divest from environmentally damaging businesses or whether to enter into a dialogue with them. This predicament now has its epicentre in Cambridge, England, where the ancient University of Cambridge faces great pressure from students and staff to respond to the threat of climate breakdown. Having already received two reports on its approach to responsible investment, the university has appointed a new chief investment officer (CIO) who, alongside University Council and the wider university community, needs to consider the question of whether to divest from or to engage with fossil-fuel firms." Chambers et al. (2020).

Shareholder activism

Exit

Case study: Church of England Pensions Board

In 2020, they engaged with 21 companies. At the end of the process, 12 companies were supposed to make sufficient progress, while 9 companies were added to the list of restricted investments. These divestments totalled £32.23 mn (wrt £3.7 bn of assets under management).

Shareholder activism

Exit

Case study: The Universities Superannuation Scheme (USS)

- USS manage about £90 bn
- In 2020, they excluded certain sectors: tobacco manufacturing; thermal coal mining (coal to be burned for electricity generation), specifically where they made up more than 25% of revenues, and certain controversial weapons
- The first exclusion was announced in May 2020
- Two years after, divestment from these sectors is completed
- Ethics for USS ⇒ USS should extend its divestment policy

Individual vs. collaborative engagement

The role of institutional investors

Impact of active ownership

Voting process

- *"The company sets the agenda for the annual shareholder meeting;*
- *The custodian confirms the identity of the shareholders and the number of shares eligible for voting – often for a specific date ahead of the meeting (record date);*
- *Shareholders receive the meeting materials from the company (may be before or after the record date);*
- *Shareholders procuring proxy advisory services receive voting recommendations;*
- *Shareholders instruct the custodian on how to vote, often through a proxy voting service provider, within a deadline ahead of the shareholder meeting (cut-off date);*
- *Voting takes place at the shareholder meeting;*
- *Shareholders receive confirmation from the service provider that their voting instructions have been carried out."*

Proxy voting

Voting policy

Statistics about ESG voting

Asset managers

Figure 104: Voting Matters series of ShareAction

2019 2020 2021 2022

Source: `https://shareaction.org`.

Statistics about ESG voting

Asset managers

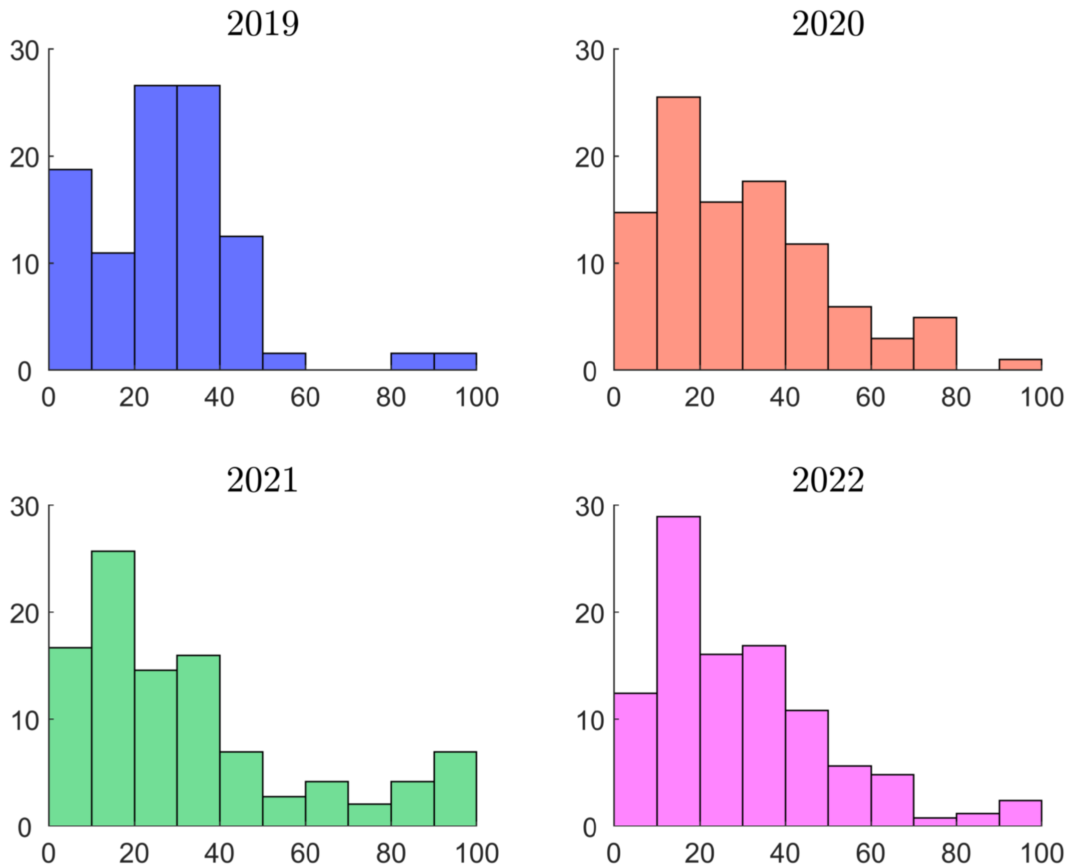
Table 71: Statistics of success rate shareholder resolutions

Year		2019	2020	2021	2022
Number of resolutions		64	102	144	249
Resolutions with majority support		3	15	29	37
Success rate (in %)		4.7	14.7	20.1	14.9
Average support rate (in %)		28.2	29.9	32.9	29.9
Percentile of support rate (in %)	10%	6.5	9.2	7.2	9.4
	25%	17.0	13.1	12.0	13.5
	75%	37.7	42.6	42.8	40.3
	90%	41.8	55.2	81.2	57.6
Average support rate (in %)	E	28.2	35.8	41.8	31.6
	S		24.5	28.8	27.4

Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting
Asset managers

Figure 105: Histogram (in %) of support rates



Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting
Asset managers

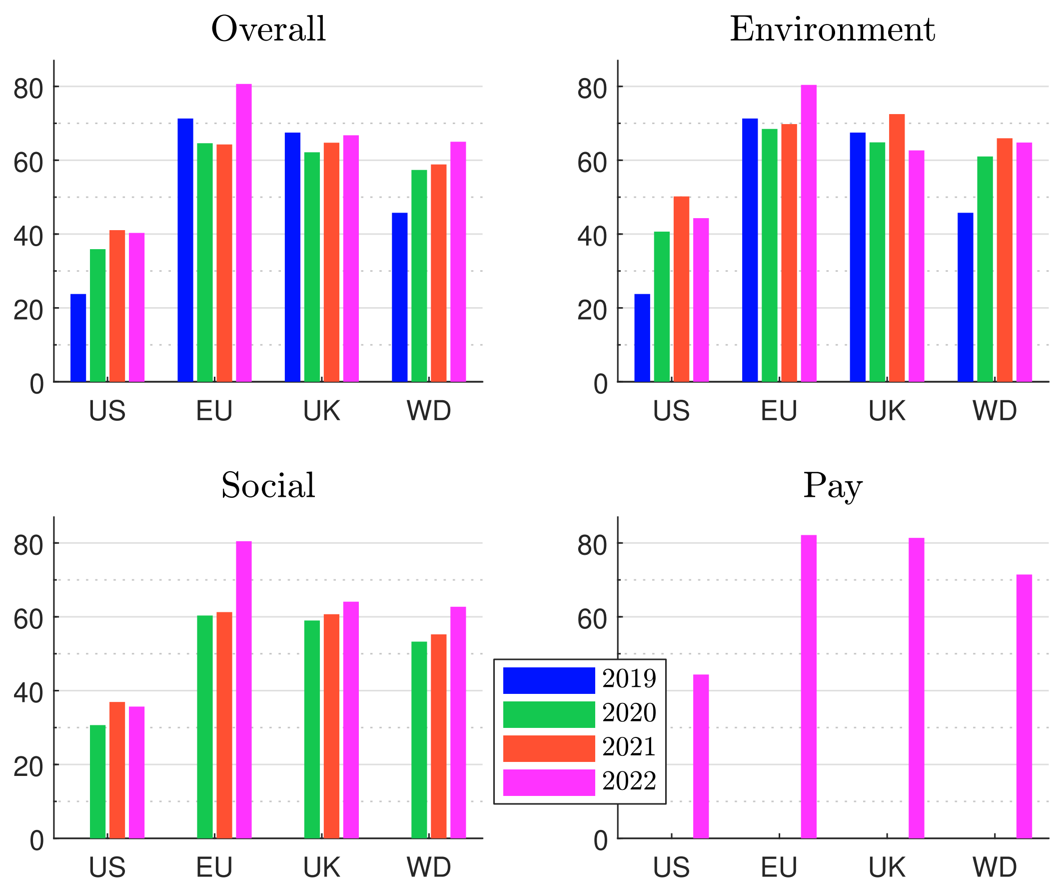
Table 72: Average support rate in % for ESG resolutions

Topic	Method	2019	2020	2021	2022
Overall	Arithmetic	45.8	57.4	58.9	65.0
	Weighted	32.7	42.1	47.6	46.5
Environment	Arithmetic	45.8	61.0	66.0	64.8
	Weighted	32.7	44.7	55.8	48.8
Social	Arithmetic		53.3	55.2	62.7
	Weighted		39.0	43.7	44.3
Pay & politics	Arithmetic				71.5
	Weighted				47.8

Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting
Asset managers

Figure 106: Arithmetic average support rate in % per country and year

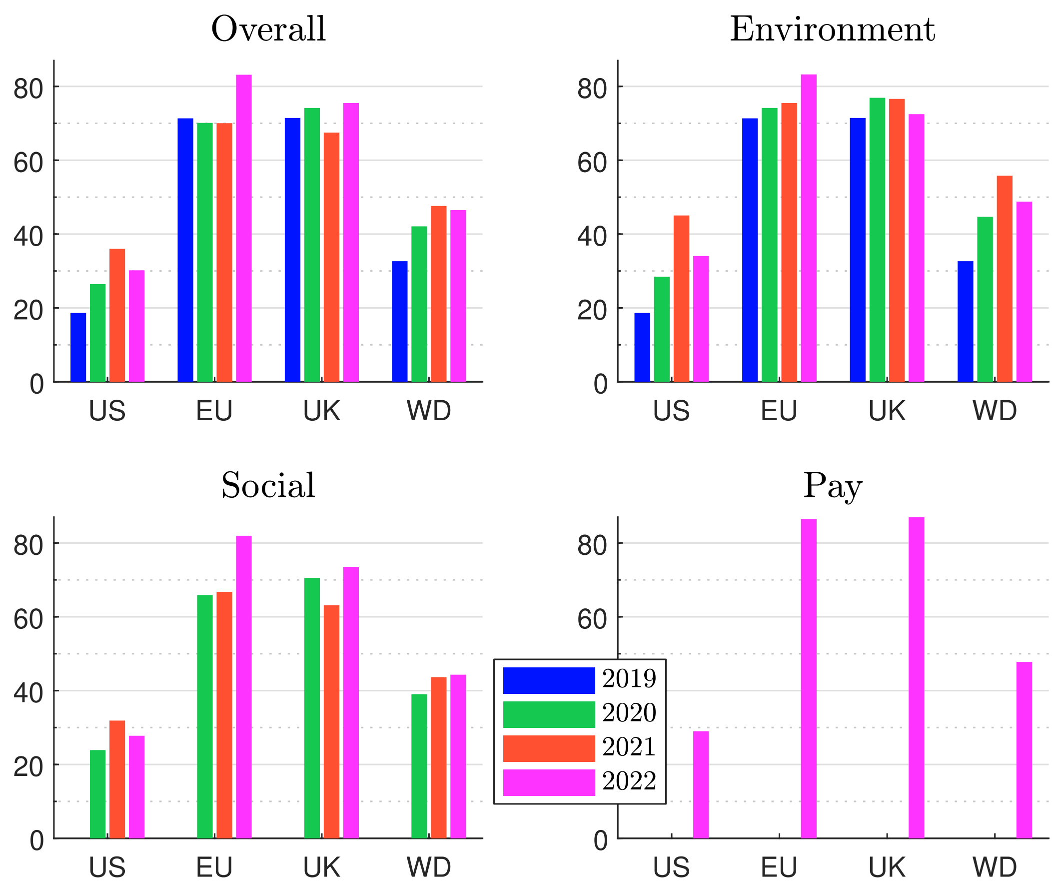


Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting

Asset managers

Figure 107: Weighted average support rate in % per country and year



Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting
Asset managers

Table 73: Best performers (2022, overall)

Rank	Name	Country	AUM	Overall	E	S	Pay
1	Achmea IM	Netherlands	251	100	100	100	100
1	Impax AM	UK	56	100	100	100	100
3	BNP PAM	France	761	99	97	100	100
3	MN	Netherlands	193	99	97	100	100
5	Candriam	Luxembourg	180	98	97	99	100
6	PGGM	Netherlands	331	97	93	100	97
7	Man	UK	149	96	98	94	98
8	Robeco	Netherlands	228	95	94	94	100
9	Aviva Investors	UK	363	93	88	96	100
10	Amundi AM	France	2 348	93	93	92	98
11	Nordea AM	Finland	333	91	93	89	90
12	Aegon AM	Netherlands	466	90	85	94	90
13	Federated Hermes	UK	672	89	88	87	90
14	Pictet AM	Switzerland	284	88	85	90	91
15	Legal & General	Switzerland	1 923	86	84	84	98

Source: ShareAction (2023) & Author's calculations.

Statistics about ESG voting

Asset managers

Table 74: Worst performers (2022, overall)

Rank	Name	Country	AUM	Overall	E	S	Pay
59	Goldman Sachs AM	US	2 218	35	56	24	24
60	Baillie Gifford	UK	455	31	29	29	45
61	SSGA	US	4 140	29	30	31	22
62	BlackRock	US	10 014	24	28	24	15
63	T. Rowe Price	US	1 642	17	26	11	18
64	Fidelity Investments	US	4 520	17	23	19	2
65	Vanguard	US	8 274	10	12	9	9
66	Dimensional Fund Advisors	US	679	4	6	5	0
67	Santander AM	Spain	220	4	0	5	6
68	Walter Scott & Partners	UK	95	3	0	6	0

Source: ShareAction (2023) & Author's calculations.

Statistics about ESG voting

Asset managers

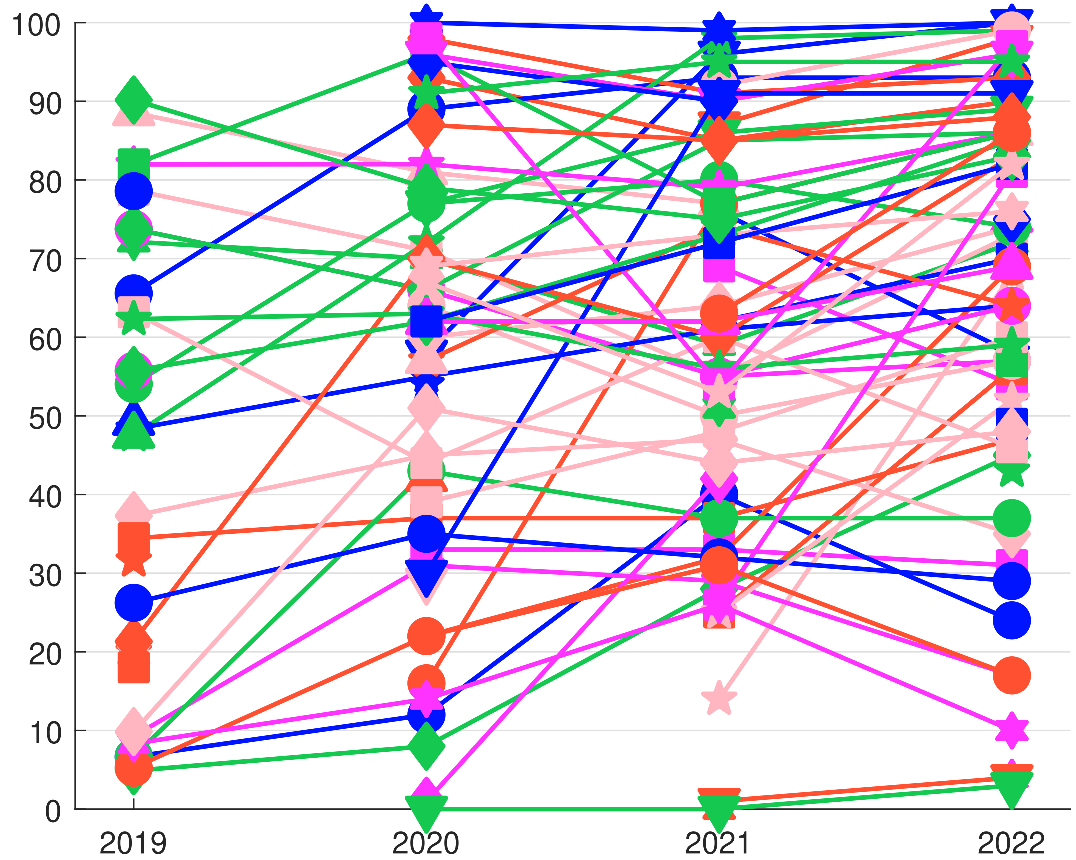
Table 75: Ranking of the 25 largest asset managers (2022, overall)

Rank	Name	Country	AUM	Overall 2019	Overall 2020	Overall 2021	Overall 2022
22	BlackRock	US	10 014	7	12	40	24
25	Vanguard	US	8 274	8	14	26	10
23	Fidelity Investments	US	4 520	9	31	29	17
21	SSGA	US	4 140	26	35	32	29
18	J.P. Morgan AM	US	2 742	7	43	37	37
16	Capital Group	US	2 716	5	8	28	45
2	Amundi AM	France	2 348	66	89	93	93
20	Goldman Sachs AM	US	2 218	37	45	47	35
3	Legal & General	UK	1 923	82	96	77	86
24	T. Rowe Price	US	1 642	5	22	31	17
15	Invesco	US	1 611	34	37	37	47
12	Morgan Stanley IM	US	1 566			55	64
14	Wellington Management	US	1 426	10	51	44	48
7	Northern Trust AM	US	1 348	21	70	60	83
13	Nuveen AM	US	1 271	62	63	56	59
8	UBS AM	Switzerland	1 216	90	79	75	83
4	DWS	Germany	1 055	74	66	85	86
10	AXA IM	France	1 009	79	71	55	73
6	Schroders	UK	991	56	62	73	85
17	AllianceBernstein	US	779				43
5	Allianz GI	Germany	766	89	81	77	86
1	BNP PAM	France	761	48	72	98	99
19	Columbia Threadneedle	US	754				37
9	Manulife IM	Canada	723				75
11	APG AM	Netherlands	721	72	70	59	72

Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting
Asset managers

Figure 108: Evolution of the support rate in % per asset manager



Source: ShareAction (2019, 2020, 2021, 2023) & Author's calculations.

Statistics about ESG voting

Asset managers

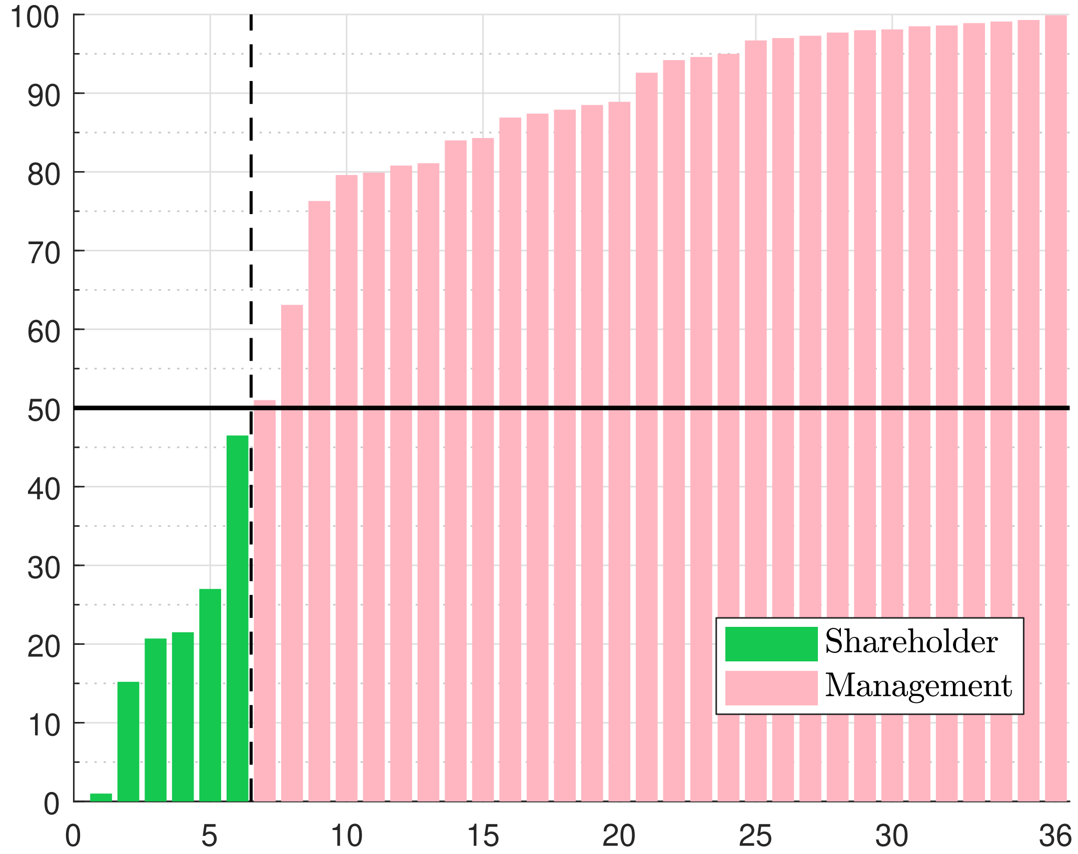
Main findings

1. *"49 additional resolutions would have received majority support if the largest asset managers had voted in favour of them.*
2. *Voting performance has been stagnant in the US and the UK compared to 2021, while European asset managers have shown a large improvement.*
3. *Asset managers across the board are hesitant to back action-oriented resolutions, which would have the most transformative impact on environmental and social issues."*

Statistics about ESG voting

Asset managers

Figure 109: Ranking of the 36 say on climate resolutions with respect to the support rate in %



Source: ShareAction (2023) & Author's calculations.

Statistics about ESG voting

Asset managers

3 case studies of Say on Climate resolutions

- Electricité de France or EDF (French energy company): 99.9%
- Barclays (British bank): 80.8%
- Woodside Energy Group Ltd. (Australian energy company): 51.03%

Statistics about ESG voting

Asset owners

Course 2022-2023 in Sustainable Finance
Lecture 6. Global Warning & Climate Change

Thierry Roncalli*

*Amundi Asset Management[17]

*University of Paris-Saclay

March 2023

[17] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Climate financial risk

Climate risks transmission channels to financial stability

- The **physical risks** that arise from the increased frequency and severity of climate and weather related events that damage property and disrupt trade
- The **liability risks** stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible
- The **transition risks** that can arise through a sudden and disorderly adjustment to a low carbon economy

Speech by Mark Carney at the International Climate Risk Conference for Supervisors, Amsterdam, April 6, 2018

Physical and transition risks ⇔ **E**

Liability risks ⇔ **S** (and **G** ?)

Climate financial risk

Risks are transversal to financial risks

- **Carbon risk** (reputational and regulation risks) $\Rightarrow$ economic, market and credit risks
- **Climate risk** (extreme weather events, natural disasters) $\Rightarrow$ economic, operational, credit and market risks

Carbon/climate risks are part of risk management

Climate financial risk

Climate risk(s)

Climate risks include transition risk and physical risks:

- Transition risk is defined as the financial risk associated with the transition to a low-carbon economy. It includes policy changes, reputational impacts, and shifts in market preferences, norms and technology
- Physical risk is defined as the financial losses due to extreme weather events and climate disasters like flooding, sea level rise, wildfires, droughts and storms

Global warming

Global warming (≈ climate change)

Global warming is the long-term heating of Earth's climate system observed since the pre-industrial period (between 1850 and 1900) due to human activities, primarily fossil fuel burning

NASA Global Climate Change — `https://climate.nasa.gov`

Global warming

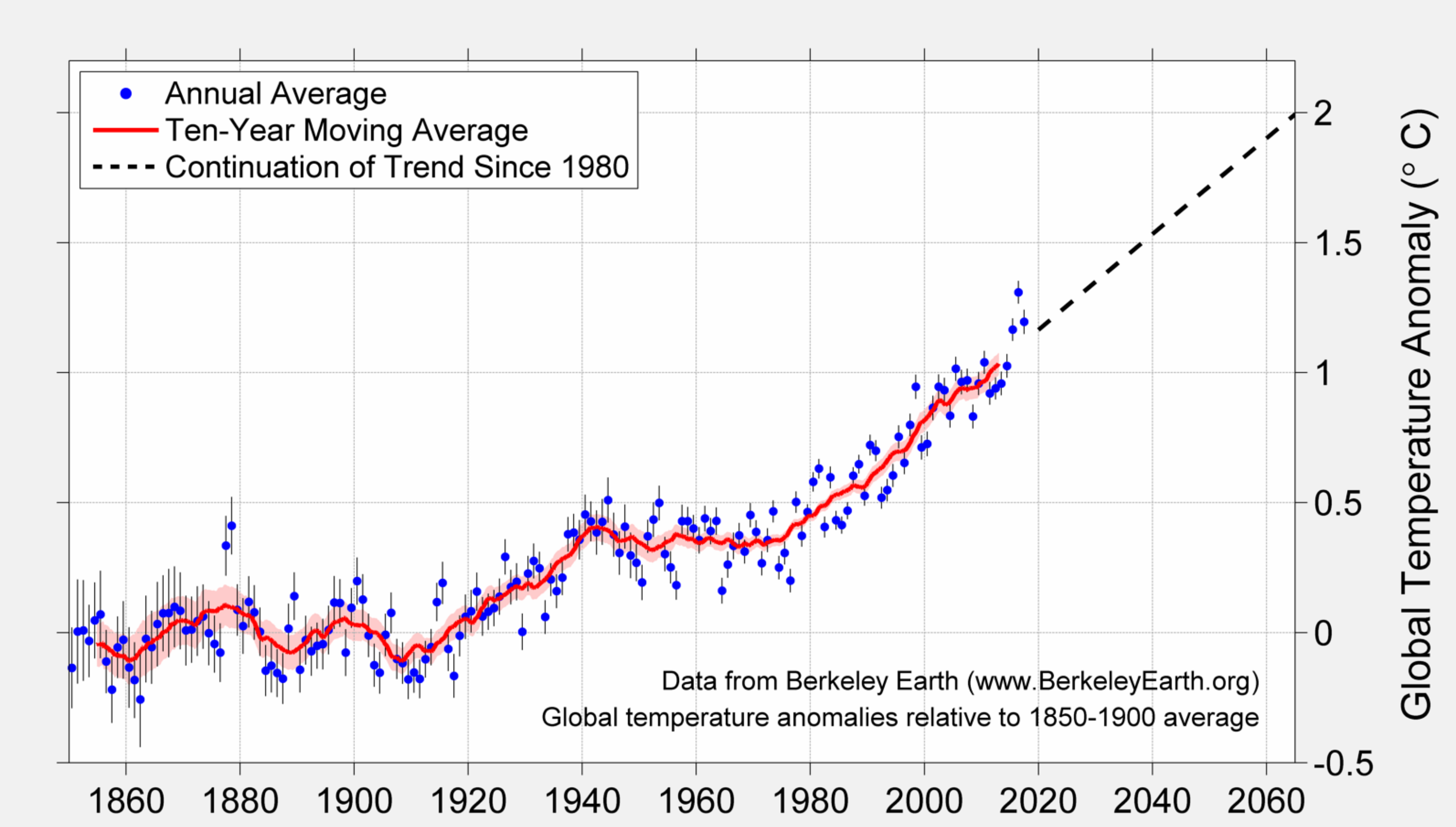


Figure 110: Global temperature anomaly

Source: Berkeley Earth (2018), `http://berkeleyearth.org`

Global warming

Carbon risk

Carbon risks correspond to the potential financial losses due to greenhouse gas (or GHG) emissions, mainly CO_2 emissions (in a strengthening regulatory context)

Global warming

GHG

Greenhouse gases absorb and emit radiation energy, causing the greenhouse effect[a]:

1. Water vapour (H_2O)
2. Carbon dioxide (CO_2)
3. Methane (CH_4)
4. Nitrous oxide (N_2O)
5. Ozone (O_3)

[a]Without greenhouse effect, the average temperature of Earth's surface would be about $-18^{\circ}\mathrm{C}$. With greenhouse effect, the current temperature of Earth's surface is about $+15^{\circ}\mathrm{C}$.

Global warming

Table 76: Pros and cons of greenhouse gases

GHG	Pros	Cons	Global warming
Water vapour	Life		
Carbon dioxide	Photosynthesis	Pollution	✓
Methane	Energy	Explosive[18]	✓
Nitrous oxide	Dentist ☺		✓
Ozone	UV rays		

[18] And dangerous for human life

Global warming

Carbon equivalent

Carbon dioxide equivalent (or $\mathrm{CO_2e}$) is a term for describing different GHG in a common unit

- A quantity of GHG can be expressed as $\mathrm{CO_2e}$ by multiplying the amount of the GHG by its global warming potential (GWP)
- 1 kg of carbone dioxide corresponds to 1 kg of CO_2
- 1 kg of methane corresponds to 28 kg of CO_2
- 1 kg of nitrous oxide corresponds to 273 kg of CO_2

CO_2 emissions

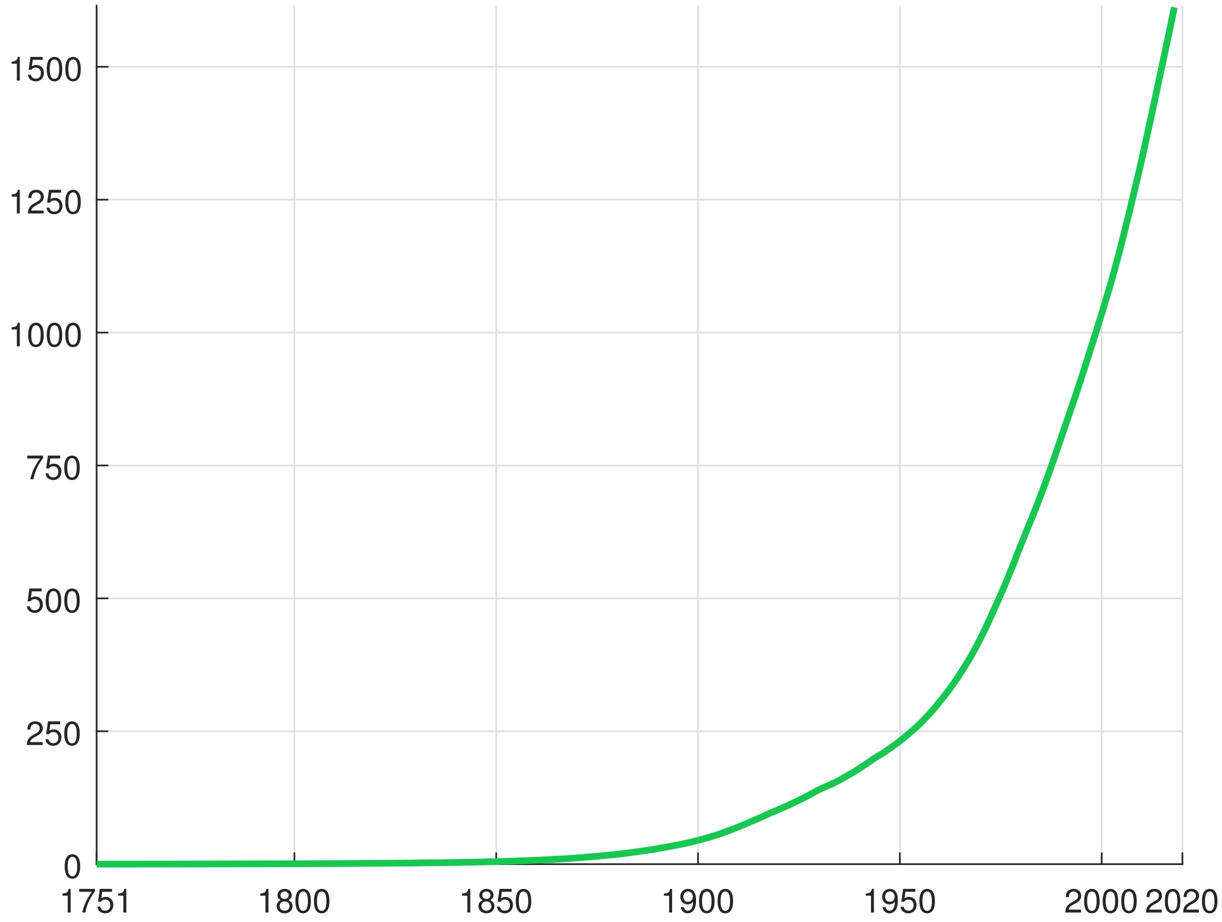


Figure 111: Cumulative CO_2e emissions (in $GtCO_2e$)

Source: Data on CO_2 and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)

CO_2 emissions

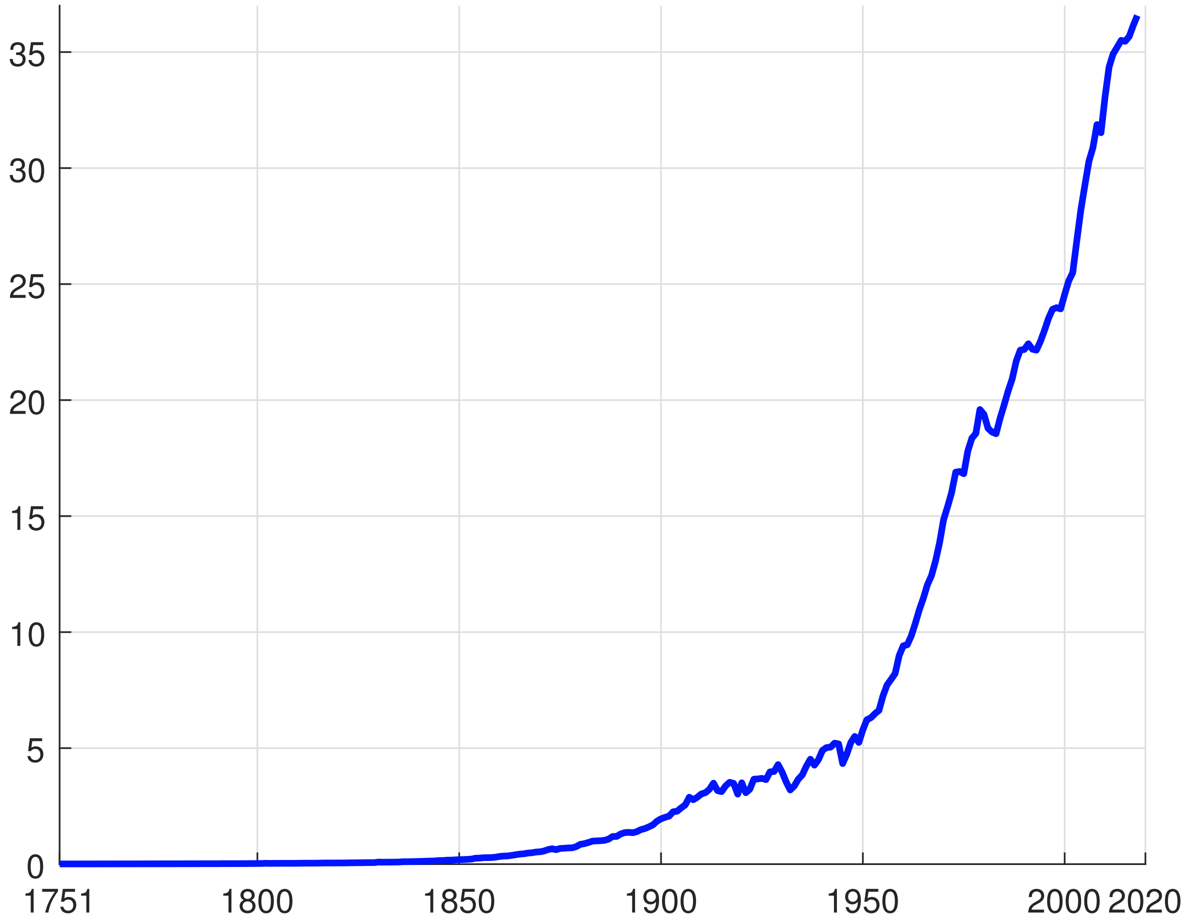


Figure 112: Annual CO_2e emissions (in $GtCO_2e$)

Source: Data on CO_2 and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)

CO_2 emissions

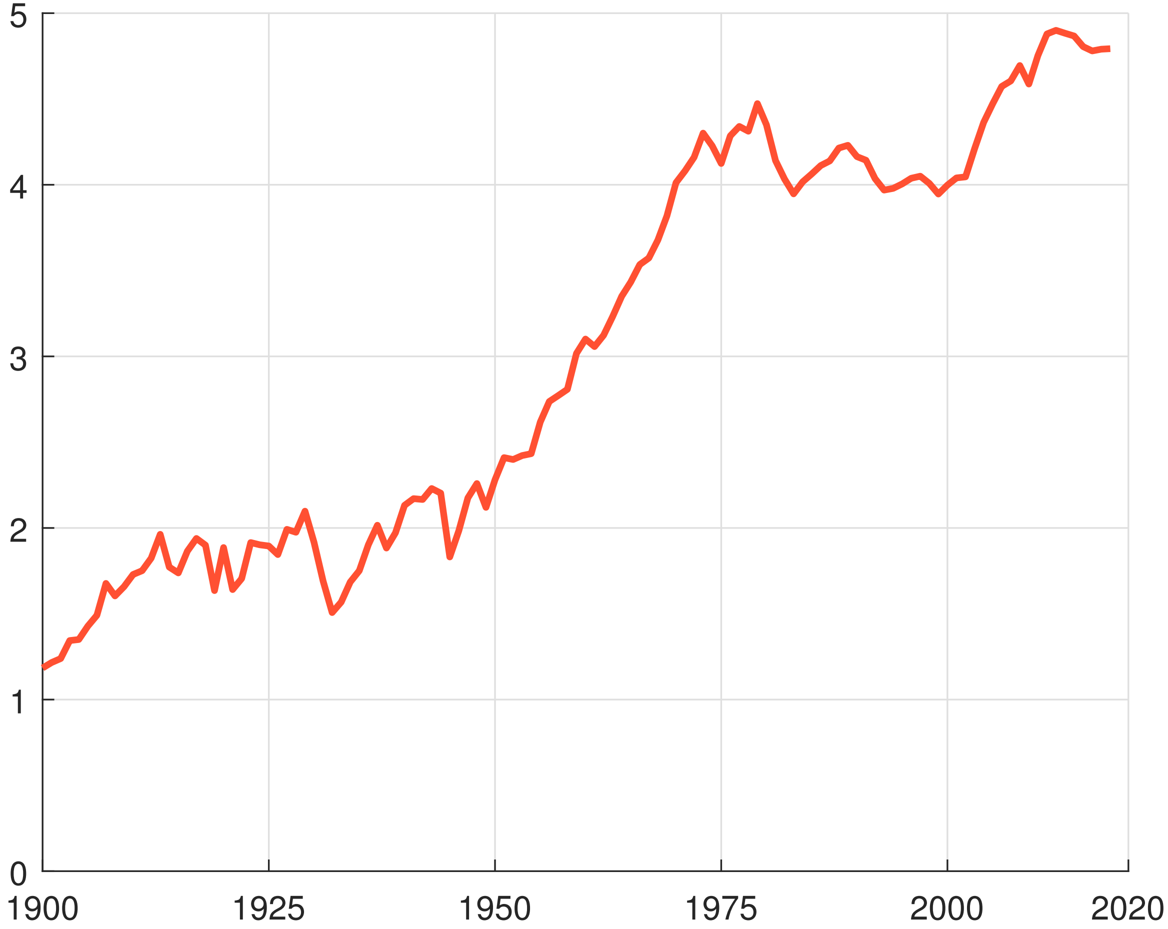


Figure 113: CO_2e emissions per capita (in tonnes per capita)

Source: Data on CO_2 and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)

CO_2 emissions

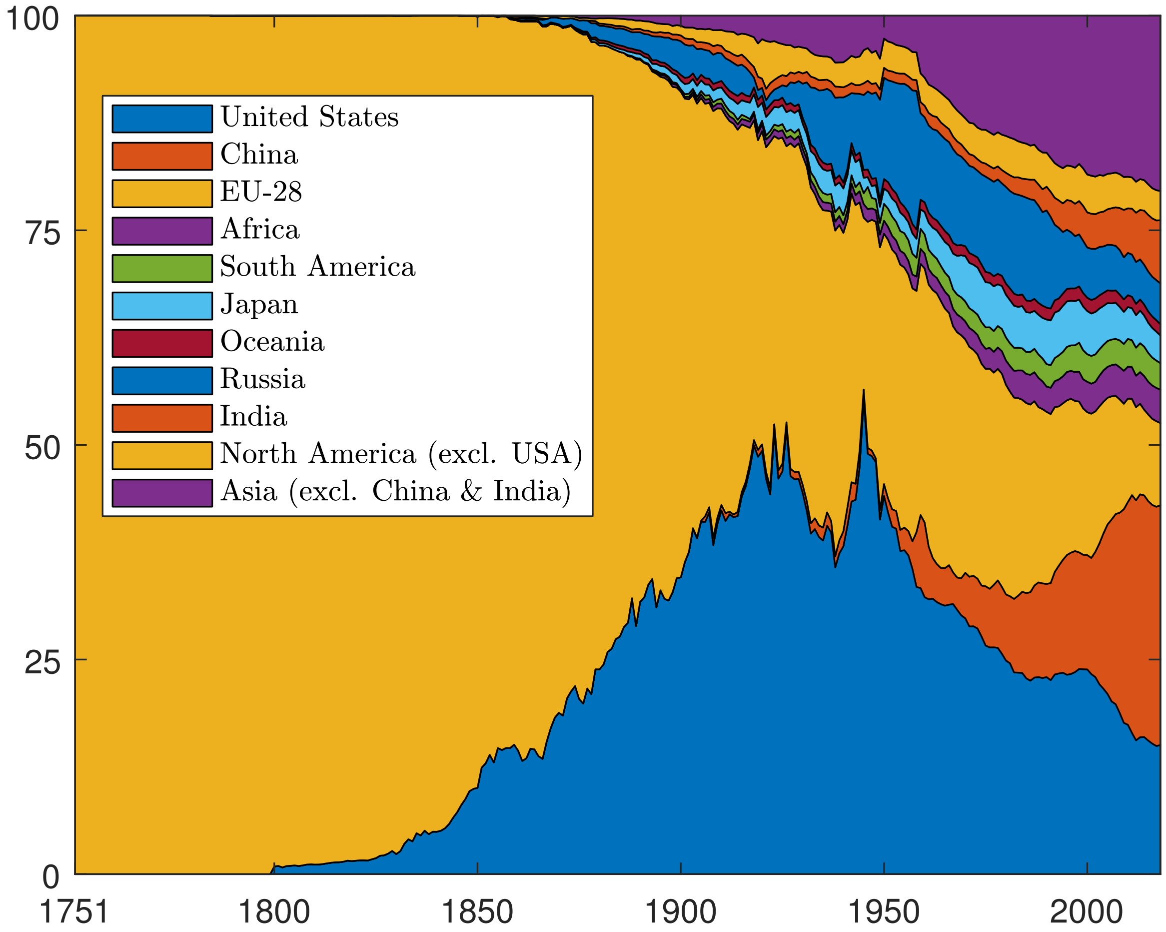
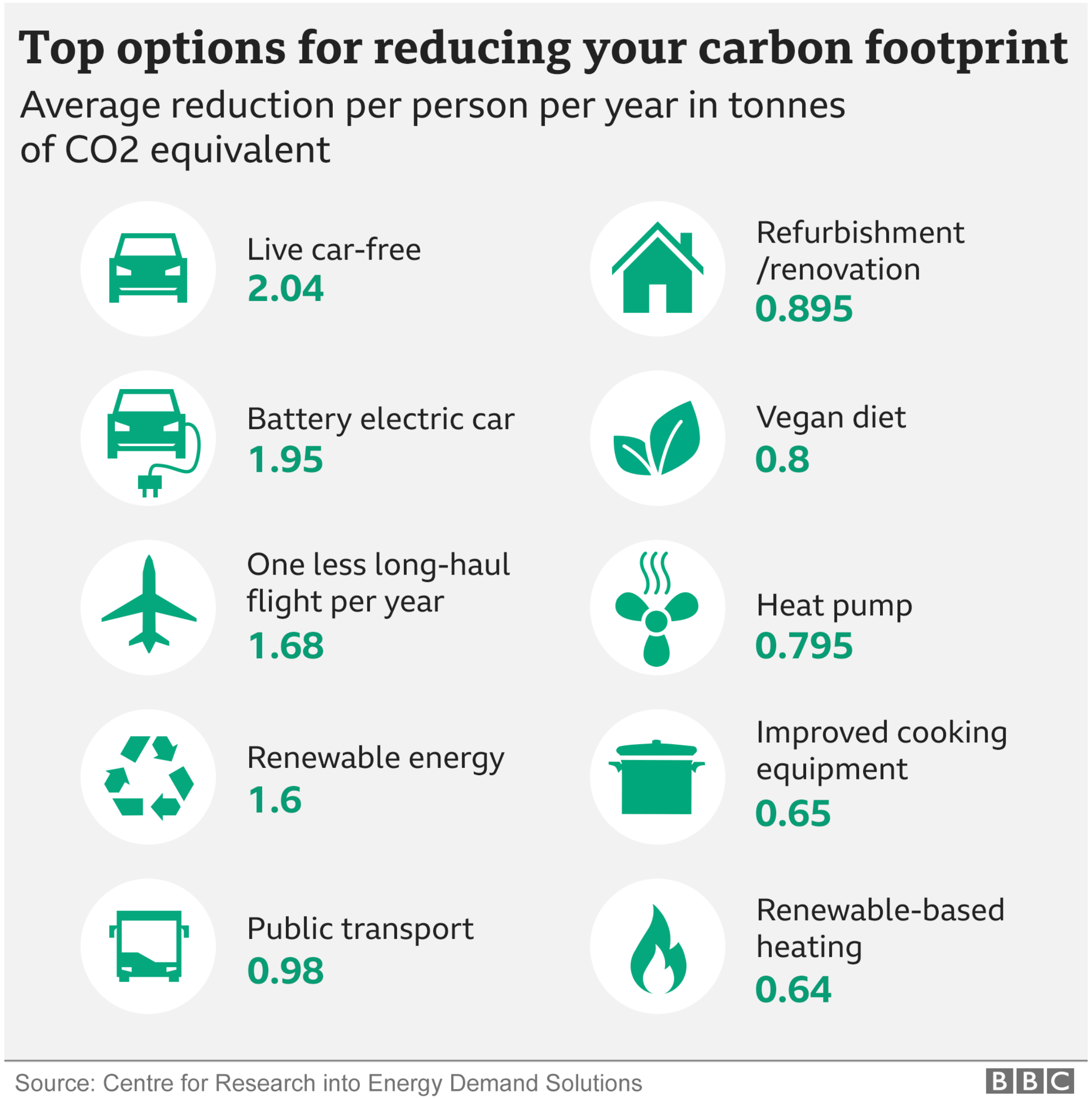


Figure 114: Share of CO_2e emissions (in %)

Source: Data on CO_2 and GHG Emissions by Our World in Data (https://github.com/owid/co2-data)

CO_2 emissions



Source: Centre for Research into Energy Demand Solutions

Scientific evidence of global warming: a rocky road

- 1824: Joseph Fourier published the scientific article "*Remarques générales sur les températures du globe terrestre et des espaces planétaires*" ⇒ the greenhouse effect
- 1863: John Tyndall published the books "*Heat Considered as a Mode of Motion*" in 1863 and "*Contributions to Molecular Physics in the Domain of Radiant Heat*" in 1872
- 1896: Svante Arrhenius published the scientific article "*On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground*" ⇒ if the quantity of carbonic acid increases in geometric progression, the augmentation of the temperature will increase nearly in arithmetic progression
- 1958: Charles David Keeling started collecting carbon dioxide samples at the Mauna Loa Observatory (Hawai) ⇒ Keeling curve
- 2021: Klaus Hasselmann and Syukuro Manabe won the Nobel Prize in Physics for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming

Scientific evidence of global warming: a rocky road

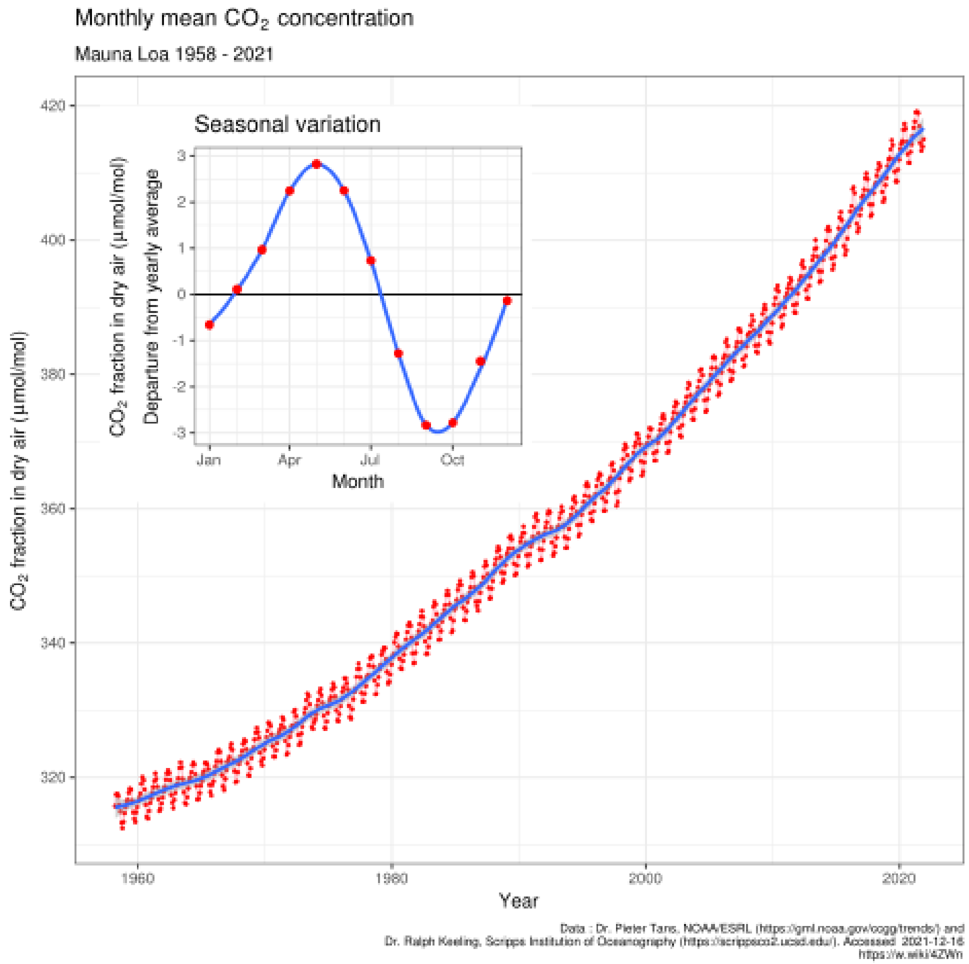


Figure 115: Keeling curve

Source: `https://en.wikipedia.org/wiki/Keeling_Curve`.

Scientific evidence of global warming

From the Holocene to the Anthropocene

The physics of climate change

IPCC

- The Intergovernmental Panel on Climate Change (IPCC) is the United Nations body for assessing the science related to climate change
- The IPCC was created to provide policymakers with regular scientific assessments on climate change, its implications and potential future risks, as well as to put forward adaptation and mitigation options
- Website: `https://www.ipcc.ch`

Remark

IPCC is known as "*Groupe d'experts intergouvernemental sur l'évolution du climat*" (GIEC) in French

⇒ Other international bodies: International Energy Agency (IEA), etc.

IPCC

Past

- Global sea level rose by 19 cm over the period 1901-2010
- Global glacier volume loss is equivalent to 400 bn tons per year since 30 years

Future

- Global sea level could increase by 82 cm by 2100
- Global glacier volume could decrease by 85% by 2100

IPCC, Climate Change Synthesis Report (2014)

IPCC

IPCC working groups

- The IPCC Working Group I (WGI) examines the physical science underpinning past, present, and future climate change
- The IPCC Working Group II (WGII) assesses the impacts, adaptation and vulnerabilities related to climate change
- The IPCC Working Group III (WGIII) focuses on climate change mitigation, assessing methods for reducing greenhouse gas emissions, and removing greenhouse gases from the atmosphere

IPCC

Some famous reports

- IPCC Fifth Assessment Report (AR5): Climate Change 2014 — www.ipcc.ch/report/ar5
- Global Warming of 1.5°C — www.ipcc.ch/sr15
- IPCC Sixth Assessment Report (AR6): Climate Change 2022 — www.ipcc.ch/report/sixth-assessment-report-cycle

IPCC scenarios

- Website: https://www.ipcc.ch/data
- AR5
- SR15
- AR6

Carbon neutrality

Carbon neutrality (or net zero) means that any CO_2 released into the atmosphere from human activity is balanced by an equivalent amount being removed

Apple Commits to Become Carbon Neutral to by 2030 (`https://www.bbc.com/news/technology-53485560`)

Carbon dioxide removal

Carbon dioxide removal (CDR)

1. Nature-based solutions
   - Afforestation (creating new forests)
   - Reforestation (multiplying trees in old forests)
   - Restoration of peat bogs
   - Restoration of coastal and marine habitats
2. Enhanced natural processes
   - Land management and no-till agriculture, which avoids carbon release through soil disturbance
   - Better wildfire management
   - Ocean fertilisation to increase its capacity to absorb CO_2 (enhanced weathering)
3. Technology solutions
   - Bioenergy with carbon capture and storage (BECCS)
   - Direct air capture (DAC)
   - Carbon mineralization

Carbon dioxide removal

The example of peatlands

- Peatlands are the largest natural terrestrial carbon store
- The term "*peatland*" refers to peat soil and wetland habitats
- They cover only 3% of the Earth's surface
- They store 600 $\mathrm{GtCO_2e}$
  - $\approx$ 45% of all soil carbon
  - $\approx$ 67% of all atmosphere carbon
- A depth of one meter corresponds to 1 000 years of carbon storage
- Natural peatlands store 0.37 $\mathrm{GtCO_2e}$ per year

Two issues:

1. Stopping the destruction
2. Restoring and rebuilding

Carbon offsetting

Carbon offsetting $\neq$ carbon emissions reduction

Definition

"Carbon offsetting consists for an entity in compensating its own carbon emissions by providing for emissions reductions outside its business boundaries [...] It allows an entity to claim carbon reductions from projects financed either directly or indirectly through carbon credits" (Créhalet, 2021).

Carbon offsetting

Carbon offsetting mechanisms:

Suppliers of carbon offsets

⬇

Carbon credits

⬇

Purchasers of carbon offsets

⇒ Many issues: carbon credit issuance, double counting, leakage, certification, etc.

Examples with **REDD+** projects:

- **R**educing **E**missions from **D**eforestation and **F**orest **D**egradation
- What will happen if the forest has burned down?
- Issues of land management (afforestation in one area can lead to a deforestation in another area)

Climate risk and missing factors

The example of permafrost

- The permafrost contains 1 700 billion tons of carbon, almost double the amount of carbon that is currently in the atmosphere.
- Arctic permafrost holds roughly 15 million gallons of mercury – at least twice the amount contained in the oceans, atmosphere and all other land combined.
- A global temperature rise of 1.5°C above current levels would be enough to start the thawing of permafrost in Siberia.
- The global warming will become out-of-control after this tipping point.
- The thawing of the permafrost also threatens to unlock disease-causing viruses long trapped in the ice.

$\Rightarrow$ The survival of Humanity becomes uncertain if the tipping point is reached

Regulation of climate risk

- UN, international bodies & coalitions
- Countries
- Cities
- Industry self-regulation
- Non-governmental organizations (NGO)
- Financial regulators

Hard regulation $\neq$ **soft regulation**

Regulation of climate risk
UN

United Nations Climate Change Conference

- Conference of the Parties (COP)
- Dealing with climate change
- COP 1: Berlin (1995)
- COP 3: Kyoto (1997) $\Rightarrow$ Kyoto Protocol (CMP)
- COP 21: Paris (2015) $\Rightarrow$ Paris Agreement (CMA)
- COP 26: Glasgow (November 2021)

Regulation of climate risk
UN

The **Kyoto Protocol** is an international treaty that commits state parties to reduce GHG emissions, based on the scientific consensus that:

1. **Global warming is occurring**
2. It is likely that **human-made CO_2 emissions have caused it**

Regulation of climate risk
UN

The **Paris Agreement** is an international treaty with the following goals:

1. Keep a global temperature rise this century well below $2^\circ\mathrm{C}$ above the pre-industrial levels
2. Pursue efforts to limit the temperature increase to $1.5^\circ\mathrm{C}$
3. Increase the ability of countries to deal with the impacts of climate change
4. Make finance flows consistent with low GHG emissions and climate-resilient pathways

$\Rightarrow$ Nationally determined contributions (NDC)

Regulation of climate risk

UN

Table 77: CO_2 emissions by country

Rank	Country	CO_2 emissions Total (in GT)	Share	CO_2 emissions Per capita (in MT)
1	China	10.06	28%	7.2
2	USA	5.41	15%	15.5
3	India	2.65	7%	1.8
4	Russia	1.71	5%	12.0
5	Japan	1.16	3%	8.9
6	Germany	0.75	2%	8.8
7	Iran	0.72	2%	8.3
8	South Korea	0.72	2%	12.1
9	Saudi Arabia	0.72	2%	17.4
10	Indonesia	0.72	2%	2.2
11	Canada	0.56	2%	15.1
15	Turkey	0.42	1%	4.7
17	United Kingdom	0.37	1%	5.8
19	France	0.33	1%	4.6
20	Italy	0.33	1%	5.3

Source: Earth System Science Data, https://earth-system-science-data.net

World Bank Open Data, https://data.worldbank.org/topic/climate-change

Regulation of climate risk
UN

Paris Agreement: where we are?

- 194 states have signed the Agreement
- They represent about 80% of GHG emissions
- USA, Iran and Turkey have not signed the Agreement

Regulation of climate risk

UN

Figure 116: Paris Agreement assessments of aviation and shipping

Source: Climate Action Tracker (CAT), `https://climateactiontracker.org`

Regulation of climate risk

Coalitions

- **The Coalition of Finance Ministers for Climate Action**

  www.financeministersforclimate.org

  - Commitment to implement fully the Paris Agreement
  - Santiago Action Plan
  - Helsinki principles (1. align, 2. share, 3. promote, 4. mainstream, 5. mobilize, 6. engage)

Regulation of climate risk
Coalitions

- **One Planet Summit**

  www.oneplanetsummit.fr

- **One Planet Sovereign Wealth Funds (OPSWF)**
  - Funding members: Abu Dhabi Investment Authority (ADIA), Kuwait Investment Authority (KIA), NZ Superannuation Fund (NZSF), Public Invesment Fund (PIF), Qatar Investment Authority (QIA), NBIM
  - New members: Bpifrance, CDP Equity, COFIDES, FONSIS, ISIF, KIC, Mubadala IC, NIIF, NIC NBK
- **One Planet Asset Managers**
  - Funding members: Amundi AM, BlackRock, BNP PAM, GSAM, HSBC Global AM, Natixis IM, Northern Trust AM, SSGA
  - New members: AXA IM, Invesco, Legal & General IM, Morgan Stanley IM, PIMCO UBS AM
- **One Planet Private Equity Funds**
  - Members: Ardian, Carlyle Group, Global Infrastructure Partners, Macquarie Infrastructure and Real Assets (MIRA), SoftBank IA

Regulation of climate risk

Countries

The example of France

- August 2015: French Energy Transition for Green Growth Law (or Energy Transition Law)
- Roadmap to mitigate climate change and diversify the energy mix

Other examples: Germany (2021 Renewable Energy Act), UK (2013 Energy Act), The Netherlands (2019 Climate Change Mitigation Act), etc.

Regulation of climate risk
Countries

Article 173 of the French Energy Transition Law

- The annual report of listed companies must include:
  - Financial risks related to the effects of climate change
  - The measures adopted by the company to reduce them
  - The consequences of climate change on the company's activities
- New requirements for investors:
  - Disclosure of climate (and ESG) criteria into investment decision making process
  - Disclosure of the contribution to the energy transition and the global warming limitation international objective
  - Reporting on climate change-related risks (including both physical risks and transition risks), and GHG emissions of assets
- Banks and credit providers shall conduct climate stress testing

Regulation of climate risk
Carbon pricing

- Polluter pays principle
  - A carbon price is a cost applied to carbon pollution to encourage polluters to reduce the amount of GHG they emit into the atmosphere
  - Negative externality
- Two instruments of carbon pricing
  1. **Carbon tax**
  2. **Cap-and-trade** (CAT) or **emissions trading scheme** (ETS)
- Some examples
  1. EU emissions trading system (2005) — `https://ec.europa.eu/clima/policies/ets_en`
  2. New Zealand ETS (2008)
  3. Chinese national carbon trading scheme (2017)

Regulation of climate risk
Carbon pricing

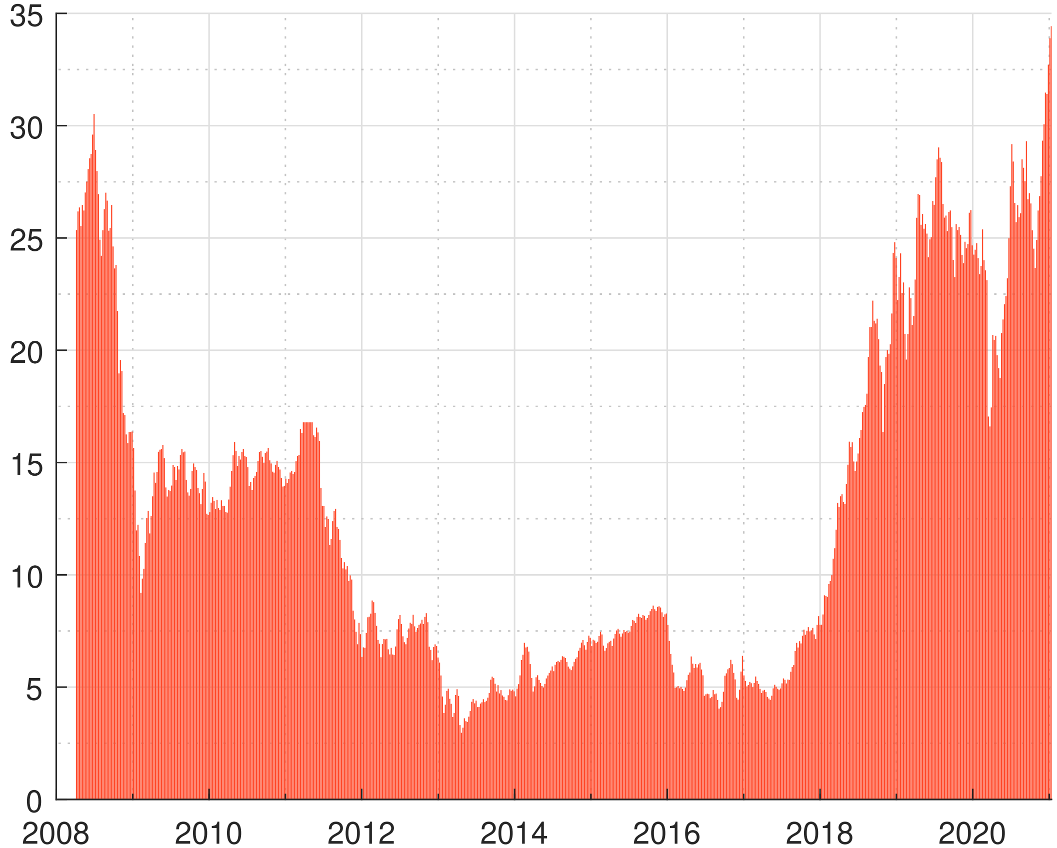


Figure 117: EU ETS carbon price* (in €/tCO_2)

(*)The carbon price reaches 34.43 euros a tonne on Monday 11, 2021

Regulation of climate risk
Carbon pricing

Table 78: Carbon tax (in $/tCO_2$)

Country	2018	2019	2020	Country	2018	2019	2020
Sweden	139.11	126.78	133.26	Latvia	5.58	5.06	10.49
Liechtenstein	100.90	96.46	105.69	South Africa			7.38
Switzerland	100.90	96.46	104.65	France	55.30	50.11	6.98
Finland	76.87	69.66	72.24	Argentina		6.24	5.94
Norway	64.29	59.22	57.14	Chile	5.00	5.00	5.00
Ireland	24.80	22.47	30.30	Colombia	5.67	5.17	4.45
Iceland	35.71	31.34	30.01	Singapore		3.69	3.66
Denmark	28.82	26.39	27.70	Mexico	3.01	2.99	2.79
Portugal	8.49	14.31	27.52	Japan	2.74	2.60	2.76
United Kingdom	25.46	23.59	23.23	Estonia	2.48	2.25	2.33
Slovenia	21.45	19.44	20.16	Ukraine	0.02	0.37	0.35
Spain	24.80	16.85	17.48	Poland	0.09	0.08	0.08

Source: World Bank Carbon Pricing Dashboard, https://carbonpricingdashboard.worldbank.org

Regulation of climate risk

Stranded assets

- Stranded Assets are assets that have suffered from unanticipated or premature write-downs, devaluations or conversion to liabilities
- For example, a $2^{\circ}\mathrm{C}$ alignment implies to keep a large proportion of existing fossil fuel reserves in the ground (30% of oil reserves, 50% of gas reserves and 80% of coal)
- Risk factors: Regulations, carbon prices, change in demand, social pressure, etc.
- Example of the covid-19 crisis $\Rightarrow$ air travel

Regulation of climate risk

Financial regulation

- Financial Stability Board (FSB)
- European Central Bank (ECB)
- The French Prudential Supervision and Resolution Authority (ACPR)
- The Prudential Regulation Authority (PRA)
- Network for Greening the Financial System (NGFS)
- Etc.

Regulation of climate risk

Financial regulation

Bolton, P., Despres, M., Pereira Da Silva, L.A., Samama, F. and Svartzman, R. (2020), *The Green Swan — Central Banking and Financial Stability in the Age of Climate Change*, BIS Publication, `https://www.bis.org/publ/othp31.htm`

Regulation of climate risk

Financial regulation

Task Force on Climate-related Financial Disclosures (TCFD)

- Established by the FSB in 2015 to develop a set of voluntary, consistent disclosure recommendations for use by companies in providing information to investors, lenders and insurance underwriters about their climate-related financial risks
- Website: `www.fsb-tcfd.org`
- Chairman: Michael R. Bloomberg (founder of Bloomberg L.P.)
- 31 members
- June 2017: Publication of the "*Recommendations of the Task Force on Climate-related Financial Disclosures*"
- October 2020: Publication of the 2020 "*Status Report: Task Force on Climate-related Financial Disclosures*"

Regulation of climate risk

Financial regulation

Recommendation	ID	Recommended Disclosure
Governance	1	Board oversight
	2	Management's role
Strategy	3	Risks and opportunities
	4	Impact on organization
	5	Resilience of strategy
Risk management	6	Risk ID and assessment processes
	7	Risk management processes
	8	Integration into overall risk management
Metrics and targets	9	Climate-related metrics
	10	Scope 1, 2, 3 GHG emissions
	11	Climate-related targets

Table 79: The 11 recommended disclosures (TCFD, 2017)

Regulation of climate risk

Financial regulation

Some key findings of the 2020 Status Report (TCFD, 2020):

- Disclosure of climate-related financial information has increased since 2017, but continuing progress is needed
- Average level of disclosure across the Task Force's 11 recommended disclosures was 40% for energy companies and 30% for materials and buildings companies
- Asset manager and asset owner reporting to their clients and beneficiaries, respectively, is likely insufficient

Climate stress testing

- ACPR (2020): Climate Risk Analysis and Supervision[19]
- Bank of England (2021): Climate Biennial Exploratory Scenario (June 2021)

Top-down approach $\neq$ bottom-up approach

Stress of risk-weighted asset: Bouchet and Le Guenedal (2020).

[19] https://acpr.banque-france.fr/en/scenarios-and-main-assumptions-acpr-pilot-climate-exercise

Climate capital requirements

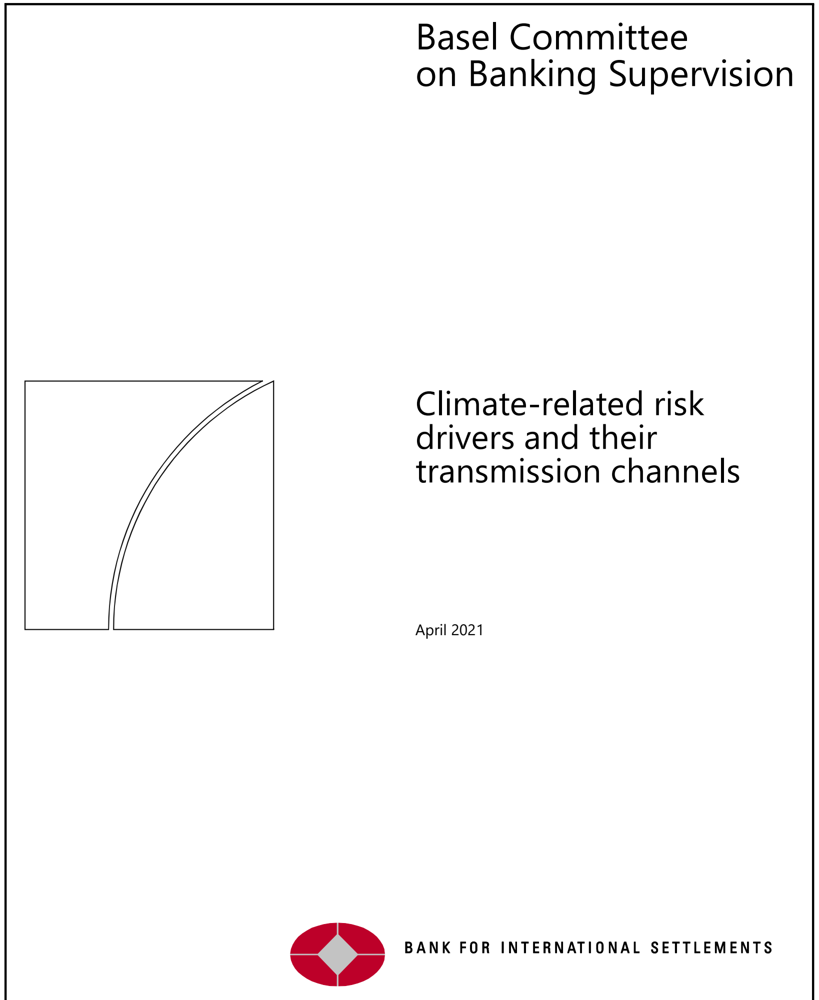
Green supporting factor

- Risk weights may depend on the green/brown nature of the credit
- Green loans
- Green supporting factor $\neq$ Brown penalising factor

Similar idea: Green Quantitative Easing (GQE)

Climate capital requirements

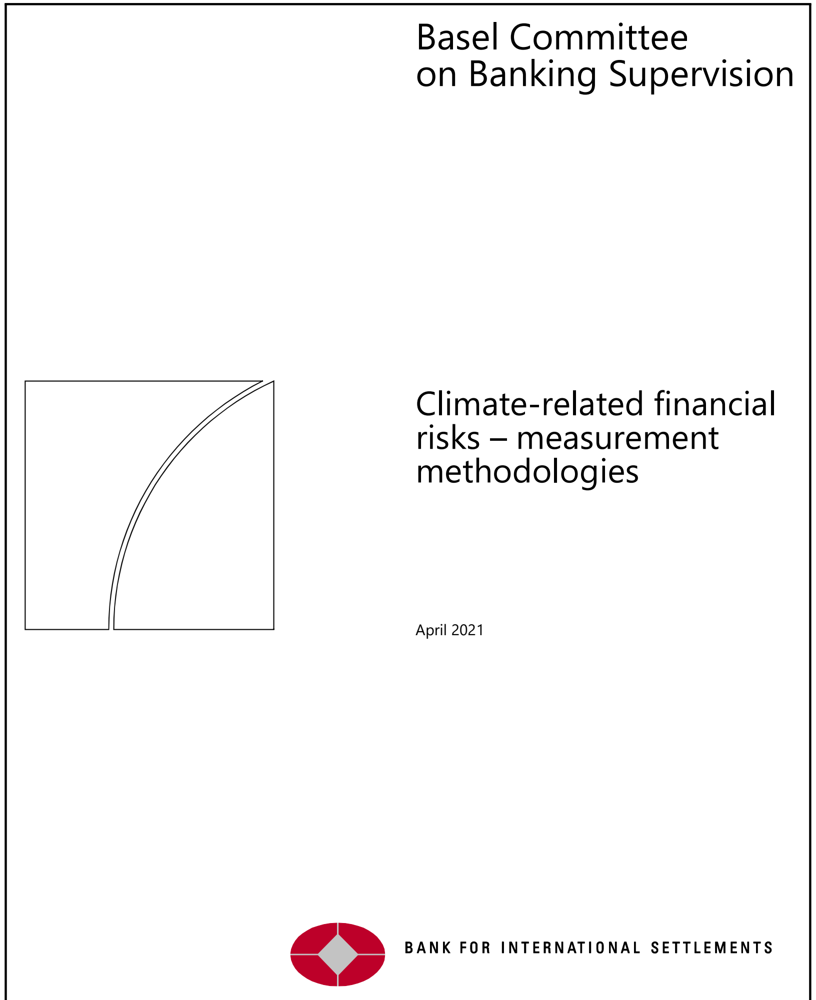
Figure 118: In April 2021, Basel Committee publishes two reports on climate risk


Basel Committee
on Banking Supervision

Climate-related risk
drivers and their
transmission channels

April 2021

BANK FOR INTERNATIONAL SETTLEMENTS


Basel Committee
on Banking Supervision

Climate-related financial
risks – measurement
methodologies

April 2021

BANK FOR INTERNATIONAL SETTLEMENTS

Climate capital requirements

In June 2022, Basel Committee publishes guidelines:

Principles for the effective management
and supervision of climate-related financial risks

Course 2022-2023 in Sustainable Finance
Lecture 7. Economic Modeling of Climate Change

Thierry Roncalli*

*Amundi Asset Management[20]

*University of Paris-Saclay

March 2023

[20] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Sustainable growth and climate change

"*There is no Plan B, because there is no Planet B*"

Ban Ki-moon, UN Secretary-General, September 2014

Is it a question of climate-related issues?

In fact, it is more an economic growth issue

"*The Golden Rule of Accumulation: A Fable for Growthmen*"

Edmund Phelps, *American Economic Review*, 1961
Nobel Prize in Economics, 2006

Sustainable growth and climate change

Environmental

Social

Gouvernance

Economy

Sustainability

Growth

Adam Smith (1776)

An Inquiry into the Nature and Causes of
The Wealth of Nations

The Solow growth model

The model

- Production function:

$$Y(t) = F(K(t), A(t)L(t))$$

where $K(t)$ is the capital, $L(t)$ is the labor and $A(t)$ is the knowledge factor

- Law of motion for the capital per unit of effective labor $k(t) = K(t)/(A(t)L(t))$:

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = s\,f(k(t)) - (g_L + g_A + \delta_K)\,k(t)$$

where s is the saving rate, δ_K is the depreciation rate of capital and g_A and g_L are the productivity and labor growth rates

The golden rule

Golden rule with the Cobb-Douglas production and Hicks neutrality

The equilibrium to respect the '*fairness*' between generations is:

$$k^\star = \left(\frac{s}{g_L + g_A + \delta_K} \right)^{\frac{1}{1-\alpha}}$$

"*Each generation in a boundless golden age of natural growth will prefer the same investment ratio, which is to say the same natural growth path*" (Phelps, 1961, page 640).

"*By a golden age I shall mean a dynamic equilibrium in which output and capital grow exponentially at the same rate so that the capital-output ratio is stationary over time*" (Phelps, 1961, page 639).

Golden rule and climate risk

What is economic growth and what is the balanced growth path?

- There is a saving rate that maximizes consumption over time and between generations ("**the fair rate to preserve future generations**")
- Economic growth corresponds to the exponential growth of capital and output to answer the needs of the growing population
- Introducing human and natural capitals add constraints and therefore **reduce growth**!

$$\text{Economic growth} \Rightarrow \begin{cases} \text{productivity} \nearrow \text{ and labor} \nearrow \\ \text{maximization of } \textbf{consumption-based utility} \text{ function} \end{cases}$$

Extension to natural capital

What are the effects of environmental constraints on growth?

Introducing a decreasing natural capital (Romer, 2006)

The balanced growth path $g_Y^{\star}$ is equal to:

$$g_Y^{\star} = g_L + g_A - \frac{g_L + g_A + \delta_{N_c}}{1 - \alpha} \vartheta$$

where δ_{N_c} is the depreciation rate of natural capital and ϑ is the elasticity of output with respect to (normalized) natural capital $N_c(t)$

"*The static-equilibrium type of economic theory which is now so well developed is plainly inadequate for an industry in which the indefinite maintenance of a steady rate of production is a physical impossibility, and which is therefore bound to decline*" (Hotteling, 1931, page 138-139)

Accounting for environment... changes the definition of economic growth

Inter-temporal utility functions

Preferences modeling (Ramsey model)

- ρ is the discount rate (time preference)
- $c(t)$ is the consumption per capita and u is the CRRA utility function:

$$u(c(t)) = \begin{cases} \dfrac{1}{1-\theta} c(t)^{1-\theta} & \text{if } \theta > 0, \quad \theta \neq 1 \\ \ln c(t) & \text{if } \theta = 1 \end{cases}$$

where θ is the risk aversion parameter

- Maximization of the welfare function:

$$\int_t^{\infty} e^{-\rho t} u(c(t)) \, \mathrm{d}t$$

The discounting issue

Does the golden rule of saving rates hold in a Keynesian approach with discounted maximization of consumption?

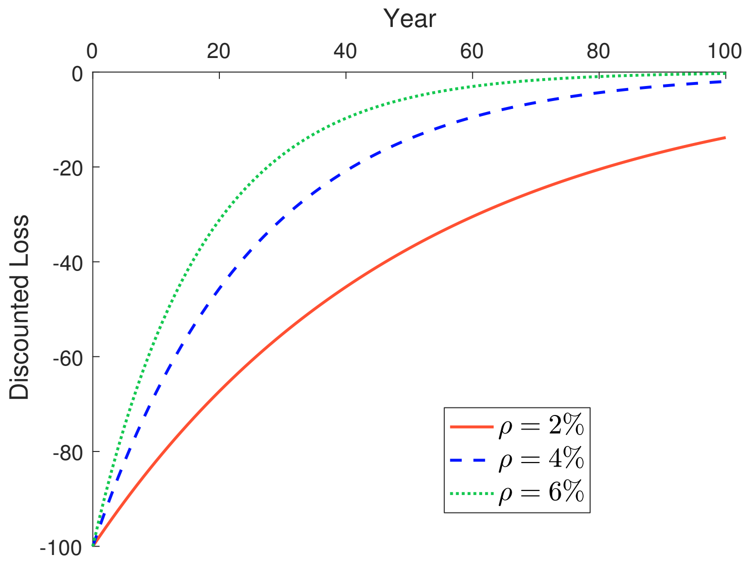


Figure 119: Discounted value of $100 loss

- "*There is still time to avoid the worst impacts of climate change, if we take strong action now*" (Stern, 2007)
- "*I got it wrong on climate change – it's far, far worse*" (Stern, 2013)

The value of a loss in 100 years almost disappears... while it is only the next generation!

Does consumption maximization make sense?

How many planets do we need?

To achieve the current levels of consumption for the world population, we need:

- US: 5 planets
- France: 3 planets
- India: 0.6 planet

Source: Global Footprint Network, `http://www.footprintcalculator.org`

Fairness between generations

Keynes

"In the long run, we are all dead"

John Maynard Keynes[a], *A Tract on Monetary Reform*, 1923.

[a] *"Men will not always die quietly"*, *The Economic Consequences of the Peace*, 1919.

Carney

"The Tragedy of the Horizon"

Mark Carney, Chairman of the Financial Stability Board, 2015

⇒ Back to the Golden Rule and the Fable for Growthmen...

Integrated assessment models (IAMs)

Main categories

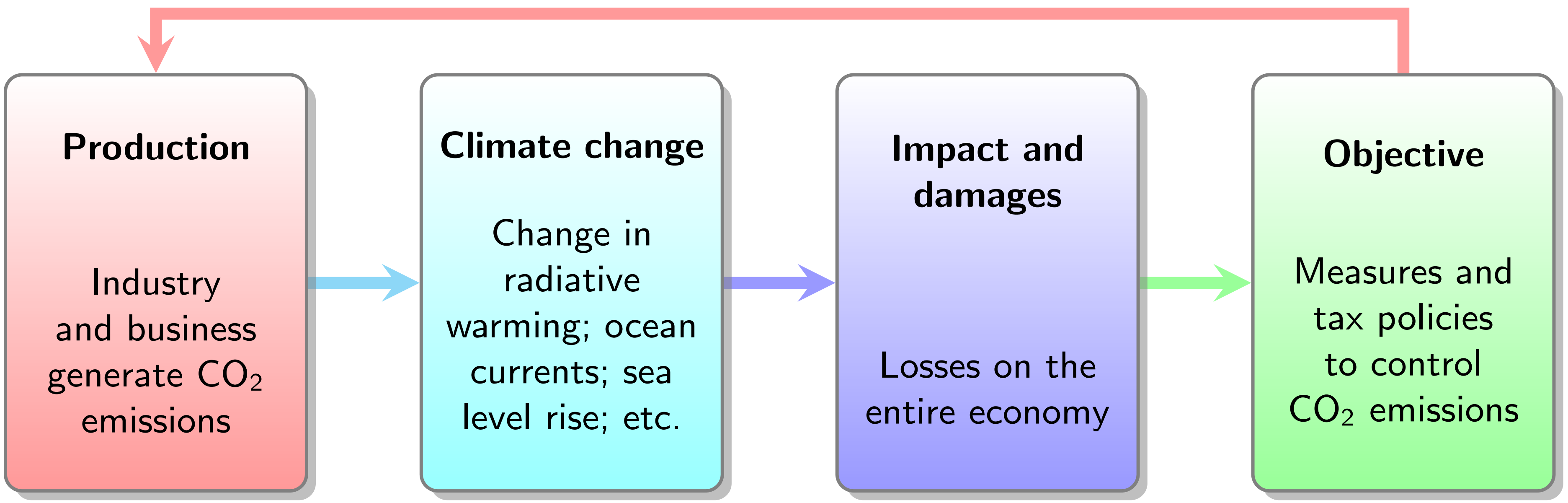
- **Optimization models**
  The inputs of these models are parameters and assumptions about the structure of the relationships between variables. The outputs provided by optimization process are scenarios depending on a set of constraints
- **Evaluation models**
  Based on exogenous scenarios, the outputs provide results from partial equilibriums between variables

Three main components of IAMs

1. Economic growth relationships
2. Dynamics of climate emissions
3. Objective function

Modeling framework

Figure 120: Economic models of climate risk



Modeling framework

1. Economic module
    1. Production function $\Longrightarrow$ GDP
    2. Impact of the climate risk on GDP (damage losses, mitigation and adaptation costs)
    3. The climate loss function depends on the temperature
2. Climate module
    1. Dynamics of GHG emissions
    2. Modeling of Atmospheric and lower ocean temperatures
3. Optimal control problem
    1. Maximization of the utility function
    2. We can test many variants

Modeling framework

The most famous IAM is the **Dynamic Integrated model of Climate and the Economy** (or DICE) developed by William Nordhaus[21]

[21] 2018 Nobel Laureate

Economic module

Production and consumption functions

- The **gross production** $Y(t)$ is given by a Cobb-Douglas function:

$$Y(t) = A(t) K(t)^{\gamma} L(t)^{1-\gamma}$$

  where:
  - $A(t)$ is the total productivity factor
  - $K(t)$ is the capital input
  - $L(t)$ is the labor input
  - $\gamma \in \left]0,1\right[$ measures the elasticity of the capital factor:
- Climate change impacts the **net output**:

$$Q(t) = \Omega_{\text{climate}}(t) Y(t) \leq Y(t)$$

- Classical identities $Q(t) = C(t) + I(t)$ and $I(t) = s(t) Q(t)$

Economic module

Production and consumption functions

- The dynamics of the state variables are:

$$\left\{\begin{array}{l}A(t) = (1 + g_A(t))\, A(t-1) \\ K(t) = (1 - \delta_K) K(t-1) + I(t) \\ L(t) = (1 + g_L(t))\, L(t-1)\end{array}\right.$$

- We have:

$$\left\{\begin{array}{l}g_A(t) = \dfrac{1}{1 + \delta_A} g_A(t-1) \\ g_L(t) = \dfrac{1}{1 + \delta_L} g_L(t-1)\end{array}\right.$$

Economic module

Labor input

Example #1

The world population was equal to 7.725 billion in 2019 and 7.805 billion in 2020. At the beginning of the 1970s, we estimate that the annual growth rate was equal to 2.045%. According to the United Nations, the global population could surpass 10 billion by 2100.

Economic module

Labor input

- In 2020, the annual growth rate was equal to:

$$g_L(2020) = \frac{L(2020)}{L(2019)} - 1 = \frac{7.805}{7.725} - 1 = 1.036\%$$

- Since we have $g_L(t) = \left(\frac{1}{1+\delta_L}\right)^{t-t_0} g_L(t_0)$, we deduce that:

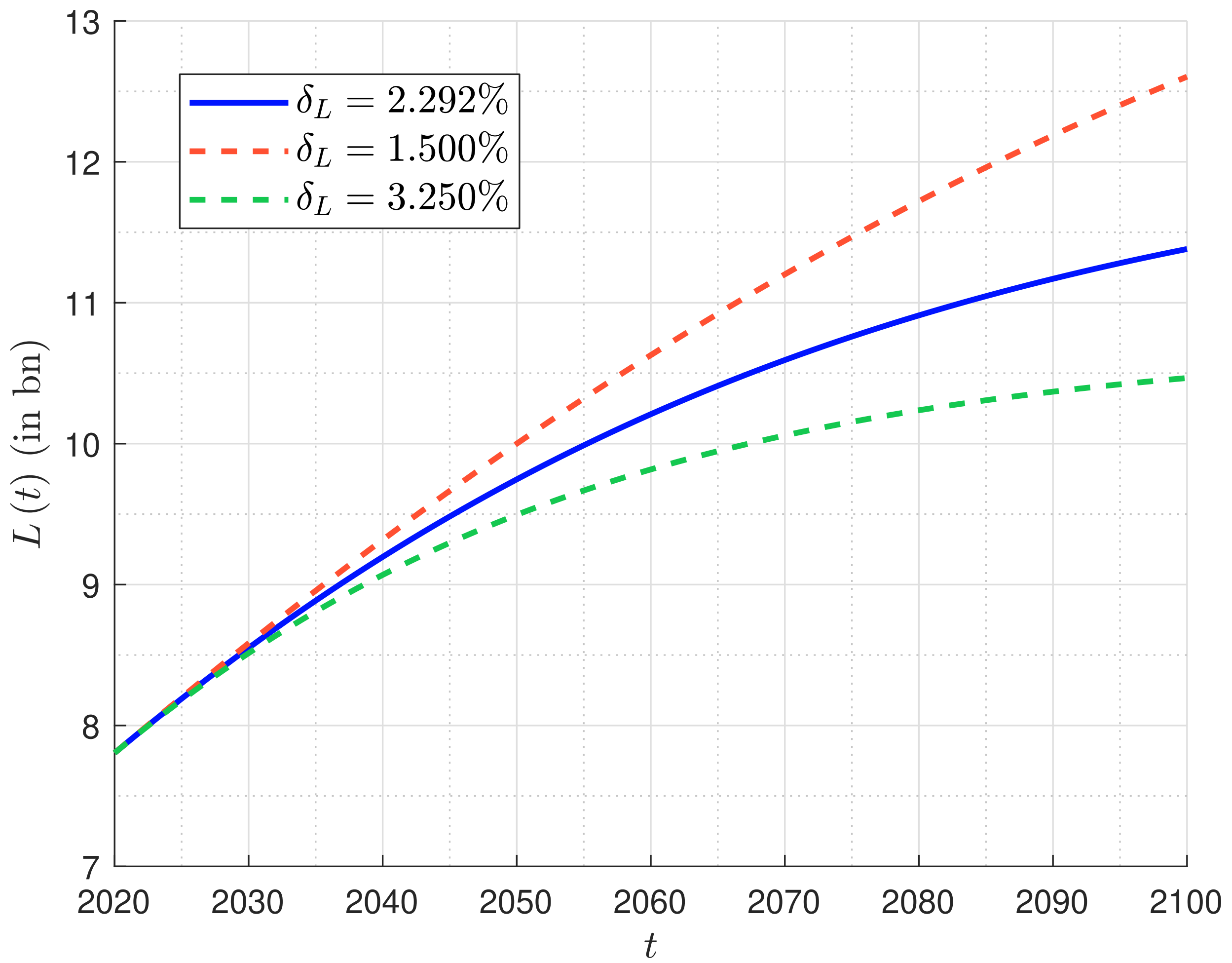
$$\delta_L = \left(\frac{g_L(t_0)}{g_L(t)}\right)^{1/(t-t_0)} - 1$$

- An estimate of δ_L is then:

$$\delta_L = \left(\frac{g_L(1970)}{g_L(2020)}\right)^{1/30} - 1 = 2.292\%$$

Economic module
Labor input

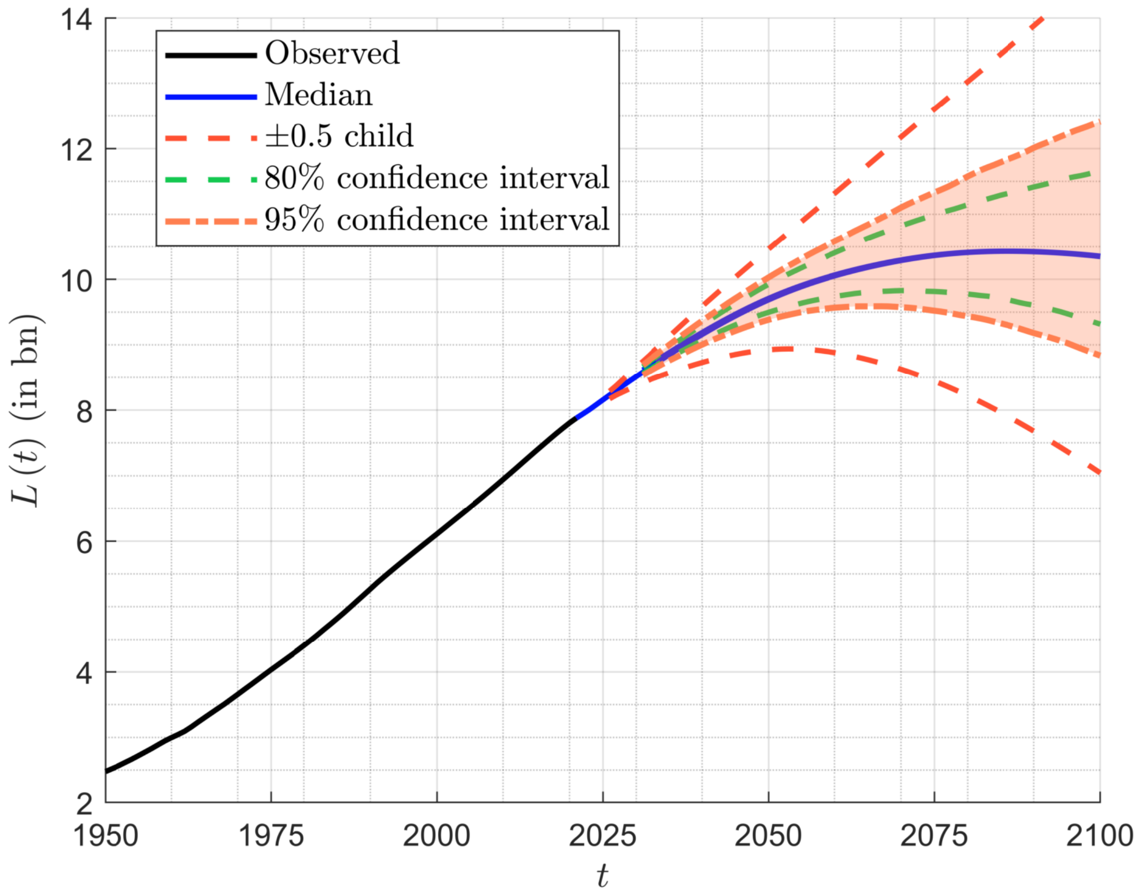
Figure 121: Evolution of the labor input $L(t)$



Economic module

Labor input

Figure 122: Projection of the world population



Source: United Nations (2022), `https://population.un.org/wpp`.

Economic module

Labor input

- AR(1) model:

$$g_L(t) = \phi g_L(t-1) + \varepsilon(t)$$

We have

$$\hat{\delta}_L = \frac{\left(1 - \hat{\phi}\right)}{\hat{\phi}}$$

- Log-linear model:

$$\ln g_L(t) = \beta_0 + \beta_1 (t - t_0) + \varepsilon(t)$$

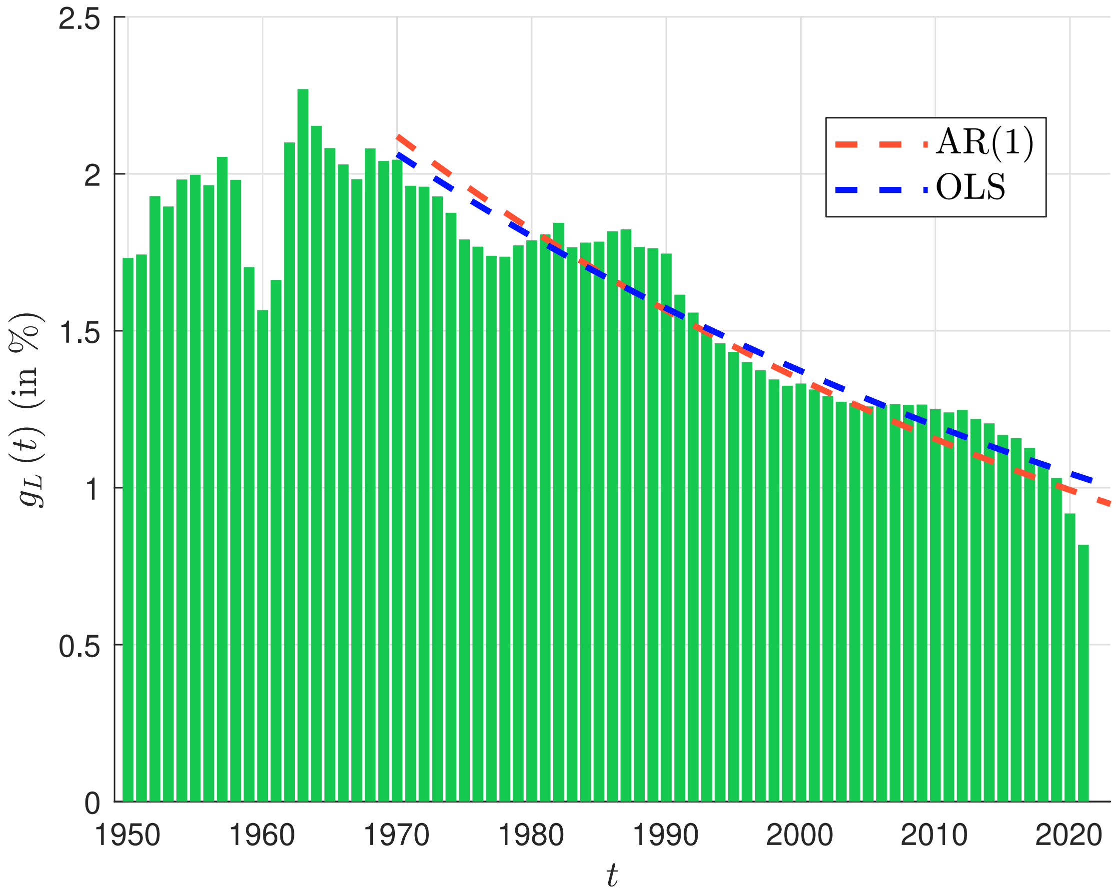
We have:

$$\hat{\delta}_L = e^{-\hat{\beta}_1} - 1$$

Economic module

Labor input

Figure 123: Population growth rate



Source: United Nations (2022), `https://population.un.org/wpp` & Author's calculations.

Economic module

Total factor productivity

Table 80: Average productivity growth rate (in %)

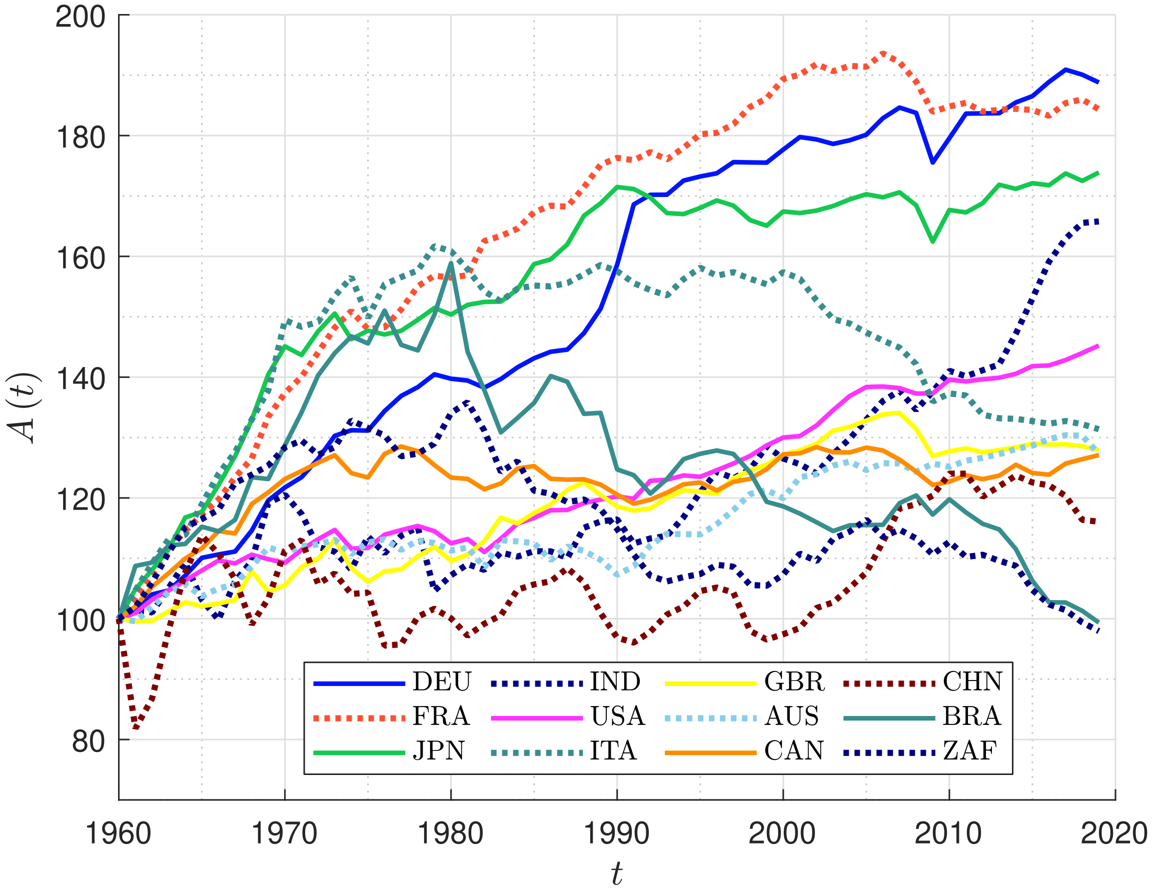
Country	1960-1970	1970-1980	1980-1990	1990-2000	2000-2010	2010-2020
AUS	1.02	0.07	−0.23	1.02	0.36	0.13
BRA	2.39	2.05	−1.04	−1.12	−0.17	−1.63
CAN	2.18	0.38	−0.25	0.21	−0.21	0.40
CHN	−0.03	−0.06	−0.04	−0.41	2.24	−0.35
FRA	3.59	1.63	1.12	0.61	−0.11	0.02
DEU	2.33	1.63	0.75	1.52	0.01	0.74
IND	2.37	−1.22	1.06	1.04	0.70	1.89
ITA	3.71	1.66	−0.19	−0.20	−1.32	−0.34
JPN	4.05	0.77	1.09	−0.22	−0.15	0.69
ZAF	2.37	0.30	−0.84	−1.11	0.50	−1.20
GBR	0.50	0.72	0.75	0.42	0.12	0.08
USA	1.00	0.42	0.46	0.73	0.65	0.56

Source: Penn World Table 10.01 (Feenstra *et al.*, 2015) & Author's calculations.

Economic module

Total factor productivity

Figure 124: Total factor productivity index (base 100 = 1960)

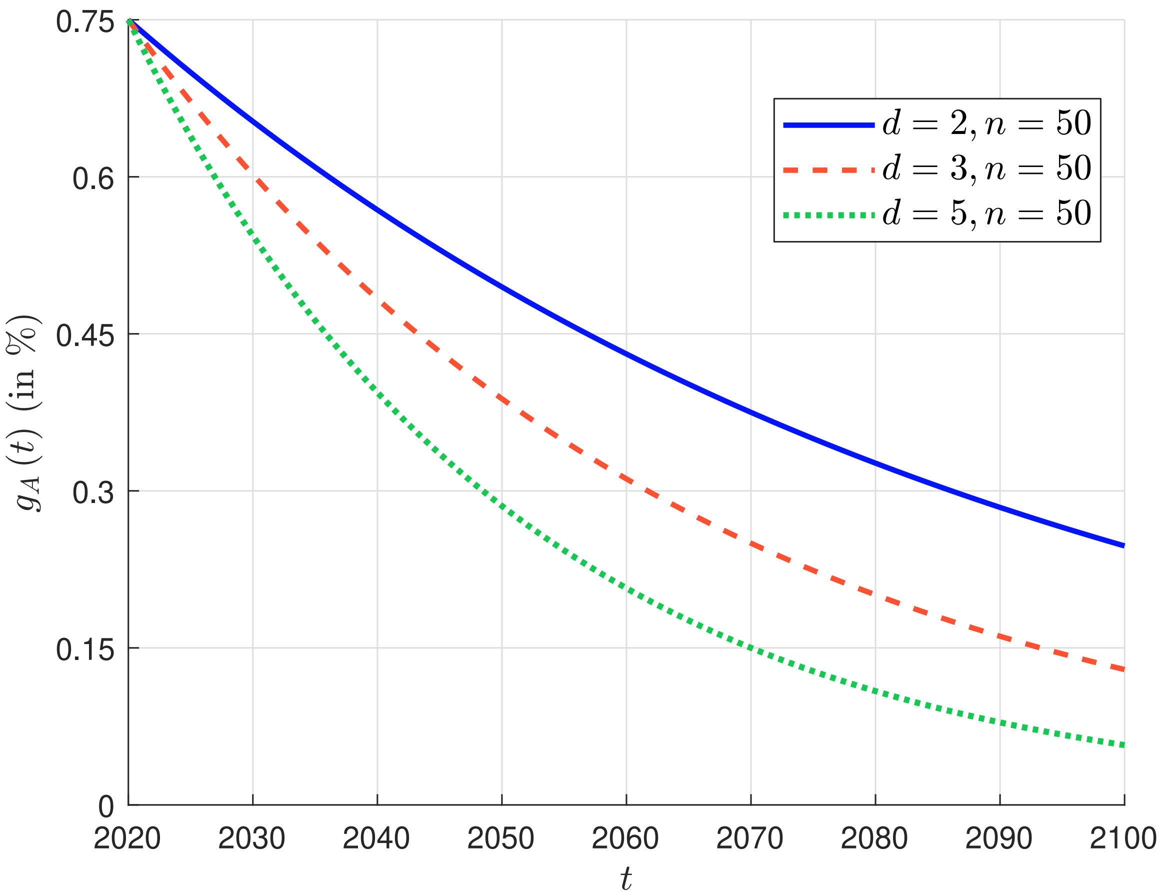


Source: Penn World Table 10.01 (Feenstra *et al.*, 2015) & Author's calculations.

Economic module

Total factor productivity

Figure 125: Dynamics of the TFP growth rate*



*We use the following calibration rule: $\delta_A = \sqrt[n]{d} - 1$

Economic module

Investment, capital stock and gross output

- Penn World Table/IMF's ICSD
- In 2019, we obtain $I(2019) = \$30.625$ tn, $K(2019) = \$318.773$ tn and $Y(2019) = \$124.418$ tn
- We also have:

$$\delta_K(t) = \frac{K(t-1) - K(t) + I(t)}{K(t-1)}$$

  and we obtain $\delta_K(2019) = 6.25\%$
- To calibrate the initial value of $A(t)$, we inverse the Coob-Douglas function:

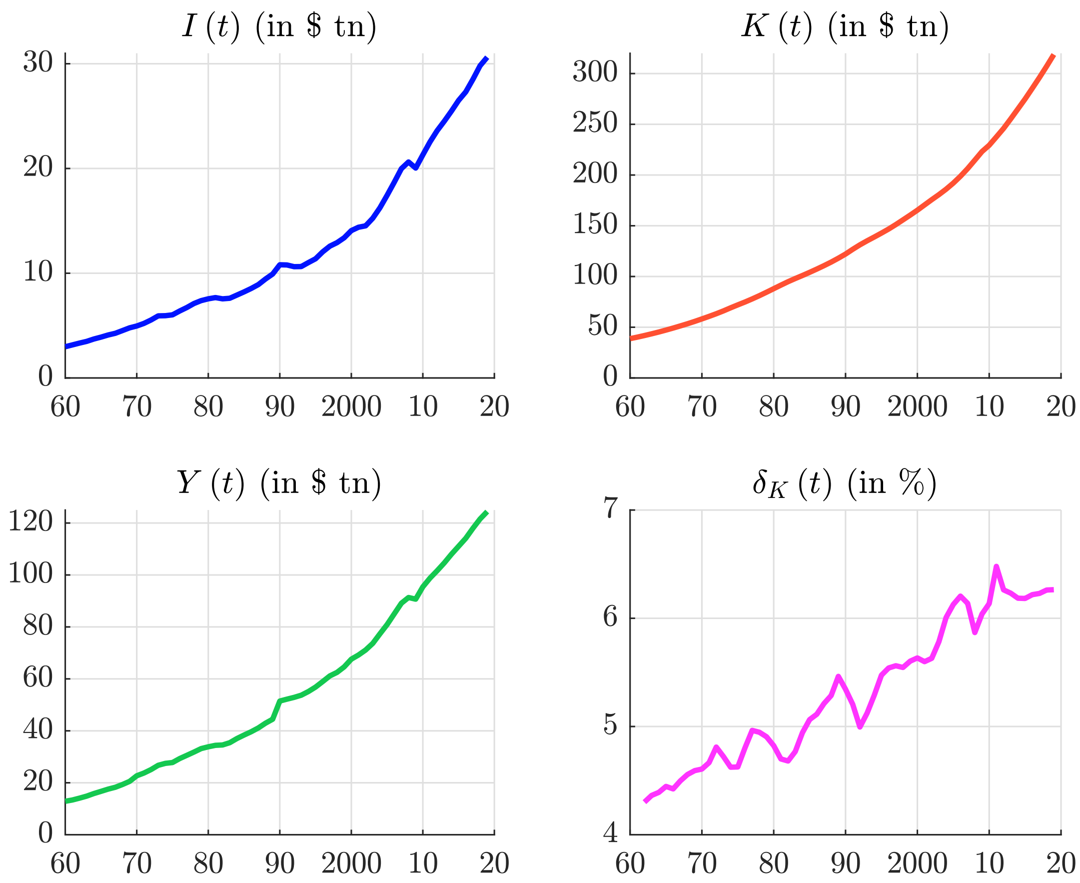
$$A(2019) = \frac{Y(t)}{K(t)^{\gamma} L(t)^{1-\gamma}} = \frac{124.418}{318.773^{0.30} \times 7.725^{0.70}} = 5.276$$

- The saving rate $s(t)$ is exogenous

Economic module

Investment, capital stock and gross output

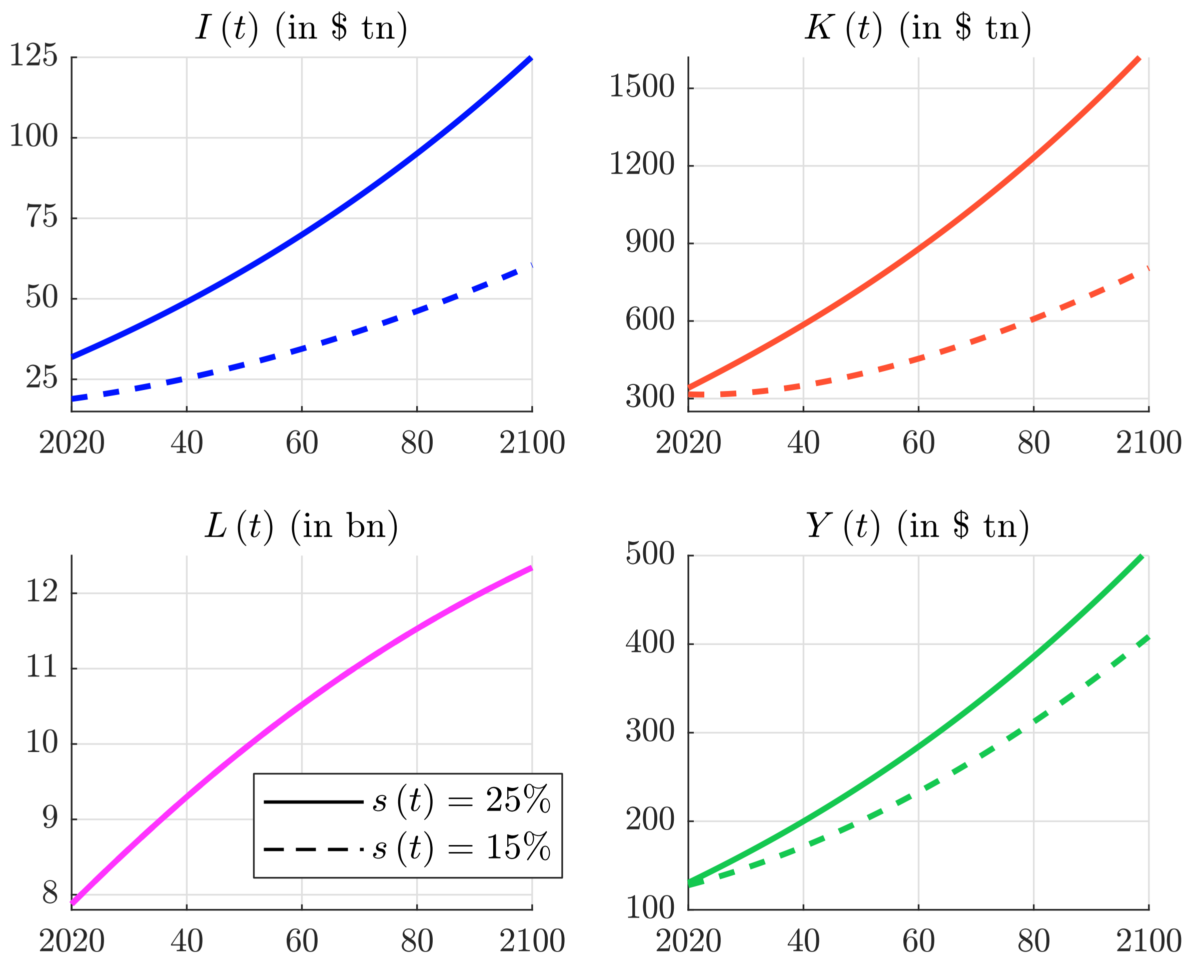
Figure 126: Historical estimates of $I(t)$, $K(t)$, $Y(t)$ and $\delta_K(t)$



Source: IMF Investment and Capital Stock Dataset (2021) & Author's calculations.

Economic module

Figure 127: Simulation of the DICE macroeconomic module



Economic module

Cost function of climate change

- The survival function is given by:

$$\Omega_{\text{climate}}(t) = \Omega_D(t)\,\Omega_\Lambda(t) = \frac{1}{1 + D(t)}(1 - \Lambda(t))$$

where:

- $D(t) \geq 0$ is the climate damage function (physical risk)
- $\Lambda(t) \geq 0$ is the mitigation or abatement cost (transition risk)

Economic module
Cost function of climate change

- The cost $D(t)$ resulting from natural disasters depends on the atmospheric temperature $\mathcal{T}_{\mathrm{AT}}(t)$:

$$D(t) = \psi_1 \mathcal{T}_{\mathrm{AT}}(t) + \psi_2 \mathcal{T}_{\mathrm{AT}}(t)^2$$

- The abatement cost function depends on the control variable $\mu(t)$:

$$\Lambda(t) = \theta_1(t)\,\mu(t)^{\theta_2}$$

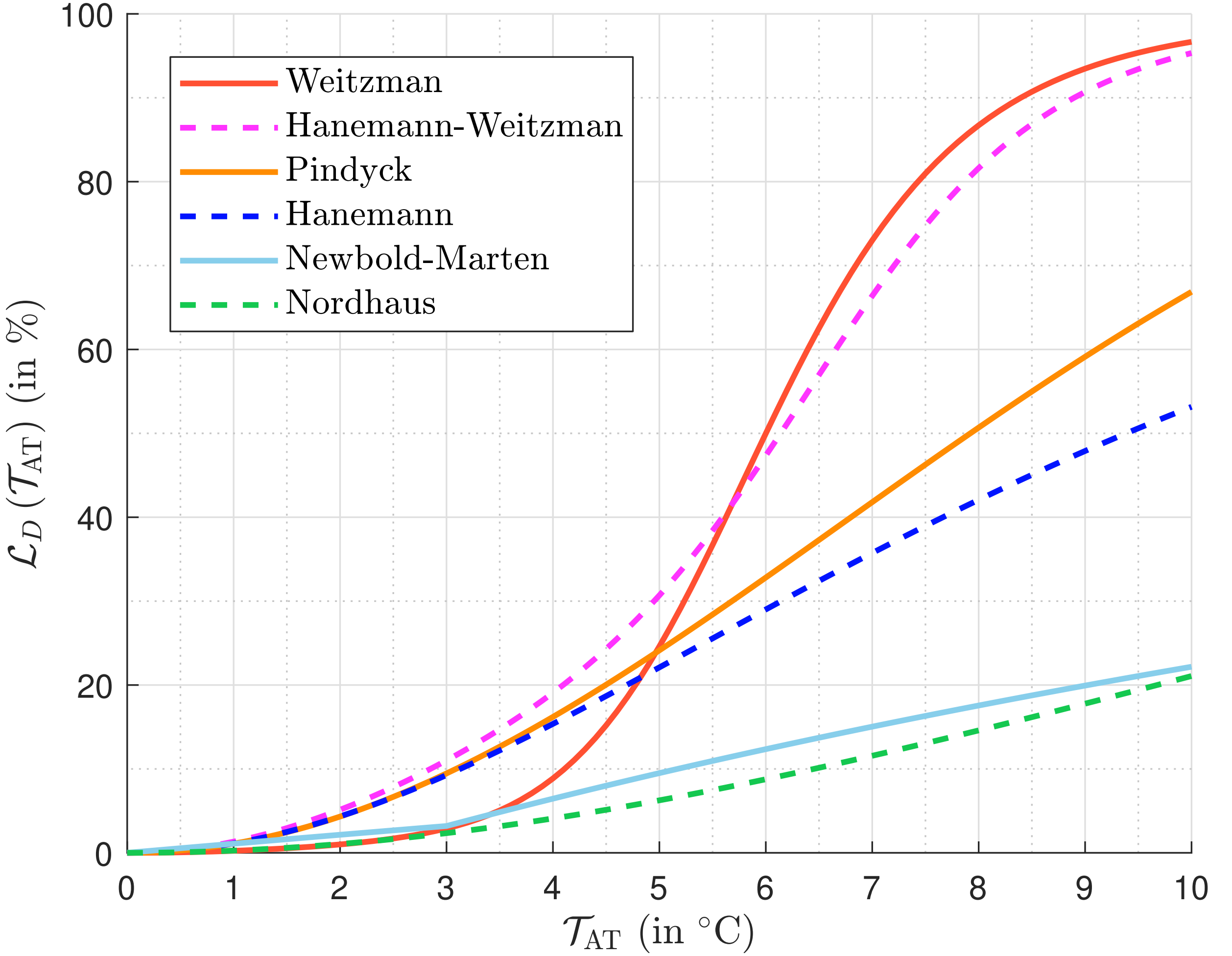
- The global impact of climate change is equal to:

$$\Omega_{\mathrm{climate}}(t) = \frac{1 - \theta_1(t)\,\mu(t)^{\theta_2}}{1 + \psi_1 \mathcal{T}_{\mathrm{AT}}(t) + \psi_2 \mathcal{T}_{\mathrm{AT}}(t)^2}$$

Economic module

Cost function of climate change

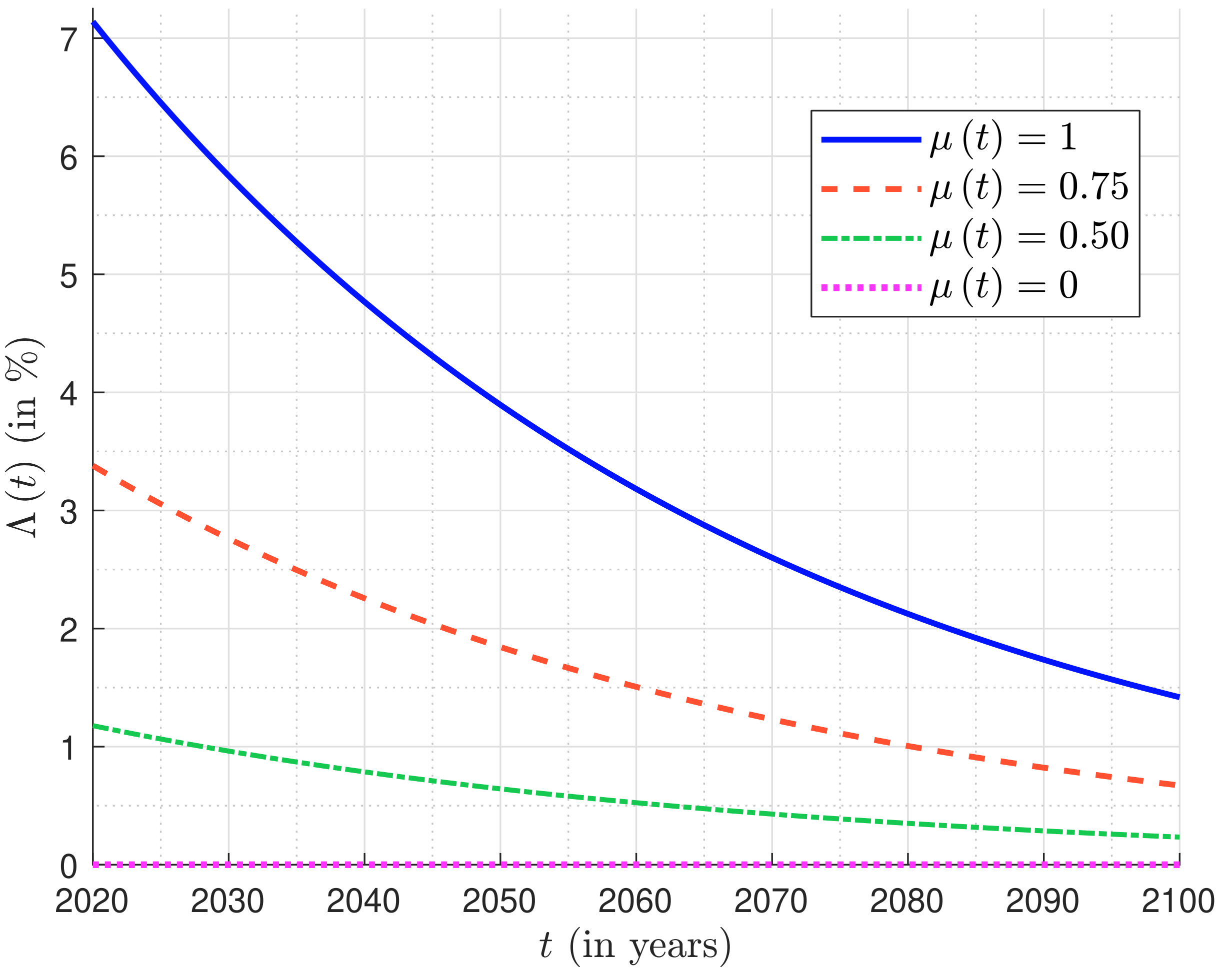
Figure 128: Loss function due to climate damage costs



Economic module

Cost function of climate change

Figure 129: Abatement cost function



Climate module

GHG emissions

- The total GHG emissions depends on the production $Y(t)$ and the land use emissions $\mathcal{CE}_{\mathrm{Land}}(t)$:

$$\begin{aligned}\mathcal{CE}(t) &= \mathcal{CE}_{\mathrm{Industry}}(t) + \mathcal{CE}_{\mathrm{Land}}(t) \\ &= (1-\mu(t))\sigma(t)Y(t) + \mathcal{CE}_{\mathrm{Land}}(t)\end{aligned}$$

- $\sigma(t)$ is the anthropogenic carbon intensity of the economy:

$$\sigma(t) = (1 + g_{\sigma}(t))\sigma(t-1)$$

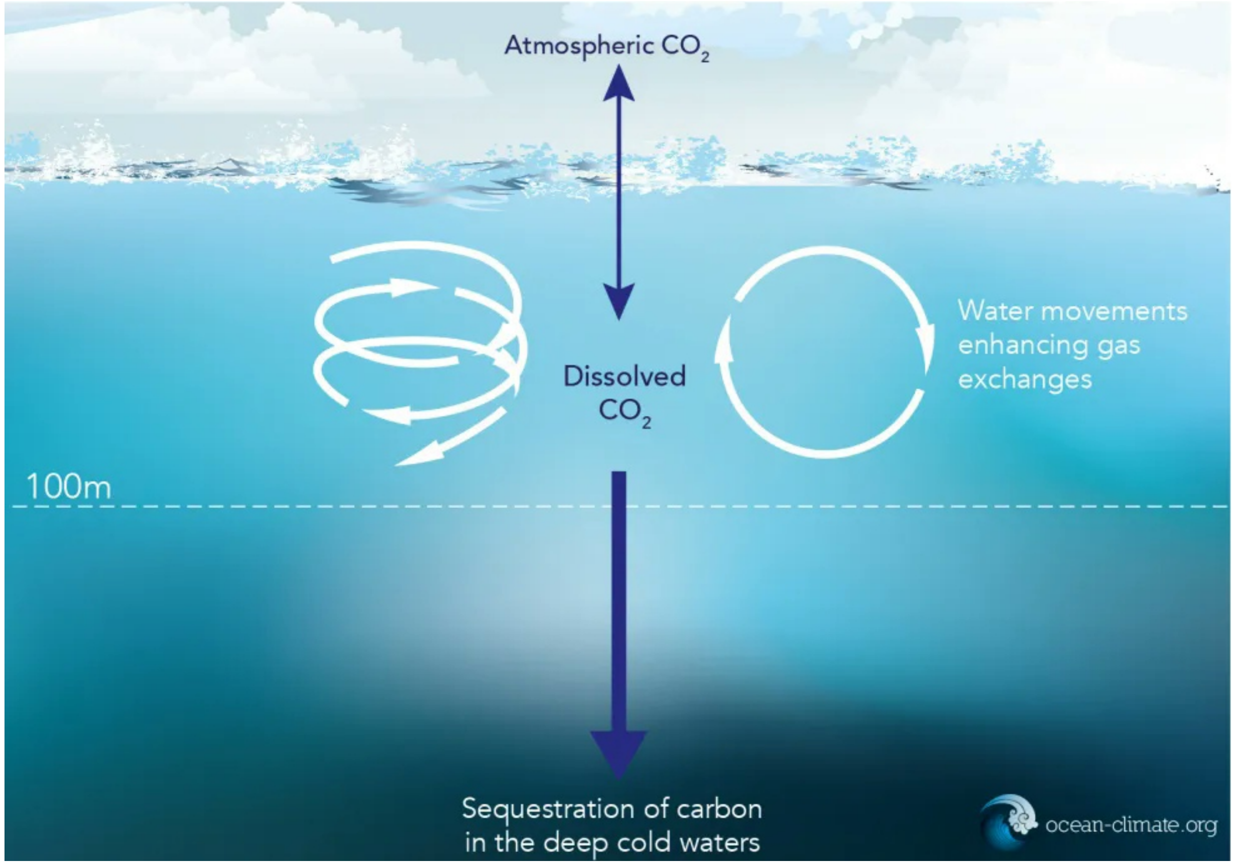
where:

$$g_{\sigma}(t) = \frac{1}{1+\delta_{\sigma}} g_{\sigma}(t-1)$$

Climate module

Temperature modeling

Figure 130: Physical carbon pump



Source: `ocean-climate.org`.

Climate module

Concentration modeling

- We have:

$$\left\{\begin{array}{l} \mathcal{CC}_{\mathrm{AT}}(t)=\phi_{1,1}\mathcal{CC}_{\mathrm{AT}}(t-1)+\phi_{1,2}\mathcal{CC}_{\mathrm{UP}}(t-1)+\phi_1\boldsymbol{\mathcal{CE}}(t) \\ \mathcal{CC}_{\mathrm{UP}}(t)=\phi_{2,1}\mathcal{CC}_{\mathrm{AT}}(t-1)+\phi_{2,2}\mathcal{CC}_{\mathrm{UP}}(t-1)+\phi_{2,3}\mathcal{CC}_{\mathrm{LO}}(t-1) \\ \mathcal{CC}_{\mathrm{LO}}(t)=\phi_{3,2}\mathcal{CC}_{\mathrm{UP}}(t-1)+\phi_{3,3}\mathcal{CC}_{\mathrm{LO}}(t-1) \end{array}\right.$$

- The dynamics of $\mathcal{CC}=(\mathcal{CC}_{\mathrm{AT}},\mathcal{CC}_{\mathrm{UP}},\mathcal{CC}_{\mathrm{LO}})$ is a VAR(1) process:

$$\mathcal{CC}(t)=\Phi_{\mathcal{CC}}\mathcal{CC}(t-1)+B_{\mathcal{CC}}\boldsymbol{\mathcal{CE}}(t)$$

Carbon cycle diffusion matrix

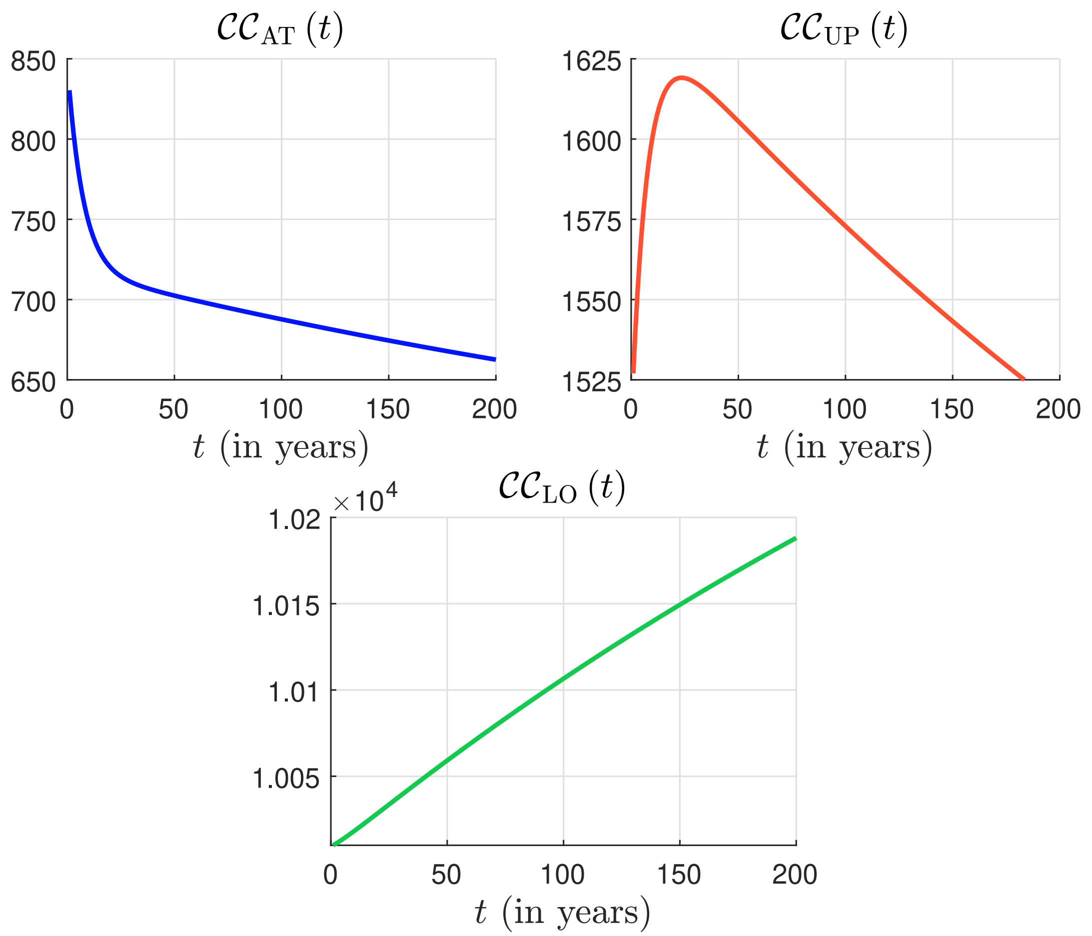
We have:

$$\Phi_{\mathcal{CC}}=\begin{pmatrix} 91.20\% & 3.83\% & 0 \\ 8.80\% & 95.92\% & 0.03\% \\ 0 & 0.25\% & 99.97\% \end{pmatrix}$$

Climate module

Concentration modeling

Figure 131: Impulse response analysis ($\Delta\mathcal{CE} = -1$ $\mathrm{GtCO_2e}$)



Climate module

Radiative forcing

We have:

$$\mathcal{F}_{\mathrm{RAD}}(t) = \frac{\eta}{\ln 2} \ln \left(\frac{\mathcal{CC}_{\mathrm{AT}}(t)}{\mathcal{CC}_{\mathrm{AT}}(1750)} \right) + \mathcal{F}_{\mathrm{EX}}(t)$$

where:

- $\mathcal{F}_{\mathrm{RAD}}(t)$ is the change in total radiative forcing of GHG emissions since 1750 (expressed in $\mathrm{W/m^2}$)
- η is the temperature forcing parameter
- $\mathcal{F}_{\mathrm{EX}}(t)$ is the exogenous forcing (other GHG emissions)

Climate module

Temperature modeling

- The climate system for temperatures is characterized by a two-layer system:

$$\left\{\begin{array}{lcl}\mathcal{T}_{\mathrm{AT}}(t) & = & \mathcal{T}_{\mathrm{AT}}(t-1)+\xi_{1}\left(\mathcal{F}_{\mathrm{RAD}}(t)-\xi_{2}\mathcal{T}_{\mathrm{AT}}(t-1)-\right. \\ & & \left.\xi_{3}\left(\mathcal{T}_{\mathrm{AT}}(t-1)-\mathcal{T}_{\mathrm{LO}}(t-1)\right)\right) \\ \mathcal{T}_{\mathrm{LO}}(t) & = & \mathcal{T}_{\mathrm{LO}}(t-1)+\xi_{4}\left(\mathcal{T}_{\mathrm{AT}}(t-1)-\mathcal{T}_{\mathrm{LO}}(t-1)\right)\end{array}\right.$$

- Let $\mathcal{T}=(\mathcal{T}_{\mathrm{AT}},\mathcal{T}_{\mathrm{LO}})$ be the temperature vector. We have:

$$\mathcal{T}(t)=\Xi_{\mathcal{T}}\mathcal{T}(t-1)+B_{\mathcal{T}}\mathcal{F}_{\mathrm{RAD}}(t)$$

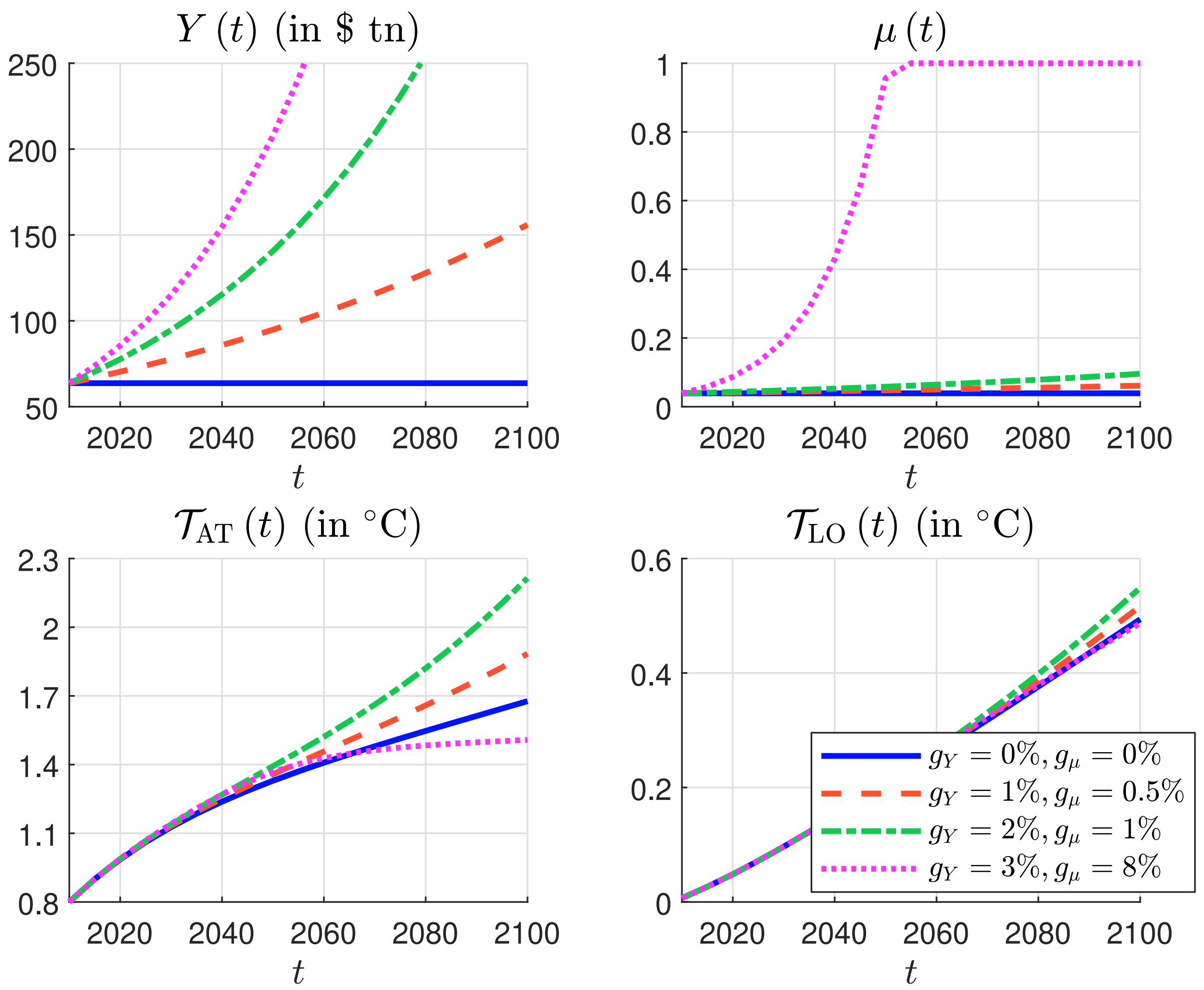
Climate module

Table 81: Output of the DICE climate module ($Y(t) = Y(t_0)$, $\mu(t) = \mu(t_0)$)

t	$\mathcal{CE}(t)$	$\sigma(t)$	$\mathcal{CC}_{\mathrm{AT}}(t)$	$\mathcal{F}_{\mathrm{RAD}}(t)$	$\mathcal{T}_{\mathrm{AT}}(t)$	$\mathcal{T}_{\mathrm{LO}}(t)$
2010	36.91	0.55	830.4	2.14	0.800	0.007
2015	36.25	0.55	825.7	2.14	0.900	0.027
2020	36.06	0.56	821.9	2.14	0.986	0.048
2025	35.97	0.57	818.9	2.14	1.061	0.072
2030	35.98	0.57	816.6	2.15	1.127	0.097
2035	36.05	0.58	814.9	2.16	1.186	0.122
2040	36.18	0.58	813.9	2.18	1.238	0.149
2045	36.36	0.59	813.3	2.20	1.286	0.176
2050	36.58	0.59	813.3	2.23	1.329	0.204
2055	36.82	0.60	813.6	2.26	1.370	0.232
2060	37.09	0.61	814.4	2.29	1.408	0.261
2065	37.39	0.61	815.4	2.32	1.445	0.289
2070	37.70	0.62	816.8	2.35	1.480	0.318
2075	38.02	0.62	818.4	2.39	1.514	0.347
2080	38.36	0.63	820.3	2.43	1.547	0.376
2085	38.71	0.64	822.4	2.46	1.580	0.406
2090	39.06	0.64	824.7	2.50	1.612	0.435
2095	39.43	0.65	827.1	2.55	1.645	0.464
2100	39.80	0.66	829.7	2.59	1.677	0.494

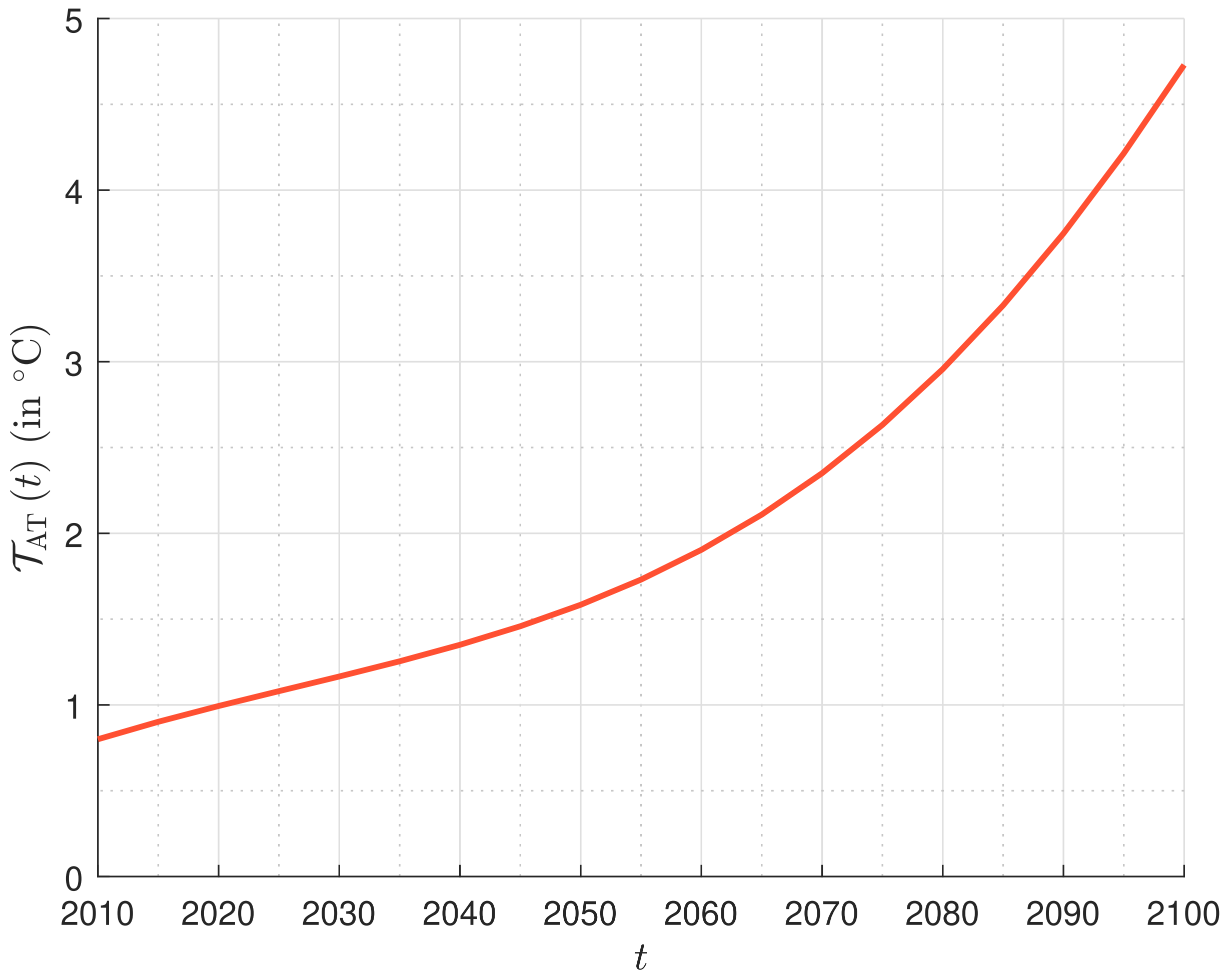
Climate module

Figure 132: Simulation of the DICE climate module



Climate module

Figure 133: The nightmare climate-economic scenario ($g_Y = 0\%$, $\mu(t) = 0$)



The optimal control problem

Optimization problem

- The social welfare function W is equal to:

$$W\left(s\left(t\right),\mu\left(t\right)\right)=\sum_{t=t_0+1}^{T}\frac{L\left(t\right)\mathcal{U}\left(c\left(t\right)\right)}{\left(1+\rho\right)^{t-t_0}}$$

  where ρ is the (generational) discount rate and $c\left(t\right)=C\left(t\right)/L\left(t\right)$ is the consumption per capita
- $\mathcal{U}\left(c\right)=\left(c^{1-\alpha}-1\right)/\left(1-\alpha\right)$ is the CRRA utility function
- The optimal control problem is then given by:

$$\begin{aligned}\left(s^{\star}\left(t\right),\mu^{\star}\left(t\right)\right) & = & \arg\max W\left(s\left(t\right),\mu\left(t\right)\right)\\ & & \text{s.t.}\quad\left\{\begin{array}{l}\text{DICE Equations}\\ \mu\left(t\right)\in\left[0,1\right]\\ s\left(t\right)\in\left[0,1\right]\end{array}\right.\end{aligned}$$

The optimal control problem

The important variables are:

- $\mathcal{T}_{\mathrm{AT}}(t)$ — Atmospheric temperature
- $\mu(t)$ — Control rate (mitigation policies)
- $\mathcal{CE}(t)$ — Total emissions of GHG
- $\mathrm{SCC}(t)$ — Social cost of carbon

Social cost of carbon (SCC)

"The most important single economic concept in the economics of climate change is the social cost of carbon (SCC). This term designates the economic cost caused by an additional tonne of carbon dioxide emissions or its equivalent. In a more precise definition, it is the change in the discounted value of economic welfare from an additional unit of COtwo-equivalent emissions. The SCC has become a central tool used in climate change policy, particularly in the determination of regulatory policies that involve greenhouse gas emissions." (Nordhaus, 2017).

Social cost of carbon (SCC)

Mathematical definition

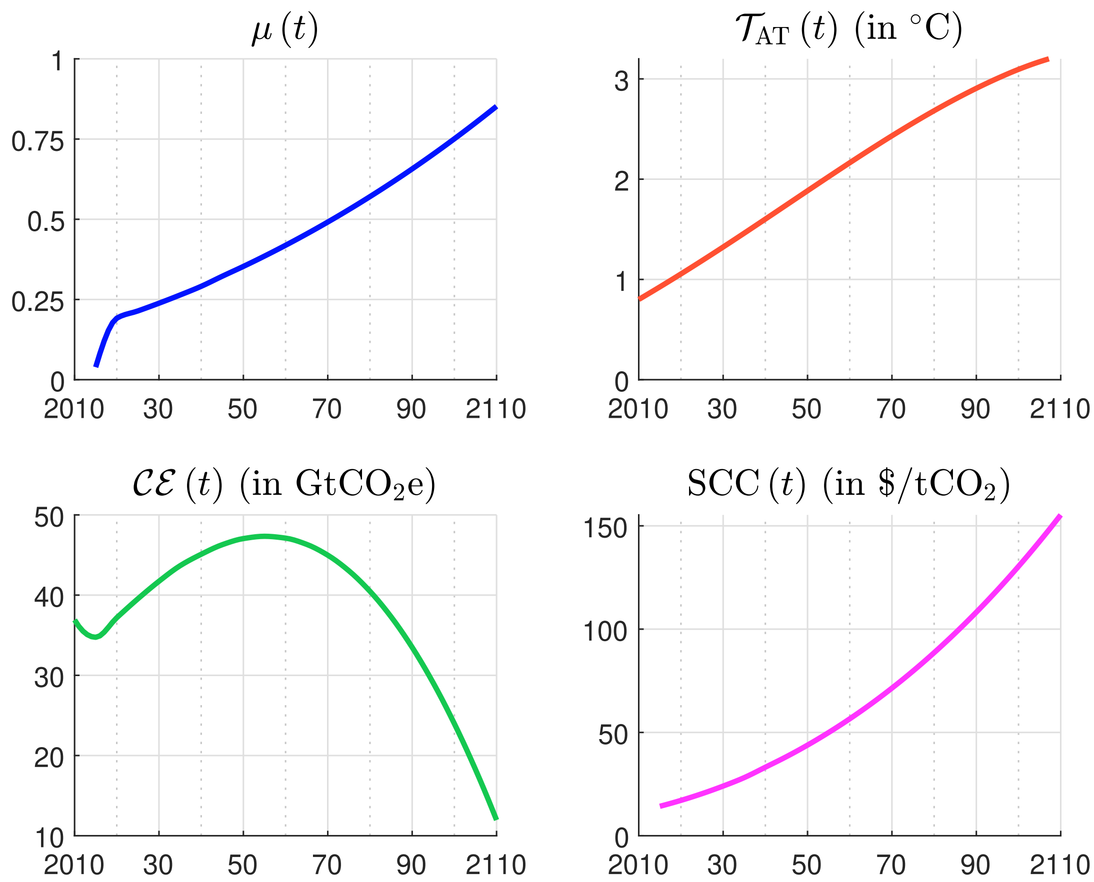
- The social cost of carbon is then defined as:

$$\mathrm{SCC}(t) = \frac{\dfrac{\partial W(t)}{\partial \mathcal{CE}(t)}}{\dfrac{\partial W(t)}{\partial C(t)}} = \frac{\partial C(t)}{\partial \mathcal{CE}(t)}$$

- It is expressed in $\$/\mathrm{tCO}_2$

Social cost of carbon (SCC)

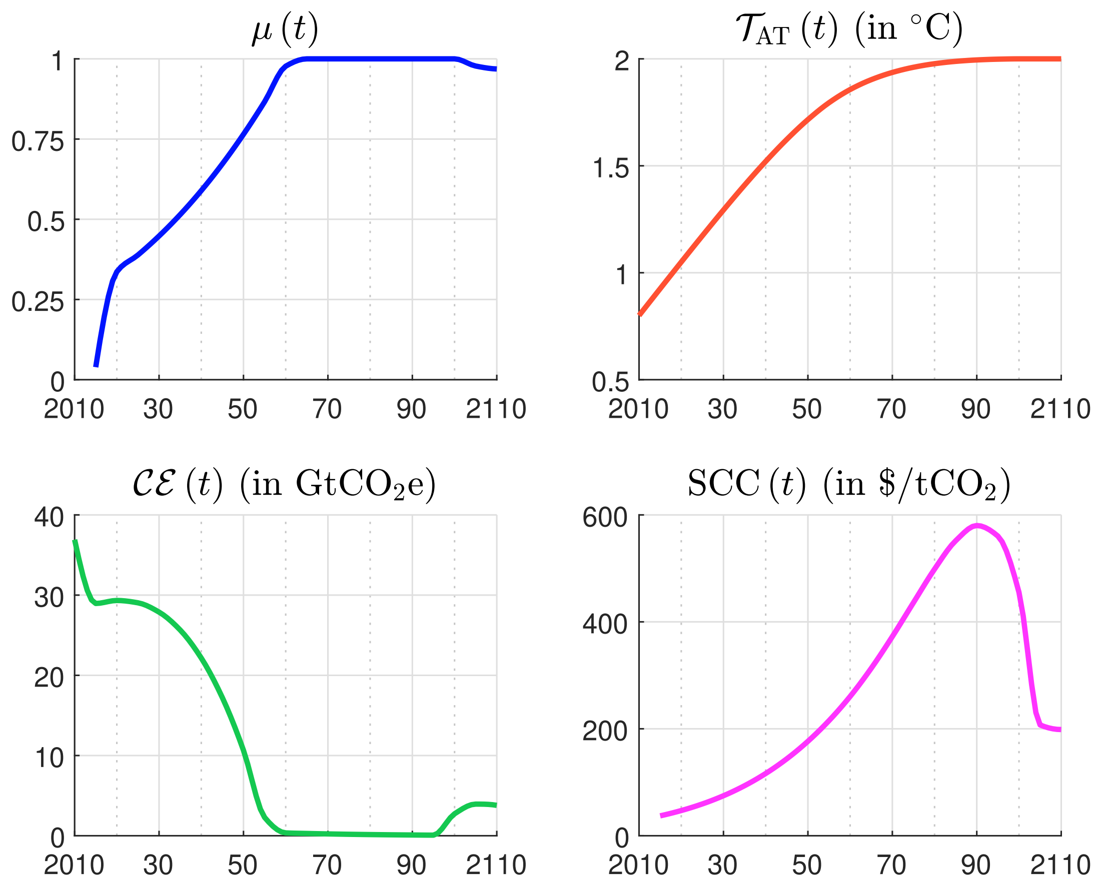
Figure 134: Optimal welfare scenario (DICE 2013R)



Source: Le Guenedal (2019).

Social cost of carbon (SCC)

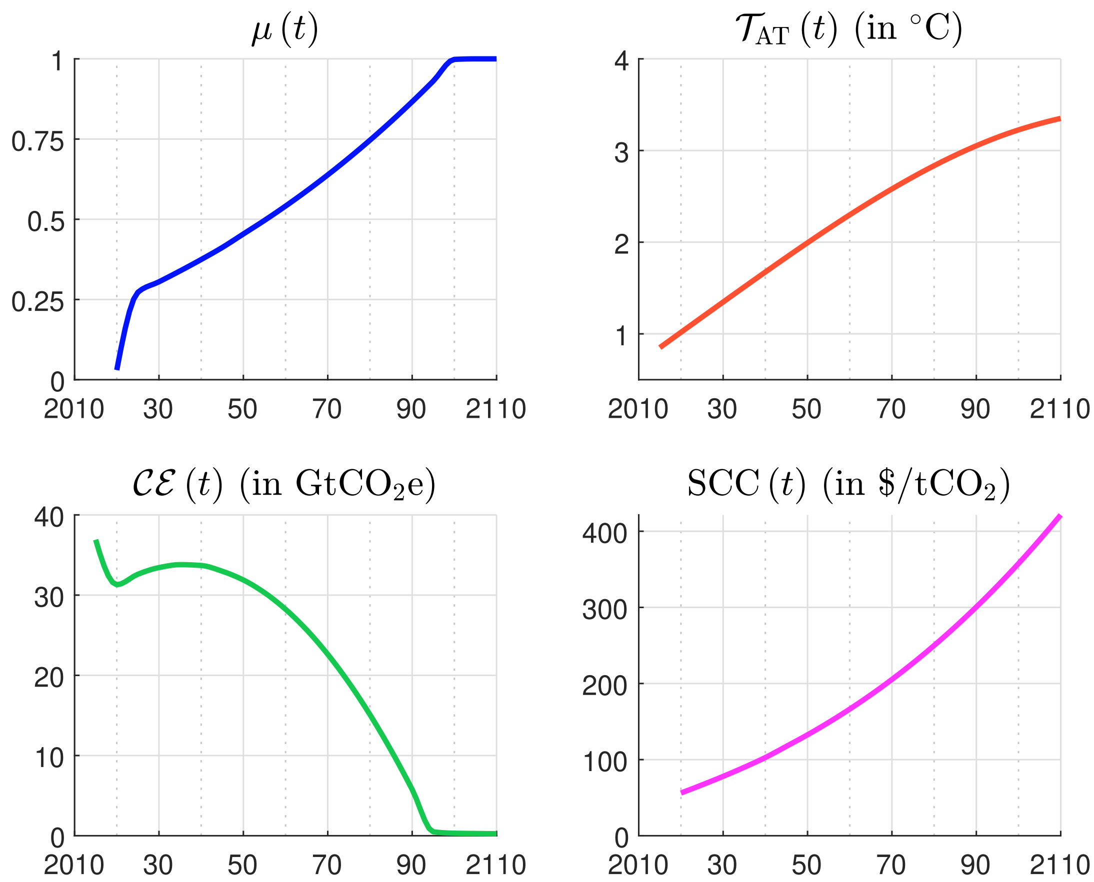
Figure 135: $2^{\circ}\mathrm{C}$ scenario (DICE 2013R)



Source: Le Guenedal (2019).

Social cost of carbon (SCC)

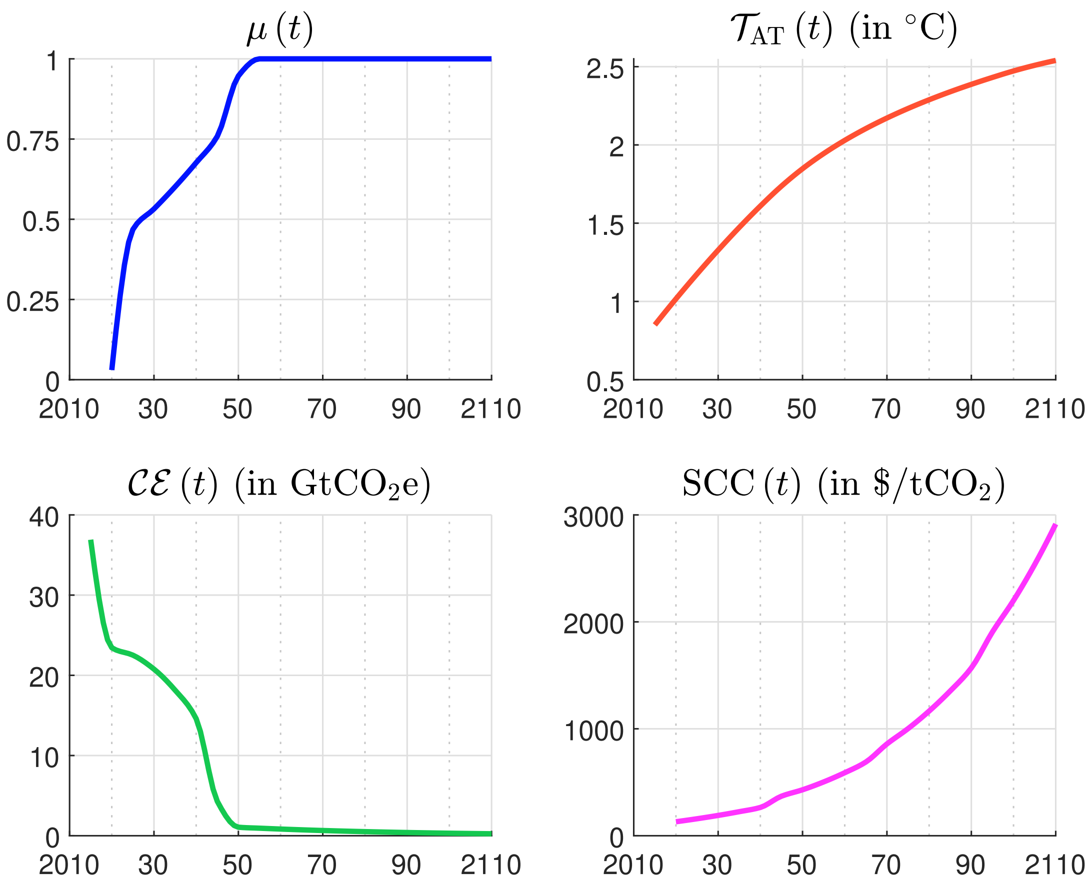
Figure 136: Optimal welfare scenario (DICE 2016R)



Source: Le Guenedal (2019).

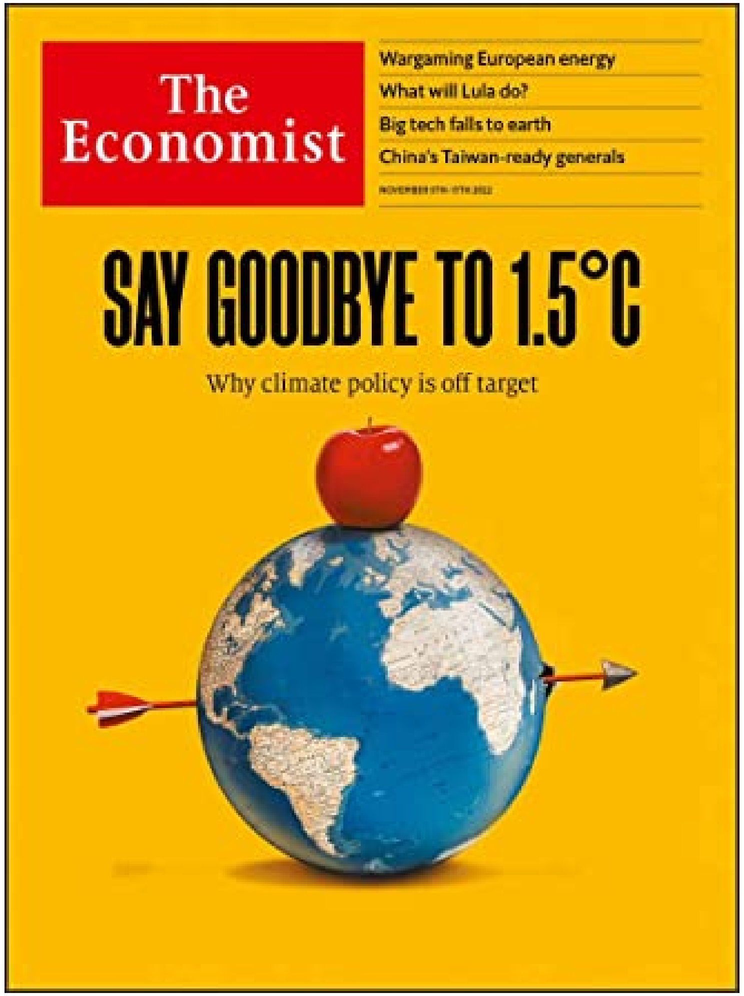
Social cost of carbon (SCC)

Figure 137: $2^{\circ}\mathrm{C}$ scenario (DICE 2016R)



Source: Le Guenedal (2019).

The tragedy of the horizon



The tragedy of the horizon

Achieving the 2°C scenario

- In 2013, the DICE model suggested to reduce drastically CO_2 emissions...
- Since 2016, **the 2°C trajectory is no longer feasible!** (minimum $\approx$ 2.6°C)
- For many models, we now have:

$$\mathbb{P}\left(\Delta T > 2^{\circ} C\right) > 95\%$$

Social cost of carbon (SCC)

Table 82: Global SCC under different scenario assumptions (in $\$/tCO_2$)

Scenario	2015	2020	2025	2030	2050	CAGR
Baseline	31.2	37.3	44.0	51.6	102.5	3.46%
Optimal	30.7	36.7	43.5	51.2	103.6	3.54%
2.5°C-max	184.4	229.1	284.1	351.0	1 006.2	4.97%
2.5°C-mean	106.7	133.1	165.1	203.7	543.3	4.76%

Source: Nordhaus (2017).

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

- In 2007, Nicholas Stern published a report called **The Economics of Climate Change: The Stern Review**
- The Stern Review called for sharp and immediate action to stabilize greenhouse gases because:

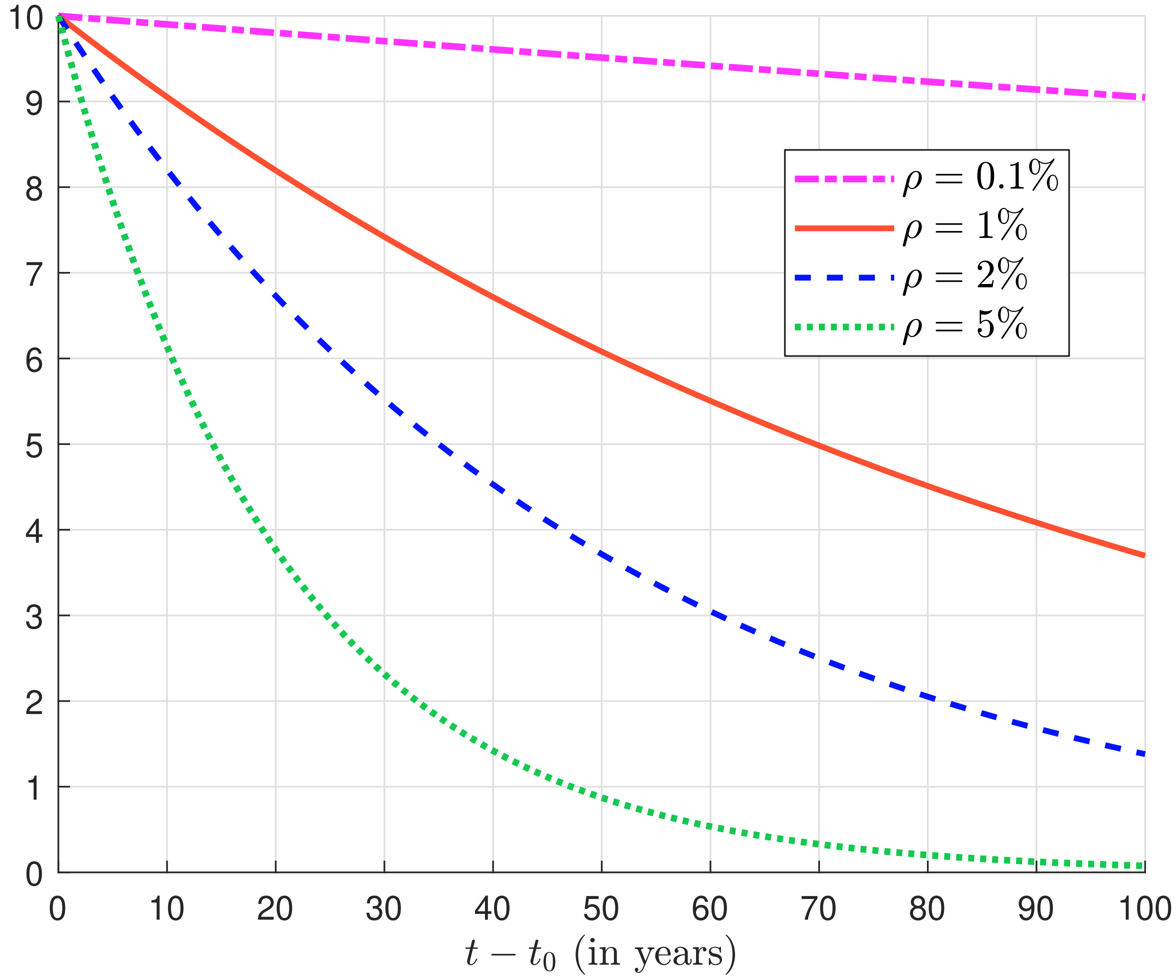
  "**the benefits of strong, early action on climate change outweighs the costs**"

- The Stern Review proposes to use $\rho = \mathbf{0.10\%}$

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

Figure 138: Discounted value of $10



Social cost of carbon (SCC)

The Stern-Nordhaus controversy

- The time (or generational) discount rate ρ is also called the pure rate of time preference
- It is related to the Ramsey rule:

$$r = \rho + \alpha g$$

where:

  - r is the real interest rate
  - $g = \partial\, c(t) / c(t)$ is the growth rate of per capita consumption
  - α is the consumption elasticity of the utility function

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

We report the computations done by Dasgupta (2008):

Model	ρ	α	g_c	r
Cline (1992)	0.0%	1.5	1.3%	2.05%
Nordhaus (2007)	3.0%	1.0	1.3%	4.30%
Stern (2007)	0.1%	1.0	1.3%	1.40%

Social cost of carbon (SCC)

The Stern-Nordhaus controversy

Table 83: Global SCC under different discount rate assumptions

Discount rate	2015	2020	2025	2030	2050	CAGR
Stern	197.4	266.5	324.6	376.2	629.2	3.37%
Nordhaus	30.7	36.7	43.5	51.2	103.6	3.54%
2.5%	128.5	140.0	152.0	164.6	235.7	1.75%
3%	79.1	87.3	95.9	104.9	156.6	1.97%
4%	36.3	40.9	45.8	51.1	81.7	2.34%
5%	19.7	22.6	25.7	29.1	49.2	2.65%

Source: Nordhaus (2017).

Some models

- AIM ______________________ RCP 6.0
- DICE/RICE
- FUND
- GCAM
- IMACLIM (CIRED)
- IMAGE ______________________ RCP 2.6
- MESSAGE ______________________ RCP 8.5
- MiniCAM ______________________ RCP 4.5
- PAGE
- REMIND
- RESPONSE (CIRED)
- WITCH

Some models

Table 84: Main integrated assessment models

Model	Reference	Name
Stylized simple models		
DICE	Nordhaus and Sztorc (2013)	Dynamic Integrated Climate-Economy
FUND	Anthoff and Tol (2014)	Climate Framework for Uncertainty, Negotiation and Distribution
PAGE	Hope (2011)	Policy Analysis of the Greenhouse Effect
Complex models		
AIM/CGE	Fujimori *et al.* (2017)	Asia-Pacific Integrated Model/Computable General Equilibrium
GCAM	Calvin *et al.* (2019)	Global Change Assessment Model
GLOBIOM	Havlik *et al.* (2018)	Global Biosphere Management Model
IMACLIM-R	Sassi *et al.* (2010)	Integrated Model to Assess Climate Change
IMAGE	Stehfest *et al.* (2014)	Integrated Model to Assess the Greenhouse Effect
MAGICC	Meinshausen *et al.* (2011)	Model for the Assessment of Greenhouse Gas Induced Climate Change
MAgPIE	Dietrich *et al.* (2019)	Model of Agricultural Production and its Impact on the Environment
MESSAGEix	Huppmann *et al.* (2019)	Model for Energy Supply Strategy Alternatives and their General Environmental Impact
REMIND	Aboumahboub *et al.* (2020)	REgional Model of INvestments and Development
WITCH	Bosetti *et al.* (2006)	World Induced Technical Change Hybrid

Source: Grubb *et al.* (2021) & Author's research.

Stylized IAMs

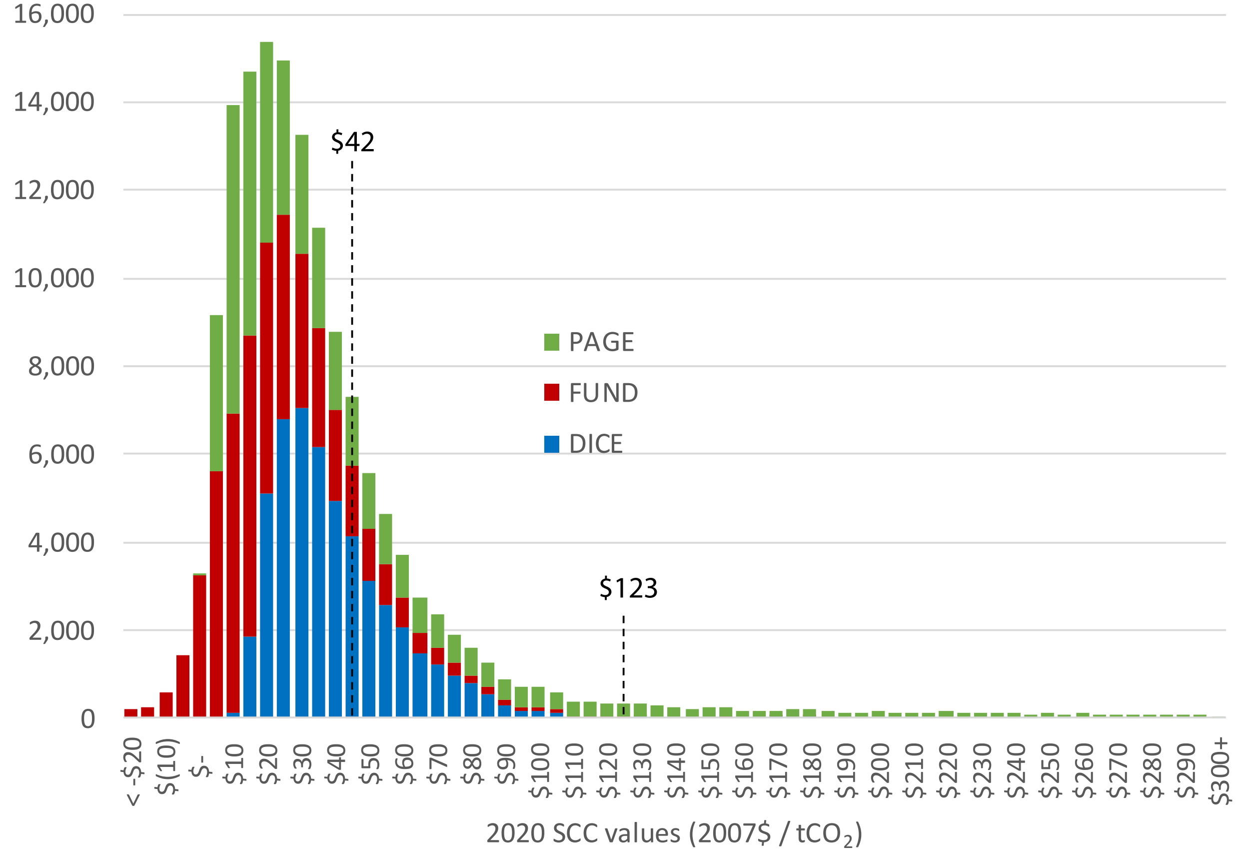
The Leaders

- DICE
- FUND
- PAGE

$\Rightarrow$ SCC: PAGE $\succ$ DICE $\succ$ FUND

Stylized IAMs

Figure 139: Histogram of the 150 000 US Government SCC estimates for 2020 with a 3% discount rate

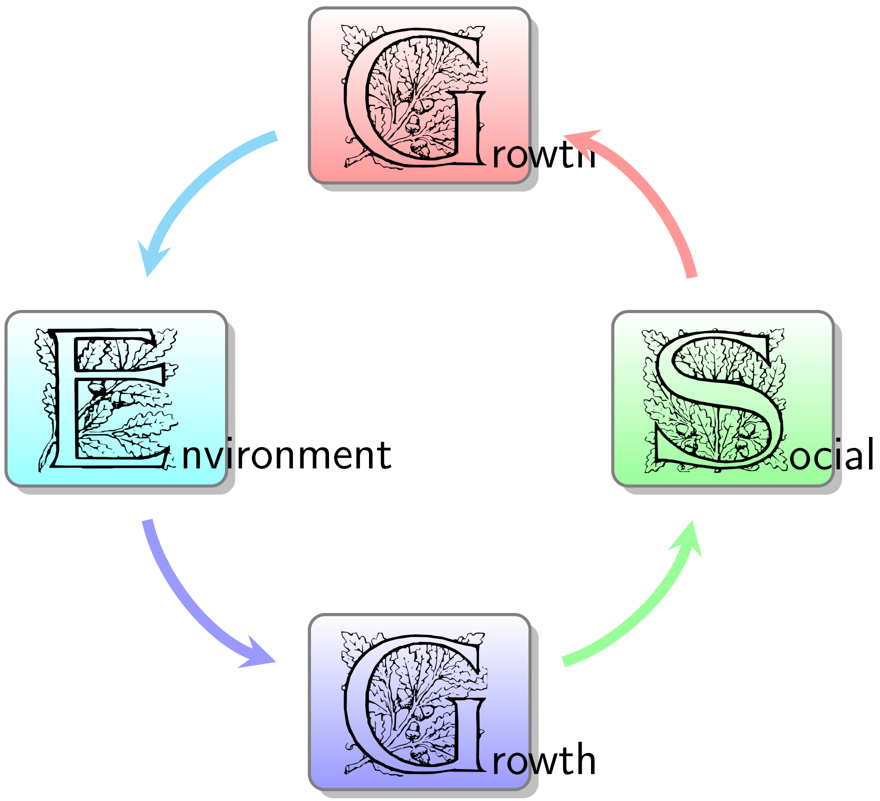


Source: Rose *et al.* (2017).

Stylized IAMs

The liability/fairness question

Liability



Aristotle (384 BC – 322 BC)
ΗΘΙΚΩΝ ΝΙΚΟΜΑΧΕΙΩΝ

Karl Marx and Friedrich Engels (1848)
The Communist Manifesto

Stylized IAMs

The liability/fairness question

Fairness

Du Contrat Social

Stylized IAMs

Climate risk and inequalities

Three types of inequalities

- Spatial (or regional) inequalities
- Social (or intra-generation) inequalities
- Time (or inter-generation) inequalities

⇒ These issues are highly related to liability risks:

> *"[...] liability risks stemming from parties who have suffered loss from the effects of climate change seeking compensation from those they hold responsible" (Mark Carney, 2018)*

- Regional inequalities ⇒ lack of cooperation between countries (e.g., Glasgow COP 26)
- Social inequalities ⇒ climate action postponing (e.g., carbon tax in France)

Stylized IAMs
Regional inequalities

The **R**egional **I**ntegrated model of **C**limate and the **E**conomy (RICE) model is a sub-regional neoclassical climate economy model (Nordhaus and Yang, 1996)

$\Rightarrow$ Sub-regional problem of welfare:

- Each region of the world has a different utility functions
- The big issue is how the most developed regions can finance the transition to a low-carbon economy of the less developed regions

Both spacial and time (inter-generation) inequalities

Stylized IAMs

Social inequalities

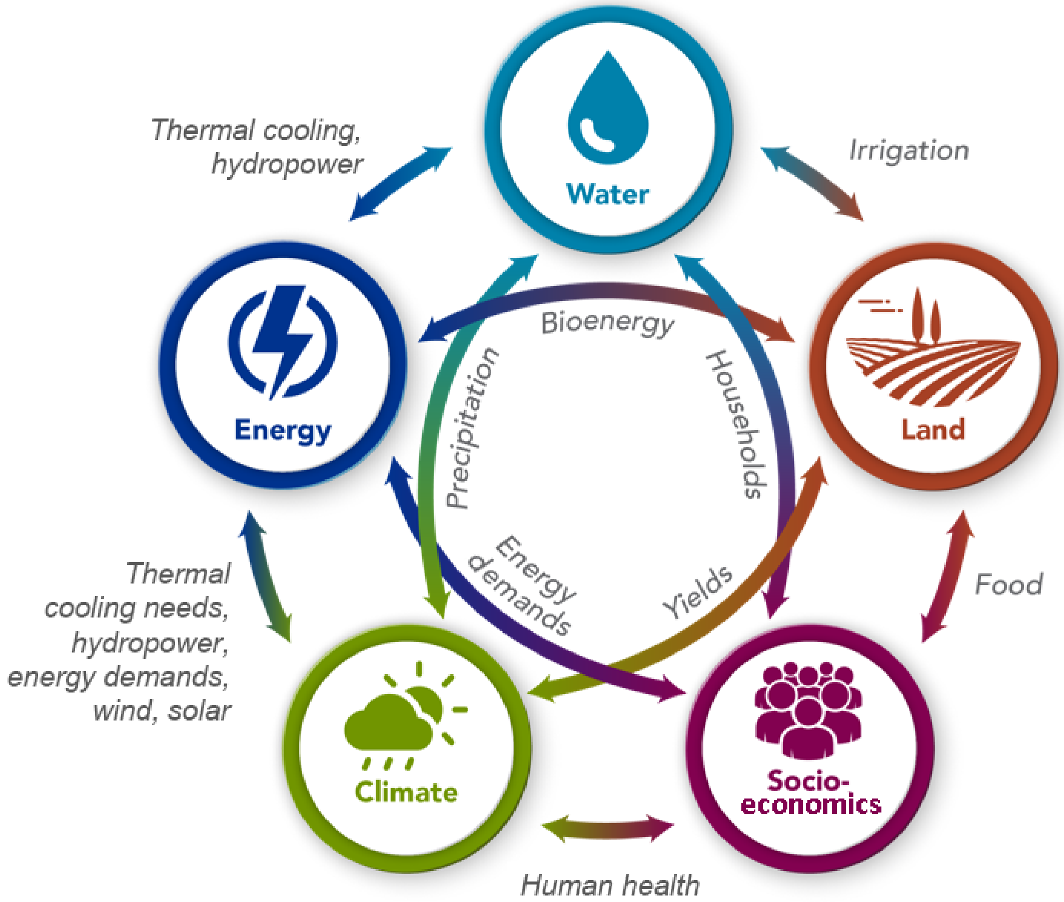
The **N**ested **I**nequalities **C**limate-**E**conomy (NICE) model integrates distributional differences of income (Dennig *et al.*, 2015)

> *"[...] If the distribution of damage is less skewed to high income than the distribution of consumption, then weak or no climate policy will result in sufficiently large damages on the lower economic strata to eventually stop their welfare levels from improving, and instead cause them to decline" (Dennig et al., 2015)*

Both social (intra-generation) and time (inter-generation) inequalities

Complex IAMs

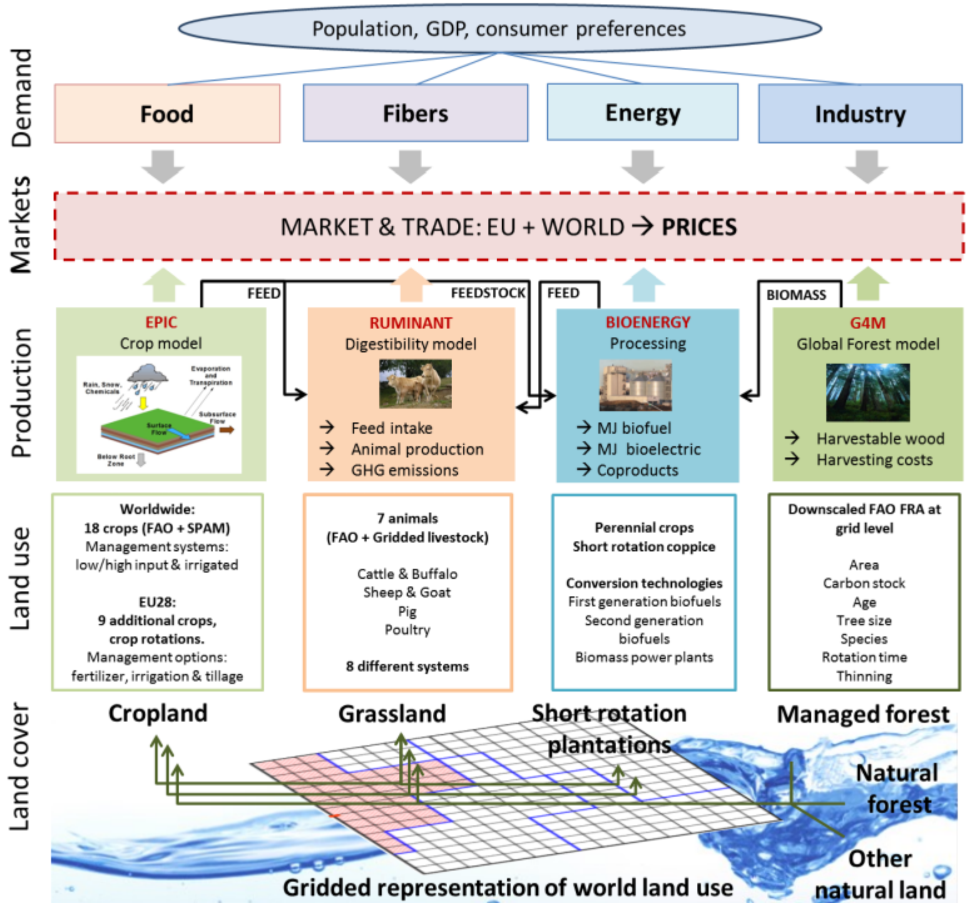
Figure 140: Linkages between the major systems in GCAM



Source: Calvin *et al.* (2019).

Complex IAMs

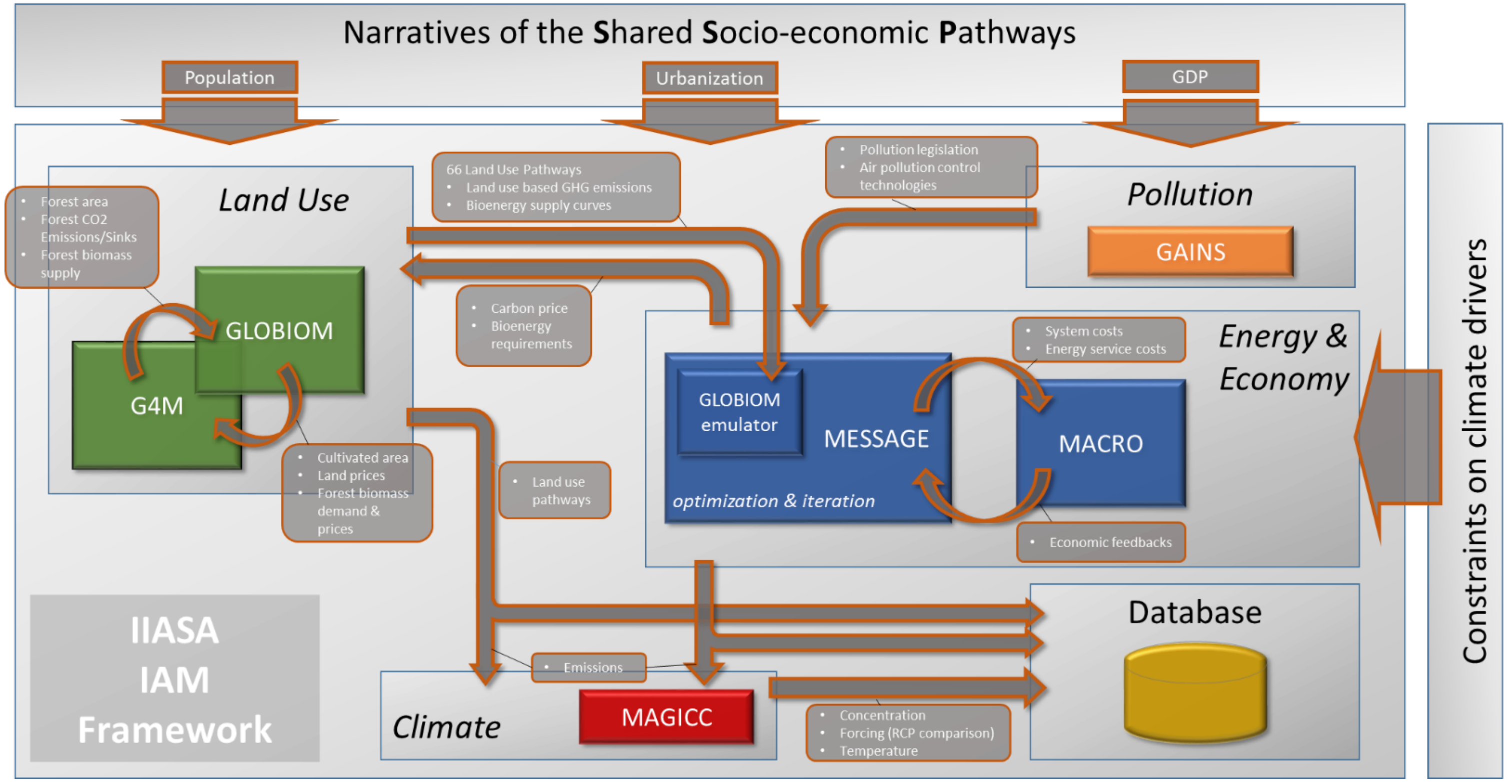
Figure 141: The main land use sectors of GLOBIOM



Source: https://iiasa.github.io/GLOBIOM.

Complex IAMs

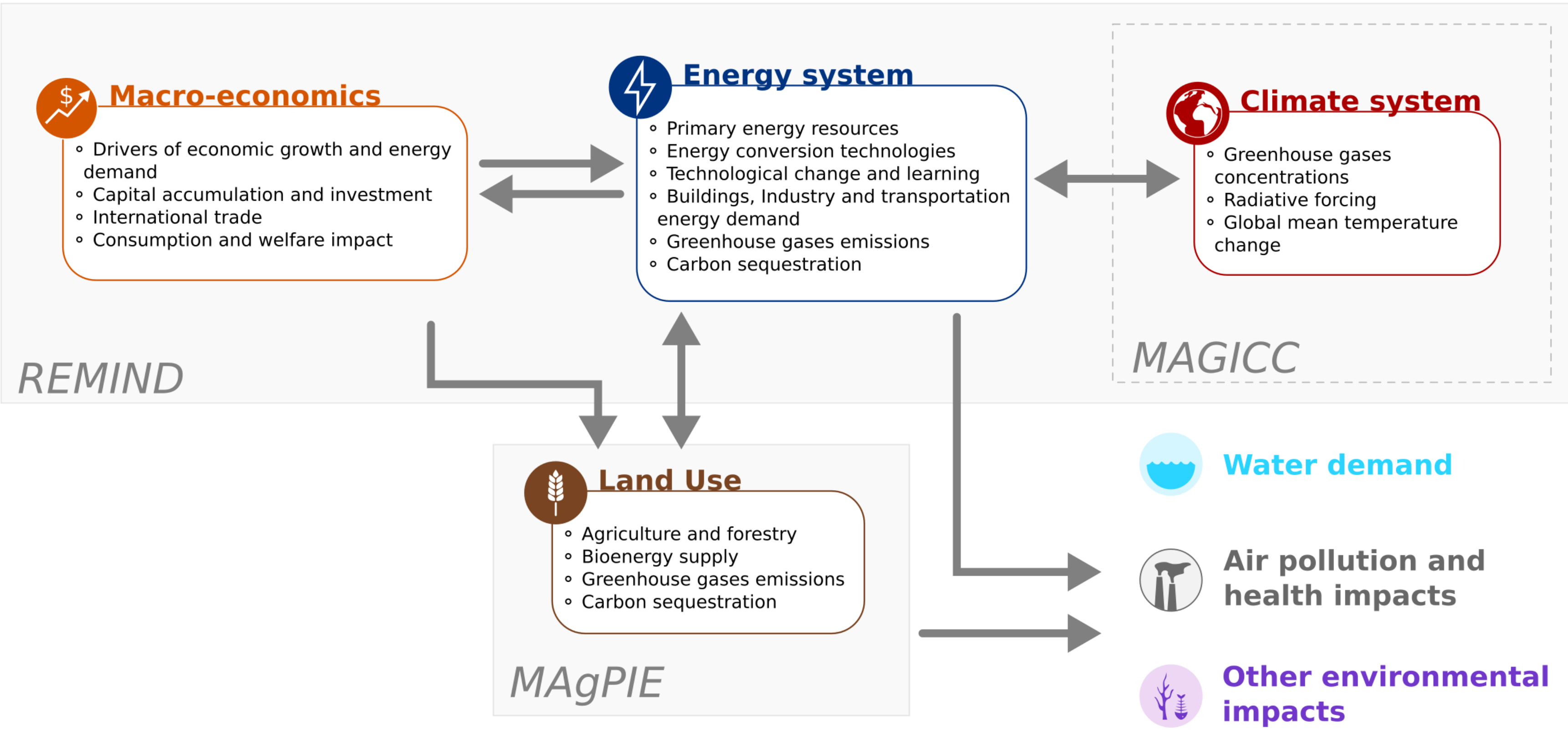
Figure 142: Overview of the IIASA IAM framework



Source: https://docs.messageix.org/projects/global/en/latest/overview/index.html.

Complex IAMs

Figure 143: The Remind-MAgPIE framework



Source: `www.pik-potsdam.de/en/institute/departments/transformation-pathways/models/remind`.

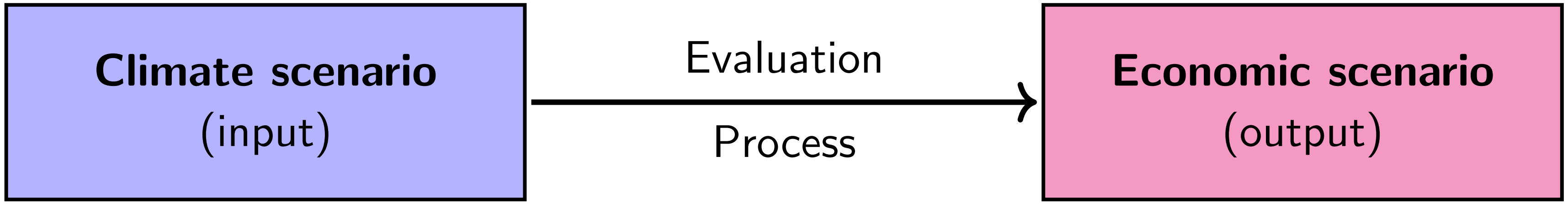
Criticisms of integrated assessment models

"IAM-based analyses of climate policy create a perception of knowledge and precision that is illusory and can fool policymakers into thinking that the forecasts the models generate have some kind of scientific legitimacy" (Pindyck, 2017)

- Certain inputs, such as the discount rate, are arbitrary
- There is a lot of uncertainty about climate sensitivity and the temperature trajectory
- Modeling damage functions is arbitrary
- IAMs are unable to consider tail risk

Scenarios

Figure 144: Scenario evaluation



Climate scenarios

- The representative concentration pathways (RCPs) — IPCC AR5
- The IEA scenarios
- The $1.5°\mathrm{C}$ scenarios — SR15
- The scenarios for the future published — IPCC AR6

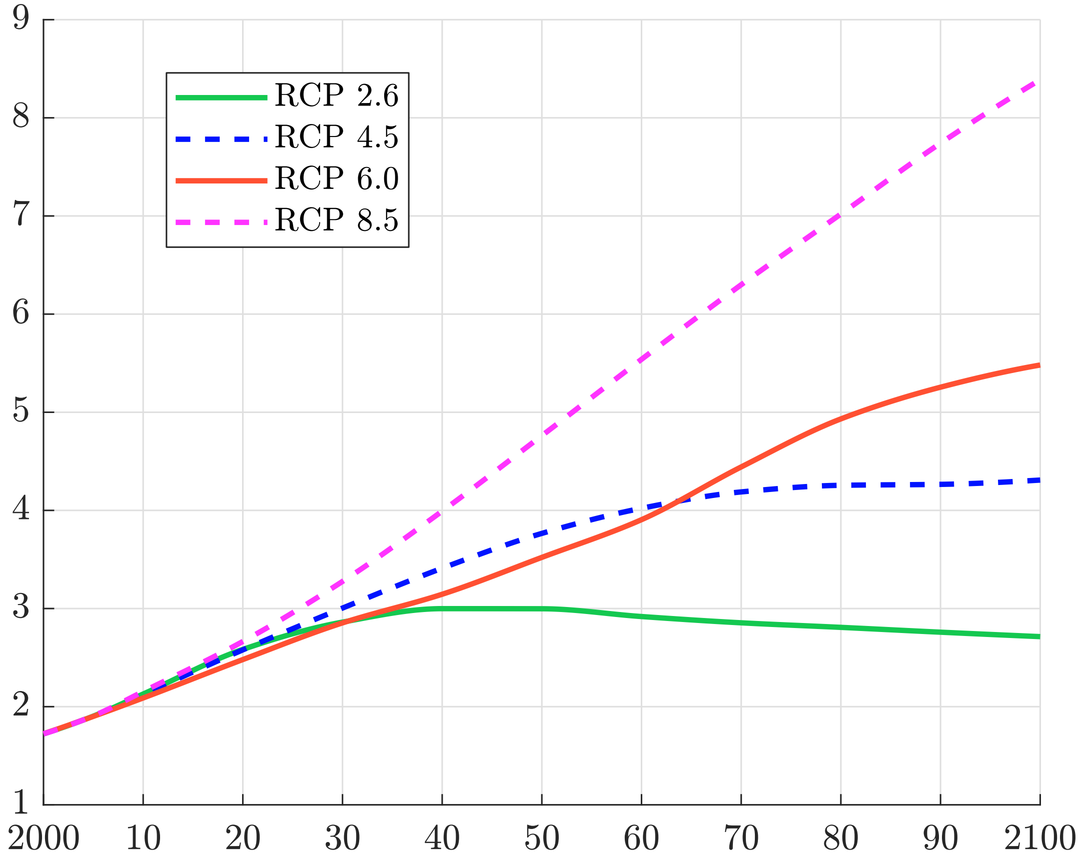
Climate scenarios
The RCP scenarios

1. RCP 2.6: GHG emissions start declining by 2020 and go to zero by 2100 (IMAGE)
2. RCP 4.5: GHG emissions peak around 2040, and then decline (MiniCAM)
3. RCP 6.0: GHG emissions peak around 2080, and then decline (AIM)
4. RCP 8.5: GHG emissions continue to rise throughout the 21st century (MESSAGE)

Climate scenarios

The RCP scenarios

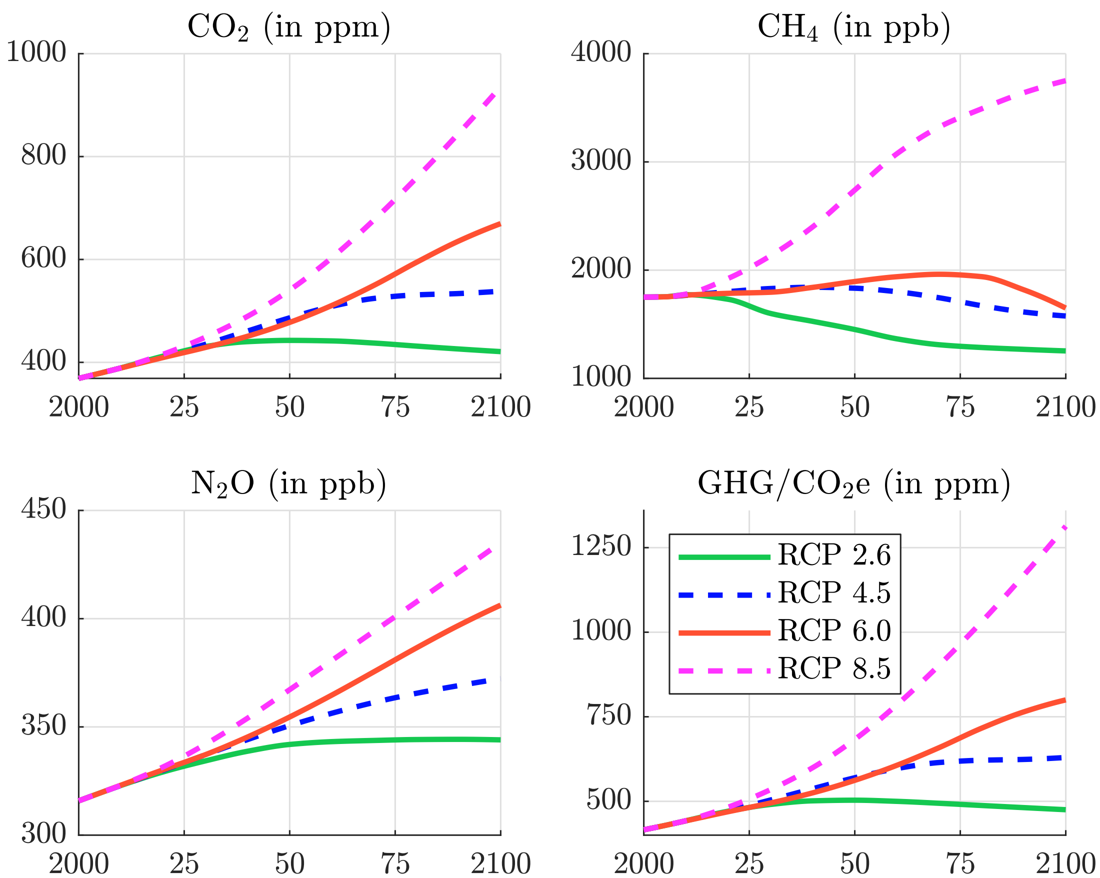
Figure 145: Total radiative forcing (in $\mathrm{W/m^2}$)



Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios
The RCP scenarios

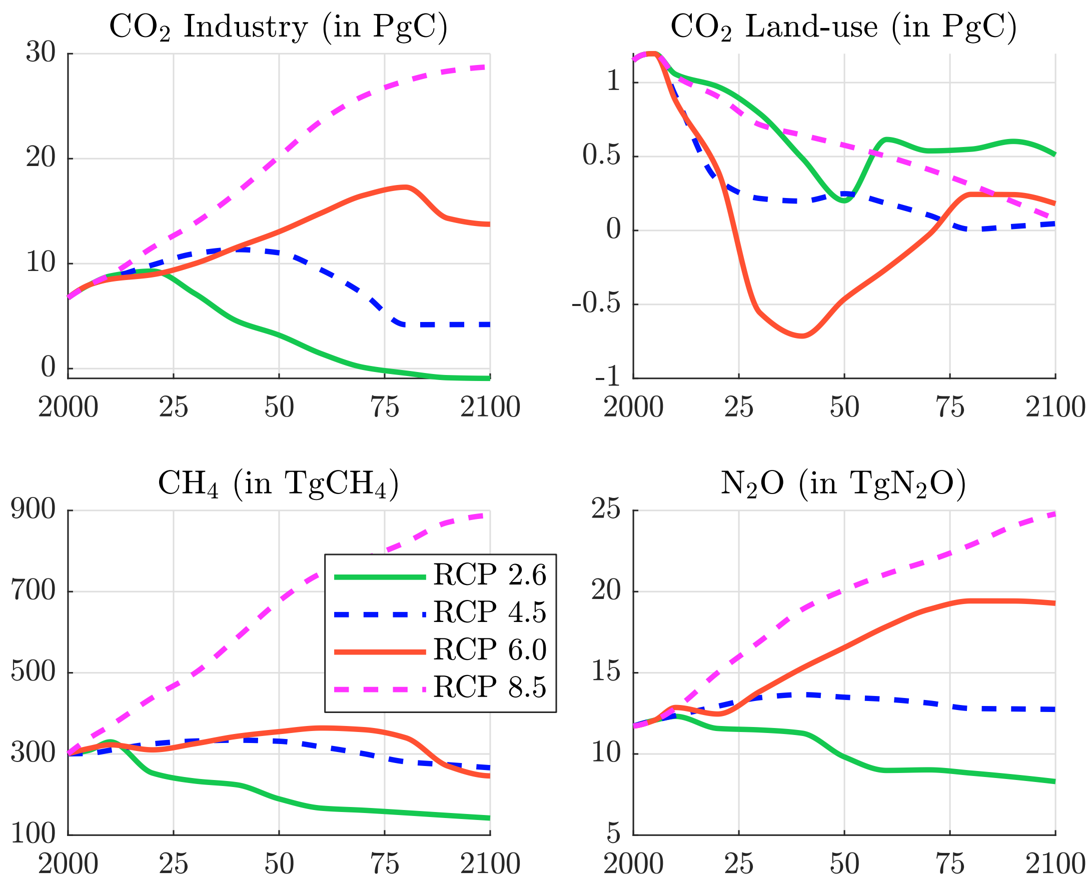
Figure 146: Greenhouse gas concentration trajectory



Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios
The RCP scenarios

Figure 147: Greenhouse gas emissions trajectory

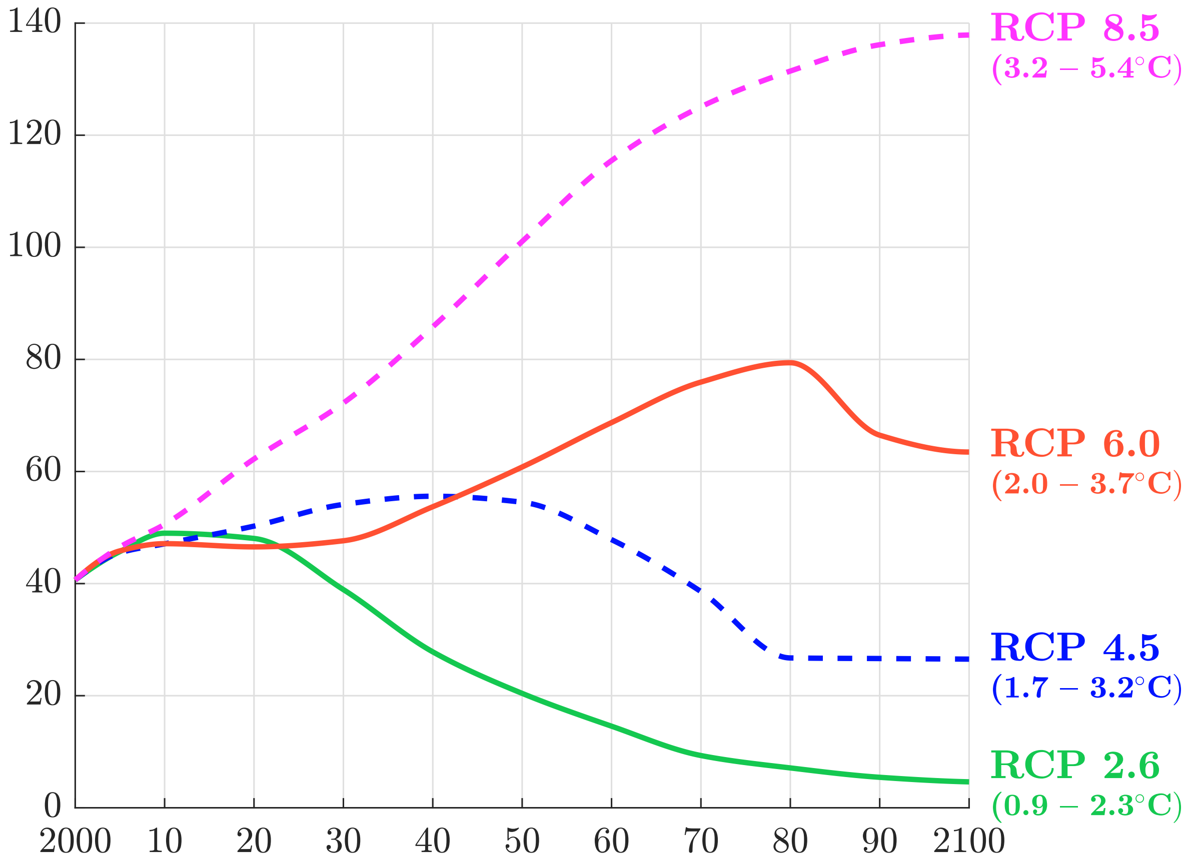


Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios

The RCP scenarios

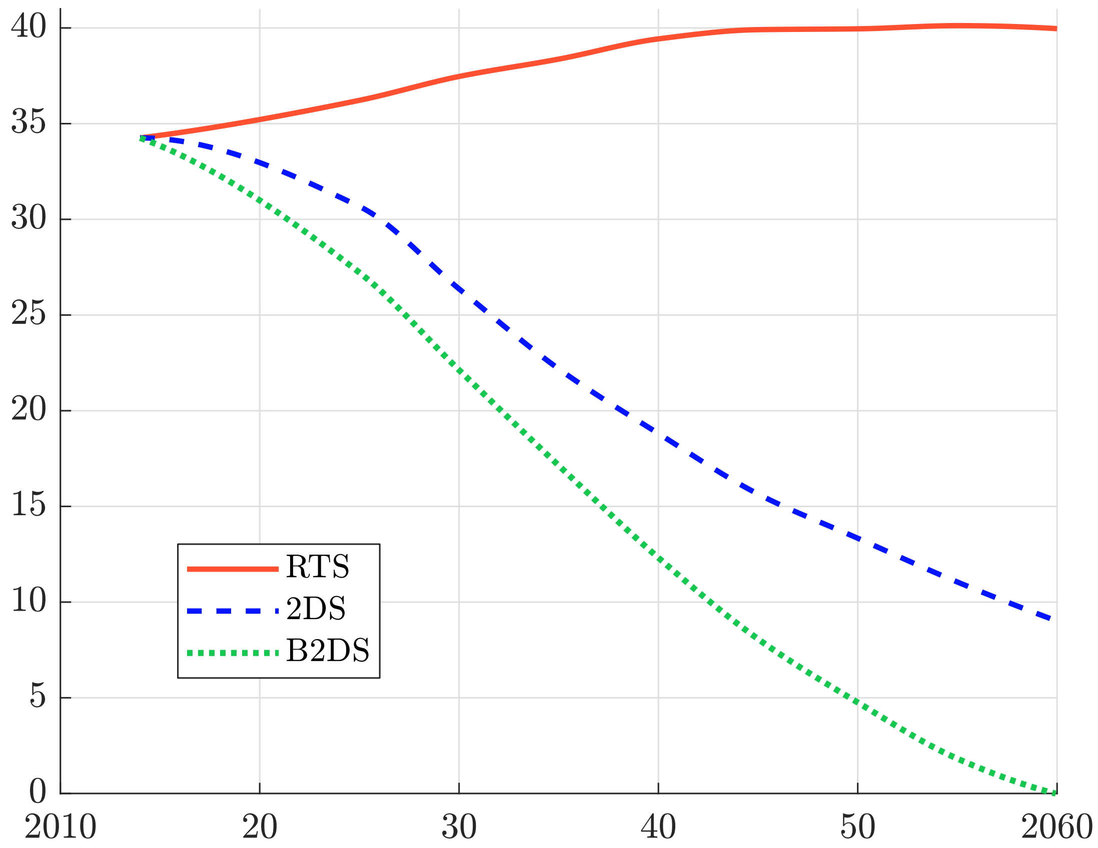
Figure 148: Total GHG emissions trajectory (in $\mathrm{GtCO_2e}$)



Source: `https://tntcat.iiasa.ac.at/RcpDb`.

Climate scenarios
The IEA scenarios

Figure 149: Direct CO_2 emissions (in Gt)

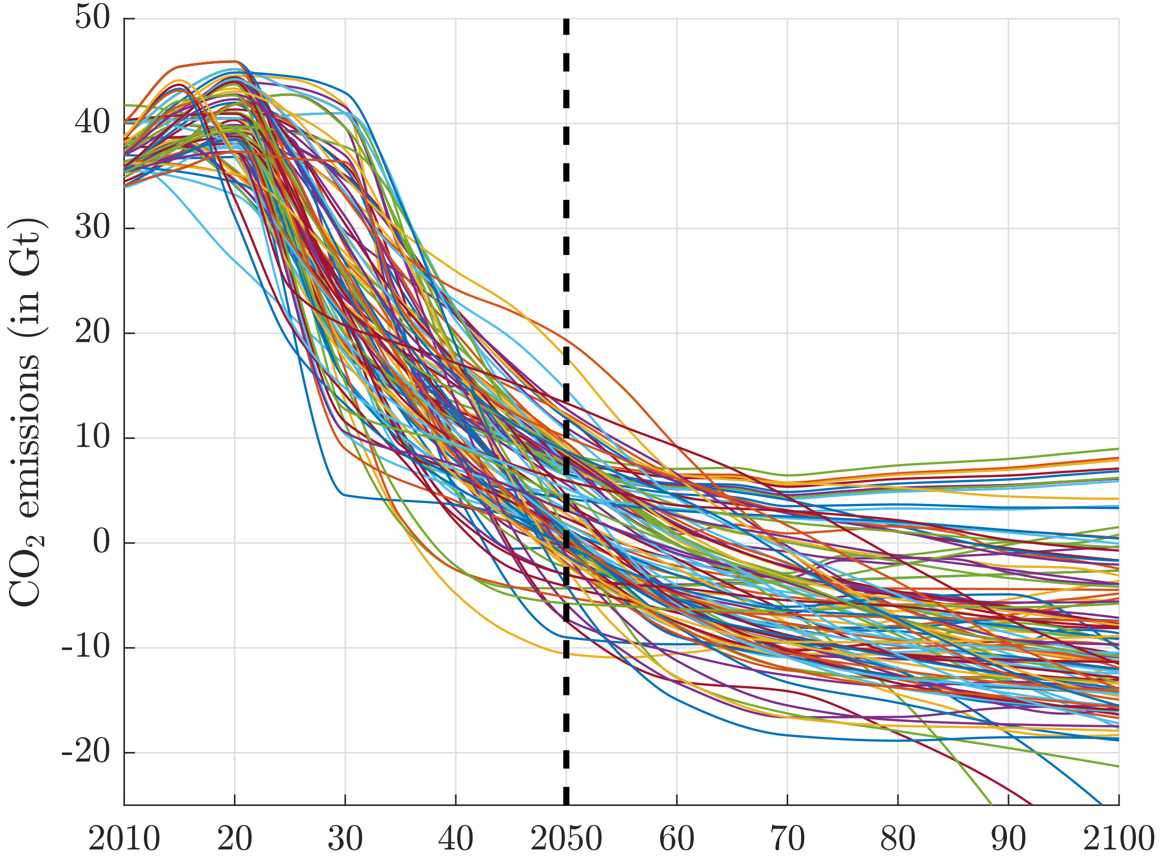


Source: IEA (2017).

Climate scenarios

The 1.5°C scenarios

Figure 150: IPCC 1.5°C scenarios of CO_2 emissions

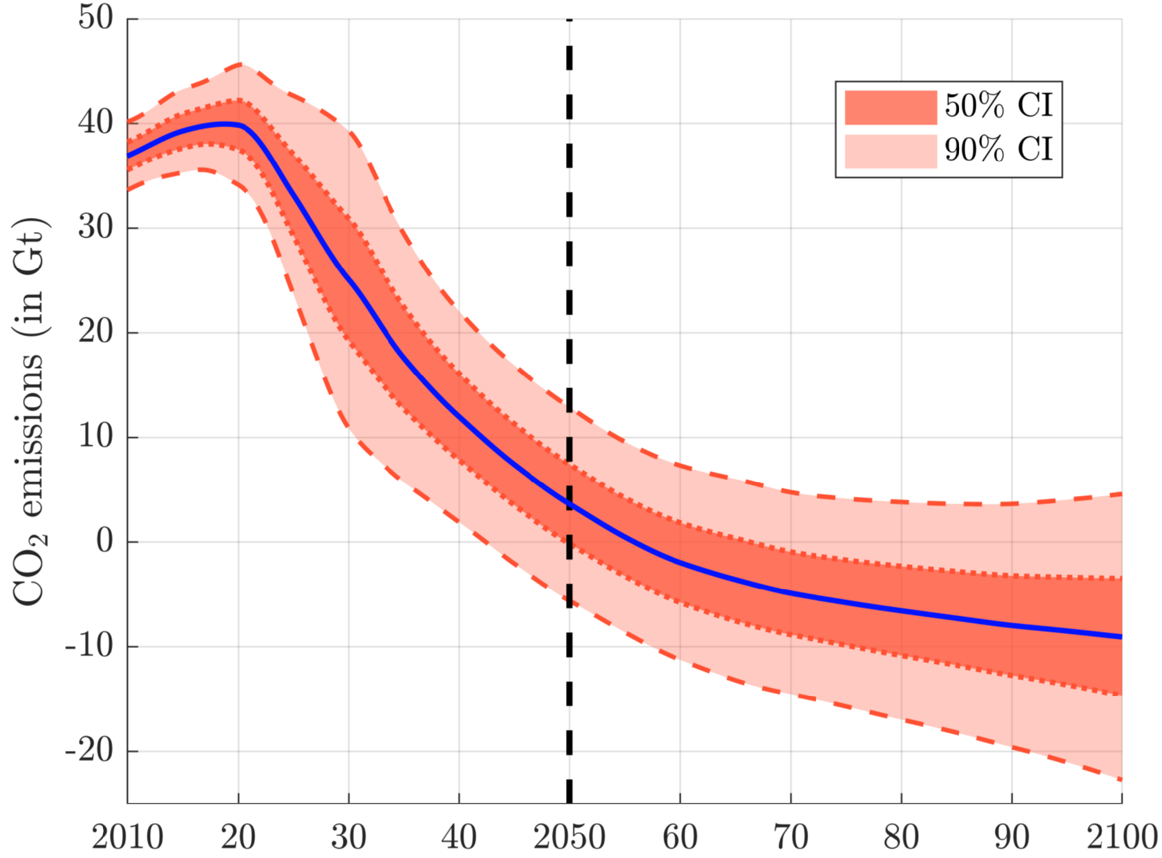


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The 1.5°C scenarios

Figure 151: Confidence interval of the average IPCC 1.5°C scenario

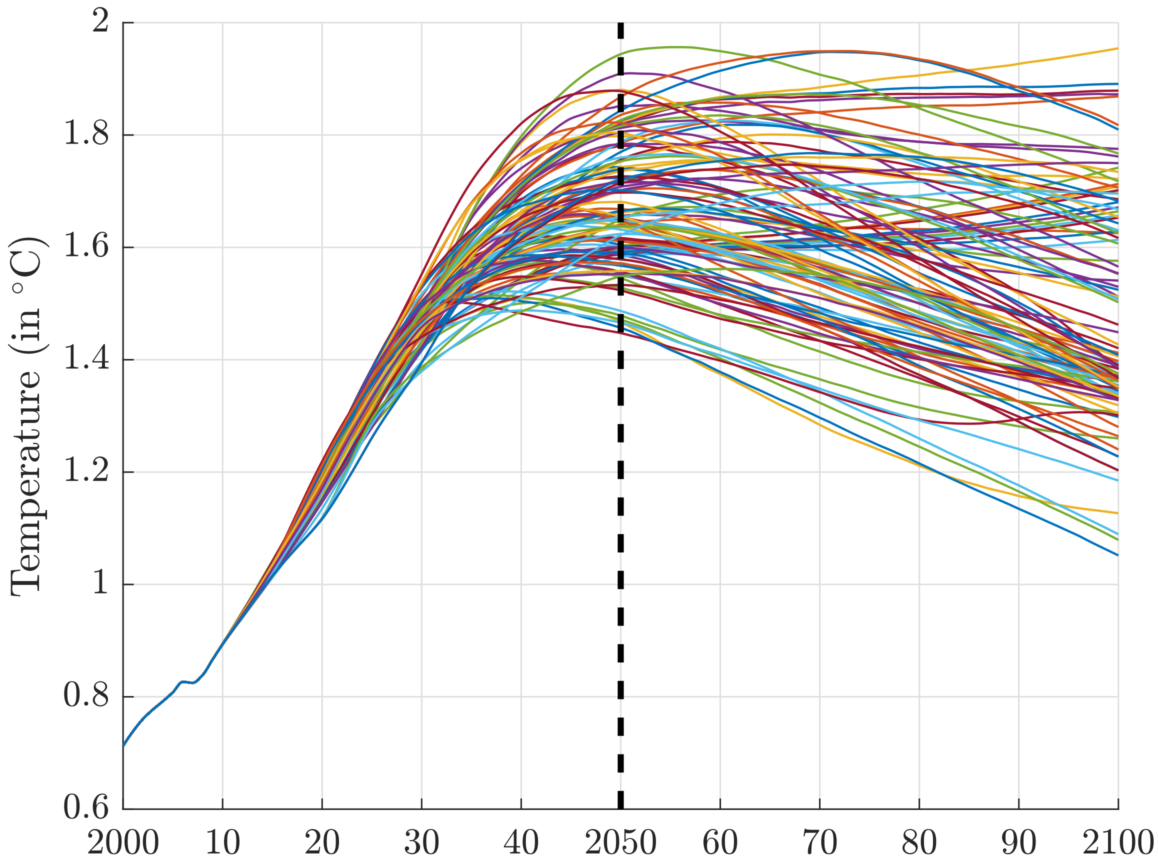


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The 1.5°C scenarios

Figure 152: IPCC 1.5°C scenarios of the global mean temperature

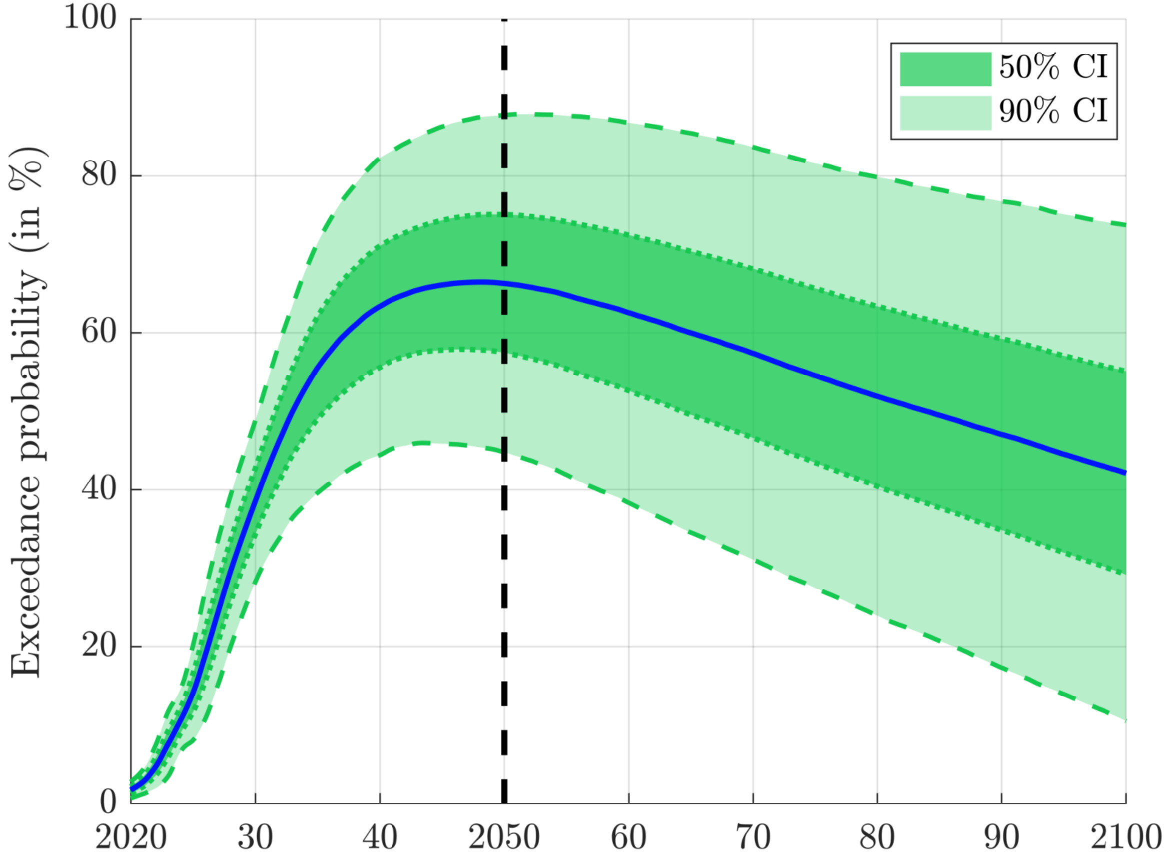


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The 1.5°C scenarios

Figure 153: Confidence interval of the exceedance probability Pr $\{\mathcal{T} > 1.5^\circ\mathrm{C}\}$

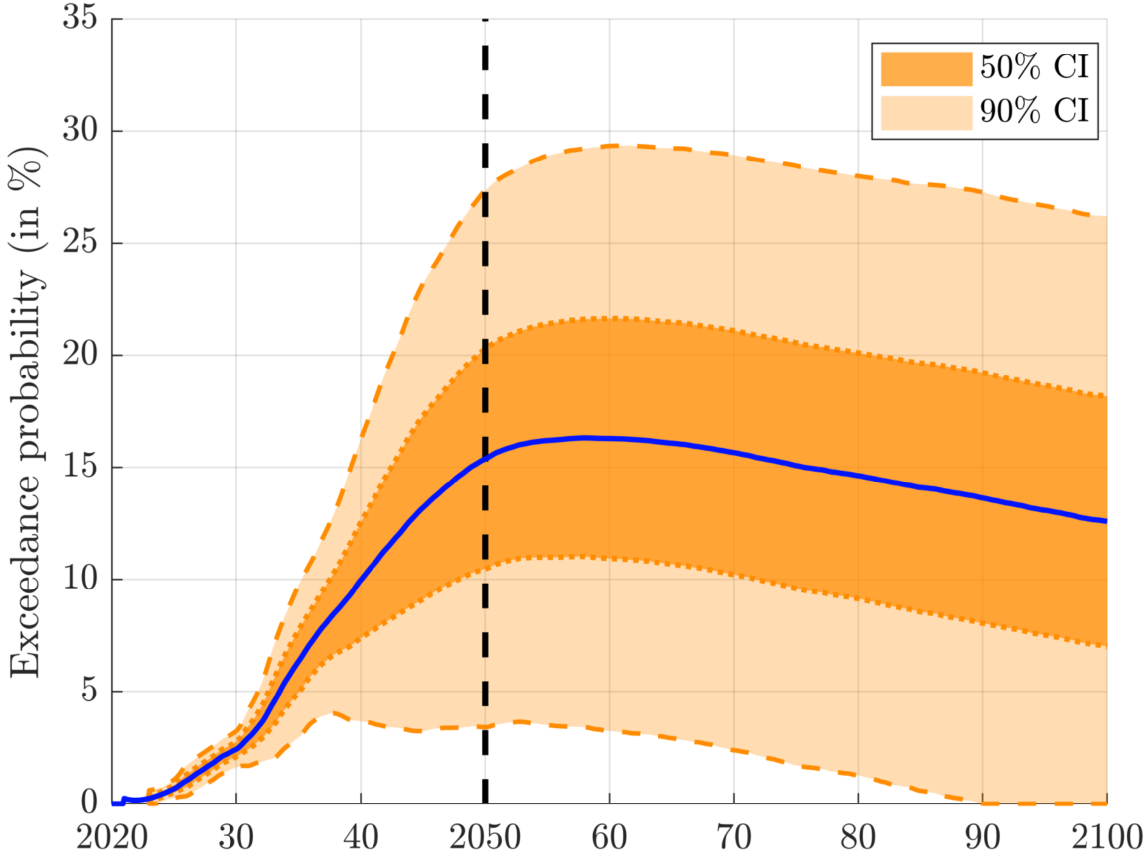


Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios

The 1.5°C scenarios

Figure 154: Confidence interval of the exceedance probability Pr $\{\mathcal{T} > 2^\circ\mathrm{C}\}$



Source: `https://data.ene.iiasa.ac.at/iamc-1.5c-explorer`.

Climate scenarios
The AR6 scenarios

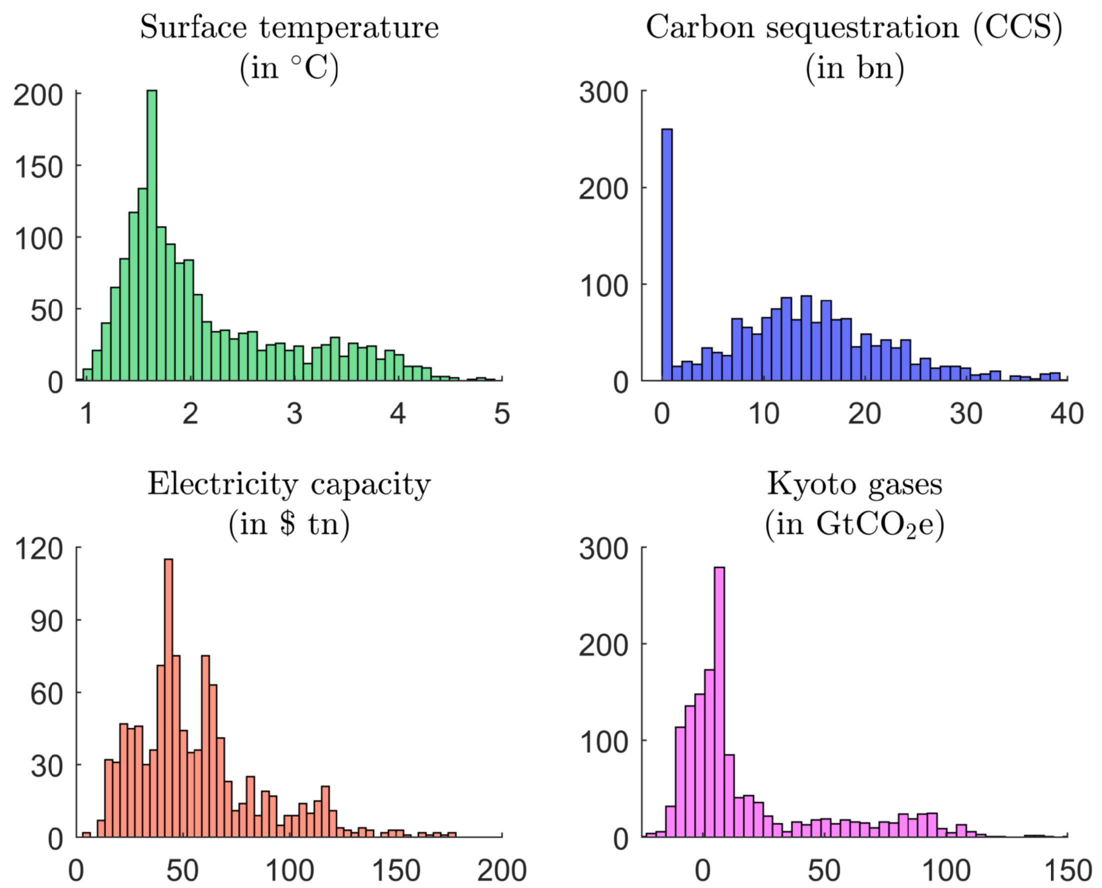
The new dataset contains 188 models, 1 389 scenarios, 244 countries and regions, and 1 791 variables, which can be split into six main categories:

- Agriculture: agricultural demand, crop, food, livestock, production, etc.
- Capital cost: coal, electricity, gas, hydro, hydrogen, nuclear, etc.
- Energy: capacity, efficiency, final energy, lifetime, OM cost, primary/secondary energy, etc.
- GHG impact: carbon sequestration, concentration, emissions, forcing, temperature, etc.
- Natural resources: biodiversity, land cover, water consumption, etc.
- Socio-economic variables: capital formation, capital stock, consumption, discount rate, employment, expenditure, export, food demand, GDP, Gini coefficient, import, inequality, interest rate, investment, labour supply, policy cost, population, prices, production, public debt, government revenue, taxes, trade, unemployment, value added, welfare, etc.

Climate scenarios

The AR6 scenarios

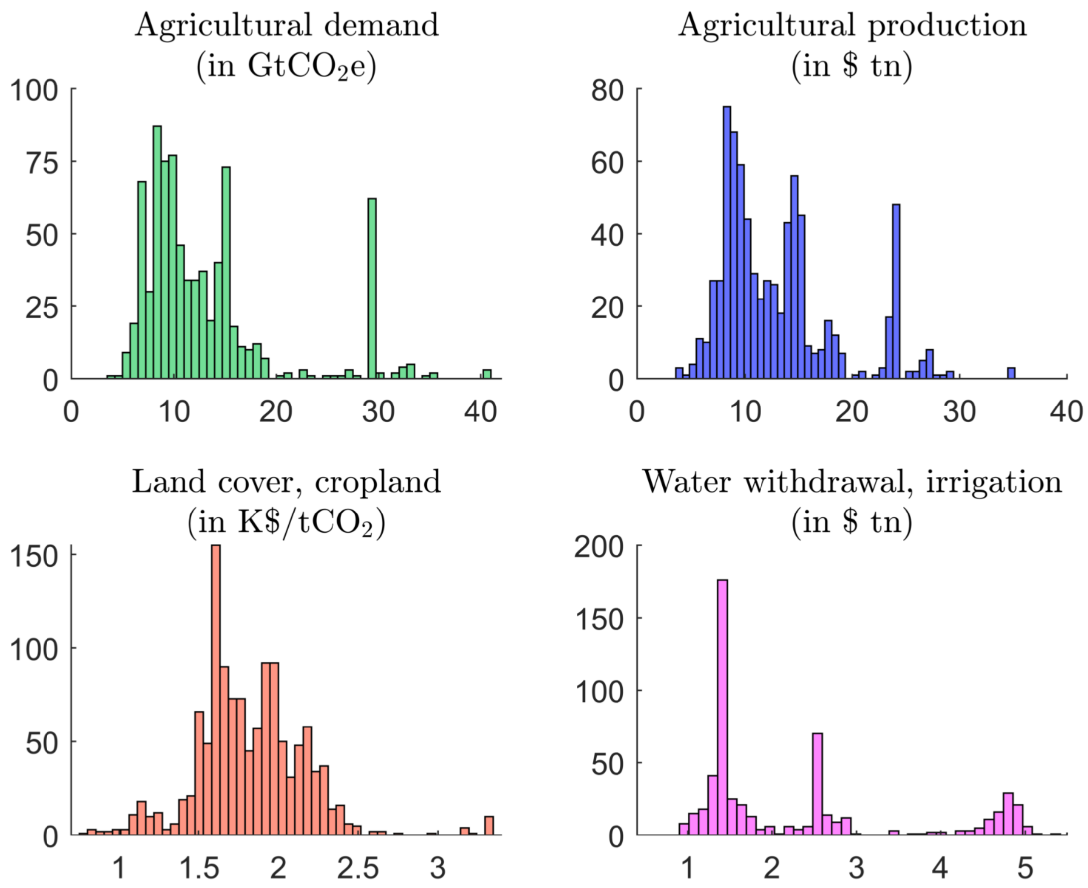
Figure 155: Histogram of some AR6 output variables by 2100



Source: `https://data.ene.iiasa.ac.at/ar6`.

Climate scenarios
The AR6 scenarios

Figure 156: Histogram of some AR6 output variables by 2100



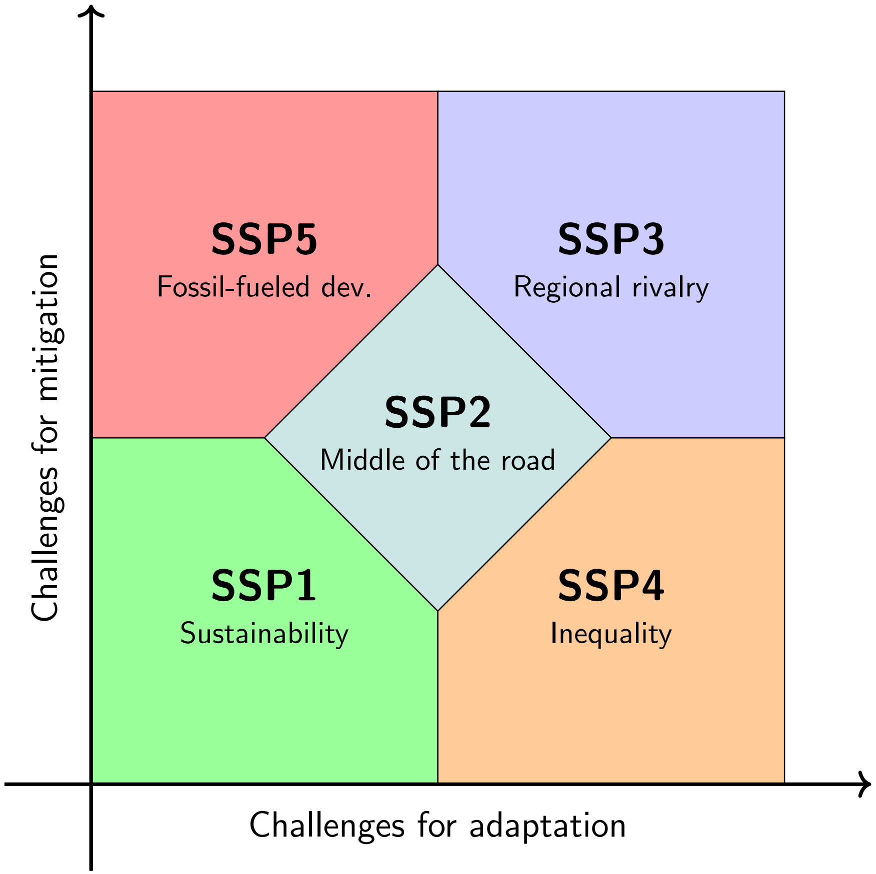
Source: `https://data.ene.iiasa.ac.at/ar6`.

Shared socioeconomic pathways

"The SSP narratives [are] a set of five qualitative descriptions of future changes in demographics, human development, economy and lifestyle, policies and institutions, technology, and environment and natural resources. [...] Development of the narratives drew on expert opinion to (1) identify key determinants of the challenges [to mitigation and adaptation] that were essential to incorporate in the narratives and (2) combine these elements in the narratives in a manner consistent with scholarship on their inter-relationships. The narratives are intended as a description of plausible future conditions at the level of large world regions that can serve as a basis for integrated scenarios of emissions and land use, as well as climate impact, adaptation and vulnerability analyses." (O'Neill et al., 2017)

Shared socioeconomic pathways

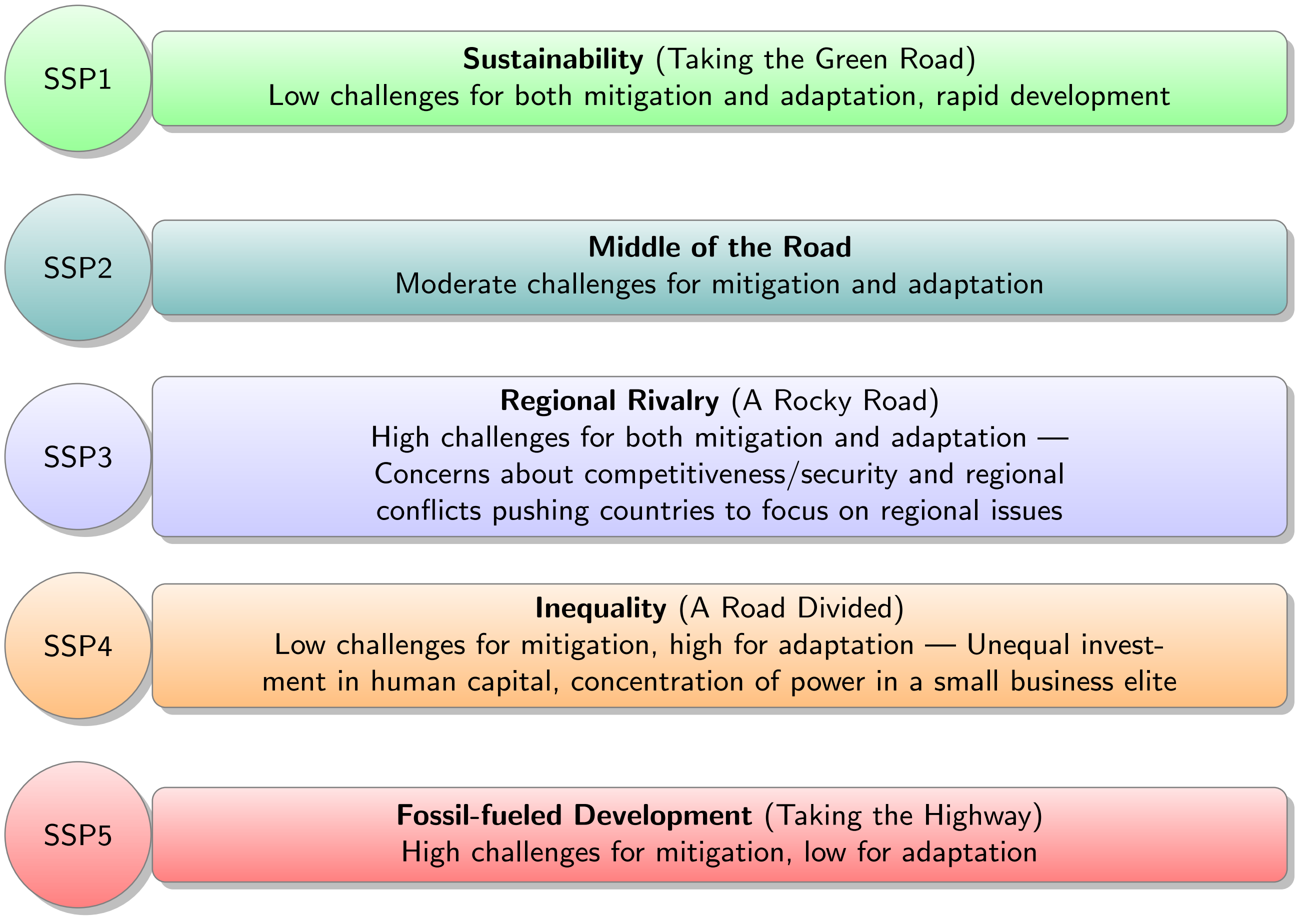
Figure 157: The shared socioeconomic pathways



Source: O'Neill *et al.* (2017).

Shared socioeconomic pathways

Figure 158: The shared socioeconomic pathways



Source: O'Neill *et al.* (2017).

Shared socioeconomic pathways

Relationship with the ESG dimensions

E The mitigation/adaptation trade-off is obviously an environmental issue, but the SSPs encompass other environmental narratives, e.g. land use, energy efficiency and green economy

S The social dimension is the central theme of SSPs, and concerns demography, wealth, inequality & poverty, health, education, employment, and more generally the evolution of society. This explains that SSPs and SDGs are highly interconnected

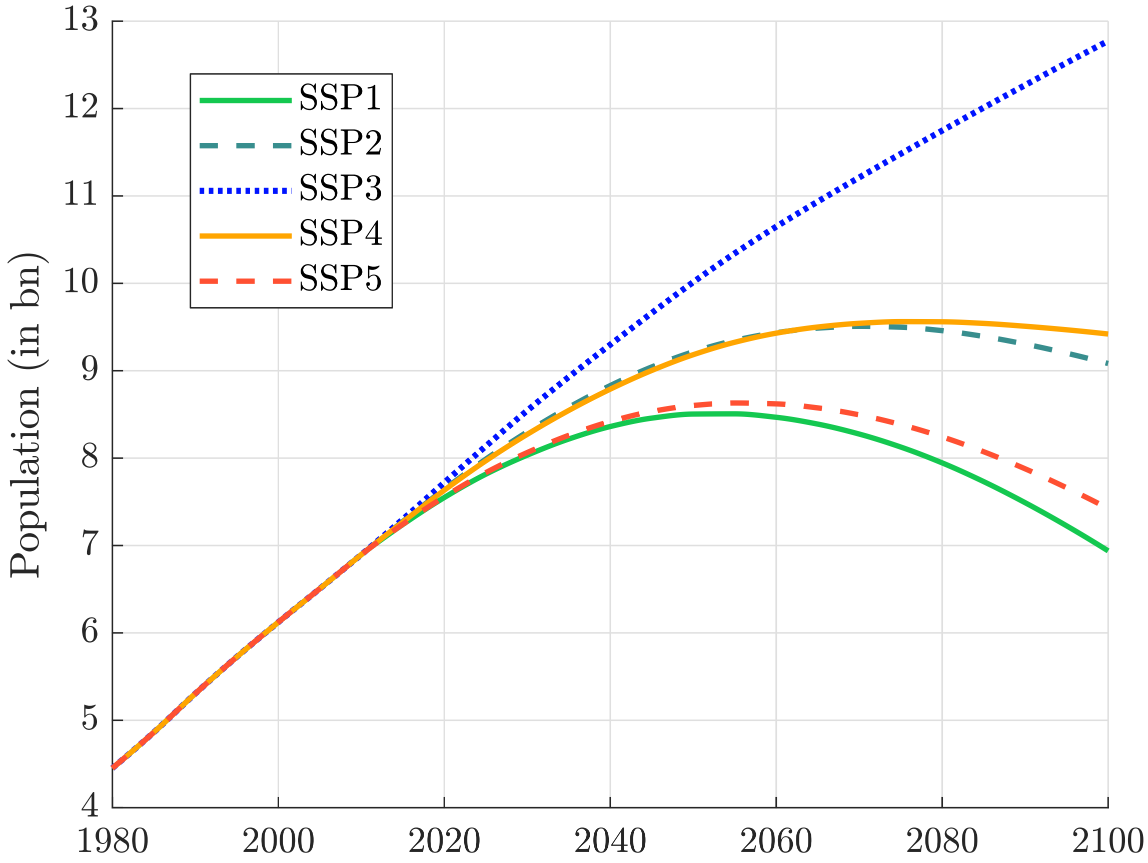
G Finally, the governance dimension is present though two major themes: international fragmentation or cooperation, and the political/economic system, including corruption, stability, rule of law, etc.

Shared socioeconomic pathways

- SSP1: IMAGE (PBL)
- SSP2: MESSAGE-GLOBIOM (IIASA)
- SSP3: AIM/CGE (NIES)
- SSP4: GCAM (PNNL)
- SSP5: REMIND-MAGPIE (PIK) and WITCH-GLOBIOM (FEEM)

Shared socioeconomic pathways

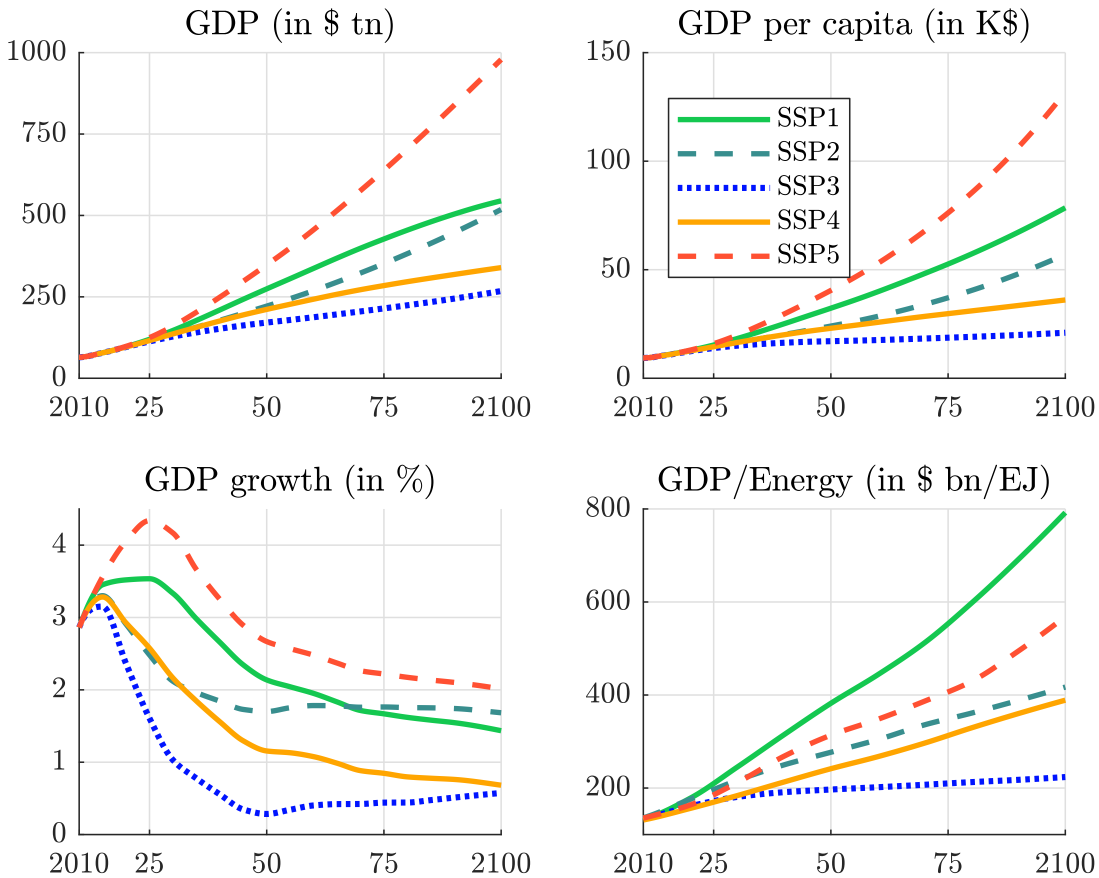
Figure 159: SSP demography projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

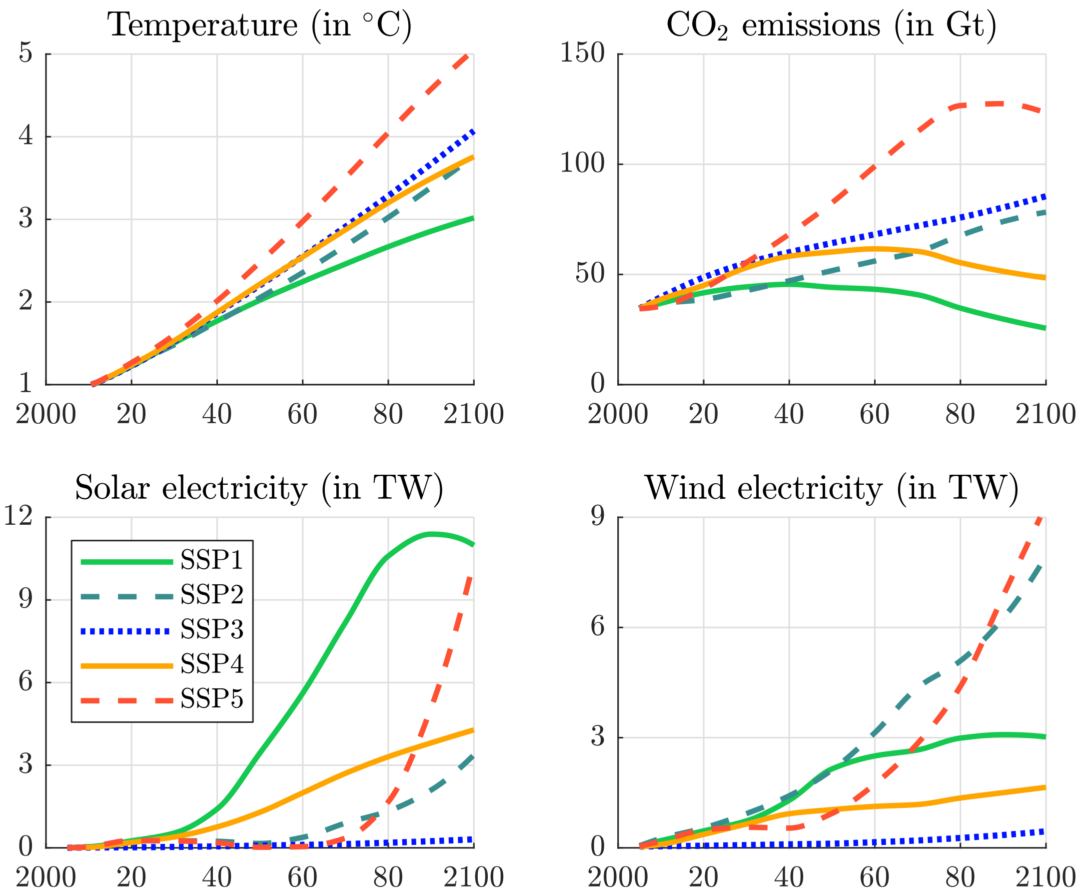
Figure 160: SSP economic projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

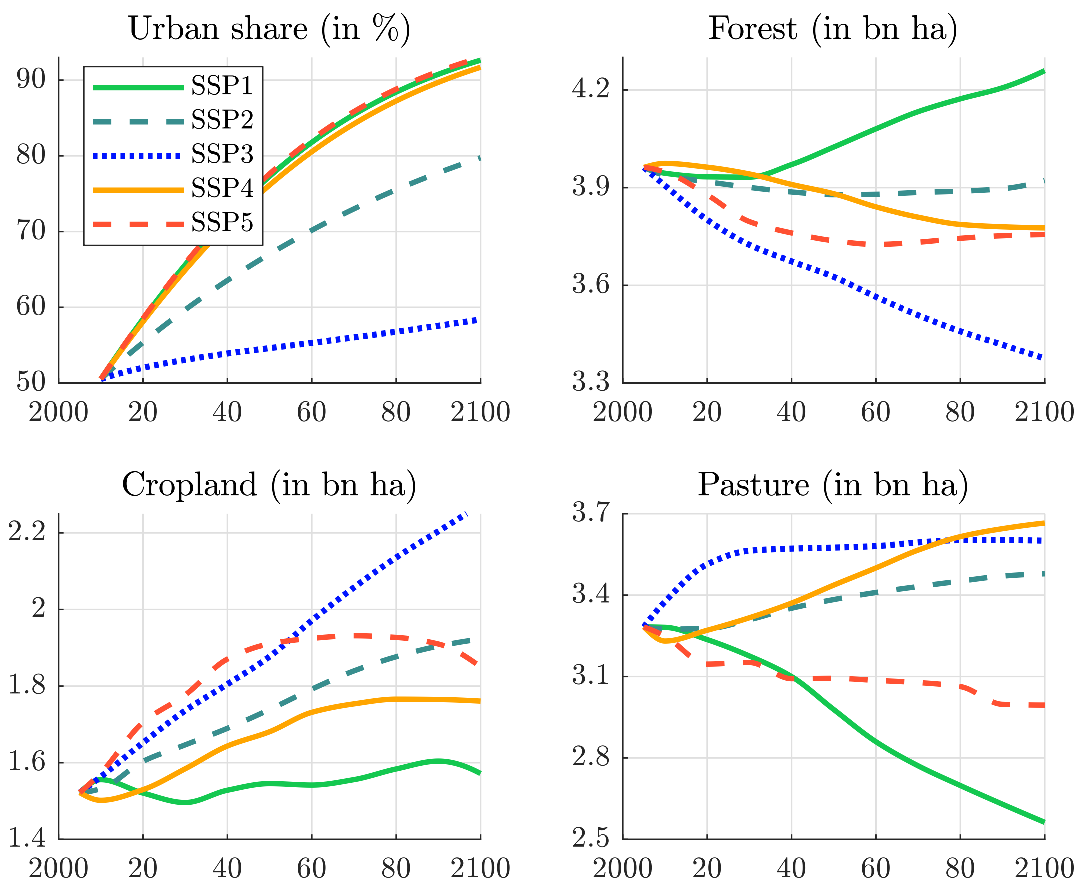
Figure 161: SSP environmental projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

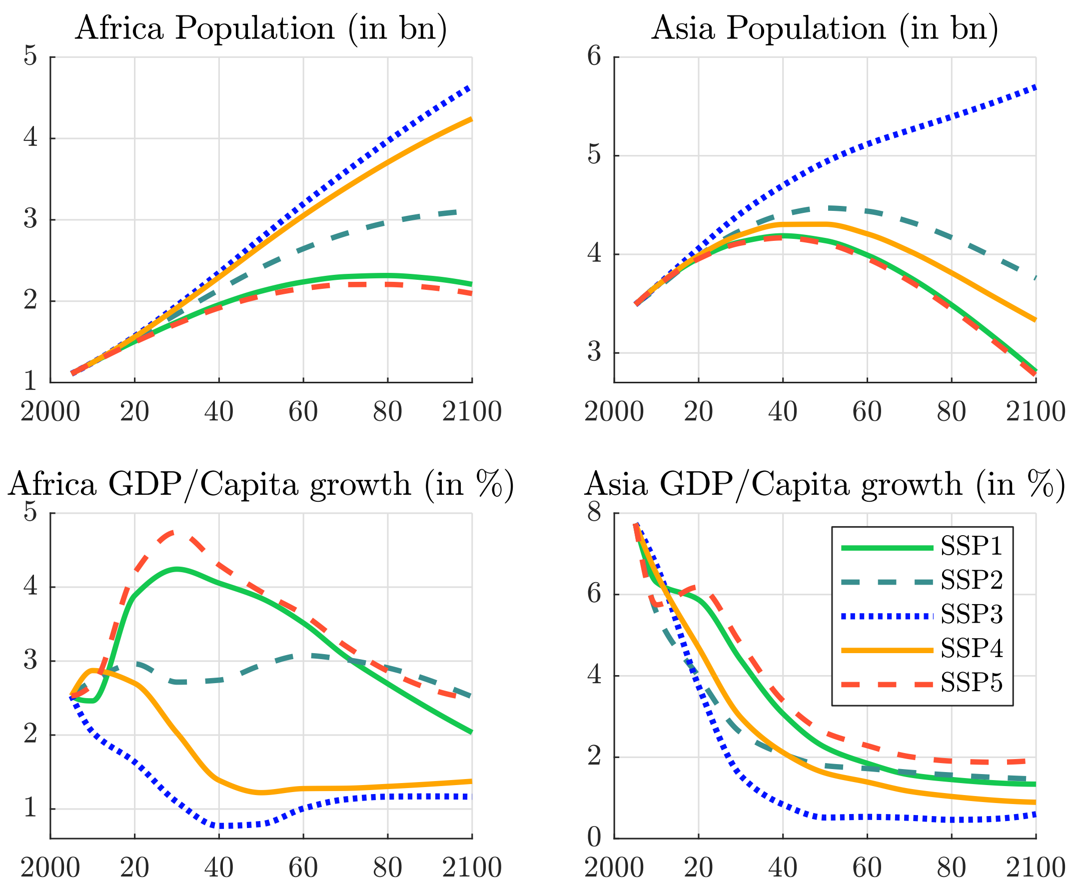
Figure 162: SSP land use projections



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

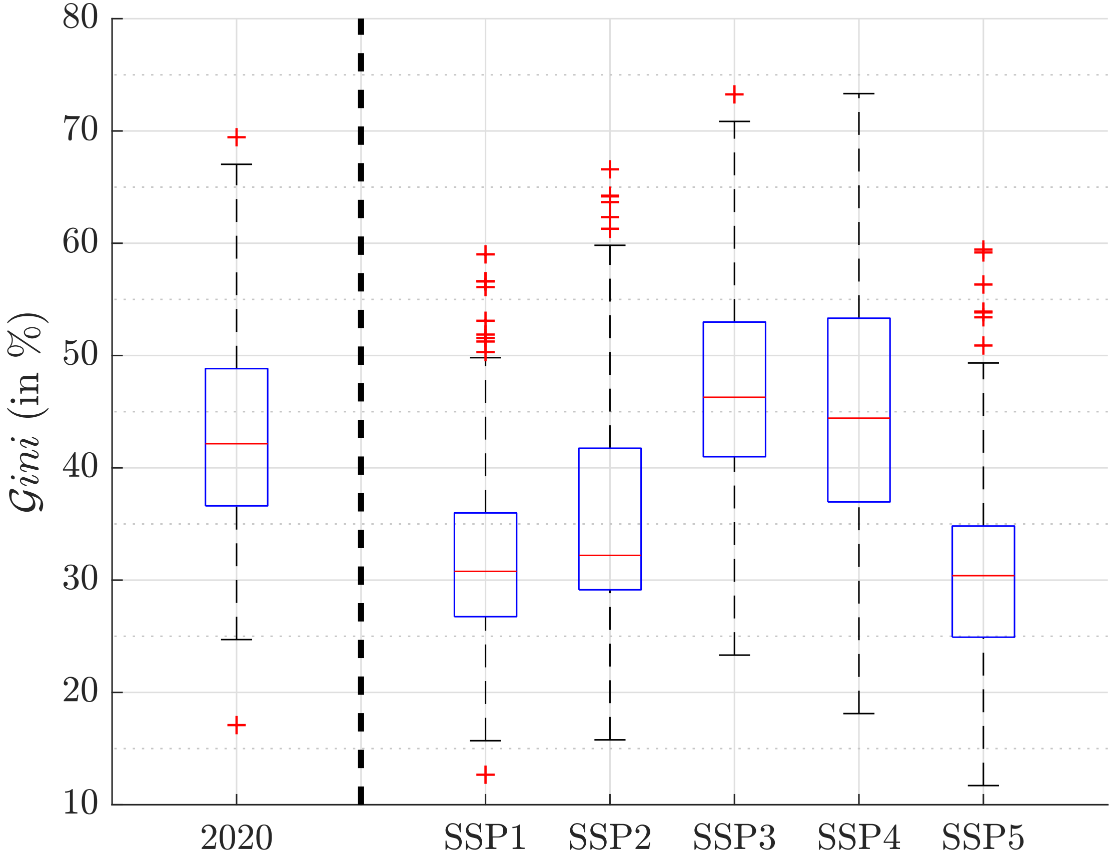
Figure 163: Example of SSP regional differences



Source: `https://tntcat.iiasa.ac.at/SspDb`.

Shared socioeconomic pathways

Figure 164: Gini coefficient projections by 2100



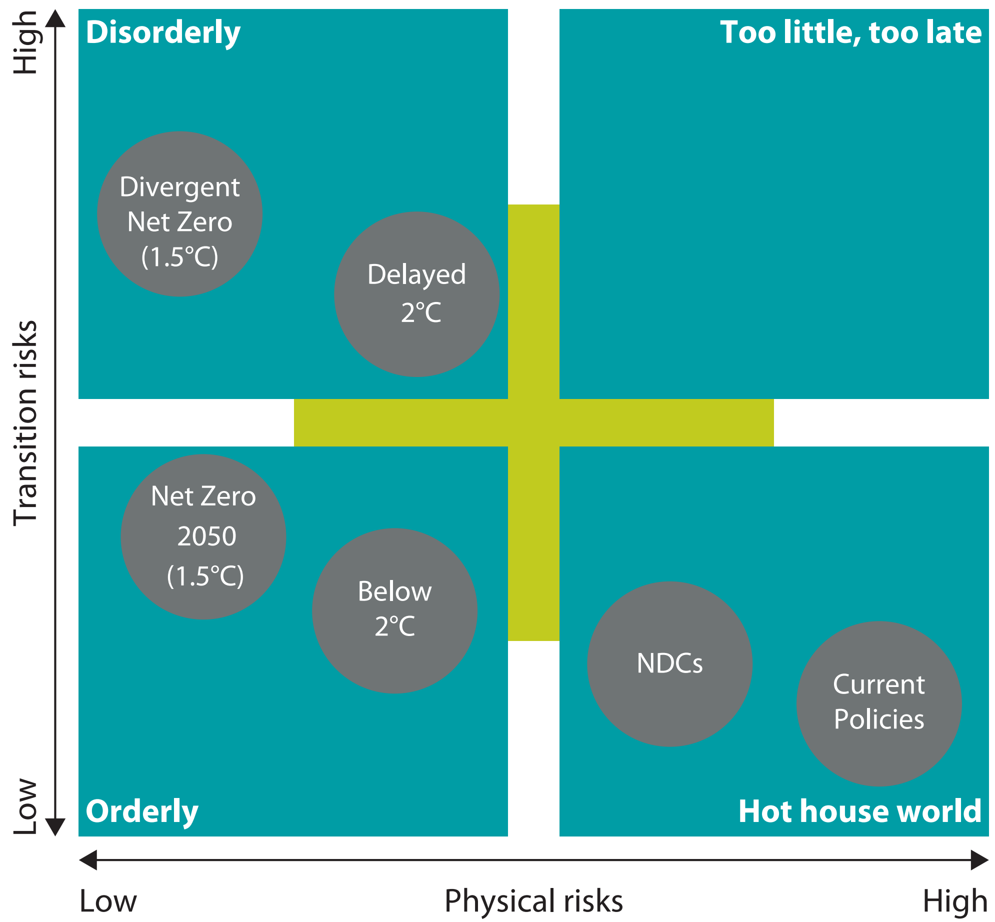
Source: `https://tntcat.iiasa.ac.at/SspDb`.

NGFS scenarios

Figure 165: Network of Central Banks and Supervisors for Greening the Financial System (NGFS)

NGFS scenarios

Figure 166: NGFS scenarios framework



NGFS scenarios

- Orderly scenarios
  - #1 Net zero 2050 (NZ)
  - #2 Below $2^\circ\mathrm{C}$ (B2D)
- Disorderly scenarios
  - #3 Divergent net zero (DNZ)
  - #4 Delayed transition (DT)
- Hot house world scenarios
  - #5 Nationally determined contributions (NDC)
  - #6 Current policies (CP)

NGFS scenarios

Figure 167: Physical and transition risk level of NGFS scenarios

		Physical risk	Transition risk			
Category	**Scenario**	**Policy ambition**	**Policy reaction**	**Technology change**	**Carbon dioxide removal** ⁻	**Regional policy variation** ⁺
Orderly	Net Zero 2050	1.4°C	Immediate and smooth	Fast change	Medium-high use	Medium variation
	Below 2°C	1.6°C	Immediate and smooth	Moderate change	Medium-high use	Low variation
Disorderly	Divergent Net Zero	1.4°C	Immediate but divergent across sectors	Fast change	Low-medium use	Medium variation
	Delayed Transition	1.6 °C	Delayed	Slow / Fast change	Low-medium use	High variation
Hot house world	Nationally Determined Contributions (NDCs)	2.6°C	NDCs	Slow change	Low-medium use	Medium variation
	Current Policies	3°C +	Non-currente policies	Slow change	Low use	Low variation

NGFS scenarios

Variables (economic)

- Central bank intervention rate
- Domestic demand
- Effective exchange rate
- Exchange rate
- Exports (goods and services)
- Gross Domestic Product (GDP)
- Gross domestic income
- Imports (goods and services)
- Inflation rate
- Long term & real interest rates
- Trend output for capacity utilisation
- Unemployment

Variables (energy)

- Coal price
- Gas price
- Oil price
- Quarterly consumption of coal
- Quarterly consumption of gas
- Quarterly consumption of oil
- Quarterly consumption of renewables
- Total energy consumption

Models (IPCC)

- Meta-model: NiGEM 1.21
- Sub-models:
  1. GCAM 5.3
  2. MESSAGE-GLOBIOM 1.1
  3. REMIND-MAgPIE 2.1-4.2

6 scenarios

1. Net Zero 2050 (NZ)
2. Below 2°C (B2D)
3. Divergent Net Zero (DNZ)
4. Delayed Transition (DT)
5. Notionally Determined Contribution (NDC)
6. Current Policies (CP)

NGFS scenarios

Table 85: Impact of climate change on the GDP loss by 2050 (GCAM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−3.09	−5.64	−2.35	−3.28	−5.15	−2.56
Transition risk	−0.75		−3.66	−1.78	−0.89	−0.88
Combined risk	−3.84	−5.64	−6.00	−5.05	−6.03	−3.44
Combined + business confidence			−6.03	−5.09		

NGFS scenarios

Table 86: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−2.05	−5.26	−1.55	−2.64	−4.78	−1.59
Transition risk	−1.46		−10.00	−10.77	−1.39	−3.26
Combined risk	−3.51	−5.26	−11.53	−13.37	−6.16	−4.84
Combined + business confidence			−11.57	−13.40		

NGFS scenarios

Table 87: Impact of climate change on the GDP loss by 2050 (REMIND-MAgPIE)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Chronic physical risk	−2.24	−6.05	−1.67	−2.65	−5.41	−1.76
Transition risk	−0.78		−3.01	−1.95	−0.33	−1.46
Combined risk	−3.02	−6.05	−4.68	−4.59	−5.73	−3.21
Combined + business confidence			−4.70	−4.63		

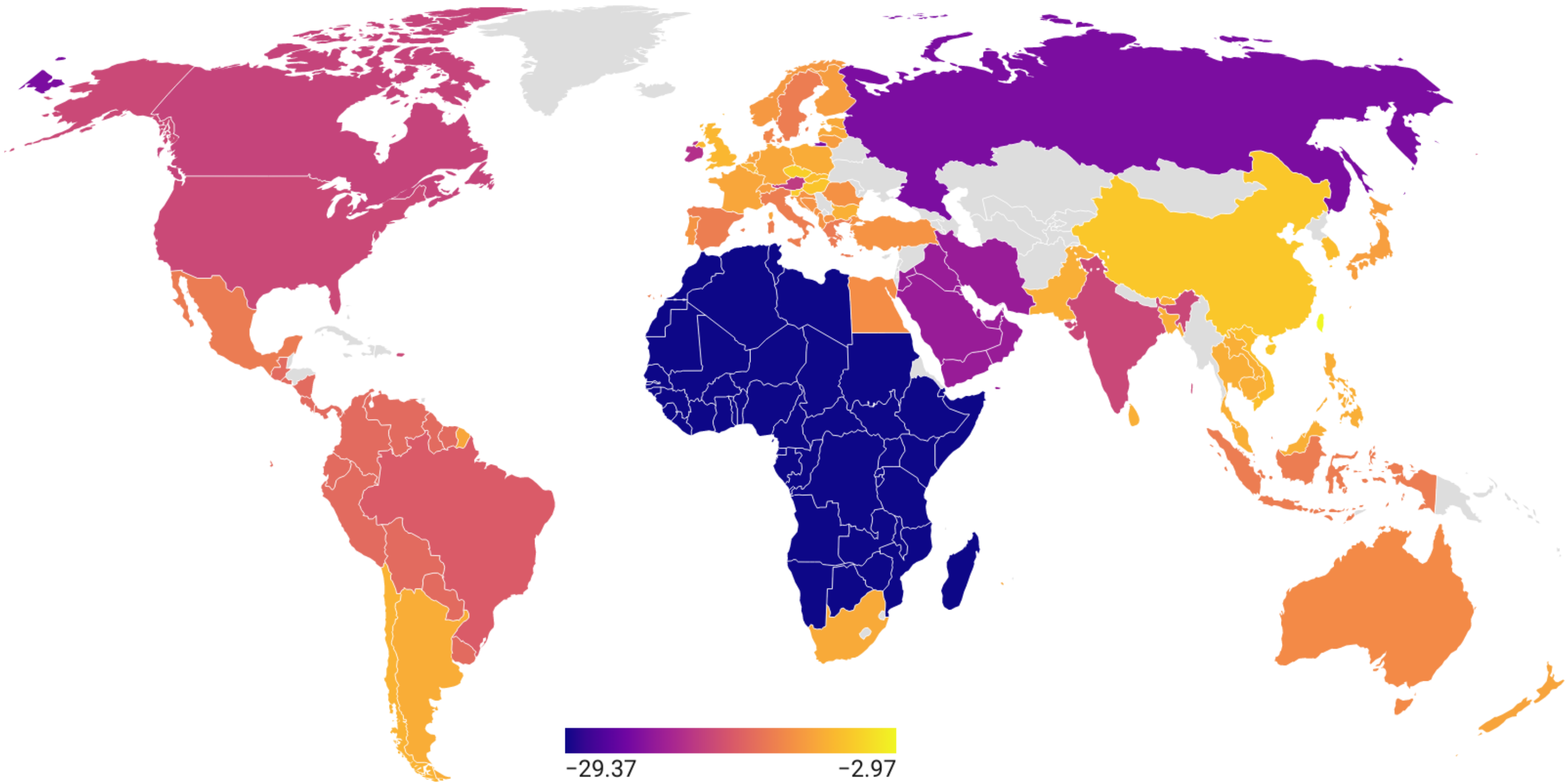
NGFS scenarios

Table 88: Impact of climate change on the GDP loss by 2050 (MESSAGEix-GLOBIOM)

Risk	B2D	CP	DNZ	DT	NDC	NZ
Africa	−13.58	−7.50	−27.35	−29.37	−11.78	−18.36
Asia	−1.50	−7.29	−5.44	−8.76	−6.78	−1.38
Australia	−4.11	−3.90	−11.03	−11.74	−5.77	−5.19
Brazil	−4.43	−5.92	−13.15	−15.90	−6.67	−6.65
Canada	−1.02	−2.37	−15.07	−18.12	−4.33	−4.87
China	−2.33	−4.97	−5.13	−6.73	−4.67	−2.76
Developing Europe	−0.28	−3.11	−0.56	−7.38	−2.73	0.39
Europe	−1.02	−2.84	−9.64	−11.02	−4.01	−1.62
France	−1.15	−2.80	−8.35	−9.48	−3.68	−1.56
Germany	−0.77	−2.38	−8.58	−9.38	−3.63	−1.21
India	−3.45	−8.61	−16.43	−17.74	−8.71	−3.86
Italy	−0.15	−3.69	−9.23	−12.88	−4.85	−0.89
Japan	−1.26	−4.14	−7.16	−10.05	−4.61	−1.40
Latam	−4.35	−6.10	−12.70	−14.58	−6.97	−5.74
Middle East	−9.97	−7.98	−22.03	−21.96	−10.28	−15.24
Russia	−12.18	−2.26	−23.46	−23.80	−7.54	−17.11
South Africa	−2.02	−5.06	−7.24	−9.16	−5.38	−3.04
South Korea	0.11	−3.49	−3.23	−7.57	−3.33	0.12
Spain	−2.41	−3.81	−12.49	−12.89	−5.41	−3.30
Switzerland	2.32	−2.25	−9.47	−10.35	−2.18	2.30
United Kingdom	−0.86	−1.90	−6.50	−8.05	−2.56	−1.33
United States	−2.67	−4.38	−15.37	−17.66	−6.31	−4.36
World	−3.51	−5.26	−11.53	−13.37	−6.16	−4.84

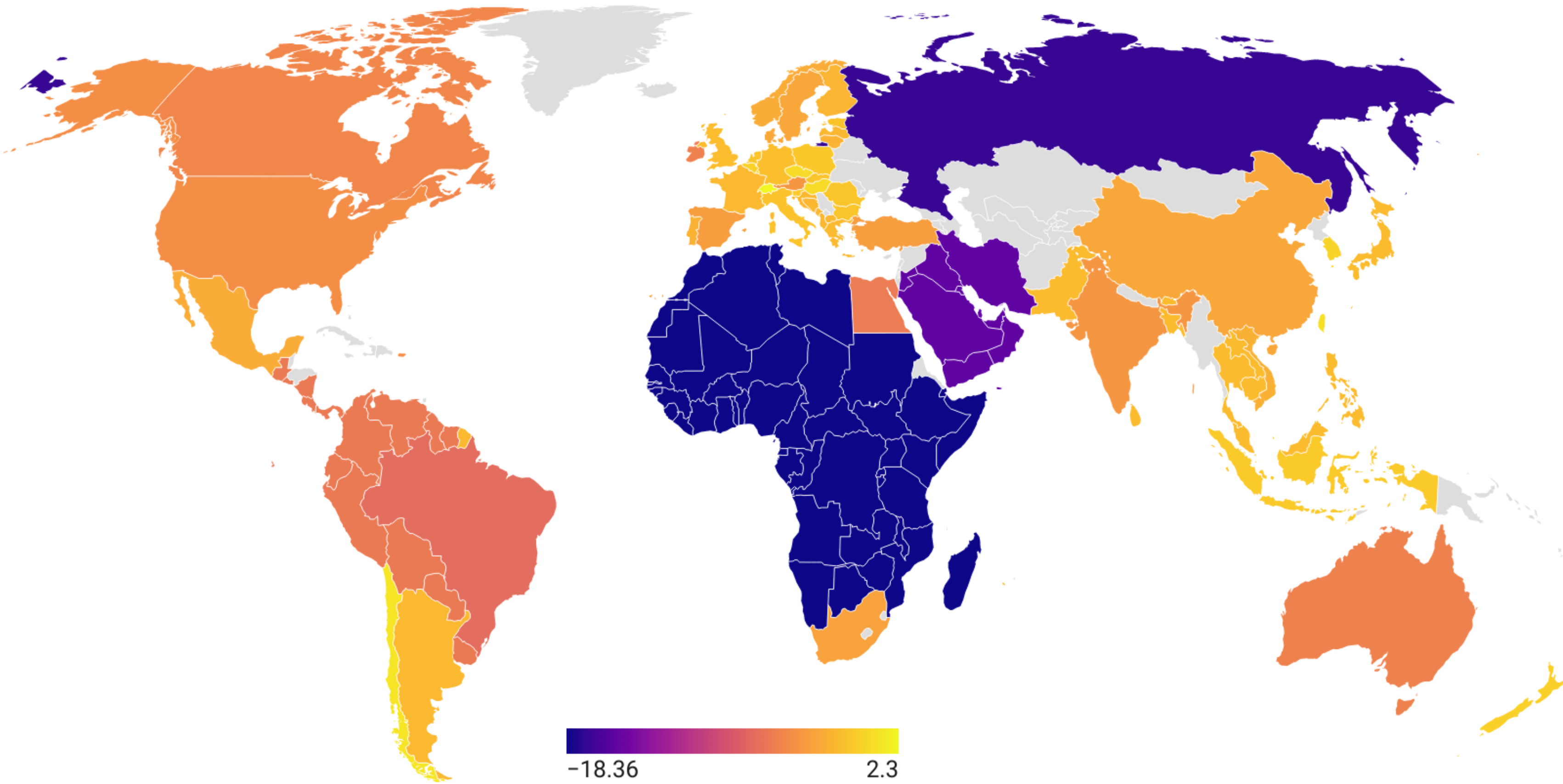
NGFS scenarios

Figure 168: GDP impact by 2050 (% change from baseline) — Delayed transition scenario



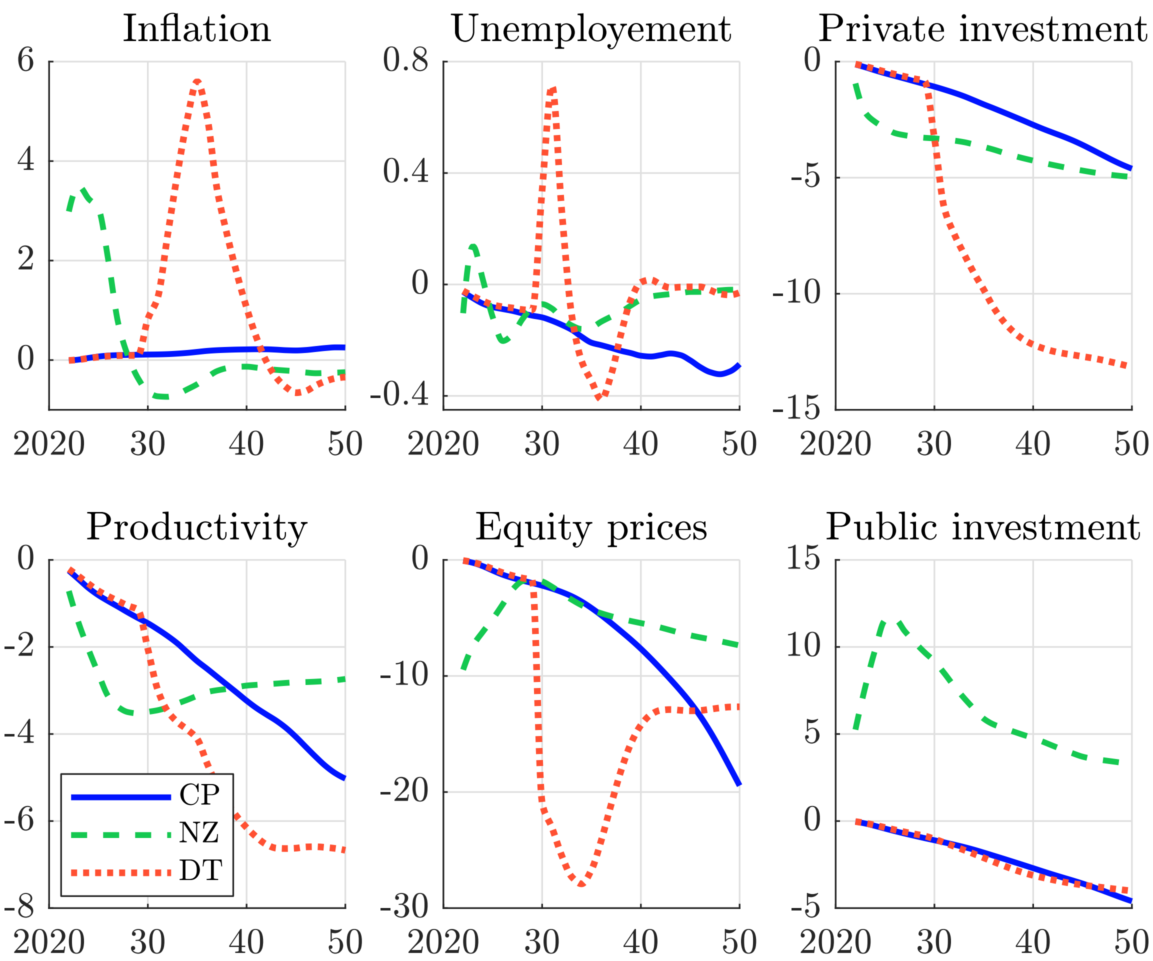
NGFS scenarios

Figure 169: GDP impact by 2050 (% change from baseline) — Net zero 2050 scenario



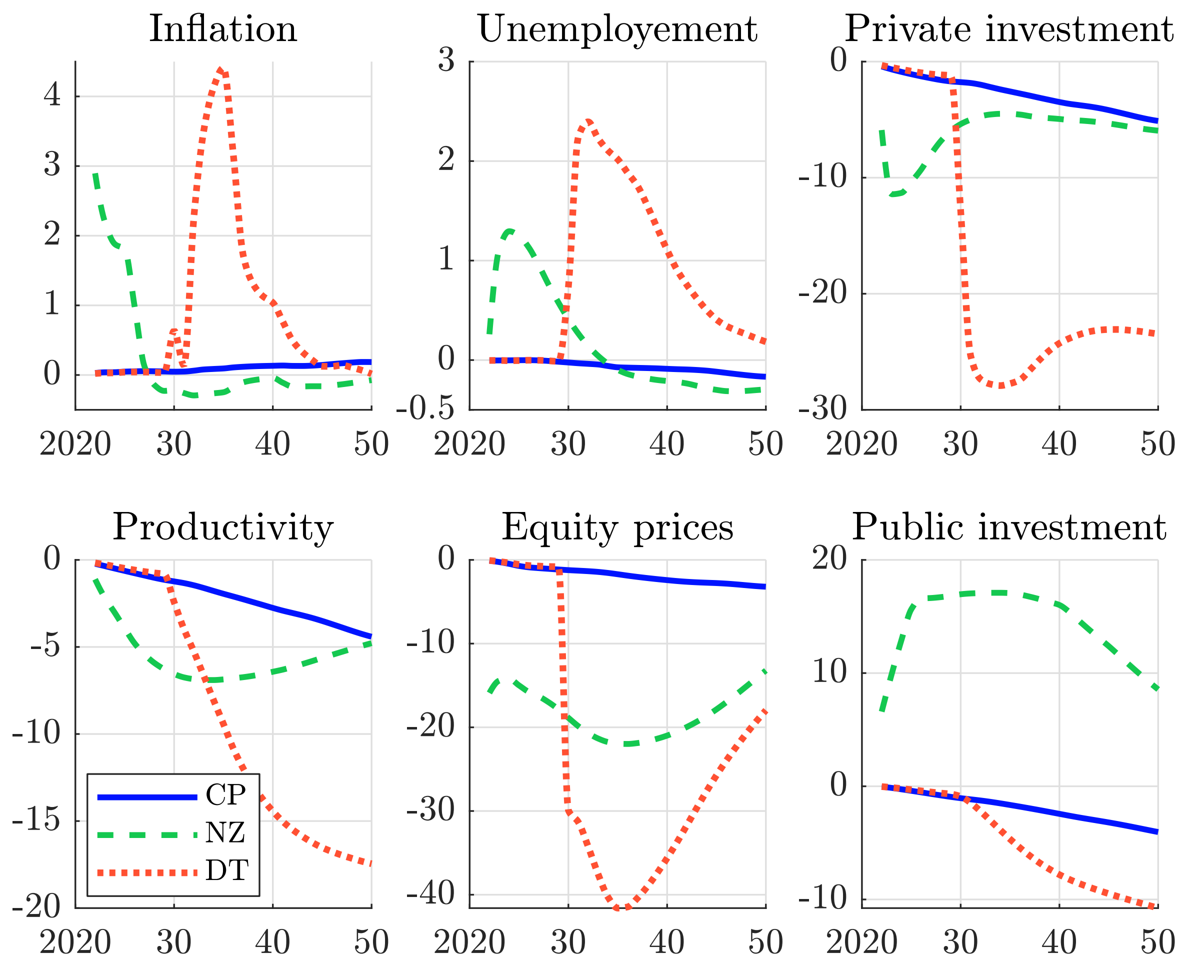
NGFS scenarios

Figure 170: Impact of climate scenarios on economics (% change from baseline) — China



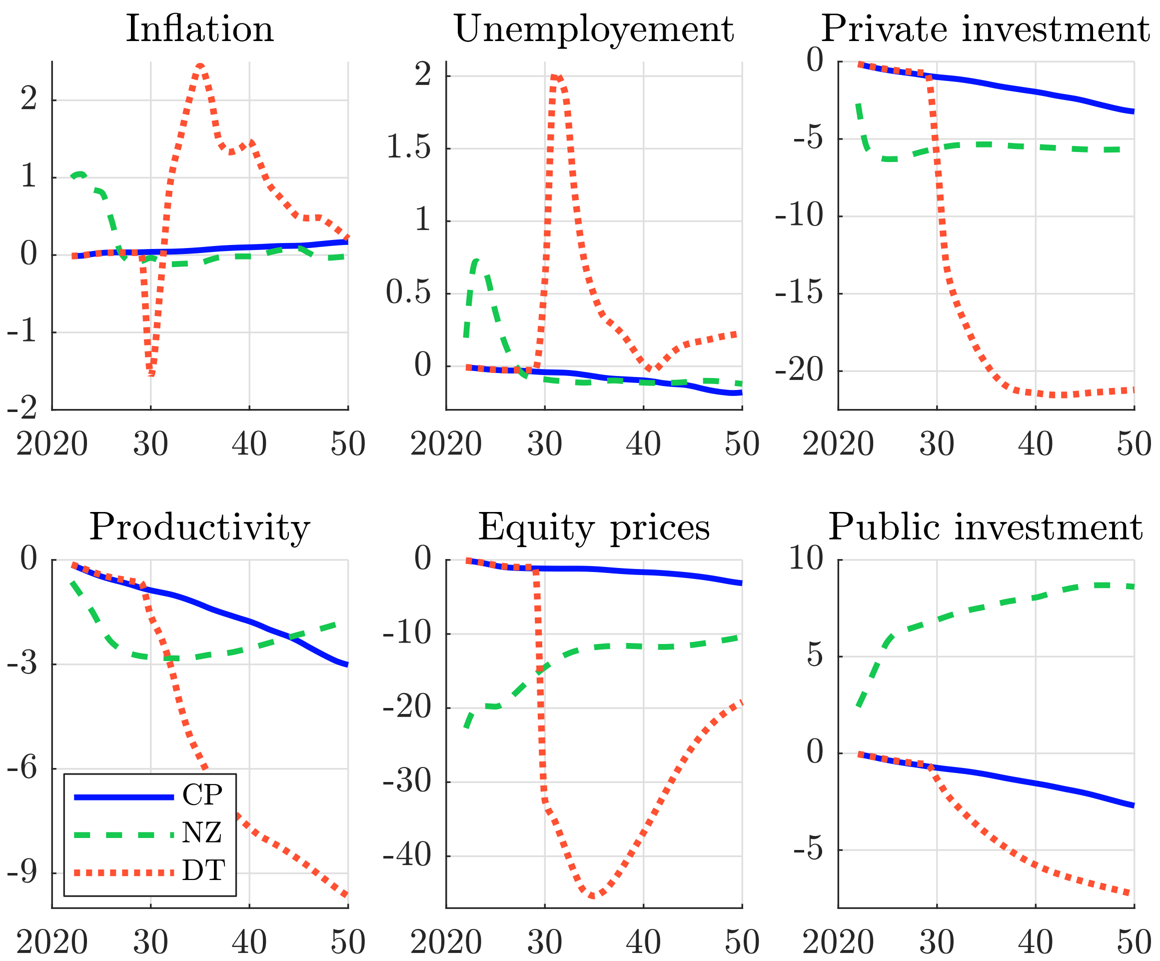
NGFS scenarios

Figure 171: Impact of climate scenarios on economics (% change from baseline) — United States



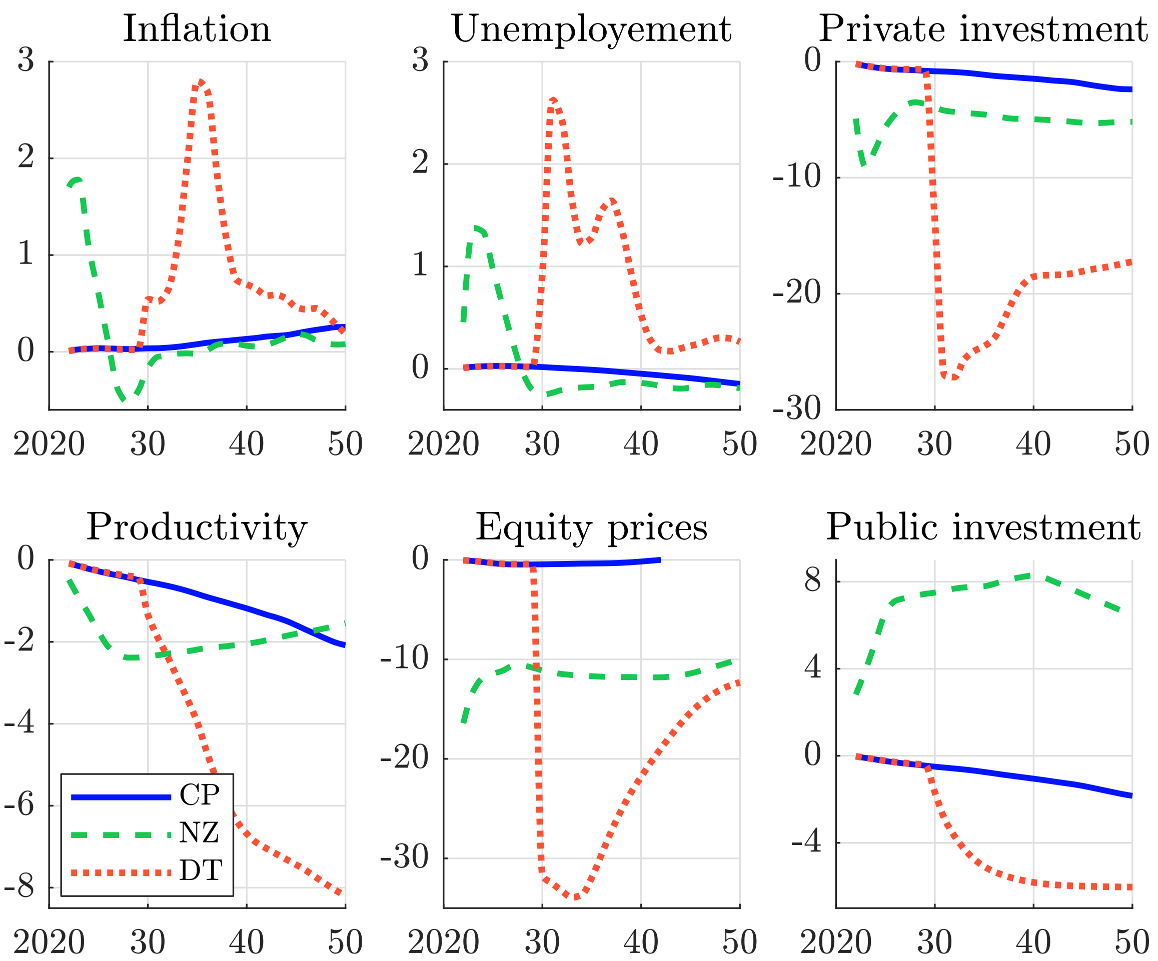
NGFS scenarios

Figure 172: Impact of climate scenarios on economics (% change from baseline) — France



NGFS scenarios

Figure 173: Impact of climate scenarios on economics (% change from baseline) — United Kingdom



Course 2022-2023 in Sustainable Finance
Lecture 8. Climate Risk Measures

Thierry Roncalli*

*Amundi Asset Management[22]

*University of Paris-Saclay

March 2023

[22]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Definition

How to define the carbon footprint?

Wackernagel and Rees (1996) published the seminal book on the ecological footprint:

> *"the carbon footprint stands for a certain amount of gaseous emissions that are relevant to climate change and associated with human production or consumption activities"*

Wiedmann and Minx (2008) proposed this definition:

> *"The carbon footprint is a measure of the exclusive total amount of carbon dioxide emissions that is directly and indirectly caused by an activity or is accumulated over the life stages of a product"*

Carbon footprint

- The carbon footprint is measured in carbon dioxide equivalent $(\mathrm{CO_2e}) \Rightarrow$ a common unit
- We have:

$$\text{equivalent mass of } CO_2 = \text{mass of the gas} \times \text{gwp of the gas}$$

- Examples (IPCC, AR5, 2013):
  - 1 kg of methane corresponds to 28 kg of CO_2
  - 1 kg of nitrous oxide corresponds to 265 kg of CO_2
- The carbon footprint is equal to:

$$m = \sum_{i=1}^{n} m_i \cdot \mathrm{gwp}_i$$

- The units are: $\mathrm{kgCO_2e}$, $\mathrm{tCO_2e}$, $\mathrm{ktCO_2e}$, $\mathrm{MtCO_2e}$ and $\mathrm{GtCO_2e}$

Carbon footprint

Example #1

We consider a company A that emits 3 017 tonnes of CO_2, 10 tonnes of CH_4 and 1.8 tonnes of N_2O. For the company B, the GHG emissions are respectively equal to 2 302 tonnes of CO_2, 32 tonnes of CH_4 and 3.0 tonnes of N_2O.

The mass of CO_2 equivalent for companies A and B is equal to:

$$m_A = 3017 \times 1 + 10 \times 28 + 1.8 \times 265 = 3\,774 \text{ tCO}_2\text{e}$$

and:

$$m_B = 2302 \times 1 + 32 \times 28 + 3.0 \times 265 = 3\,993 \text{ tCO}_2\text{e}$$

Estimation of the global warming potential

- According to IPCC (2007), GWP is defined as "*the cumulative radiative forcing, both direct and indirect effects, over a specified time horizon resulting from the emission of a unit mass of gas related to some reference gas*".
- Each gas differs in their capacity to absorb the energy (radiative efficiency) and how long it stays in the atmosphere (lifetime)
- The impact of a gas on global warming depends on the combination of radiative efficiency and lifetime

Estimation of the global warming potential

The mathematics of GWP

- The mathematical definition of the global warming potential is:

$$\mathrm{gwp}_i(t) = \frac{\mathrm{Agwp}_i(t)}{\mathrm{Agwp}_0(t)} = \frac{\int_0^t RF_i(s)\,\mathrm{d}s}{\int_0^t RF_0(s)\,\mathrm{d}s} = \frac{\int_0^t A_i(s)\,\mathbf{S}_i(s)\,\mathrm{d}s}{\int_0^t A_0(s)\,\mathbf{S}_0(s)\,\mathrm{d}s}$$

  where $A_i(t)$ is the radiative efficiency value of gas i, $\mathbf{S}_i(t)$ is the decay function and $i = 0$ is the reference gas (e.g, CO_2)
- We assume that:

$$\mathbf{S}_i(t) = \sum_{j=1}^{m} a_{i,j} e^{-\lambda_{i,j} t}$$

  where $\sum_{j=1}^{m} a_{i,j} = 1$
- We obtain:

$$\mathrm{gwp}_i(t) = \frac{A_i \sum_{j=1}^{m} a_{i,j} \lambda_{i,j}^{-1} \left(1 - e^{-\lambda_{i,j} t}\right)}{A_0 \sum_{j=1}^{m} a_{0,j} \lambda_{0,j}^{-1} \left(1 - e^{-\lambda_{0,j} t}\right)}$$

Estimation of the global warming potential

- Carbon dioxide
  - $A_{\mathrm{CO}_2} = 1.76 \times 10^{-18}$
  - The impulse response function is:

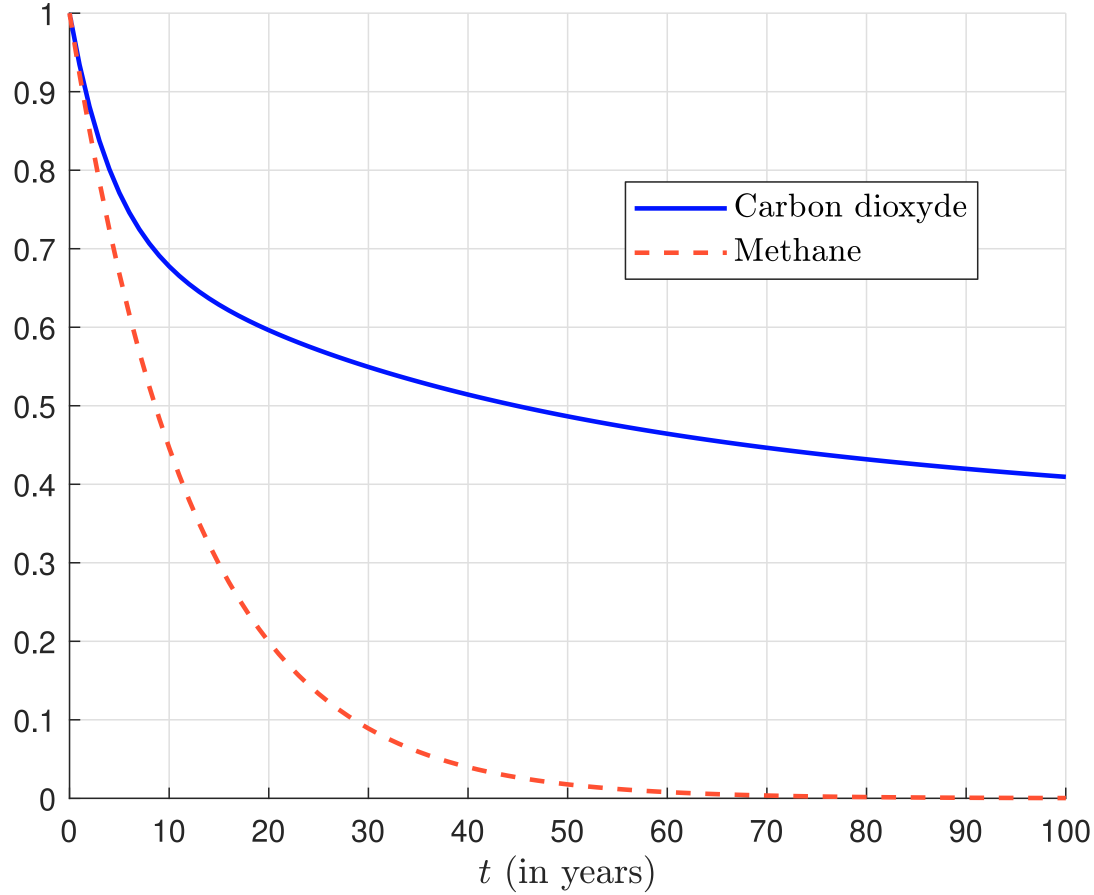
$$\begin{aligned} \mathbf{S}_{\mathrm{CO}_2}(t) &= 0.2173 + \\ & 0.2240 \cdot \exp\left(-\frac{t}{394.4}\right) + \\ & 0.2824 \cdot \exp\left(-\frac{t}{36.54}\right) + \\ & 0.2763 \cdot \exp\left(-\frac{t}{4.304}\right) \end{aligned}$$

- Methane
  - $A_{\mathrm{CH}_4} = 2.11 \times 10^{-16}$
  - The impulse response function is:

$$\mathbf{S}_{\mathrm{CH}_4}(t) = \exp\left(-\frac{t}{12.4}\right)$$

Estimation of the global warming potential

Figure 174: Fraction of gas remaining in the atmosphere



Source: Kleinberg(2020) & Author's calculations.

Estimation of the global warming potential

Remark

- The decay function is a survival function
- The density function is equal to $f_i(t) = -\partial_t \mathbf{S}_i(t)$
- Let τ_i be random time that the gas remains in the atmosphere
- In the case of the exponential distribution $\mathcal{E}(\lambda)$, we have

$$\begin{aligned}\mathbf{S}_i(t) &= e^{-\lambda t} \\ f_i(t) &= \lambda e^{-\lambda t} \\ \mathbb{E}[\tau_i] &= \frac{1}{\lambda}\end{aligned}$$

$\Rightarrow$ The survival function of the CH_4 gas is exponential with a mean time equal to 12.4 years ($\lambda = 1/12.4$)

Estimation of the global warming potential

- In the general case, the probability density function is equal to:

$$f_i(t) = -\partial_t \mathbf{S}_i(t) = \sum_{j=1}^{m} a_{i,j}\lambda_{i,j}e^{-\lambda_{i,j}t}$$

- The mean time $\mathcal{T}_i$ is given by:

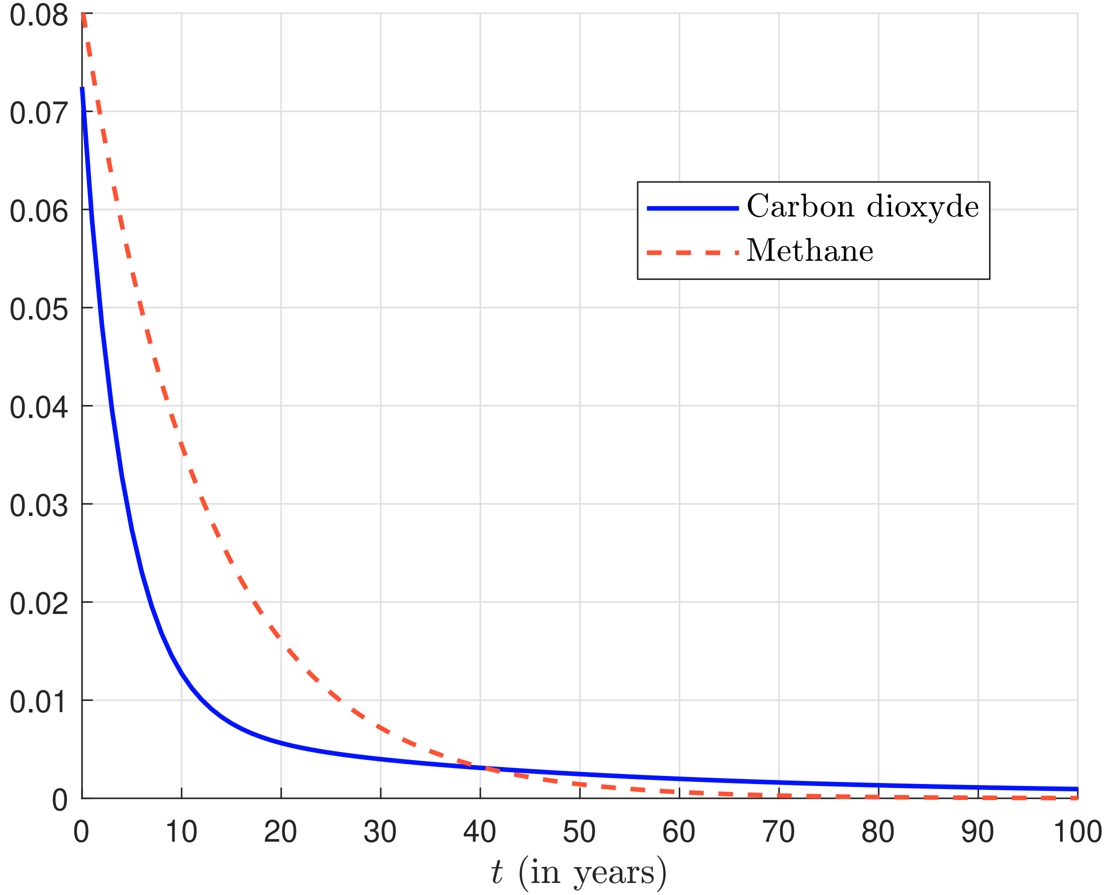
$$\begin{aligned} \mathcal{T}_i := \mathbb{E}[\tau_i] &= \int_0^\infty s f_i(s)\,\mathrm{d}s \\ &= \sum_{j=1}^{m} a_{i,j}\int_0^\infty \lambda_{i,j} s e^{-\lambda_{i,j}s}\,\mathrm{d}s \\ &= \sum_{j=1}^{m} \frac{a_{i,j}}{\lambda_{i,j}} \end{aligned}$$

Remark

We have $\mathcal{T}_{\mathrm{CH_4}} = 12.4$ years, but $\mathcal{T}_{\mathrm{CO_2}} = \infty$

Estimation of the global warming potential

Figure 175: Probability density function of the random time



Source: Kleinberg (2020) & Author's calculations.

Estimation of the global warming potential

Remark

- $f_i(t)$ is an exponential mixture distribution where m is the number of mixture components
- $\mathcal{E}(\lambda_{i,j})$ is the probability distribution associated with the j^{th} component
- $a_{i,j}$ is the mixture weight of the j^{th} component

We have:

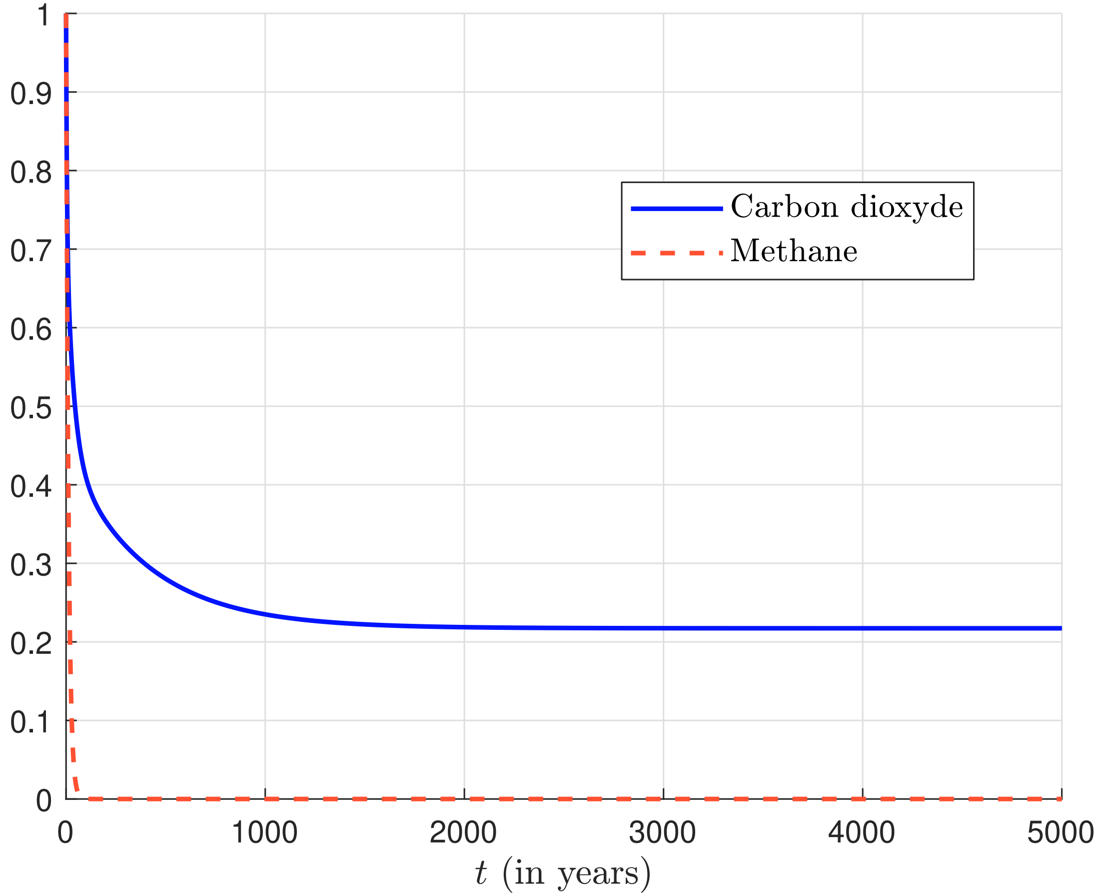
$$\mathcal{T}_i = \mathbb{E}[\tau_i] = \sum_{j=1}^{m} a_{i,j}\mathbb{E}[\tau_{i,j}] = \sum_{j=1}^{m} a_{i,j}\mathcal{T}_{i,j}$$

For the CO_2 gas, the exponential mixture distribution is defined by the following parameters:

j	1	2	3	4
$a_{i,j}$	0.2173	0.2240	0.2824	0.2763
$\lambda_{i,j}$ $(\times 10^3)$	0.00	2.535	27.367	232.342
$\mathcal{T}_{i,j}$ (in years)	∞	394.4	36.54	4.304

Estimation of the global warming potential

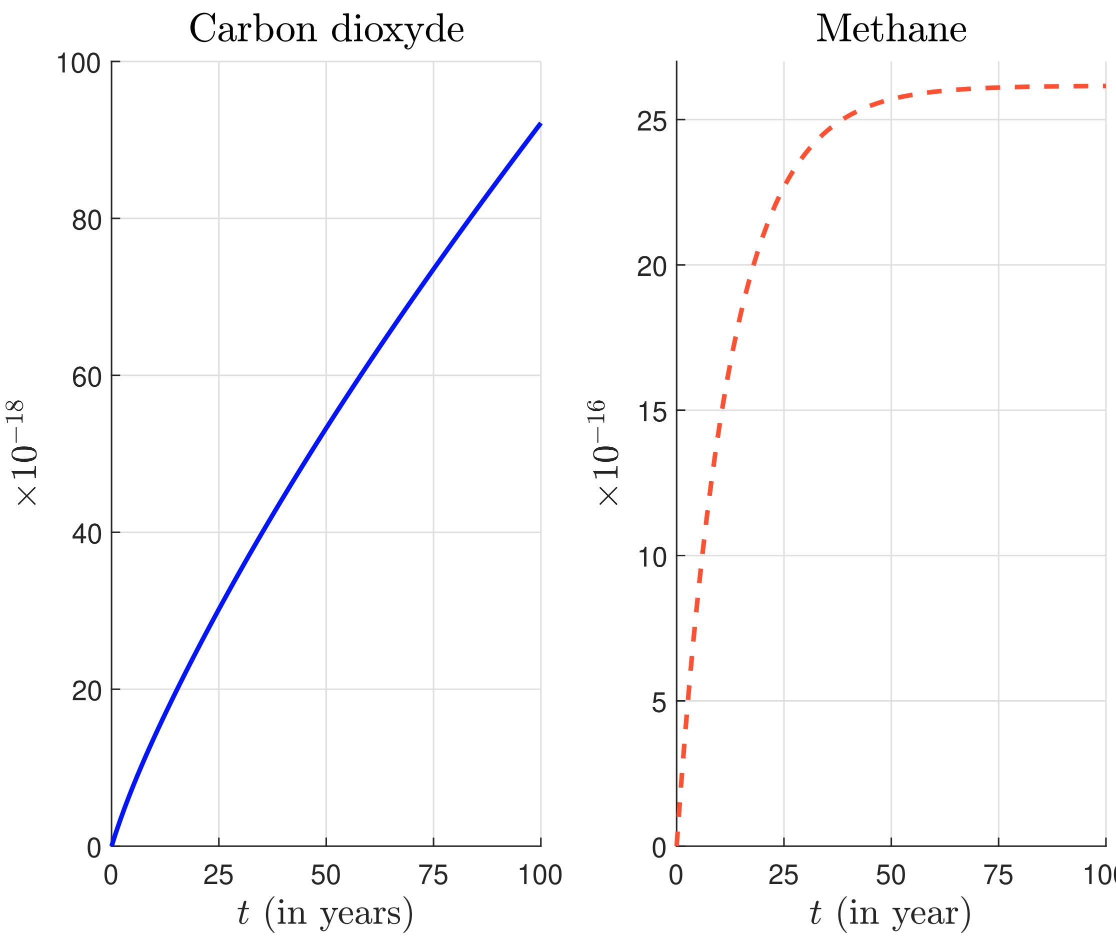
Figure 176: Survival function



We have $\mathbf{S}_{\mathrm{CO}_2}(\infty) = 21.73\%$!

Estimation of the global warming potential

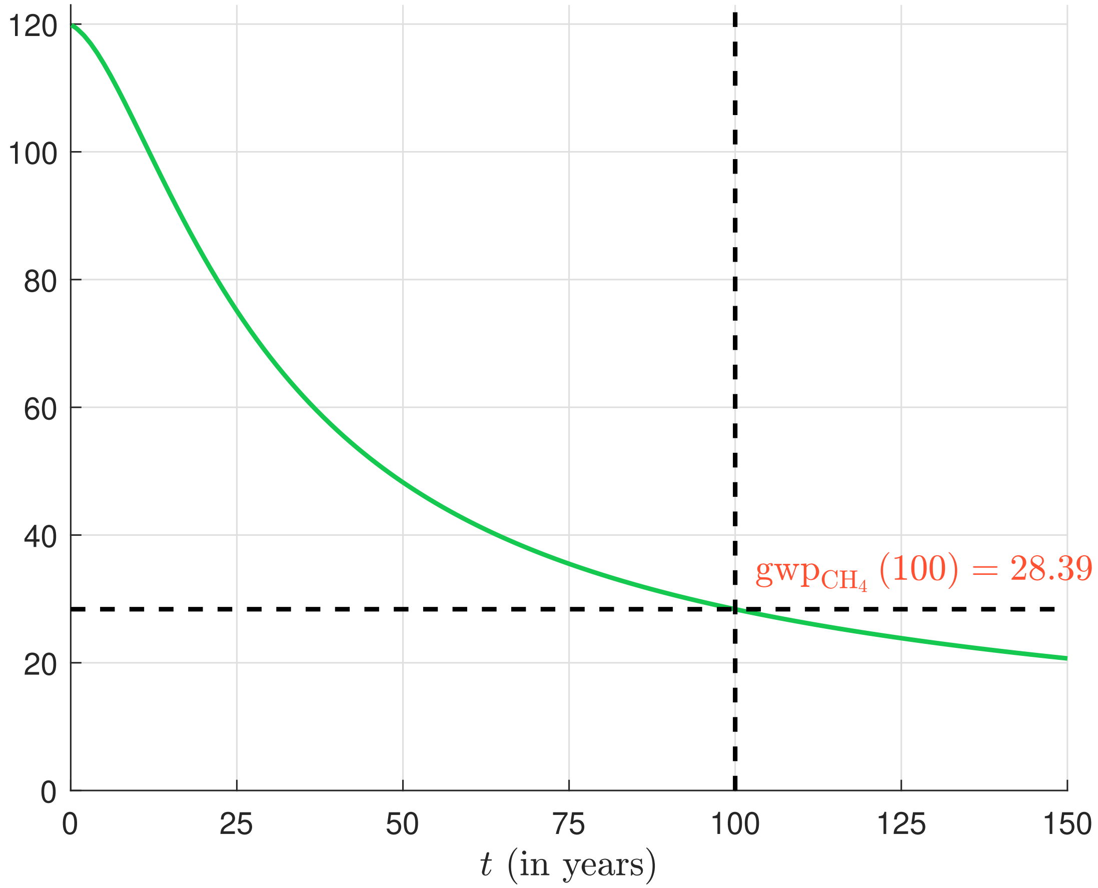
Figure 177: Absolute global warming potential



Source: Kleinberg (2020) & Author's calculations.

Estimation of the global warming potential

Figure 178: Global warming potential for methane



Source: Kleinberg (2020) & Author's calculations.

Estimation of the global warming potential

We have:

- $\mathrm{Agwp}_{\mathrm{CO}_2}(\infty) = \infty$
- $\mathrm{Agwp}_{\mathrm{CH}_4}(\infty) = A_{\mathrm{CH}_4} \times \mathcal{T}_{\mathrm{CH}_4} \propto 2.11 \times 12.4 = 26.164$
- The instantaneous global warming potential of the methane is equal to:

$$\mathrm{gwp}_{\mathrm{CH}_4}(0) = \frac{A_{\mathrm{CH}_4}}{A_{\mathrm{CO}_2}} = \frac{2.11 \times 10^{-16}}{1.76 \times 10^{-18}} \approx 119.9$$

- After 100 years, we obtain:

$$\boxed{\mathrm{gwp}_{\mathrm{CH}_4}(100) = 28.3853}$$

  This is the IPCC value!
- Because of the persistant regime of the carbon dioxyde, we have $\mathrm{gwp}_{\mathrm{CH}_4}(\infty) = 0$
- We have:

$$\mathrm{gwp}_{\mathrm{CH}_4}(t) \leq 1 \Leftrightarrow t \geq 6\,382 \text{ years}$$

Estimation of the global warming potential

Table 89: GWP values for 100-year time horizon

Name	Formula	AR2	AR4	AR5
Carbon dioxide	CO_2	1	1	1
Methane	CH_4	21	25	28
Nitrous oxide	N_2O	310	298	265
Sulphur hexafluoride	SF_6	23 900	22 800	23 500
Hydrofluorocarbons (HFC)	CHF_3	11 700	14 800	12 400
	CH_2F_2	650	675	677
	Etc.			
Perfluorocarbons (PFC)	CF_4	6 500	7 390	6 630
	C_2F_6	9 200	12 200	11 100
	Etc.			

Consolidation accounting at the company level

Two approaches:

1. Equity share approach
2. Control approach
    1. Financial control
    2. Operational control

Consolidation accounting at the company level

Table 90: Percent of reported GHG emissions under each consolidation method

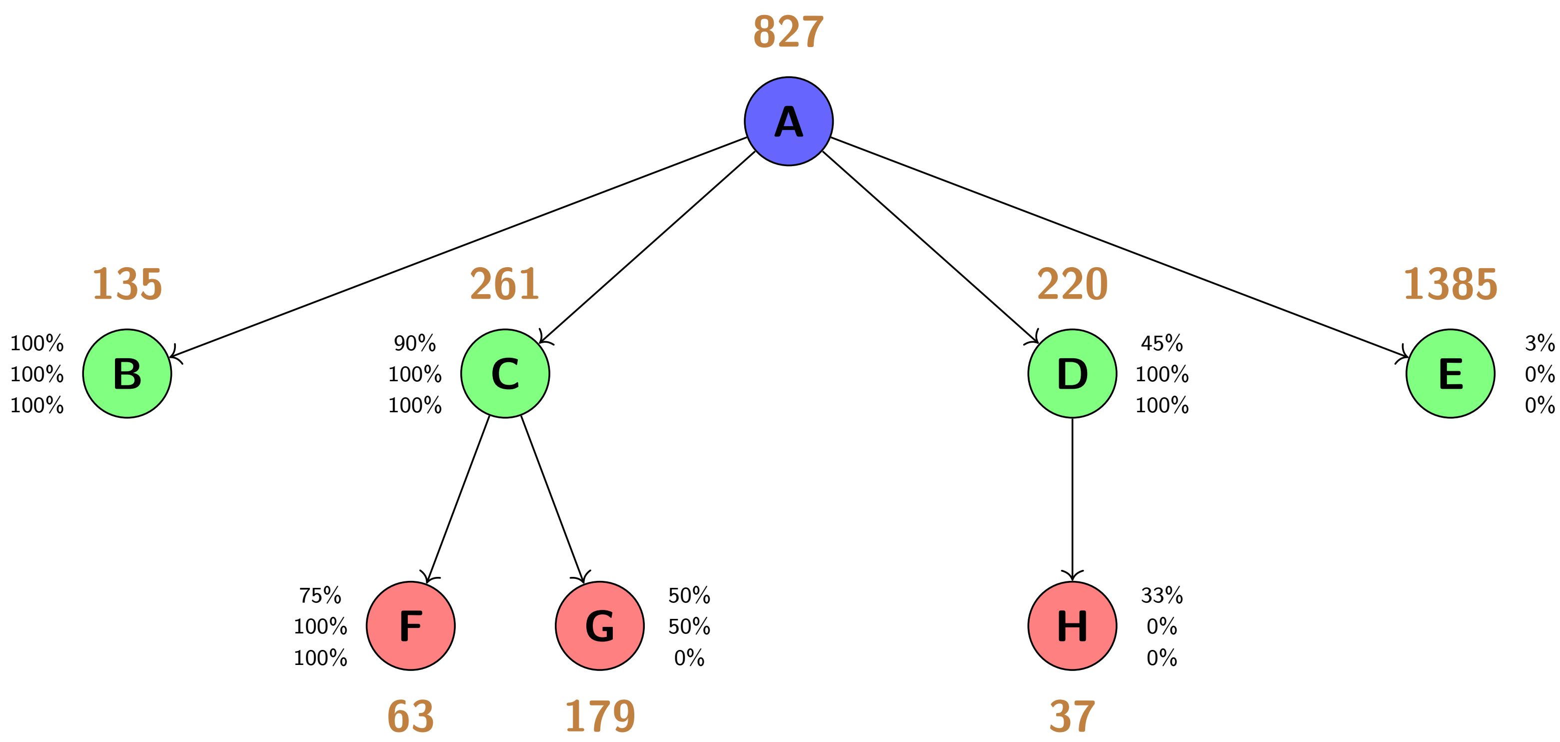
Accounting categories	GHG accouting based on		
	equity share	financial control	operational control
Wholly owned asset	100%	100%	100%
Group companies/subsidiaries	OWNR	100%	100%
Associated/affiliated companies	OWNR	0%	0%/100%
Joint ventures/partnerships	OWNR	OWNR	0%/100%
Fixed asset investments	0%	0%	0%
Franchises	0%	0%	0%
	OWNR	100%	100%

Source: GHG Protocol (2004, Table 1, page 19).

OWNR = Ownership ratio

Consolidation accounting at the company level

Figure 179: Defining the organizational boundary of company *A*



For each company, the brown number corresponds to the carbon emissions in $\mathrm{tCO_2e}$. The three figures at the right or left of the node corresponds respectively to the equity share, the financial control and the operational control

Consolidation accounting at the company level

- Equity share approach:

$$\begin{aligned}\mathcal{CE}_A &= 827 + 100\% \times 135 + 90\% \times 261 + 45\% \times 220 + 0\% \times 1\,385 + \\ &\quad 90\% \times 75\% \times 63 + 90\% \times 50\% \times 179 + 45\% \times 33\% \times 37 \\ &= 1\,424.4 \mathrm{tCO_2e}\end{aligned}$$

- Financial control approach:

$$\begin{aligned}\mathcal{CE}_A &= 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1\,385 + \\ &\quad 100\% \times 100\% \times 63 + 100\% \times 50\% \times 179 + 100\% \times 0\% \times 37 \\ &= 1\,595.50 \mathrm{tCO_2e}\end{aligned}$$

- Operational control approach:

$$\begin{aligned}\mathcal{CE}_A &= 827 + 100\% \times 135 + 100\% \times 261 + 100\% \times 220 + 0\% \times 1385 + \\ &\quad 100\% \times 100\% \times 63 + 100\% \times 0\% \times 179 + 100\% \times 0\% \times 37 \\ &= 1\,506.00 \mathrm{tCO_2e}\end{aligned}$$

Scope 1, 2 and 3 of carbon emissions

GHG Protocol (`www.ghgprotocol.org/corporate-standard`)

- Scope 1 denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- Scope 2 corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- Scope 3 are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories[a]:
    - Upstream scope 3 emissions are defined as indirect carbon emissions related to purchased goods and services.
    - Downstream scope 3 emissions are defined as indirect carbon emissions related to sold goods and services.

[a] The upstream value chain includes all activities related to the suppliers whereas the downstream value chain refers to post-manufacturing activities.

Scope 1, 2 and 3 of carbon emissions

Table 91: Examples of CDP reporting ($\mathcal{CE}$ in $\mathrm{tCO_2e}$, year 2020)

Scope	Category	Sub-category	Amazon	Danone	ENEL	Pfizer	Netflix	Walmart
1			9 623 138	668 354	45 255 000	654 460	30 883	7 236 499
2	Location-based (2a)		9 019 786	864 710	4 990 685	551 577	28 585	11 031 800
	Market-based (2b)		5 265 089	479 210	7 855 954	542 521	141	9 190 337
3	Upstream	Purchased goods and services	16 683 423	19 920 918		2 526 537	765 208	130 200 000
		Capital goods	13 202 065			191 894	116 366	645 328
		Fuel and energy related activities	1 248 847	283 764	1 061 268	203 093	12 287	3 327 874
		Upstream transportation and distribution	8 563 695	321 558	112 358	723 558	64 693	342 577
		Waste generated in operations	16 628	152 789	3 161	14 940		869 927
		Business travel	313 043			35 128	41 439	37 439
		Employee commuting	306 033			48 414	19 116	3 500 000
		Upstream leased assets	1 223 903			30 522	131	
	Downstream	Downstream transportation and distribution	2 785 676	1 627 090		7 295		5 099
		Processing of sold products						
		Use of sold products	1 426 543	1 885 548	46 524 860		952	32 211 000
		End-of-life treatment of sold products	0	782 649				130
		Downstream leased assets					349	130 000
		Franchises						
		Investments				36 839		
Total	Scope 1 + 2a		18 642 924	1 533 064	50 245 685	1 206 037	59 468	18 268 299
	Scope 1 + 2b		14 888 227	1 147 564	53 110 954	1 196 981	31 024	16 426 836
	Scope 3 upstream		41 557 637	20 679 029	1 176 787	3 774 086	1 019 240	138 923 145
	Scope 3 downstream		4 212 219	4 295 287	46 524 860	44 134	1 301	32 346 229
	Scope 3		45 769 856	24 974 316	47 701 647	3 818 220	1 020 541	171 269 374
	Scope 1 + 2a + 3		64 412 780	26 507 380	97 947 332	5 024 257	1 080 009	189 537 673
	Scope 1 + 2b + 3		60 658 083	26 121 880	100 812 601	5 015 201	1 051 565	187 696 210

Source: CDP database as of 01/07/2022 & Author's computation.

Scope 1, 2 and 3 of carbon emissions

CDP questionnaire for corporates

- www.cdp.net/en/guidance/guidance-for-companies
- HTML, Word and PDF formats
- 129 pages and 16 sections: $\mathcal{SC}_1$ (§C6.1), $\mathcal{SC}_2$ (§C6.3) and $\mathcal{SC}_3$ emissions (§C6.5) — emissions intensities (§C6.10)

Computation of scope 1 emissions

- We allocate the activities to the three scopes
- Then, we apply an emission factor to each activity and each gas:

$$E_{g,h} = A_h \cdot \mathcal{EF}_{g,h}$$

  where A_h is the h^{th} activity rate (also called activity data) and $\mathcal{EF}_{g,h}$ is the emission factor for the h^{th} activity and the g^{th} gas
  - A_h can be measured in volume, weight, distance, duration, surface, etc.
  - $E_{g,h}$ is expressed in tonne
  - $\mathcal{EF}_{g,h}$ is measured in tonne per activity unit
- For each gas, we calculate the total emissions:

$$E_g = \sum_{h=1}^{n_A} E_{g,h} = \sum_{h=1}^{n_A} A_h \cdot \mathcal{EF}_{g,h}$$

- Finally, we estimate the carbon emissions by applying the right GWP:

$$\mathcal{CE} = \sum_{g=1}^{n_G} \mathrm{gwp}_g \cdot E_g$$

Tier methods

The choice of data inputs is codified by IPCC (2019):

- Tier 1 methods use global default emission factors;
- Tier 2 methods use country-level or region-specific emission factors;
- Tier 3 methods use directly monitored or site-specific emission factors.

$\Rightarrow$ IPCC Emission Factor Database, National Inventory Reports (NIRs), country emission factor databases, etc.

France

- The database of emission factors is managed by **ADEME** (Agence de l'Environnement et de la Maîtrise de l'Energie)
- It contains about 5 300 validated emission factors
- `https://bilans-ges.ademe.fr`

Reporting of scope 1 emissions

GHG inventory document of Enel (2021)

- Scope 1 emissions expressed in $\mathrm{ktCO_2e}$:

	CO_2	CH_4	N_2O	NF_3	SF_6	HFCs	Total
Electricity power generation	50 643.54	385.25	98.14	0.014	31.15	10.22	51 168.32
Electricity distribution	208.33	0.24	0.45		111.62		320.64
Real estate	79.87	0.22	1.24				81.30
Total	50 931.72	385.71	99.83	0.014	142.77	10.22	51 750.26

- The scope 1 emissions of Enel is equal to 51.75 $\mathrm{MtCO_2e}$

Scope 1 emissions

Table 92: Examples of emission factors (EFDB, IPCC)

Category	Description	Gas	Region	Value	Unit
Iron and steel production	Integrated facility	CO_2	Canada	1.6	t/tonne
	Electrode consumption from steel produced in electric arc furnaces	CO_2	Global	5.0	kg/tonne
	Steel processing (rolling mills)	N_2O	Global	40	g/tonne
Manufacture of solid fuels	Metallurgical coke production	CO_2	Global	0.56	t/tonne
		CH_4	Global	0.1	g/tonne
Fuel combustion activities	Crude oil	CO_2	Global	20	tCarbon/TeraJoule
	Natural gas	CO_2	Global	15.3	tCarbon/TeraJoule
	Ethane	CO_2	Global	16.8	tCarbon/TeraJoule
Integrated circuit or semiconductor	Semiconductor manufacturing (silicon)	CF_4	Global	0.9	kg/m^2
Cement production	Cement production	CO_2	Global	0.4985	t/tonne
Horses	Enteric fermentation	CH_4	Global	18	kg/head/year
	Manure management (annual average temperature is less than 15oC)	CH_4	Developed countries	1.4	kg/head/year
	Manure management (annual average temperature is between 15oC and 25oc)	CH_4	Developed countries	2.1	kg/head/year
Buffalo	Enteric fermentation	CH_4	Global	55	kg/head/year
Poultry	Manure management (annual average temperature is less than 15oC)	CH_4	Developed countries	0.078	kg/head/year
	Manure management (annual average temperature is between 15oC and 25oc)	CH_4	Developed countries	0.117	kg/head/year
	Manure management (annual average temperature is greater than 25oC)	CH_4	Developed countries	0.157	kg/head/year
	Manure management (annual average temperature is greater than 25oC)	CH_4	Developing countries	0.023	kg/head/year

Source: EFDB, `www.ipcc-nggip.iges.or.jp/EFDB`.

Scope 2 emissions

Definition

Scope 2 is "*an indirect emission category that includes GHG emissions from the purchased or acquired electricity, steam, heat, or cooling consumed*" (GHG Protocol, 2015):

- Electricity
  People use electricity for operating machines, lighting, heating, cooling, electric vehicle charging, computers, electronics, public transportation systems, etc.
- Steam
  Industries use steam for mechanical work, heating, propulsion, driven turbines in electric power plants, etc.
- Heat
  Buildings use heat to control inside temperature and heat water, while the industrial sector uses heat for washing, cooking, sterilizing, drying, etc. Heat may be produced from electricity, solar heat processes or thermal combustion.
- Cooling
  It is produced from electricity or though the processes of forced air, conduction, convection, etc.

Scope 2 emissions

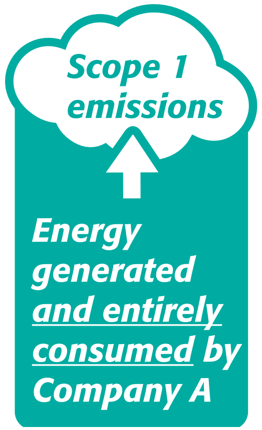


Figure 180: Energy production and consumption from owned/operated generation

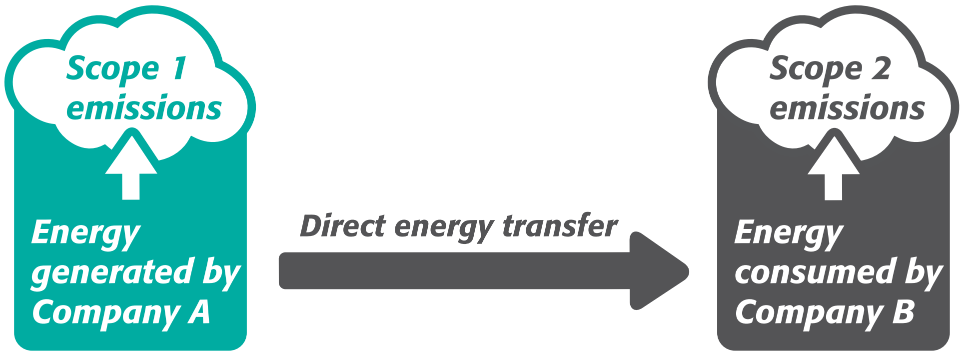
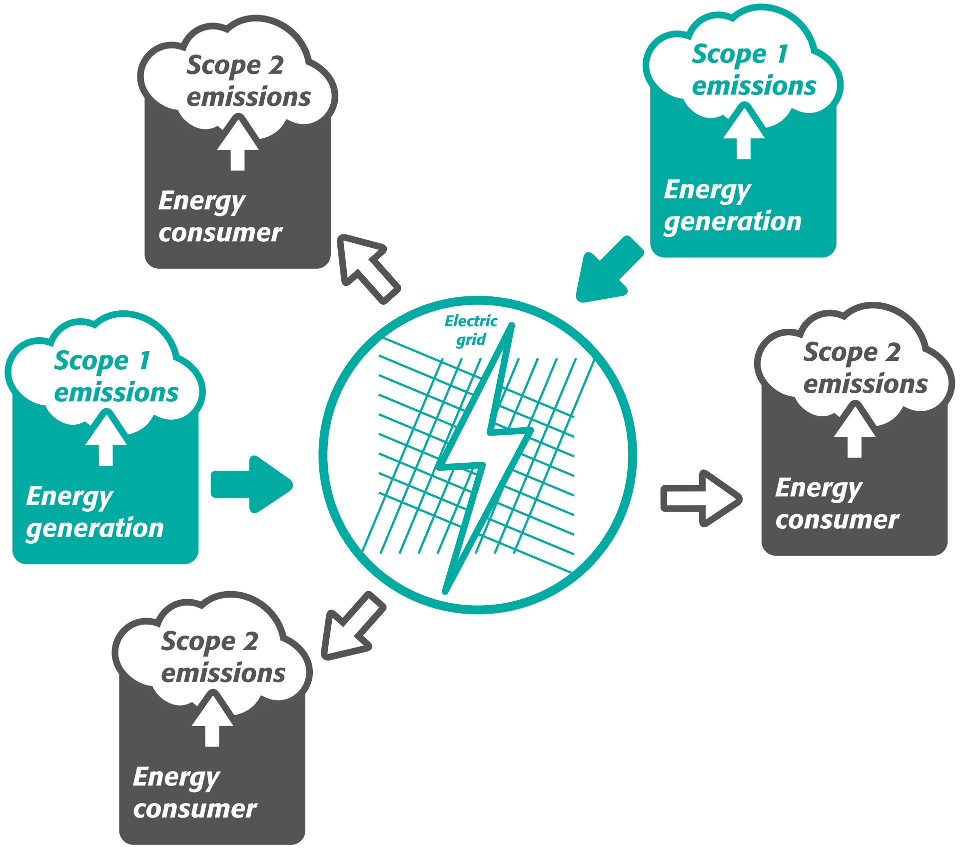


Figure 181: Direct line energy transfer

Source: GHG Protocol (2015, Figures 5.1 and 5.2, pages 35-36).

Scope 2 emissions

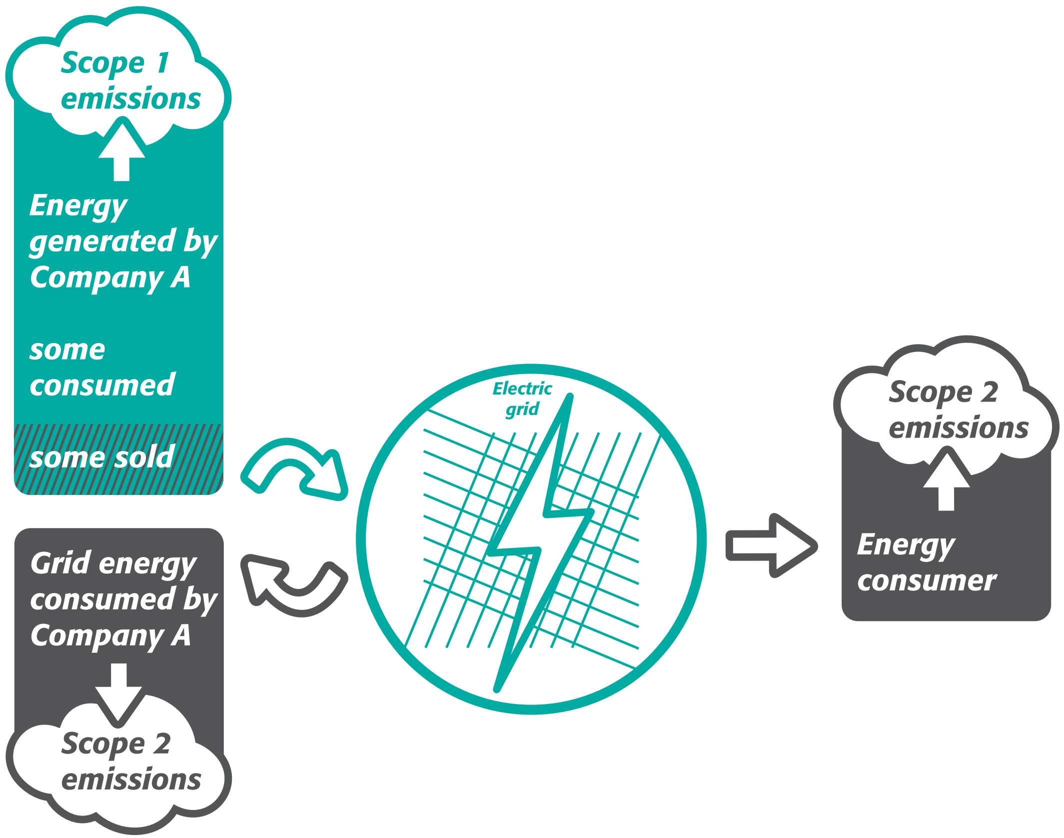
Figure 182: Electricity production on a grid



Source: GHG Protocol (2015, Figure 5.4, page 38).

Scope 2 emissions

Figure 183: Facility consuming both energy generated on-site and purchased from the grid



Source: GHG Protocol (2015, Figure 5.3, page 37).

Computation of scope 2 emissions

Scope 2 emissions are calculated using activity data and emission factors expressed in MWh and $\mathrm{tCO_2e}$/MWh:

$$\mathcal{CE} = \sum_s A_s \cdot \mathcal{EF}_s$$

where:

- A_s is the amount of purchased electricity for the energy generation source s
- $\mathcal{EF}_s$ is the emission factor of the source s

Computation of scope 2 emissions

Example #2

We consider a company, whose electricity consumption is equal to 2 000 MWh per year. The electricity comes from two sources: 60% from a direct line with an electricity supplier (source S_1) and 40% from the country grid (source S_2). The emission factors are respectively equal to 200 and 350 $\mathrm{gCO_2e}$/kWh.

Computation of scope 2 emissions

- The electricity consumption from source S_1 is equal to $60\% \times 2\,000 = 1\,200$ MWh or $1\,200\,000$ kWh
- We deduce that the carbon emissions from this source is:

$$\mathcal{CE}(S_1) = \left(1.2 \times 10^6\right) \times 200 = 240 \times 10^6 \text{ gCO}_2\text{e} = 240 \text{ tCO}_2\text{e}$$

- For the second source, we obtain:

$$\mathcal{CE}(S_2) = \left(0.8 \times 10^6\right) \times 350 = 280 \times 10^6 \text{ gCO}_2\text{e} = 280 \text{ tCO}_2\text{e}$$

- We deduce that the Scope 2 carbon emissions of this company is equal to 520 tCO_2e

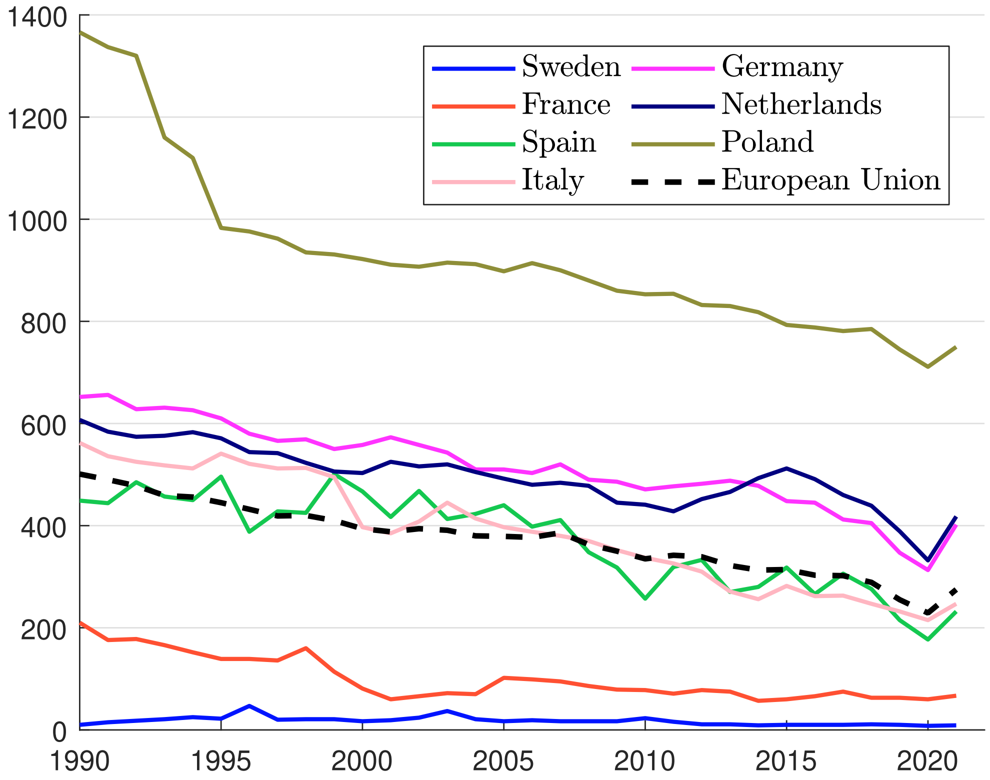
Scope 2 emissions accounting

Two main methods:

- **Location-based method**
  In this approach, the company uses the average emission factor of the region or the country. For instance, if the electricity consumption is located in France, the company can use the emission intensity of the French energy mix;
- **Market-based method**
  This approach reflects the GHG emissions from the electricity that the company has chosen in the market. This means that the scope 2 carbon emissions will depend on the scope 1 carbon intensity of the electricity supplier

Scope 2 emission factors

Figure 184: Emission factor in gCO_2e/kWh of electricity generation (European Union, 1990 – 1992)



Source: European Environment Agency (2022), `www.eea.europa.eu/data-and-maps` & Author's calculations.

Scope 2 emission factors

Table 93: Emission factor in $\mathrm{gCO_2e}$/kWh of electricity generation in the world

Region	$\mathcal{EF}$	Country	$\mathcal{EF}$	Country	$\mathcal{EF}$	Country	$\mathcal{EF}$
Africa	484	Australia	531	Germany	354	Portugal	183
Asia	539	Canada	128	India	637	Russia	360
Europe	280	China	544	Iran	492	Spain	169
North America	352	Costa Rica	33	Italy	226	Switzerland	47
South America	204	Cuba	575	Japan	479	United Kingdom	270
World	442	France	58	Norway	26	United States	380

Source: `https://ourworldindata.org/grapher/carbon-intensity-electricity`

Computation of scope 2 emissions

Example #3

We consider a French bank, whose activities are mainly located in France and the Western Europe. Below, we report the energy consumption (in MWh) by country:

Belgium	125 807	France	1 132 261
Germany	71 890	Ireland	125 807
Italy	197 696	Luxembourg	33 069
Netherlands	18 152	Portugal	12 581
Spain	61 106	Switzerland	73 148
UK	124 010	World	37 742

Computation of scope 2 emissions

- If we consider a Tier 1 approach, we can estimate the scope 2 emissions of the bank by computing the total activity data and multiplying by the global emission factor
- Since we have twelve sources, we obtain:

$$A = \sum_{s=1}^{12} A_s = 125\,807 + 1\,132\,261 + \ldots + 37\,742 = 2\,013\,269 \text{ MWh}$$

and:

$$\begin{aligned} \mathcal{CE} &= A \cdot \mathcal{EF}_{World} \\ &= \left(2\,013{,}269 \times 10^3\right) \times 442 \\ &= 889\,864\,898\,000 \ \mathrm{gCO_2e} \\ &= 889.86 \ \mathrm{ktCO_2e} \end{aligned}$$

Computation of scope 2 emissions

- Another Tier 1 approach is to consider the emission factor of the European Union, because the rest of the world represents less than 2% of the electricity consumption. Using $\mathcal{EF}_{EU} = 275$, we obtain $\mathcal{CE} = 553.65 \text{ ktCO}_2\text{e}$

Computation of scope 2 emissions

- The third approach uses a Tier 2 method by considering the emission factor of each country
- We use the previous figures and the following emission factors: Belgium (143); Ireland (402); Luxembourg (68) and Netherlands (331)
- We deduce that:

$$\begin{aligned}
\mathcal{CE} &= \sum_{s=1}^{12} A_s \cdot \mathcal{EF}_s \\
&= (125\,807 \times 143 + 1\,132\,261 \times 58 + \ldots \\
&\quad +124\,010 \times 270 + 37\,742 \times 442) \times \frac{10^3}{10^9} \\
&= 278.85 \text{ ktCO}_2\text{e}
\end{aligned}$$

$\Rightarrow$ **The estimated scope 2 emissions of this bank are sensitive to the approach**

Computation of scope 2 emissions

Example #4

We consider a Norwegian company, whose current electricity consumption is equal to 1 351 Mwh. 60% of the electricity comes from the Norwegian hydroelectricity and the GO system guarantees that this green electricity emits 1 $\mathrm{gCO_2e/kWh}$.

If we assume that the remaining 40% of the electricity consumption comes from the Norwegian grid[23], the market based scope 2 emissions of this company are equal to:

$$\begin{aligned} \mathcal{CE} &= \frac{10^6 \times 60\% \times 1 + 10^6 \times 40\% \times 26}{10^6} \\ &= 11 \ \mathrm{ktCO_2e} \end{aligned}$$

[23] The emission factor for Norway is 26 $\mathrm{gCO_2e/kWh}$.

Computation of scope 2 emissions

Table 94: Emission factor in $\mathrm{gCO_2e}/\mathrm{KWh}$ from electricity supply technologies (IPCC, 2014; UNECE, 2022)

Technology	Characteristic	IPCC		UNECE	
		Mean	Min–Max	Mean	Min–Max
Wind	Onshore	11	7–56	12	8–16
	Offshore	12	8–35	13	13–23
Nuclear		12	3–110	6	
Hydro power		24	1–2200	11	6–147
Solar power	CSP	27	9–63	32	14–122
	Rooftop (PV)	41	26–60	22	9–83
	Utility/Ground (PV)	48	18–180	20	8–82
Geothermal		38	6–79		
Biomass	Dedicated	230	130–420		
Gas	CCS	169	90-370	130	92–221
	Combined cycle	490	410–650	430	403–513
Fuel oil			510–1170		
Coal	CCS	161	70–290	350	190–470
	PC	820	740–650	1 000	912–1095

CSP: concentrated solar power; PV: photovoltaic power; CCS: carbon capture and storage; PC: pulverized coal.

Reporting of scope 2 emissions

GHG inventory document of Enel (2021)

- The scope 2 emissions expressed in $\mathrm{ktCO_2e}$ are:

	Electricity purchased from the grid	Losses on the distribution grid	Total
Location-based	1 336.67	2 966.52	4 303.18
Market-based	2 351.00	4 763.15	7 114.15

Location-based versus market-based scope 2 emissions

Table 95: Statistics of CDP scope 2 emissions (2020)

	$\mathcal{CE}_{\mathrm{loc}} = 0$	$\mathcal{CE}_{\mathrm{loc}} = \mathcal{CE}_{\mathrm{mkt}} = 0$	$\mathcal{CE}_{\mathrm{mkt}} = 0$
Frequency	0.89%	0.39%	8.78%
	$\mathcal{CE}_{\mathrm{loc}} > \mathcal{CE}_{\mathrm{mkt}}$	$\mathcal{CE}_{\mathrm{loc}} = \mathcal{CE}_{mkt}$	$\mathcal{CE}_{\mathrm{loc}} < \mathcal{CE}_{\mathrm{mkt}}$
Frequency	70.43%	9.48%	20.09%
Mean variation ratio	+43.89%	0.00%	−22.04%

Source: CDP database as of 01/07/2022 & Author's computation.

Scope 3 categories

Upstream

1. Purchased goods and services
2. Capital goods
3. Fuel and energy related activities
4. Upstream transportation and distribution
5. Waste generated in operations
6. Business travel
7. Employee commuting
8. Upstream leased assets
9. **Other upstream**

Downstream

1. Downstream transportation and distribution
2. Processing of sold products
3. Use of sold products
4. End-of-life treatment of sold products
5. Downstream leased assets
6. Franchises
7. Investments
8. **Other downstream**

Scope 3 emissions

Scope 3 emissions are all the indirect emissions in the company's value chain, apart from indirect emissions which are reported in scope 2:

1. **Purchased goods and services (not included in categories 2-8)**
   Extraction, production, and transportation of goods and services purchased or acquired by the company
2. **Capital goods**
   Extraction, production, and transportation of capital goods purchased or acquired by the company
3. **Fuel- and energy-related activities (not included in scopes 1 or 2)**
   Extraction, production, and transportation of fuels and energy purchased or acquired by the company
4. **Upstream transportation and distribution**
   Transportation and distribution of products purchased by the company between the company's tier 1 suppliers and its own operations; Transportation and distribution services purchased by the company, including inbound logistics, outbound logistics (e.g., sold products), and transportation and distribution between the company's own facilities

Scope 3 emissions

5. **Waste generated in operations**
   Disposal and treatment of waste generated in the company's operations
6. **Business travel**
   Transportation of employees for business-related activities
7. **Employee commuting**
   Transportation of employees between their homes and their work sites
8. **Upstream leased assets**
   Operation of assets leased by the company (lessee)

Scope 3 emissions

9. **Downstream transportation and distribution**
   Transportation and distribution of products sold by the company between the company's operations and the end consumer (if not paid for by the company)
10. **Processing of sold products**
    Processing of intermediate products sold by downstream companies (e.g., manufacturers)
11. **Use of sold products**
    End use of goods and services sold by the company
12. **End-of-life treatment of sold products**
    Waste disposal and treatment of products sold by the company at the end of their life

Scope 3 emissions

13. **Downstream leased assets**
    Operation of assets owned by the company (lessor) and leased to other entities
14. **Franchises**
    Operation of franchises reported by franchisor
15. **Investments**
    Operation of investments (including equity and debt investments and project finance)

Scope 3 emissions

Table 96: Scope 3 emission factors for business travel and employee commuting (United States)

Vehicle type	CO_2 (kg/unit)	CH_4 (g/unit)	N_2O (g/unit)	Unit
Passenger car	0.332	0.0070	0.0070	vehicle-mile
Light-duty truck	0.454	0.0120	0.0090	vehicle-mile
Motorcycle	0.183	0.0700	0.0070	vehicle-mile
Intercity rail (northeast corridor)	0.058	0.0055	0.0007	passenger-mile
Intercity rail (other routes)	0.150	0.0117	0.0038	passenger-mile
Intercity rail (national average)	0.113	0.0092	0.0026	passenger-mile
Commuter rail	0.139	0.0112	0.0028	passenger-mile
Transit rail (subway, tram)	0.099	0.0084	0.0012	passenger-mile
Bus	0.056	0.0210	0.0009	passenger-mile
Air travel (short haul, $<$ 300 miles)	0.207	0.0064	0.0066	passenger-mile
Air travel (medium haul, 300-2300 miles)	0.129	0.0006	0.0041	passenger-mile
Air travel (long haul, $>$ 2300 miles)	0.163	0.0006	0.0052	passenger-mile

Source: US EPA (2020), Table 10, `www.epa.gov`, `ghg-emission-factors-hub.xlsx`.

These factors are intended for use in the distance-based method defined in the Scope 3 Calculation Guidance. If fuel data are available, then the fuel-based method should be used.

Scope 3 emissions

Table 97: Examples of monetary scope 3 emission factors

Category	S3E	ADEME	Category	S3E	ADEME
Agriculture	2 500	2 300	Air transport	1 970	1 190
Construction	810	360	Education	310	120
Financial intermediation	140	110	Health and Social Work	300	500
Hotels and restaurants	560	320	Rubber and plastics	1 270	800
Telecommunications	300	170	Textiles	1 100	600

Source: Scope 3 Evaluator (S3E), `https://quantis-suite.com/Scope-3-Evaluator` & ADEME, `https://bilans-ges.ademe.fr`.

Carbon emissions of investment portfolios

Two methods for measuring the carbon footprint of an investment portfolio:

1. Financed emissions approach
2. Ownership approach

Carbon emissions of investment portfolios

Financed emissions approach

- The investor calculates the carbon emissions that are financed across both equity and debt
- EVIC is used to estimate the value of the enterprise. It is "*the sum of the market capitalization of ordinary and preferred shares at fiscal year end and the book values of total debt and minorities interests*" (TEG, 2019)
- Let W be the wealth invested in the company, the financed emissions are equal to:

$$\mathcal{CE}(W) = \frac{W}{\mathrm{EVIC}} \cdot \mathcal{CE}$$

- In the case of a portfolio $(W_1, \ldots, W_n)$ where W_i is the wealth invested in company i, we have:

$$\mathcal{CE}(W) = \sum_{i=1}^{n} \mathcal{CE}_i(W_i) = \sum_{i=1}^{n} \frac{W_i}{\mathrm{EVIC}_i} \cdot \mathcal{CE}_i$$

- $\mathcal{CE}(W)$ is expressed in $\mathrm{tCO_2e}$

Carbon emissions of investment portfolios

Ownership approach

- We break down the carbon emissions between the stockholders of the company
- We have:

$$\mathcal{CE}(W) = \sum_{i=1}^{n} \frac{W_i}{\mathrm{MV}_i} \cdot \mathcal{CE}_i = \sum_{i=1}^{n} \varpi_i \cdot \mathcal{CE}_i$$

  where:
  - MV_i is the market value of company i
  - ϖ_i is the ownership ratio of the investor

Carbon emissions of investment portfolios

Ownership approach

- Let $W = \sum_{i=1}^{n} W_i$ be the portfolio value
- The portfolio weight of asset i is given by:

$$w_i = \frac{W_i}{W}$$

- We deduce that:

$$\varpi_i = \frac{W_i}{\mathrm{MV}_i} = \frac{w_i \cdot W}{\mathrm{MV}_i}$$

- It follows that:

$$\mathcal{CE}(W) = \sum_{i=1}^{n} \frac{w_i \cdot W}{\mathrm{MV}_i} CE_i = W \left(\sum_{i=1}^{n} w_i \cdot \frac{\mathcal{CE}_i}{\mathrm{MV}_i} \right) = W \left(\sum_{i=1}^{n} w_i \cdot \mathcal{CI}_i^{\mathrm{MV}} \right)$$

where $\mathcal{CI}_i^{\mathrm{MV}}$ is the market value-based carbon intensity:

$$\mathcal{CI}_i^{\mathrm{MV}} = \frac{\mathcal{CE}_i}{\mathrm{MV}_i}$$

- $\mathcal{CE}(W)$ is generally computed with $W = \$1$ mn and is expressed in $\mathrm{tCO_2e}$ (per \$ mn invested)

Carbon emissions of investment portfolios

Ownership approach

Remark

The ownership approach is valid only for equity portfolios. To compute the market value (or the total market capitalization), we use the following approximation:

$$\mathrm{MV} = \frac{\mathrm{MC}}{\mathcal{FP}}$$

where MC *and* $\mathcal{FP}$ *are the free float market capitalisation and percentage of the company.*

Carbon emissions of investment portfolios

Example #5

We consider a $100 mn investment portfolio with the following composition: $63.1 mn in company A, $16.9 mn in company B and $20.0 mn in company C. The data are the following:

Issuer	Market capitalization (in $ bn)		
	31/12/2021	31/12/2022	31/01/2023
A	12.886	10.356	10.625
B	7.005	6.735	6.823
C	3.271	3.287	3.474

Issuer	Debt (in $ bn)	$\mathcal{FP}$ (in %)	$\mathcal{SC}_{1-2}$ (in $\mathrm{ktCO_2e}$)
A	1.112	99.8	756.144
B	0.000	39.3	23.112
C	0.458	96.7	454.460

Carbon emissions of investment portfolios

- As of 31 January 2023, the EVIC value for company A is equal to:

$$\mathrm{EVIC}_A = \frac{10\,356}{0.998} + 1\,112 = \$11489 \text{ mn}$$

- We deduce that the financed emissions are equal to:

$$\mathcal{CE}_A\,(\$63.1 \text{ mn}) = \frac{63.1}{11\,489} \times 756.144 = 4.153 \text{ ktCO}_2\text{e}$$

Carbon emissions of investment portfolios

- If we assume that the investor has no bond in the portfolio, we can use the ownership approach:

$$\varpi_A = \frac{63.1}{(10\,625/0.998)} = 59.2695 \text{ bps}$$

- The carbon emissions of the investment in company A is then equal to:

$$\mathcal{CE}_A\,(\$63.1 \text{ mn}) = 59.2695 \times 10^{-4} \times 756.144 = 4.482 \text{ ktCO}_2\text{e}$$

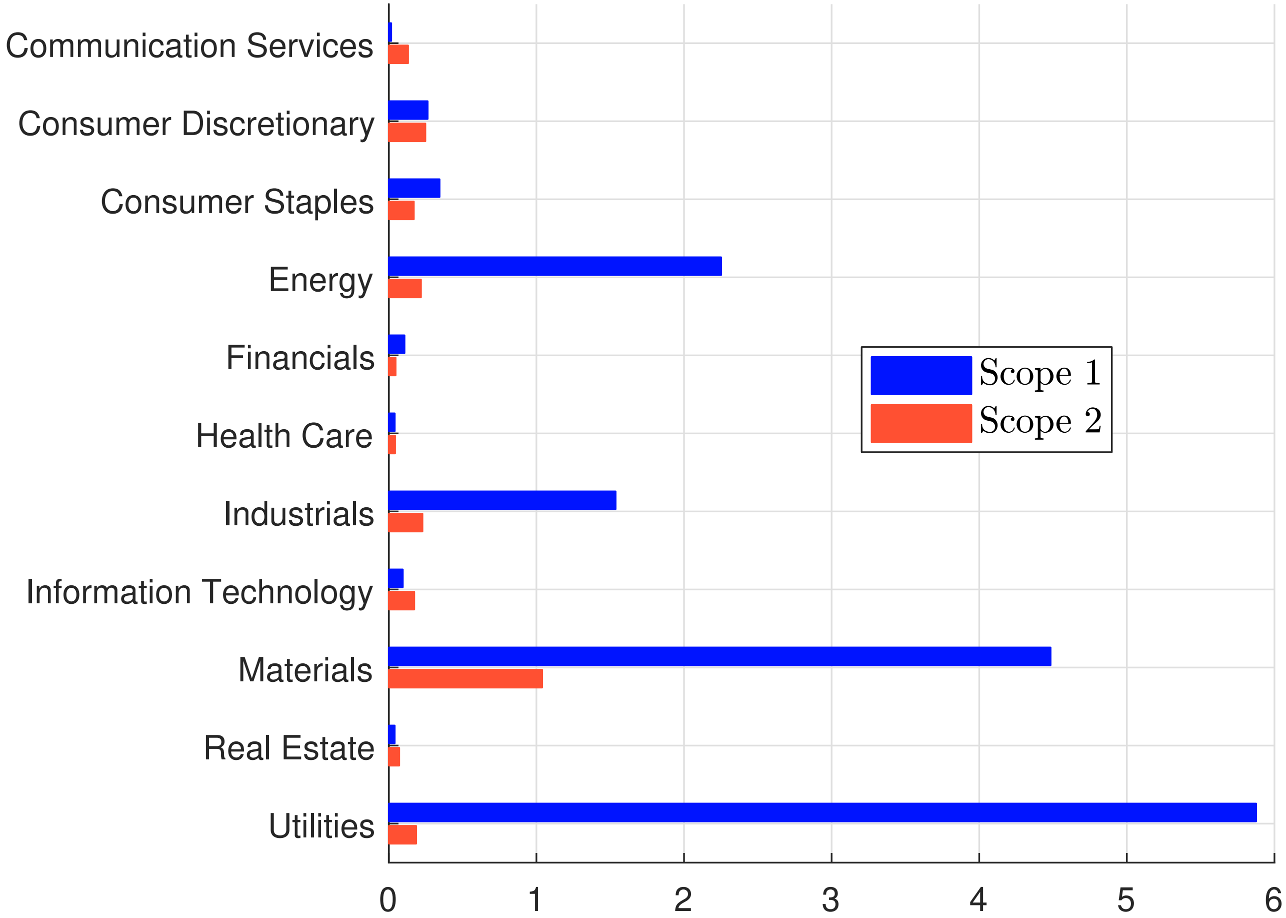
Carbon emissions of investment portfolios

Finally, we obtain the following results:

	Financed emissions	Carbon emissions
Company A	4.153	4.482
Company B	0.023	0.022
Company C	2.356	2.530
Portfolio	6.532	7.034

Statistics

Figure 185: 2019 carbon emissions per GICS sector in $\mathrm{GtCO_2e}$ (scopes 1 & 2)



Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

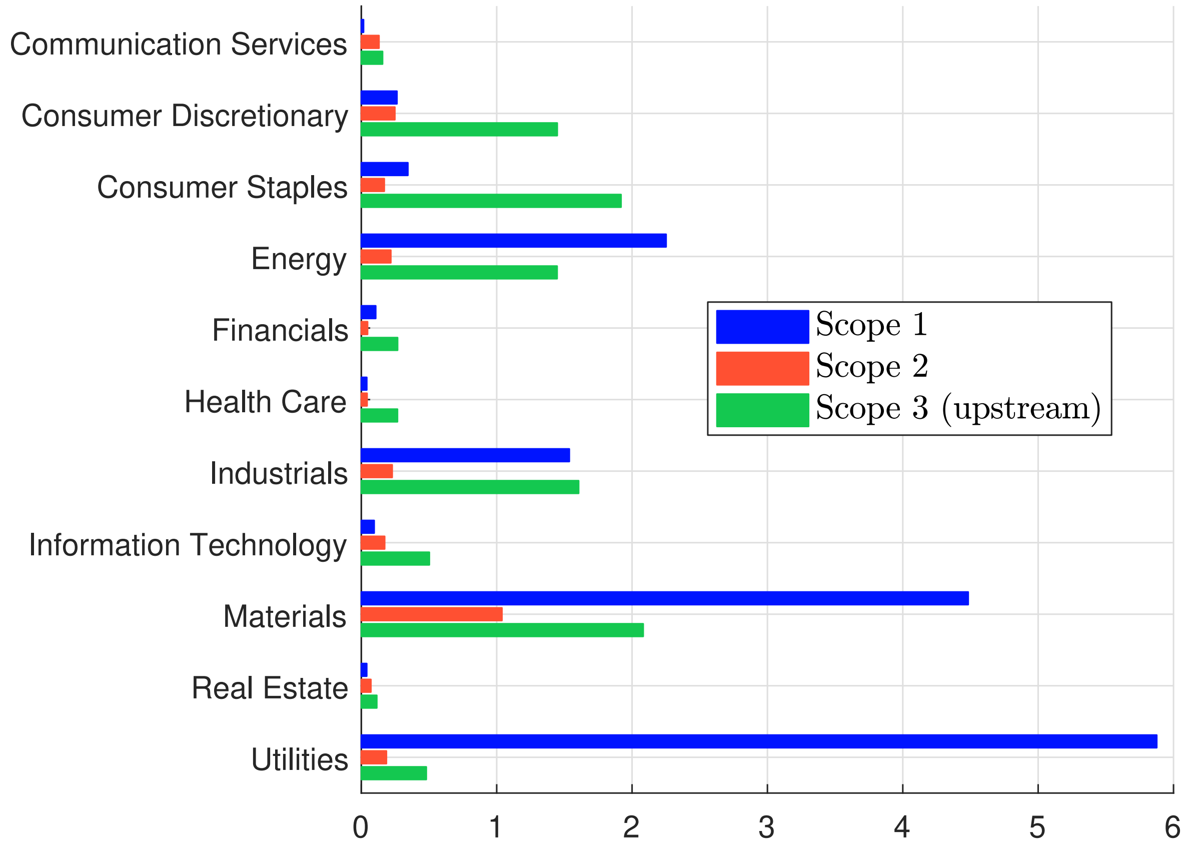
Table 98: Breakdown (in %) of carbon emissions in 2019

Sector	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_3^{\mathrm{up}}$	$\mathcal{SC}_3^{\mathrm{down}}$	$\mathcal{SC}_3$	$\mathcal{SC}_{1-3}$
Communication Services	0.1	5.1	0.8	1.5	0.2	0.4	0.5
Consumer Discretionary	1.7	9.7	2.9	14.1	10.2	10.8	9.1
Consumer Staples	2.3	6.7	2.9	18.6	1.6	4.4	4.1
Energy	15.0	8.5	14.0	14.1	40.1	36.0	31.2
Financials	0.7	1.8	0.9	2.6	1.8	2.0	1.7
Health Care	0.3	1.7	0.5	2.6	0.2	0.6	0.6
Industrials	10.2	8.9	10.0	15.6	24.2	22.8	20.0
Information Technology	0.6	6.8	1.5	4.9	2.3	2.7	2.5
Materials	29.8	40.7	31.4	20.2	13.5	14.6	18.2
Real Estate	0.3	2.8	0.6	1.1	1.0	1.0	0.9
Utilities	39.0	7.3	34.4	4.7	4.8	4.8	11.2
Total (in $\mathrm{GtCO_2e}$)	15.1	2.6	17.6	10.3	53.7	64.0	81.6

Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

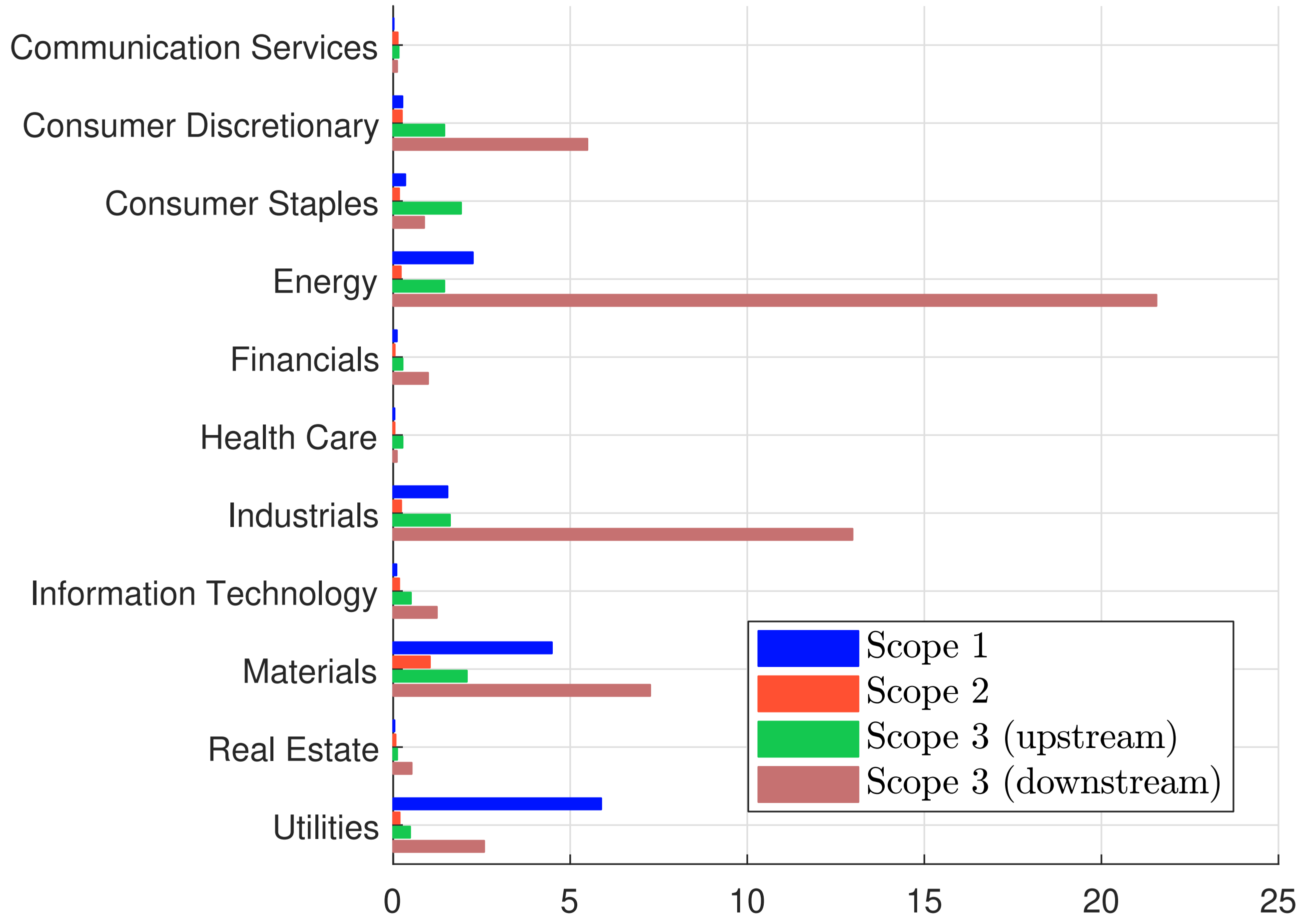
Figure 186: 2019 carbon emissions per GICS sector in $GtCO_2e$ (scopes 1, 2 & 3 upstream)



Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

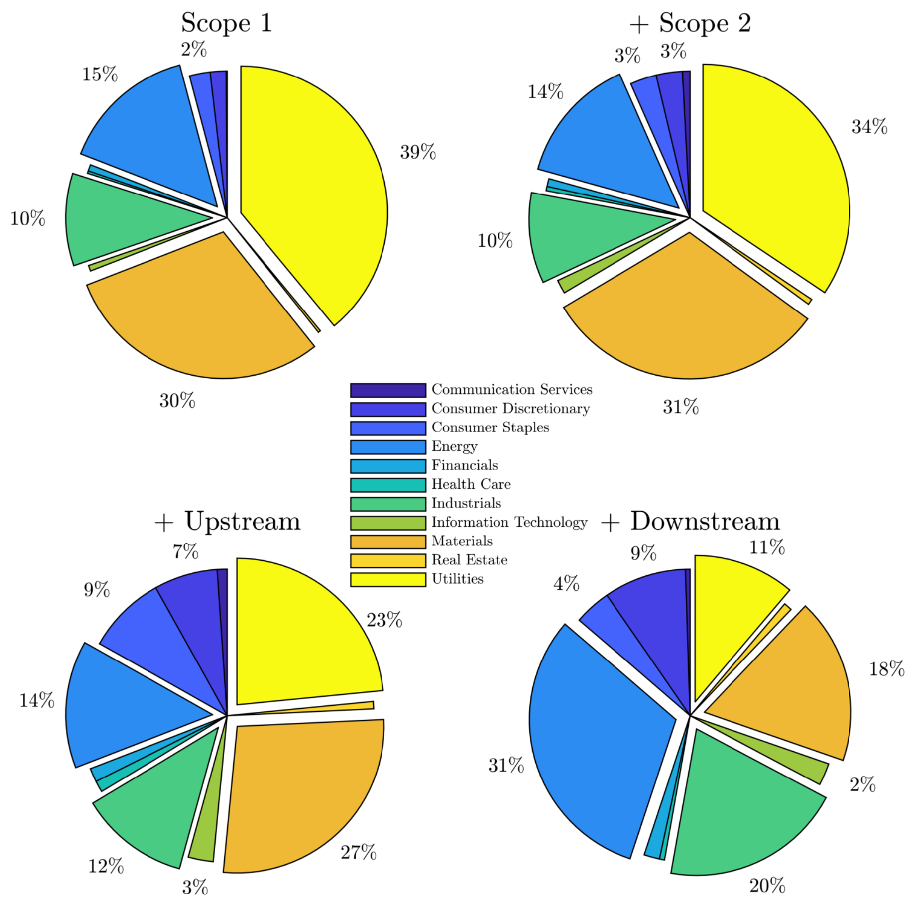
Figure 187: 2019 carbon emissions per GICS sector in $\mathrm{GtCO_2e}$ (scopes 1, 2 & 3)



Source: Trucost (2022) & Barahhou *et al.* (2022)

Statistics

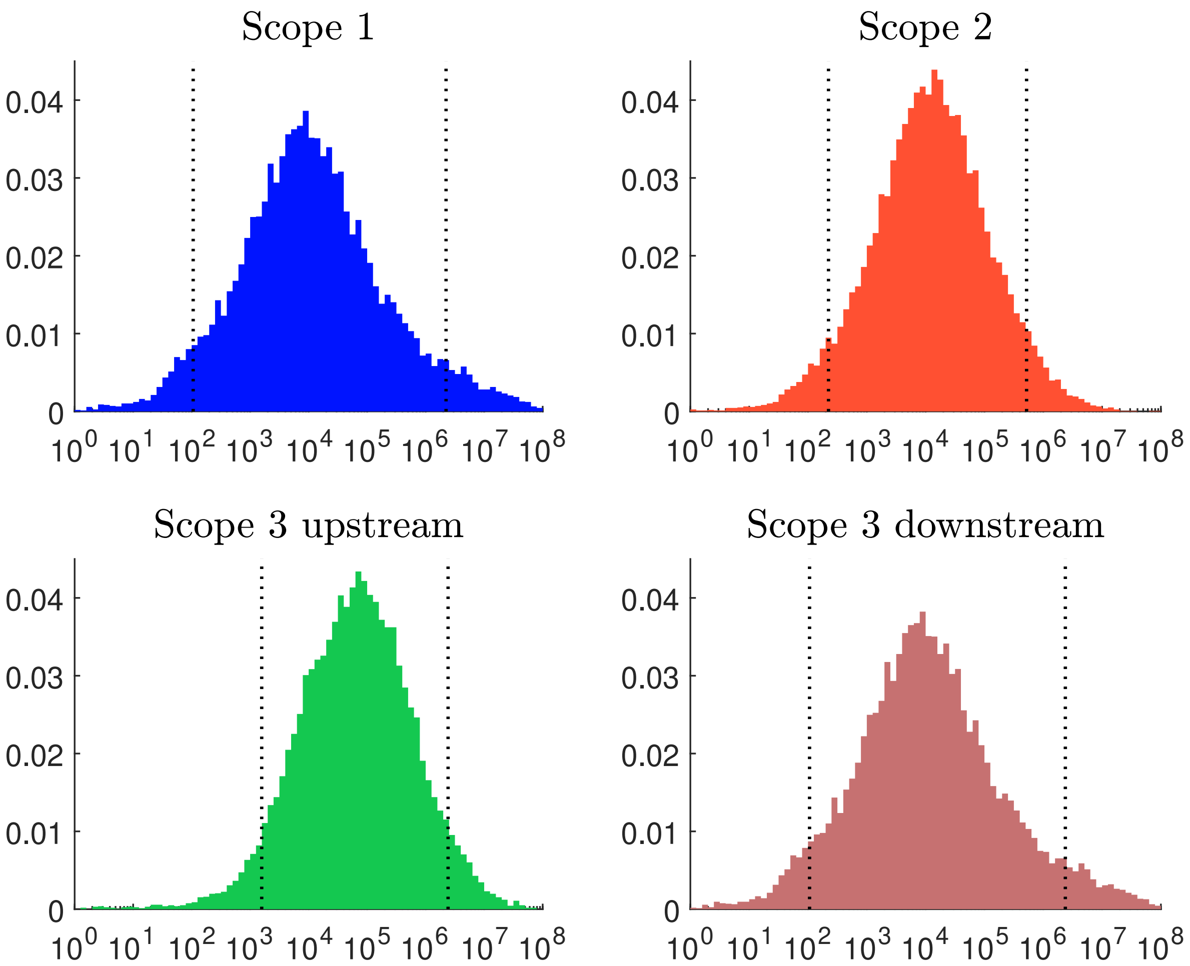
Figure 188: Sector contribution in %



Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

Figure 189: Histogram of 2019 carbon emissions (logarithmic scale, tCO_2e)



Source: Trucost (2022) & Barahhou *et al.* (2022).

Negative and avoided emissions

Carbon intensity

- Carbon emissions = absolute carbon footprint in an absolute value
- Carbon intensity = relative carbon footprint

⇒ **we normalize the carbon emissions by a size or activity unit**

Carbon intensity

We can measure the carbon footprint of:

- countries by $\mathrm{tCO_2e}$ per capita
- watching television by $\mathrm{CO_2e}$ emissions per viewer-hour
- washing machines by $\mathrm{kgCO_2e}$ per wash
- cars by $\mathrm{kgCO_2e}$ per kilometer driven
- companies by $\mathrm{ktCO_2e}$ per $1 mn revenue
- etc.

Physical intensity ratios

Product carbon footprint (PCF)

- The product carbon footprint measures the relative carbon emissions of a product throughout its life cycle
- Life cycle assessment (LCA), distinguishes two methods:
  1. **Cradle-to-gate** refers to the carbon footprint of a product from the moment it is produced (including the extraction of raw materials) to the moment it enters the store
  2. **Cradle-to-grave** covers the entire life cycle of a product, including the use-phase and recycling

Physical intensity ratios

Table 99: Examples of product carbon footprint (in $\mathrm{kgCO_2e}$ per unit)

Product	Category	Cradle-to-gate	Cradle-to-grave
Screen	21.5 inches	222	236
	23.8 inches	248	265
Computer	Laptop	156	169
	Desktop	169	189
	High performance	295	394
Smartphone	Classical	16	16
	5 inches	33	32
Oven	Built-in electric	187	319
	Professional (combi steamer)	734	12 676
Washing machine	Capacity 5kg	248	468
	Capacity 7kg	275	539
Shirt	Coton	10	13
	Viscose	9	12
Balloon	Football	3.4	5.1
	Basket-ball	3.6	5.9

Source: Lhotellier *et al* (2018, Annex 4, pages 212-215)

Physical intensity ratios

Corporate carbon footprint (CCF)

- Extension of the PCF to companies
- The CCF of a cement manufacturer is measured by the amount of GHG emissions per tonne of cement
- The CCF of airlines is measured by the amount of GHG emissions per RPK (revenue passenger kilometers, which is calculated by multiplying the number of paying passengers by the distance traveled)

Sector	Unit	Description
Transport sector (aviation)	CO_2e/RPK	Revenue passenger kilometers)
Transport sector (shipping)	CO_2e/RTK	Revenue tonne kilometers
Industry (cement)	CO_2e/t cement	Tonne of cement
Industry (steel)	CO_2e/t steel	Tonne of steel
Electricity	CO_2e/MWh	Megawatt hour
Buildings	CO_2e/SQM	Square meter

Monetary intensity ratios

Problem

- How to aggregate carbon footprint?
- Portfolio managers use monetary intensity ratios, which are defined as:

$$\mathcal{CI} = \frac{\mathcal{CE}}{Y}$$

where $\mathcal{CE}$ is the company's carbon emissions and Y is a monetary variable measuring its activity

Monetary intensity ratios

For instance, we can use revenues, sales, etc. to normalize carbon emissions:

Revenue	Sales	EVIC	MV
$\mathcal{CI}^{\text{Revenue}} = \dfrac{\mathcal{CE}}{\text{Revenue}}$	$\mathcal{CI}^{\text{Sales}} = \dfrac{\mathcal{CE}}{\text{Sales}}$	$\mathcal{CI}^{\text{EVIC}} = \dfrac{\mathcal{CE}}{\text{EVIC}}$	$\mathcal{CI}^{\text{MV}} = \dfrac{\mathcal{CE}}{\text{MV}}$

Remark

The previous carbon emission metrics based on EVIC and market value can be viewed as carbon intensity metrics

Additivity property of $\mathcal{CI}$

- If we consider the EVIC-based approach, the carbon intensity of the portfolio is given by:

$$\begin{aligned}
\mathcal{CI}^{\mathrm{EVIC}}(w) &= \frac{\mathcal{CE}^{\mathrm{EVIC}}(W)}{W} \\
&= \frac{1}{W}\sum_{i=1}^{n}\frac{W_i}{\mathrm{EVIC}_i}\cdot\mathcal{CE}_i \\
&= \sum_{i=1}^{n}\frac{W_i}{W}\cdot\frac{\mathcal{CE}_i}{\mathrm{EVIC}_i} \\
&= \sum_{i=1}^{n} w_i\cdot\mathcal{CI}_i^{\mathrm{EVIC}}
\end{aligned}$$

  where $w=(w_1,\ldots,w_n)$ is the vector of portfolio weights
- In a similar way, we obtain:

$$\mathcal{CI}^{\mathrm{MV}}(w)=\sum_{i=1}^{n} w_i\cdot\mathcal{CI}_i^{\mathrm{MV}}$$

Non-additivity property of $\mathcal{CI}$

- We consider the revenue-based carbon intensity (also called the economic carbon intensity)
- The carbon intensity of the portfolio is:

$$\mathcal{CI}^{\text{Revenue}}(w) = \frac{\mathcal{CE}(w)}{Y(w)}$$

where:

- $\mathcal{CE}(w)$ measures the carbon emissions of the portfolio:

$$\mathcal{CE}(w) = \sum_{i=1}^{n} W_i \cdot \frac{\mathcal{CE}_i}{\mathrm{MV}_i} = W \sum_{i=1}^{n} \frac{w_i}{\mathrm{MV}_i} \cdot \mathcal{CE}_i$$

- $Y(w)$ is the total revenue of the portfolio:

$$Y(w) = \sum_{i=1}^{n} W_i \cdot \frac{Y_i}{\mathrm{MV}_i} = W \sum_{i=1}^{n} \frac{w_i}{\mathrm{MV}_i} \cdot Y_i$$

Non-additivity property of $\mathcal{CI}$

- We deduce that:

$$\mathcal{CI}^{\mathrm{Revenue}}(w) = \frac{\sum_{i=1}^{n} \dfrac{w_i}{\mathrm{MV}_i} \cdot \mathcal{CE}_i}{\sum_{i=1}^{n} \dfrac{w_i}{\mathrm{MV}_i} \cdot Y_i} = \sum_{i=1}^{n} w_i \cdot \omega_i \cdot \mathcal{CI}_i^{\mathrm{Revenue}}$$

  where ω_i is the ratio between the revenue per market value of company i and the weighted average revenue per market value of the portfolio:

$$\omega_i = \frac{\dfrac{Y_i}{\mathrm{MV}_i}}{\sum_{k=1}^{n} w_k \cdot \dfrac{Y_k}{\mathrm{MV}_k}}$$

- We conclude that:

$$\mathcal{CI}^{\mathrm{Revenue}}(w) \neq \sum_{i=1}^{n} w_i \cdot \mathcal{CI}_i^{\mathrm{Revenue}}$$

WACI

In order to avoid the previous problem, we generally use the weighted average carbon intensity (WACI) of the portfolio:

$$\mathcal{CI}^{\mathrm{Revenue}}(w) = \sum_{i=1}^{n} w_i \cdot \mathcal{CI}_i^{\mathrm{Revenue}}$$

This method is the standard approach in portfolio management

Additivity property of $\mathcal{CI}$

Carbon intensity is always additive when we consider a given issuer:

$$\begin{aligned}\mathcal{CI}_i\left(\mathcal{SC}_{1-3}\right) &= \frac{\mathcal{CE}_i\left(\mathcal{SC}_1\right)+\mathcal{CE}_i\left(\mathcal{SC}_2\right)+\mathcal{CE}_i\left(\mathcal{SC}_3\right)}{Y_i} \\ &= \mathcal{CI}_i\left(\mathcal{SC}_1\right)+\mathcal{CI}_i\left(\mathcal{SC}_2\right)+\mathcal{CI}_i\left(\mathcal{SC}_3\right)\end{aligned}$$

Illustration

Example #6

We assume that $\mathcal{CE}_1 = 5 \times 10^6$ $\mathrm{CO_2e}$, $Y_1 = \$0.2 \times 10^6$, $\mathrm{MV}_1 = \$10 \times 10^6$, $\mathcal{CE}_2 = 50 \times 10^6$ $\mathrm{CO_2e}$, $Y_2 = \$4 \times 10^6$ and $\mathrm{MV}_2 = \$10 \times 10^6$. We invest $W = \$10$ mn.

Illustration

- We deduce that:

$$\mathcal{CI}_1 = \frac{5 \times 10^6}{0.2 \times 10^6} = 25.0 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

and

$$\mathcal{CI}_2 = 12.5 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

- We have:

$$\left\{ \begin{array}{l} \mathcal{CE}(w) = W \left(w_1 \dfrac{\mathcal{CE}_1}{\text{MV}_1} + w_2 \dfrac{\mathcal{CE}_2}{\text{MV}_2} \right) \\ Y(w) = W \left(w_1 \dfrac{Y_1}{\text{MV}_1} + w_2 \dfrac{Y_2}{\text{MV}_2} \right) \\ \mathcal{CI}(w) = w_1 \mathcal{CI}_1 + w_2 \mathcal{CI}_2 \end{array} \right.$$

Illustration

- We obtain the following results:

w_1	w_2	$\mathcal{CE}(w)$ $(\times 10^6\ CO_2e)$	$Y(w)$ $(\times \$10^6)$	$\dfrac{\mathcal{CE}(w)}{Y(w)}$	$\mathcal{CI}(w)$
0%	100%	50.00	4.00	12.50	12.50
10%	90%	45.50	3.62	12.57	13.75
20%	80%	41.00	3.24	12.65	15.00
30%	70%	36.50	2.86	12.76	16.25
50%	50%	27.50	2.10	13.10	18.75
70%	30%	18.50	1.34	13.81	21.25
80%	20%	14.00	0.96	14.58	22.50
90%	10%	9.50	0.58	16.38	23.75
100%	0%	5.00	0.20	25.00	25.00

- We notice that the weighted average carbon intensity can be very different than the economic carbon intensity

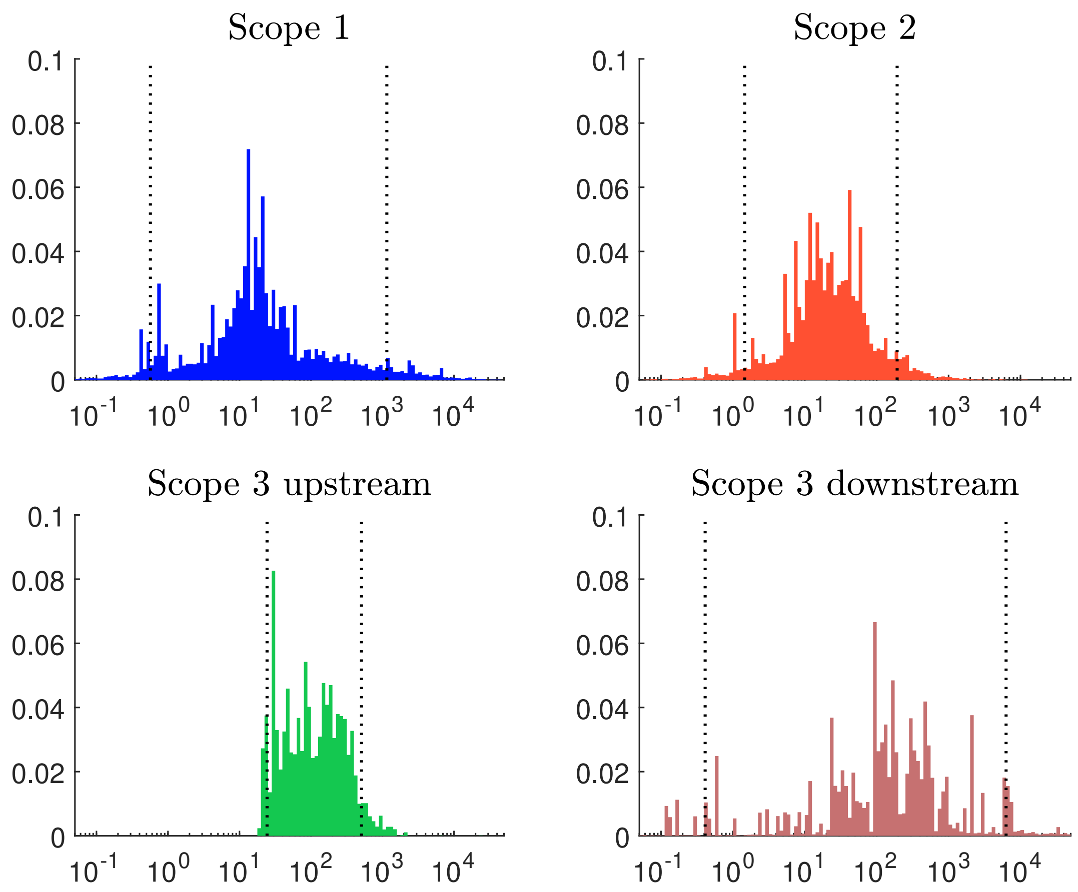
The case of sovereign issuers

Remark

For sovereign issuers, the economic carbon intensity is measured in mega-tonnes of CO_2e *per million dollars of GDP while the physical carbon intensity unit is* tCO_2e *per capita*

Statistics

Figure 190: Histogram of 2019 carbon intensities (logarithmic scale, tCO_2e/\$ mn)



Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

Table 100: Examples of 2019 carbon emissions and intensities

Company	Carbon emissions (in tCO_2e)				Revenue	Intensity (in $tCO_2e/\$$ mn)			
	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{up}$	$\mathcal{SC}_3^{down}$	(in $ mn)	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{up}$	$\mathcal{SC}_3^{down}$
Airbus	576 705	386 674	12 284 183	23 661 432	78 899	7.3	4.9	155.7	299.9
Allianz	46 745	224 315	3 449 234	3 904 000	135 279	0.3	1.7	25.5	28.9
Alphabet	111 283	5 118 152	7 142 566		161 857	0.7	31.6	44.1	
Amazon	5 760 000	5 500 000	20 054 722	10 438 551	280 522	20.5	19.6	71.5	37.2
Apple	50 549	862 127	27 624 282	5 470 771	260 174	0.2	3.3	106.2	21.0
BNP Paribas	64 829	280 789	1 923 307	1 884	78 244	0.8	3.6	24.6	0.0
Boeing	611 001	871 000	9 878 431	22 959 719	76 559	8.0	11.4	129.0	299.9
BP	49 199 999	5 200 000	103 840 194	582 639 687	276 850	177.7	18.8	375.1	2 104.5
Caterpillar	905 000	926 000	15 197 607	401 993 744	53 800	16.8	17.2	282.5	7 472.0
Danone	722 122	944 877	28 969 780	4 464 773	28 308	25.5	33.4	1 023.4	157.7
Enel	69 981 891	5 365 386	8 726 973	53 774 821	86 610	808.0	61.9	100.8	620.9
Exxon	111 000 000	9 000 000	107 282 831	594 131 943	255 583	434.3	35.2	419.8	2 324.6
JPMorgan Chase	81 655	692 299	3 101 582	15 448 469	115 627	0.7	6.0	26.8	133.6
Juventus	6 665	15 739	35 842	77 114	709	9.4	22.2	50.6	108.8
LVMH	67 613	262 609	11 853 749	942 520	60 083	1.1	4.4	197.3	15.7
Microsoft	113 414	3 556 553	5 977 488	4 003 770	125 843	0.9	28.3	47.5	31.8
Nestle	3 291 303	3 206 495	61 262 078	33 900 606	93 153	35.3	34.4	657.6	363.9
Netflix	38 481	145 443	1 900 283	2 192 255	20 156	1.9	7.2	94.3	108.8
NVIDIA	2 767	65 048	2 756 353	1 184 981	11 716	0.2	5.6	235.3	101.1
PepsiCo	3 552 415	1 556 523	32 598 029	14 229 956	67 161	52.9	23.2	485.4	211.9
Pfizer	734 638	762 840	4 667 225	133 468	51 750	14.2	14.7	90.2	2.6
Roche	288 157	329 541	5 812 735	347 437	64 154	4.5	5.1	90.6	5.4
Samsung Electronics	5 067 000	10 998 000	33 554 245	60 978 947	197 733	25.6	55.6	169.7	308.4
TotalEnergies	40 909 135	3 596 127	49 817 293	456 993 576	200 316	204.2	18.0	248.7	2 280.0
Toyota	2 522 987	5 227 844	66 148 020	330 714 268	272 608	9.3	19.2	242.6	1 213.2
Volkswagen	4 494 066	5 973 894	65 335 372	354 913 446	282 817	15.9	21.1	231.0	1 254.9
Walmart	6 101 641	13 057 352	40 651 079	32 346 229	514 405	11.9	25.4	79.0	62.9

Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

Table 101: Examples of 2019 carbon intensities

Company	Intensity (in $tCO_2e/\$ mn$)			
	$\mathcal{SC}_1$	$\mathcal{SC}_2$	$\mathcal{SC}_3^{up}$	$\mathcal{SC}_3^{down}$
Amazon	20.5	19.6	71.5	37.2
Apple	**0.2**	**3.3**	**106.2**	21.0
BNP Paribas	0.8	3.6	24.6	0.0
BP	177.7	18.8	375.1	2 104.5
Caterpillar	16.8	17.2	282.5	7 472.0
Danone	25.5	33.4	**1 023.4**	**157.7**
Exxon	434.3	35.2	419.8	2 324.6
JPMorgan Chase	0.7	6.0	26.8	133.6
LVMH	1.1	4.4	197.3	15.7
Microsoft	0.9	28.3	47.5	31.8
Nestle	35.3	34.4	**657.6**	**363.9**
Pfizer	14.2	14.7	90.2	2.6
Samsung Electronics	**25.6**	**55.6**	169.7	308.4
Volkswagen	15.9	21.1	231.0	1 254.9
Walmart	11.9	25.4	79.0	62.9

Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

Table 102: Carbon intensity in $\text{tCO}_2\text{e}/\$ \text{ mn}$ per GICS sector and sector contribution in % (MSCI World, June 2022)

Sector	b_i (in %)	Carbon intensity				Risk contribution			
		$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\text{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	7.58	2	28	134	172	0.14	1.31	3.30	1.31
Consumer Discretionary	10.56	23	65	206	590	1.87	4.17	6.92	6.21
Consumer Staples	7.80	28	55	401	929	1.68	2.66	10.16	7.38
Energy	4.99	632	698	1 006	6 823	24.49	21.53	16.33	34.37
Financials	13.56	13	19	52	244	1.33	1.58	2.28	3.34
Health Care	14.15	10	22	120	146	1.12	1.92	5.54	2.12
Industrials	9.90	111	130	298	1 662	8.38	7.83	9.43	16.38
Information Technology	21.08	7	23	112	239	1.13	3.03	7.57	5.06
Materials	4.28	478	702	1 113	2 957	15.89	18.57	15.48	12.93
Real Estate	2.90	22	101	167	571	0.48	1.81	1.57	1.65
Utilities	3.21	1 744	1 794	2 053	2 840	43.47	35.59	21.41	9.24
MSCI World		130	163	310	992				
MSCI World EW		168	211	391	1 155				

Source: Trucost (2022) & Barahhou *et al.* (2022).

Statistics

- Let $b = (b_1, \ldots, b_n)$ be the weights of the assets that belong to a benchmark
- Its weighted average carbon intensity is given by:

$$\mathcal{CI}(b) = \sum_{i=1}^{n} b_i \cdot \mathcal{CI}_i$$

  where $\mathcal{CI}_i$ is the carbon intensity of asset i

- If we focus on the carbon intensity for a given sector, we use the following formula:

$$\mathcal{CI}(\mathcal{S}ector_j) = \frac{\sum_{i \in \mathcal{S}ector_j} b_i \cdot \mathcal{CI}_i}{\sum_{i \in \mathcal{S}ector_j} b_i}$$

Carbon budget

Definition

- The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period $[t_0, t]$
- From a mathematical point of view, it corresponds to the signed area of the region bounded by the function $\mathcal{CE}(t)$:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} \mathcal{CE}(s) \, \mathrm{d}s$$

Carbon budget

Example #7

Below, we report the historical data of carbon emissions from 2010 to 2020. Moreover, the company has announced his carbon targets for the years until 2050

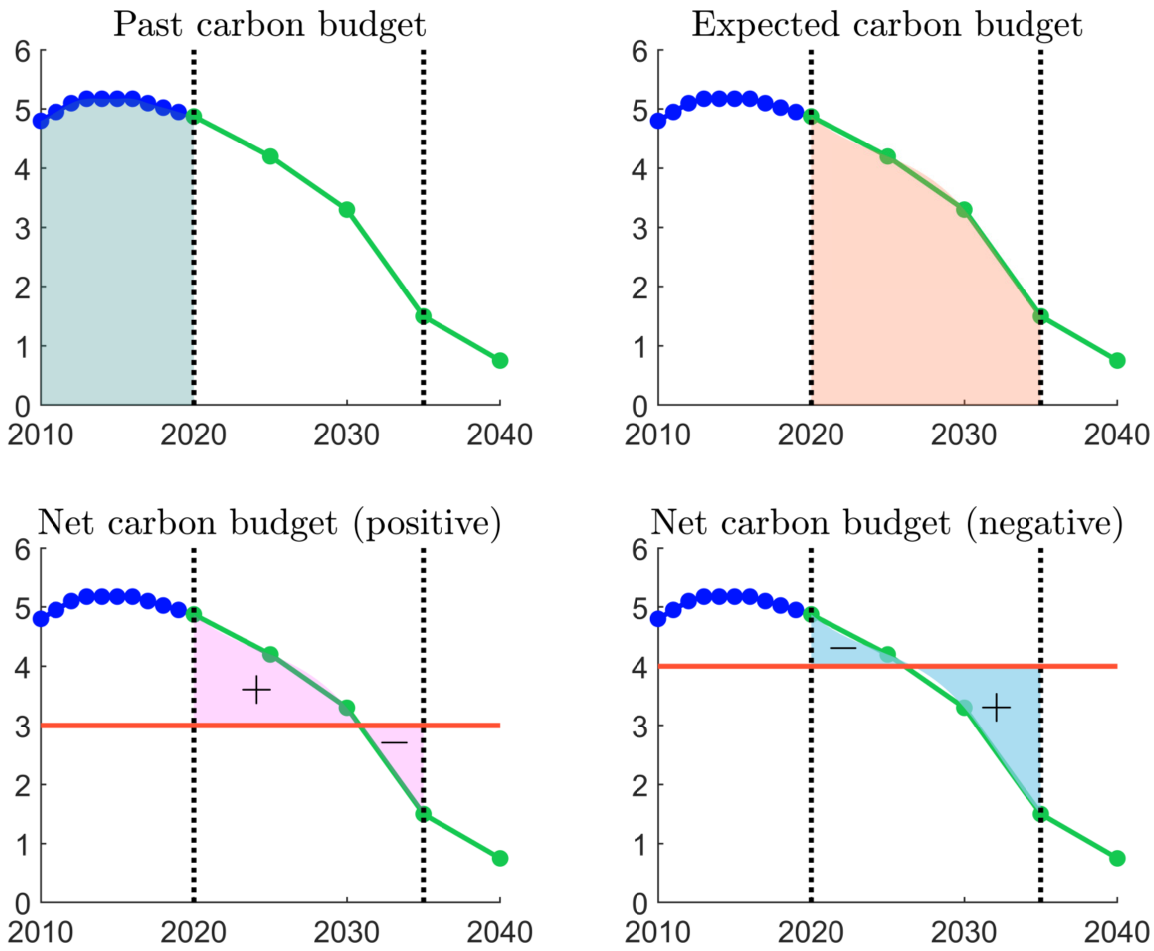
Table 103: Carbon emissions in MtCO_2e

t	2010	2011	2012	2013	2014	2015	2016	2017
$\mathcal{CE}(t)$	4.800	4.950	5.100	5.175	5.175	5.175	5.175	5.100
t	2018	2019	2020	2025*	2030*	2035*	2040*	2050*
$\mathcal{CE}(t)$	5.025	4.950	4.875	4.200	3.300	1.500	0.750	0.150

The asterisk * indicates that the company has announced a carbon target for this year

Carbon budget

Figure 191: Past, expected and net carbon budgets (Example #7)



Computation of the carbon budget

Numerical solution

- We consider the equally-spaced partition $\{[t_0, t_0 + \Delta t], \ldots, [t - \Delta t, t]\}$ of $[t_0, t]$
- Let $m = \dfrac{t - t_0}{\Delta t}$ be the number of intervals
- We set $\mathcal{CE}_k = \mathcal{CE}(t_0 + k\Delta t)$
- The right Riemann approximation is:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} \mathcal{CE}(s)\,\mathrm{d}s \approx \sum_{k=1}^{m} \mathcal{CE}(t_0 + k\Delta t)\,\Delta t = \Delta t \sum_{k=1}^{m} \mathcal{CE}_k$$

- The left Riemann sum is:

$$\mathcal{CB}(t_0, t) \approx \Delta t \sum_{k=0}^{m-1} \mathcal{CE}_k$$

- The midpoint rule is:

$$\mathcal{CB}(t_0, t) \approx \Delta t \sum_{k=1}^{m} \mathcal{CE}\left(t_0 + \frac{k}{2}\Delta t\right)$$

Computation of the carbon budget

Analytical solution: the case of a constant reduction rate

- If we use a constant linear reduction rate $\mathcal{R}(t_0, t) = \mathcal{R}(t - t_0)$, we obtain the following analytical expression:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} \left(\mathcal{CE}(t_0) - \mathcal{R}(s - t_0)\right) \mathrm{d}s = (t - t_0)\,\mathcal{CE}(t_0) - \frac{(t - t_0)^2}{2}\mathcal{R}$$

- In the case of a constant compound reduction rate:

$$\mathcal{CE}(t) = (1 - \mathcal{R})^{(t - t_0)}\,\mathcal{CE}(t_0)$$

we obtain:

$$\mathcal{CB}(t_0, t) = \mathcal{CE}(t_0) \int_{t_0}^{t} (1 - \mathcal{R})^{(s - t_0)}\,\mathrm{d}s = \frac{(1 - \mathcal{R})^{(t - t_0)} - 1}{\ln(1 - \mathcal{R})}\mathcal{CE}(t_0)$$

Computation of the carbon budget

Analytical solution: the case of a constant reduction rate

- If we assume that $\mathcal{CE}(t) = e^{-\mathcal{R}(t-t_0)}\mathcal{CE}(t_0)$, we have:

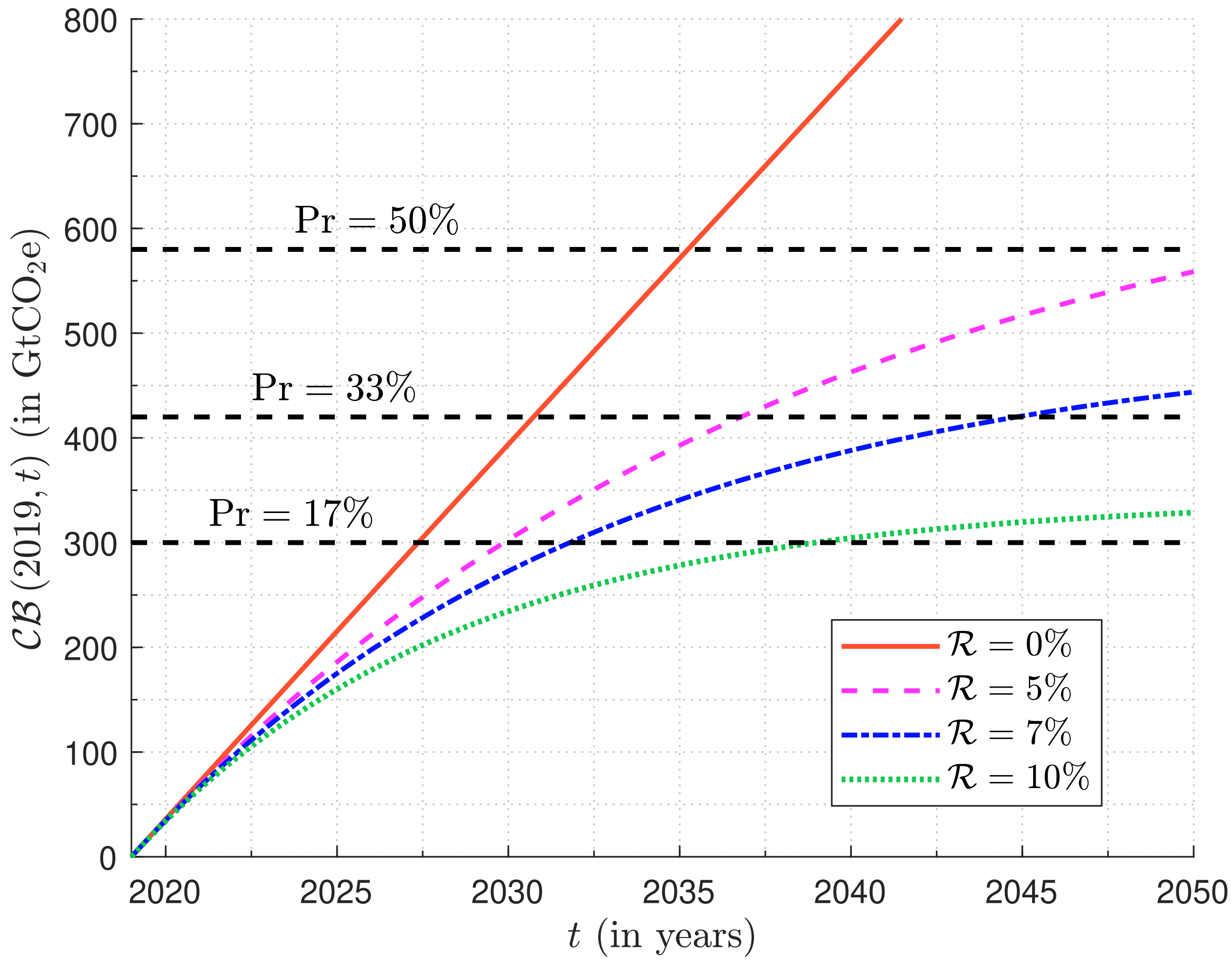
$$\mathcal{CB}(t_0, t) = \mathcal{CE}(t_0)\left[-\frac{e^{-\mathcal{R}(s-t_0)}}{\mathcal{R}}\right]_{t_0}^{t} = \mathcal{CE}(t_0)\frac{\left(1 - e^{-\mathcal{R}(t-t_0)}\right)}{\mathcal{R}}$$

Remark

If the carbon emissions increase at a positive growth rate g, we set $\mathcal{R} = -g$.

Carbon budget and global warming

Figure 192: Probability to reach 1.5°C



IPCC (2018)

The remaining carbon budget $\mathcal{CB}(2019, t)$ is:

- 580 $GtCO_2e$ for a 50% probability of limiting warming to 1.5°C
- 420 $GtCO_2e$ for a 66% probability
- 300 $GtCO_2e$ for a 83% probability

Computation of the carbon budget

Analytical solution: the case of a Linear function

- If we assume that $\mathcal{CE}(t) = \beta_0 + \beta_1 t$, we deduce that:

$$\begin{aligned} \mathcal{CB}(t_0, t) &= \int_{t_0}^{t} (\beta_0 + \beta_1 s) \, \mathrm{d}s \\ &= \left[\beta_0 s + \frac{1}{2} \beta_1 s^2 \right]_{t_0}^{t} \\ &= \beta_0 (t - t_0) + \frac{1}{2} \beta_1 \left(t^2 - t_0^2 \right) \end{aligned}$$

- We can extend this formula to a piecewise linear function:

$$\mathcal{CB}(t_0, t) = \ldots$$

Net zero emissions scenario (IEA)

Table 104: IEA NZE scenario (in $\mathrm{GtCO_2e}$)

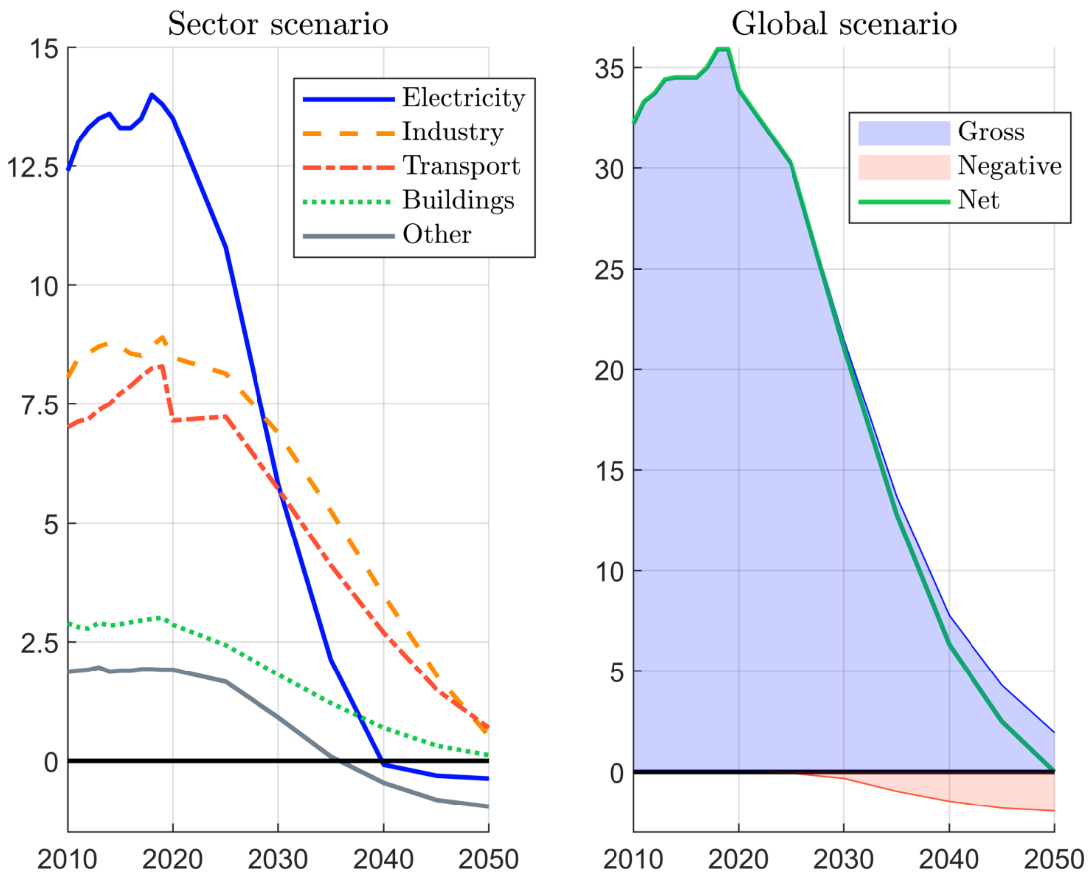
Sector	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Electricity	12.4	13	13.3	13.5	13.6	13.3	13.3	13.5	14	13.8
Buildings	2.89	2.81	2.78	2.9	2.84	2.87	2.91	2.95	2.98	3.01
Transport	7.01	7.13	7.18	7.37	7.5	7.72	7.88	8.08	8.25	8.29
Industry	8.06	8.47	8.57	8.71	8.78	8.71	8.56	8.52	8.72	8.9
Other	1.87	1.89	1.91	1.96	1.87	1.89	1.89	1.92	1.92	1.91
Gross emissions	32.2	33.3	33.7	34.4	34.5	34.5	34.5	35	35.9	35.9
BECCS/DACCS	0	0	0	0	0	0	0	0	0	0
Net emissions	32.2	33.3	33.7	34.4	34.5	34.5	34.5	35	35.9	35.9

Sector	2020	2025	2030	2035	2040	2045	2050
Electricity	13.5	10.8	5.82	2.12	−0.08	−0.31	−0.37
Buildings	2.86	2.43	1.81	1.21	0.69	0.32	0.12
Transport	7.15	7.23	5.72	4.11	2.69	1.5	0.69
Industry	8.48	8.14	6.89	5.25	3.48	1.8	0.52
Other	1.91	1.66	0.91	0.09	−0.46	−0.82	−0.96
Gross emissions	33.9	30.3	21.5	13.7	7.77	4.3	1.94
BECCS/DACCS	0	−0.06	−0.32	−0.96	−1.46	−1.8	−1.94
Net emissions	33.9	30.2	21.1	12.8	6.32	2.5	0.00

Source: IEA (2021, Figure 2.3, page 55)

Net zero emissions scenario (IEA)

Figure 193: CO_2 emissions by sector in the IEA NZE scenario (in $\mathrm{GtCO_2e}$)



Source: IEA (2021) & Author's calculations

Net zero emissions scenario (IEA)

Table 105: Carbon budget in the IEA NZE scenario (in $\mathrm{GtCO_2e}$)

t	Electricity	Buildings	Transport	Industry	Other	Gross emissions
2025	74.4	50.2	43.7	16.2	10.8	195.4
2030	115.9	87.8	76.0	26.8	17.3	324.9
2040	140.9	140.0	117.6	39.1	18.8	466.6
2045	139.9	153.2	128.1	41.6	15.6	496.8
2050	138.2	159.0	133.6	42.7	11.2	512.4

Source: IEA (2021) & Author's calculations

Carbon trend

Linear trend model

Linear trend model

- The linear trend model is defined by:

$$\mathcal{CE}(t) = \beta_0 + \beta_1 t + u(t)$$

  where $u(t) \sim \mathcal{N}\left(0, \sigma_u^2\right)$
- OLS estimation
- The projected carbon trajectory is given by:

$$\mathcal{CE}^{\mathcal{T}rend}(t) = \widehat{\mathcal{CE}}(t) = \hat{\beta}_0 + \hat{\beta}_1 t$$

Carbon trend

Linear trend model

- We have:

$$\widehat{\mathcal{CE}}(0) = \hat{\beta}_0$$

- Base year: t_0
- The linear trend model becomes:

$$\mathcal{CE}(t) = \beta'_0 + \beta'_1 (t - t_0) + u(t)$$

- We have the following relationships:

$$\begin{cases} \beta'_0 = \beta_0 + \beta_1 t_0 \\ \beta'_1 = \beta_1 \end{cases}$$

Carbon trend

Linear trend model

Example #8

Below, we report the evolution of scope $1 + 2$ carbon emissions for company A:

Table 106: Carbon emissions in $\mathrm{MtCO_2e}$ (company A)

Year	2007	2008	2009	2010	2011	2012	2013
$\mathcal{CE}(t)$	57.8	58.4	57.9	55.1	51.6	48.3	47.1
Year	2014	2015	2016	2017	2018	2019	2020
$\mathcal{CE}(t)$	46.1	44.4	42.7	41.4	40.2	41.9	45.0

Carbon trend

Linear trend model

We obtain the following estimates:

- $\hat{\beta}_0 = 2\,970.43$, $\hat{\beta}_1 = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2007$, $\hat{\beta}'_0 = 57.85$, $\hat{\beta}'_1 = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- $t_0 = 2020$, $\hat{\beta}'_0 = 38.99$, $\hat{\beta}'_1 = -1.4512$ and $\hat{\sigma}_u = 2.5844$
- The two estimated models are coherent:

$$\begin{aligned} \mathcal{CE}^{\mathcal{T}rend}(t) &= 38.99 - 1.4512 \times (t - 2020) \\ &= 2\,970.43 - 1.4512 \times t \end{aligned}$$

- We have:

$$\mathcal{CE}^{\mathcal{T}rend}(2025) = 38.99 - 1.4512 \times 5 = 31.73 \text{ MtCO}_2\text{e}$$

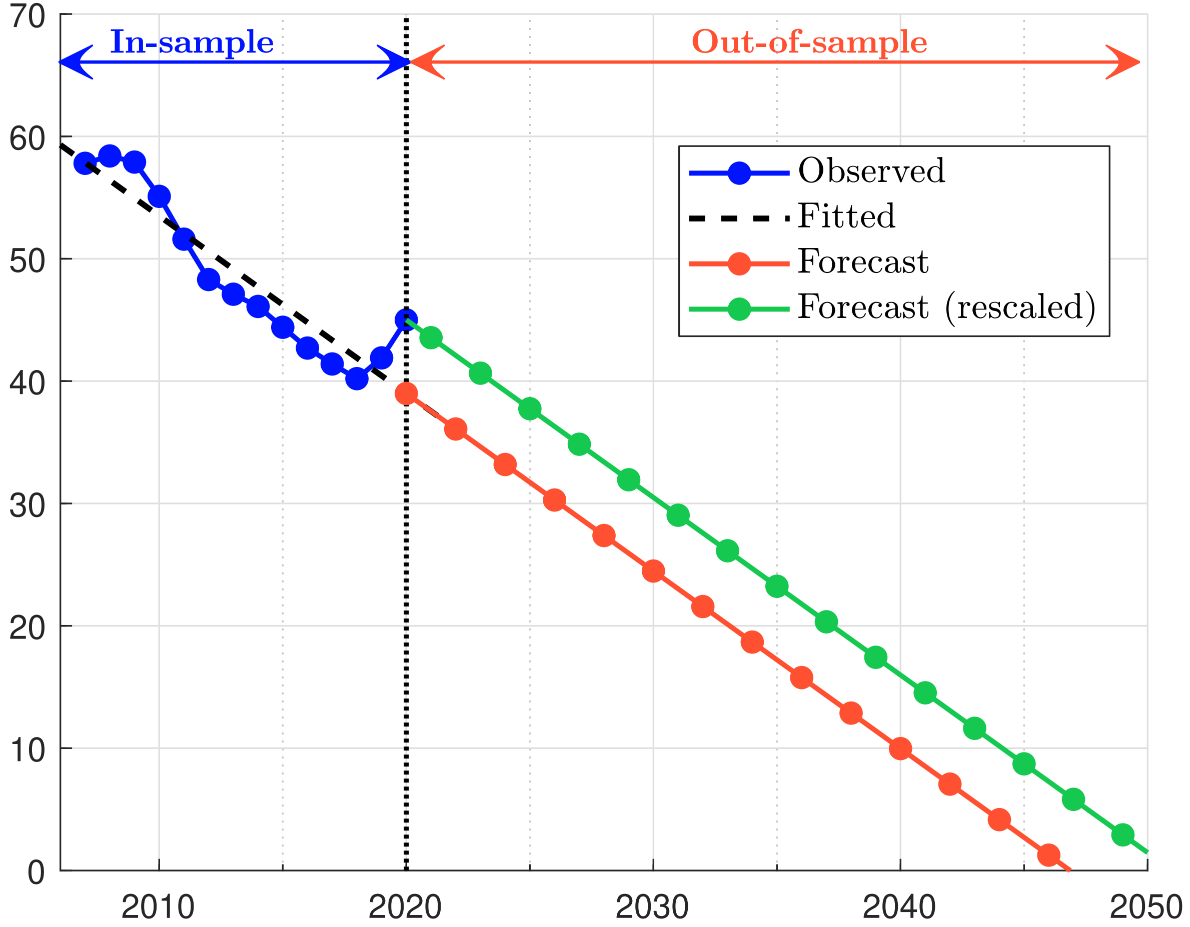
- We have $\mathcal{CE}(2020) = 45.0 \gg \widehat{\mathcal{CE}}(2020) = 38.99$
- The rescaled model has the following expression:

$$\mathcal{CE}^{\mathcal{T}rend}(t) = 45 - 1.4512 \times (t - 2020)$$

Carbon trend

Linear trend model

Figure 194: Linear carbon trend (Example #8)



Carbon trend

Log-linear trend model

Log-linear trend model

- The log-linear trend model is:

$$\ln \mathcal{CE}(t) = \gamma_0 + \gamma_1 (t - t_0) + v(t)$$

- Let $Y(t) = \ln \mathcal{CE}(t)$ be the logarithmic transform of the carbon emissions
- OLS estimation using $Y(t)$

Carbon trend

Log-linear trend model

- We have:

$$\widehat{\mathcal{CE}}(t) = \exp\left(\hat{Y}(t)\right) = \exp\left(\hat{\gamma}_0 + \hat{\gamma}_1(t - t_0)\right) = \widehat{\mathcal{CE}}(t_0)\exp\left(\hat{\gamma}_1(t - t_0)\right)$$

where $\widehat{\mathcal{CE}}(t_0) = \exp(\hat{\gamma}_0)$

- The mathematical expectation of $\mathcal{CE}(t)$ is equal to:

$$\begin{aligned}
\mathbb{E}\left[\mathcal{CE}(t)\right] &= \mathbb{E}\left[e^{Y(t)}\right] \\
&= \mathbb{E}\left[\mathcal{LN}\left(\gamma_0 + \gamma_1(t - t_0), \sigma_v^2\right)\right] \\
&= \exp\left(\gamma_0 + \gamma_1(t - t_0) + \frac{1}{2}\sigma_v^2\right) \\
&= \widehat{\mathcal{CE}}(t_0)\exp\left(\hat{\gamma}_1(t - t_0)\right)
\end{aligned}$$

where $\widehat{\mathcal{CE}}(t_0) = \exp\left(\hat{\gamma}_0 + \frac{1}{2}\hat{\sigma}_v^2\right)$

- The rescaled log-linear trend model is:

$$\mathcal{CE}^{\mathcal{T}rend}(t) = \mathcal{CE}(t_0)\exp\left(\hat{\gamma}_1(t - t_0)\right)$$

Interpretation of the slope

- β_1 is the absolute variation of carbon emissions:

$$\frac{\partial\,\mathcal{CE}\,(t)}{\partial\,t} = \beta_1$$

implying that the relative variation of carbon emissions is:

$$\frac{\dfrac{\partial\,\mathcal{CE}\,(t)}{\partial\,t}}{\mathcal{CE}\,(t)} = \frac{\beta_1}{\mathcal{CE}\,(t)}$$

- γ_1 is the relative variation of carbon emissions:

$$\frac{\dfrac{\partial\,\mathcal{CE}\,(t)}{\partial\,t}}{\mathcal{CE}\,(t)} = \frac{\partial\,\ln\mathcal{CE}\,(t)}{\partial\,t} = \gamma_1$$

Carbon trend

Log-linear trend model

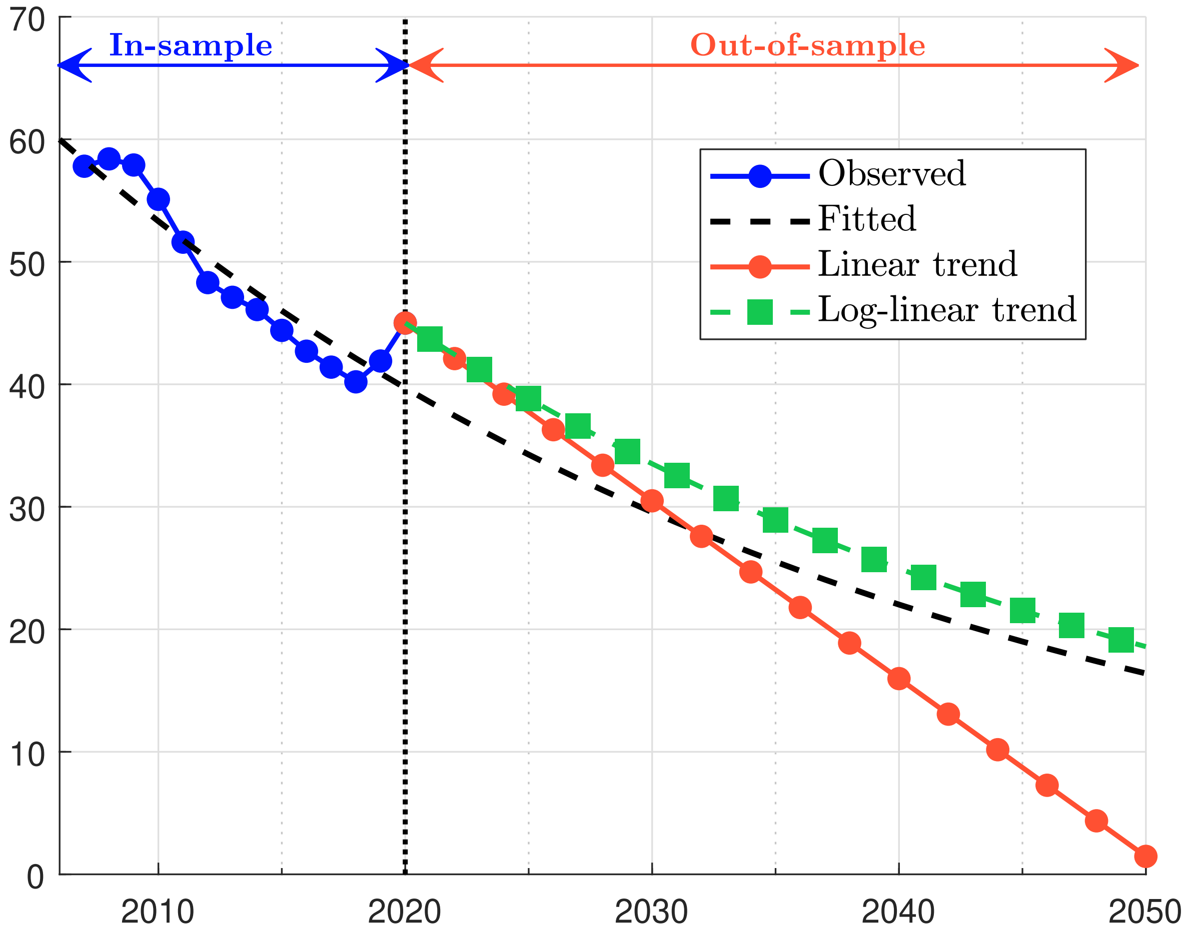
Example #8:

- We obtain the following results: $\hat{\gamma}_0 = 3.6800$, $\hat{\gamma}_1 = -2.95\%$ and $\hat{\sigma}_v = 0.0520$
- $\widehat{\mathcal{CE}}(2020) = 39.65 \text{ MtCO}_2\text{e}$ without the correction of the variance bias
- $\widehat{\mathcal{CE}}(2020) = 39.70 \text{ MtCO}_2\text{e}$ with the correction of the variance bias

Carbon trend

Log-linear trend model

Figure 195: Log-linear carbon trend (Example #8)



Linear vs. log-linear trend model

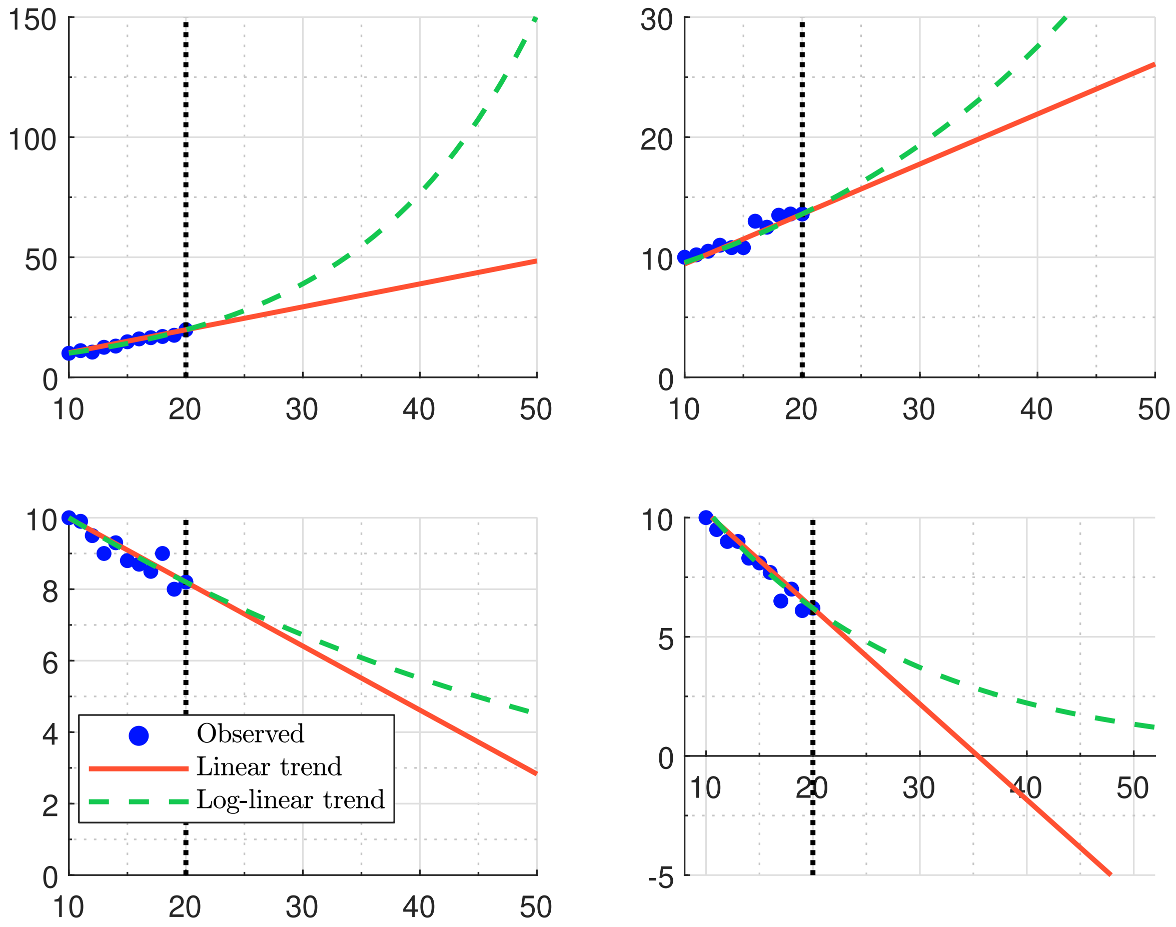
Example #9

We consider several historical trajectories of scope 1 carbon emissions:

Year	#1	#2	#3	#4
2010	10.0	10.0	10.0	10.0
2011	11.1	10.2	9.9	9.5
2012	10.5	10.5	9.5	9.0
2013	12.5	11.0	9.0	9.0
2014	13.0	10.8	9.3	8.3
2015	14.8	10.8	8.8	8.1
2016	16.0	13.0	8.7	7.7
2017	16.5	12.5	8.5	6.5
2018	17.0	13.5	9.0	7.0
2019	17.5	13.6	8.0	6.1
2020	19.8	13.6	8.2	6.2

Linear vs. log-linear trend model

Figure 196: Log-linear vs. linear carbon trend (Example #9)



Carbon trend

Stochastic trend model

Stochastic trend model

- The linear trend model can be written as:

$$\left\{ \begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1 \end{array} \right.$$

where $u(t) \sim \mathcal{N}\left(0, \sigma_u^2\right)$

- We have $y(t) = \beta_0 + \beta_1 t + u(t)$ where $\beta_0 = \mu(t_0) - \beta_1 t_0$
- The local linear trend model is defined as:

$$\left\{ \begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1 (t-1) + \eta(t) \\ \beta_1 (t) = \beta_1 (t-1) + \zeta(t) \end{array} \right.$$

where $\eta(t) \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ and $\zeta(t) \sim \mathcal{N}\left(0, \sigma_\zeta^2\right)$

- The stochastic trend $\mu(t)$ and slope $\beta_1(t)$ are estimated with KF

Carbon trend

Stochastic trend model

Example #8

- We estimate the parameters $(\sigma_u, \sigma_\eta, \sigma_\zeta)$ by maximizing the Whittle log-likelihood function
- We obtain $\hat{\sigma}_u = 0.7022$, $\hat{\sigma}_\eta = 0.7019$ and $\hat{\sigma}_\zeta = 0.8350$
- The standard deviation of the stochastic slope variation $\beta_1(t) - \beta_1(t-1)$ is then equal to $0.8350 \ \mathrm{MtCO_2e}$

Carbon trend

Stochastic trend model

Table 107: Kalman filter estimation of the stochastic trend (Example #8)

t	$\mathcal{CE}(t)$	$\hat{\beta}_1(t)$ (RLS)	$\beta_1(t)$ (KF)	$\mu(t)$ KF)
2007	57.80		0.0000	57.80
2008	58.40		0.2168	58.25
2009	57.90	0.0500	−0.0441	58.00
2010	55.10	−0.8600	−1.3941	55.56
2011	51.60	−1.5700	−2.6080	52.01
2012	48.30	−2.0200	−3.1288	48.47
2013	47.10	−2.0929	−2.2977	46.82
2014	46.10	−2.0321	−1.5508	45.85
2015	44.40	−1.9817	−1.5029	44.38
2016	42.70	−1.9406	−1.5887	42.73
2017	41.40	−1.8891	−1.4655	41.36
2018	40.20	−1.8329	−1.3202	40.15
2019	41.90	−1.6824	0.1339	41.41
2020	45.00	−1.4512	1.7701	44.45

Carbon momentum

- We have:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_1(t)}{\mathcal{CE}(t)}$$

or:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \hat{\gamma}_1(t)$$

Statistics

Table 108: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}(t)$ (MSCI World index, 1995 – 2021, linear trend)

Statistics	Carbon emissions			Carbon intensity		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$
Median	0.0	1.6	2.3	−4.8	−2.4	−1.3
Negative	49.9	41.1	29.4	76.0	69.6	75.6
Positive	50.1	58.9	70.6	24.0	30.4	24.4
$< -10\%$	23.4	15.8	5.8	36.0	25.0	5.7
$< -5\%$	32.1	22.2	10.6	48.6	36.7	13.4
$> +5\%$	22.9	27.5	23.6	6.2	7.3	2.7
$> +10\%$	9.2	9.5	8.0	2.3	2.6	1.0

Source: Trucost database (2022) & Authors' calculations.

Statistics

Table 109: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}(t)$ (MSCI World index, 1995 – 2021, log-linear trend)

Statistics	Carbon emissions			Carbon intensity		
	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$
Median	−0.1	1.7	2.8	−3.6	−1.9	−1.2
Negative	50.6	40.3	29.0	76.3	69.0	75.8
Positive	49.4	59.7	71.0	23.7	31.0	24.2
$< -10\%$	13.6	8.0	2.8	20.8	12.3	2.1
$< -5\%$	26.6	16.9	7.5	42.3	29.0	8.4
$> +5\%$	29.8	35.9	37.1	9.0	10.1	4.0
$> +10\%$	16.9	19.4	19.2	4.0	4.1	1.6

Source: Trucost database (2022) & Authors' calculations.

The $\mathcal{PAC}$ framework

Participation

Ambition

Credibility

Carbon target and decarbonization scenario

The $\mathcal{PAC}$ framework requires three time series:

- The historical pathway of carbon emission
- The reduction targets announced by the company

$$\mathbb{CT} = \left\{ \boldsymbol{\mathcal{R}}^{\mathcal{T}arget}\left(t_0, t_k\right), k = 1, \ldots, n_T \right\}$$

- The market-based sector scenario associated to the company that defines the decarbonization pathway

$$\mathbb{CS} = \left\{ \boldsymbol{\mathcal{R}}^{\mathcal{S}cenario}\left(t_0, t_k\right), k = 1, \ldots, n_S \right\}$$

The $\mathcal{PAC}$ framework

Table 110: Reduction rates of the IEA NZE scenario (base year = 2020)

Year	Electricity	Industry	Transport	Buildings	Other	Global
2025	20.0	4.0	−1.1	15.0	13.1	10.6
2030	56.9	18.8	20.0	36.7	52.4	36.6
2035	84.3	38.1	42.5	57.7	95.3	59.6
2040	100.0	59.0	62.4	75.9	100.0	77.1
2045	100.0	78.8	79.0	88.8	100.0	87.3
2050	100.0	93.9	90.3	95.8	100.0	94.3

Source: IEA (2021) & Author's calculations.

The $\mathcal{PAC}$ framework

The 3 questions of the $\mathcal{PAC}$ framework

1. Is the trend of the issuer in line with the scenario?
2. Is the commitment of the issuer to fight climate change ambitious?
3. Is the target setting of the company relevant and robust, or is it a form of greenwashing?

The $\mathcal{PAC}$ framework

Example #10

- We consider Example #8
- Company A has announced the following targets:
  1. $\mathcal{R}^{\mathcal{T}arget}(2020, 2025) = 40\%$
  2. $\mathcal{R}^{\mathcal{T}arget}(2020, 2030) = 50\%$
  3. $\mathcal{R}^{\mathcal{T}arget}(2020, 2035) = 75\%$
  4. $\mathcal{R}^{\mathcal{T}arget}(2020, 2040) = 80\%$
  5. $\mathcal{R}^{\mathcal{T}arget}(2020, 2050) = 90\%$
- Company A is an utility corporation $\Rightarrow$ we use the IEA NZE scenario for the sector Electricity

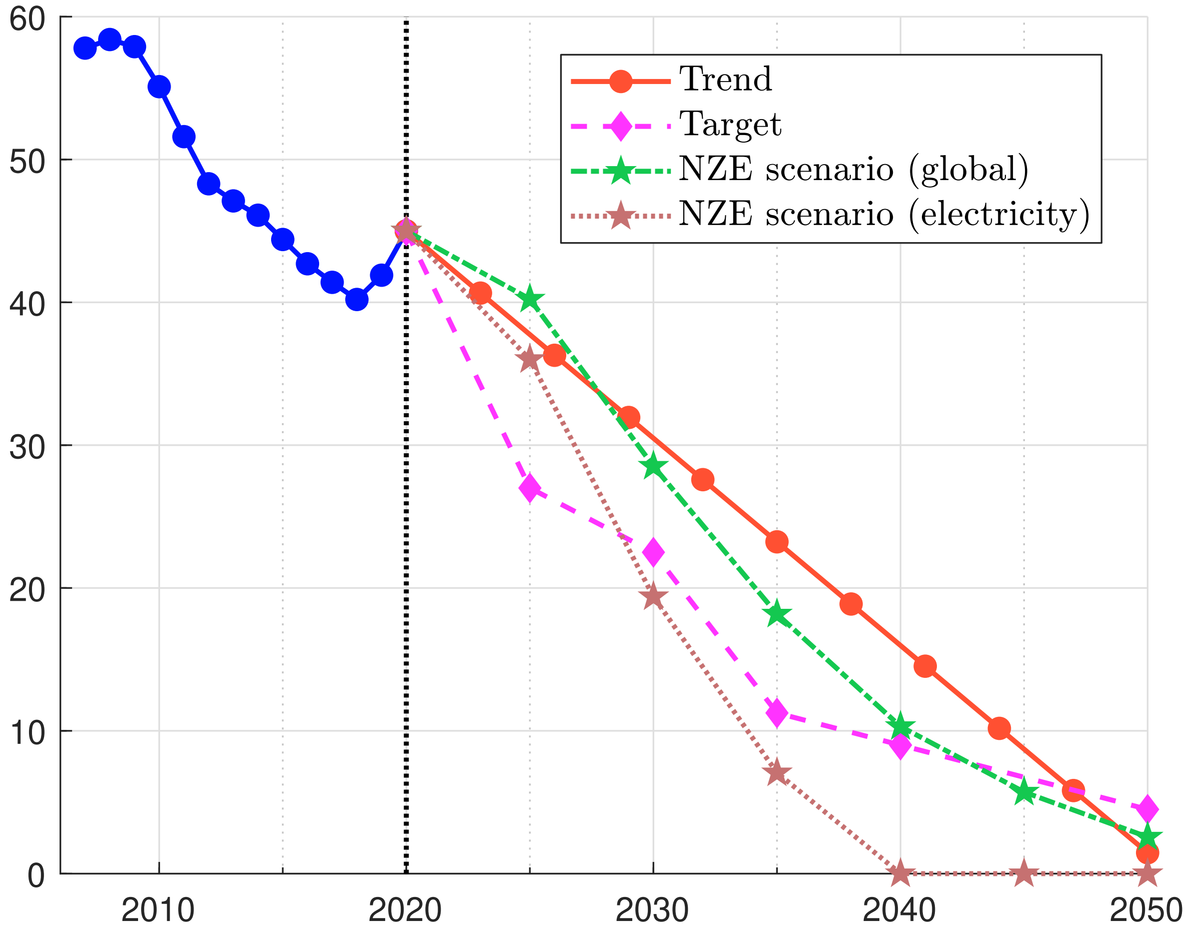
The $\mathcal{PAC}$ framework

Table 111: Comparison of carbon budgets (Example #10, base year = 2020)

Year	Trend (linear)	Trend (log-linear)	Target	Scenario (global)	Scenario (electricity)
2025	207	209	180	213	203
2030	377	390	304	385	341
2035	512	546	388	502	407
2040	610	680	439	573	425
2045	671	796	478	613	425
2050	697	896	506	634	425

The $\mathcal{PAC}$ framework

Figure 197: Carbon trend, targets and NZE scenario of company A



Source: IEA (2021) & Author's calculations.

Assessment of the $\mathcal{PAC}$ pillars

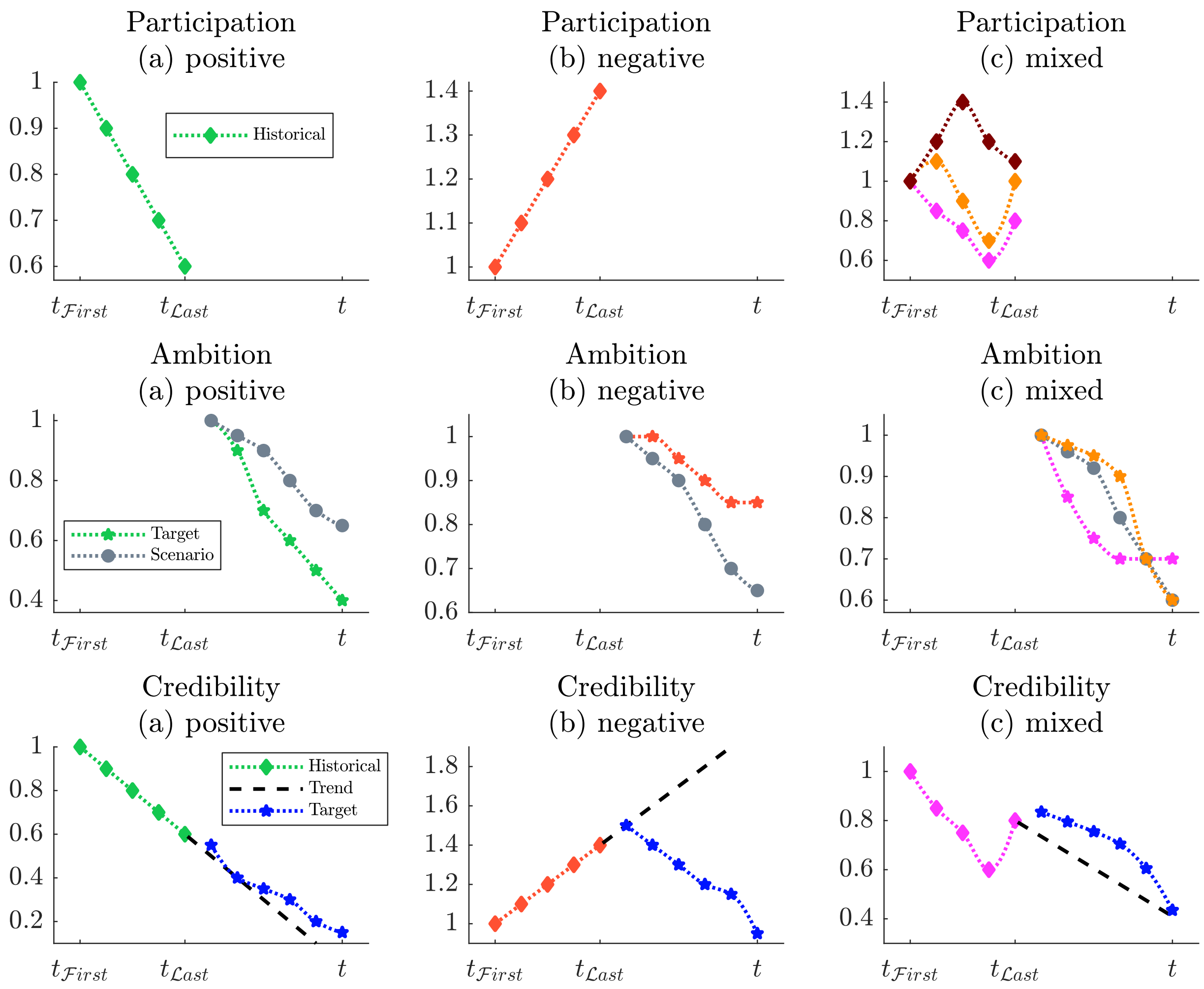
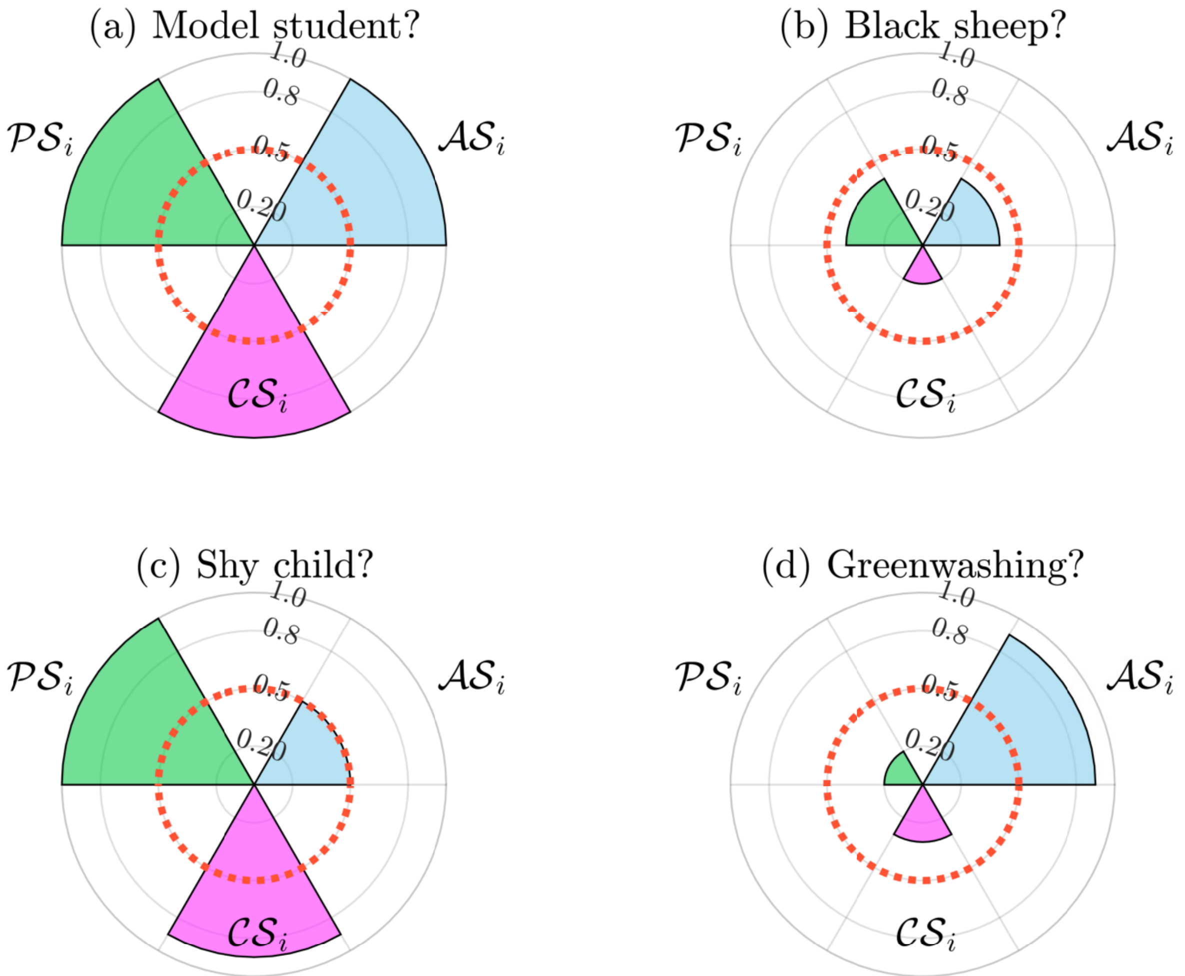


Figure 198: Illustration of the participation, ambition and credibility pillars

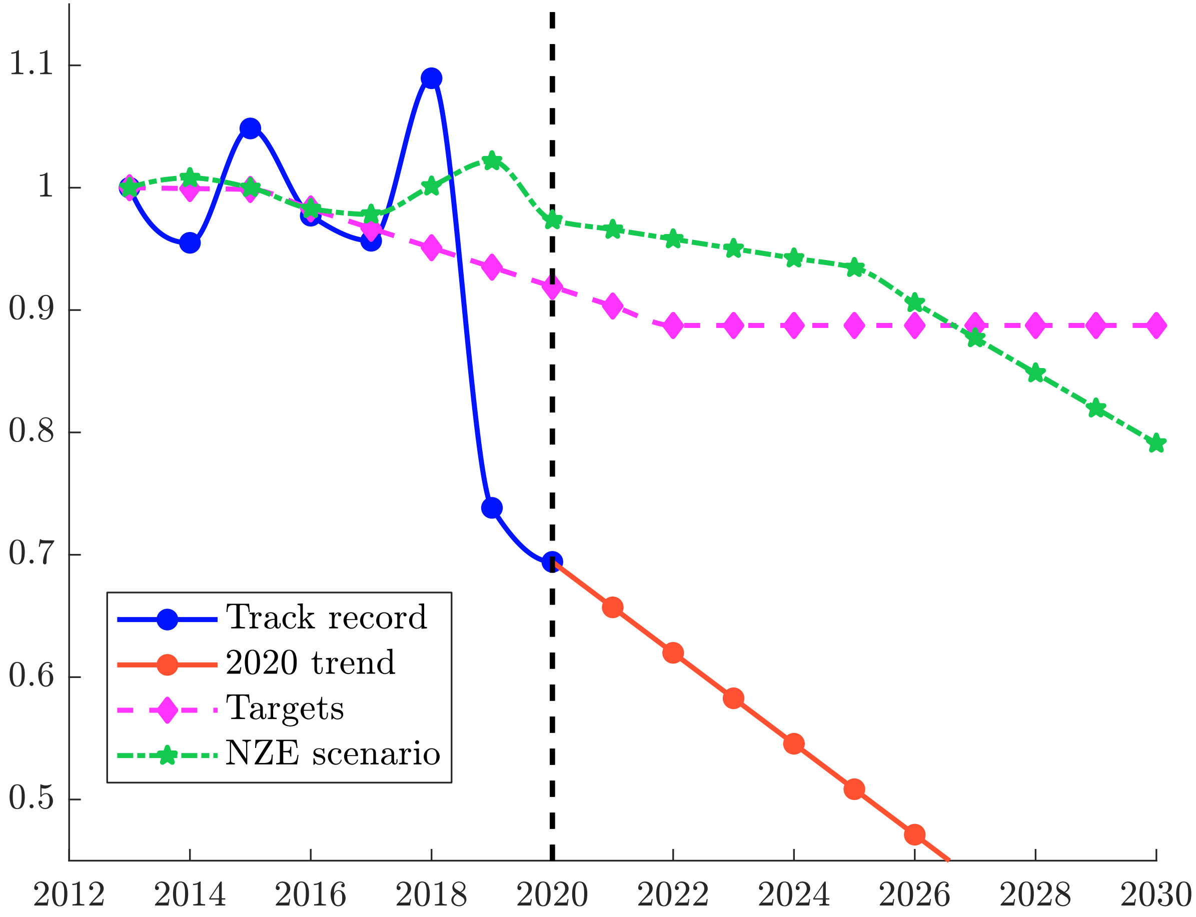
Temperature scoring system

Figure 199: The $\mathcal{PAC}$ scoring system



Illustration

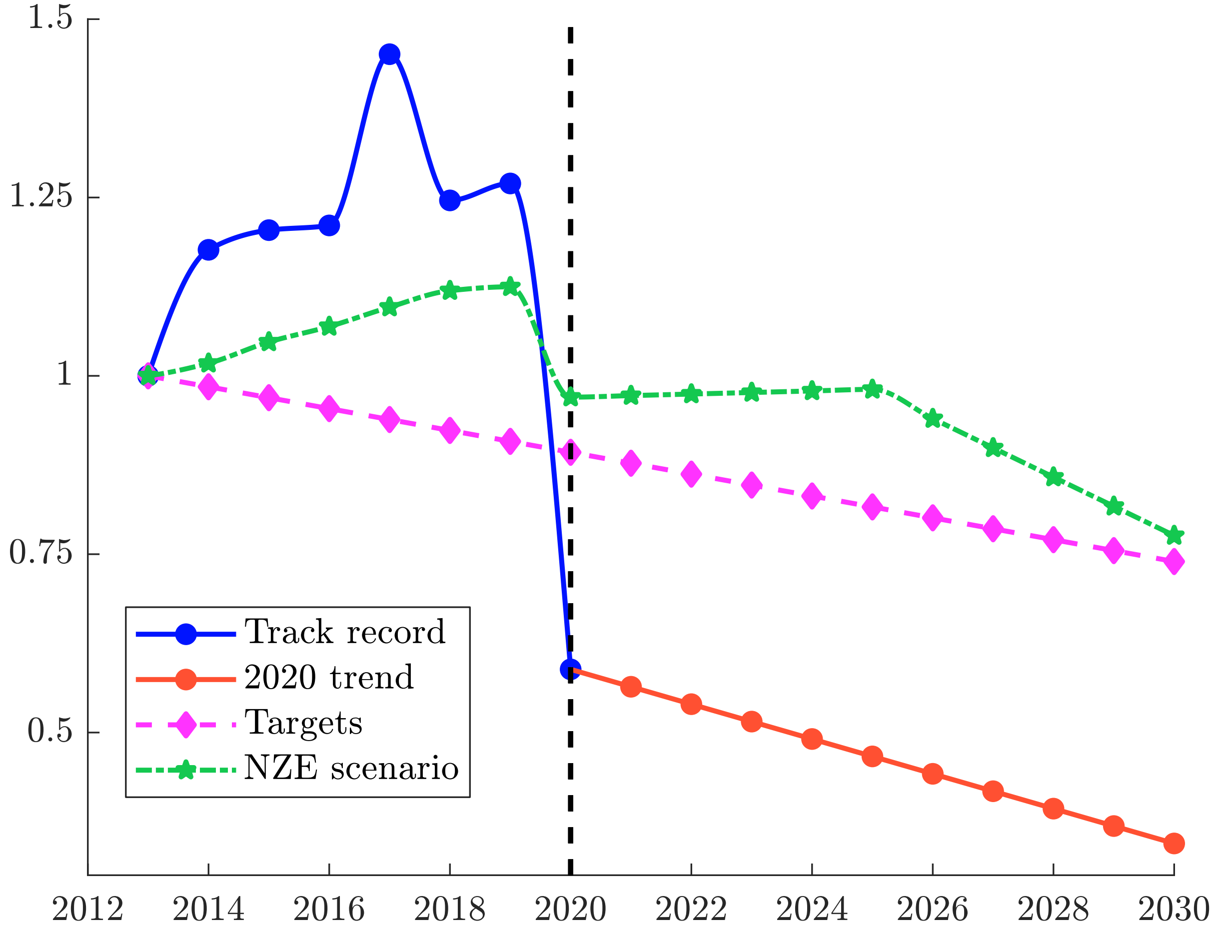
Figure 200: Carbon emissions, trend, targets and NZE scenario (Company *B*)



Source: CDP database (2021), IEA (2021) & Leguenedal *et al.* (2022)

Illustration

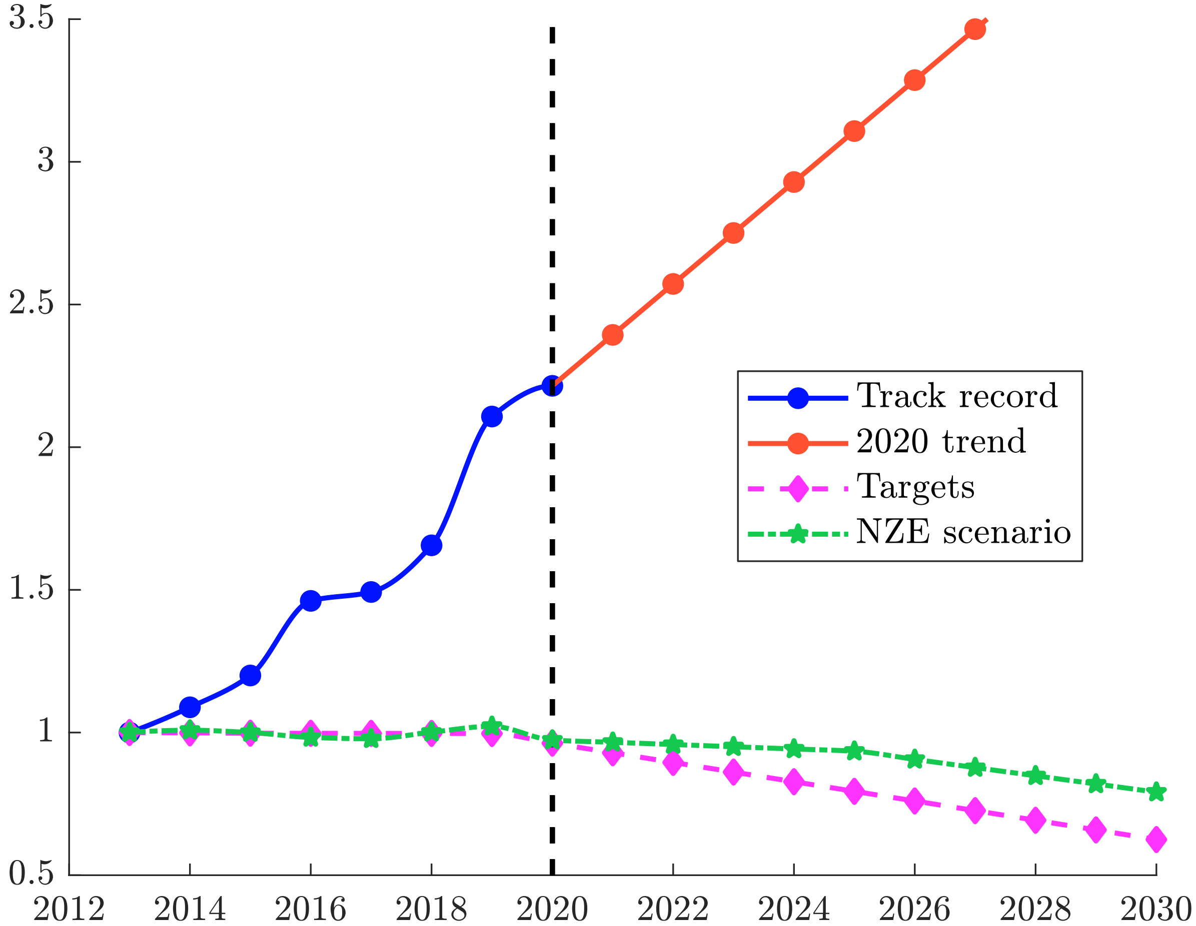
Figure 201: Carbon emissions, trend, targets and NZE scenario (Company C)



Source: CDP database (2021), IEA (2021) & Leguenedal *et al.* (2022)

Illustration

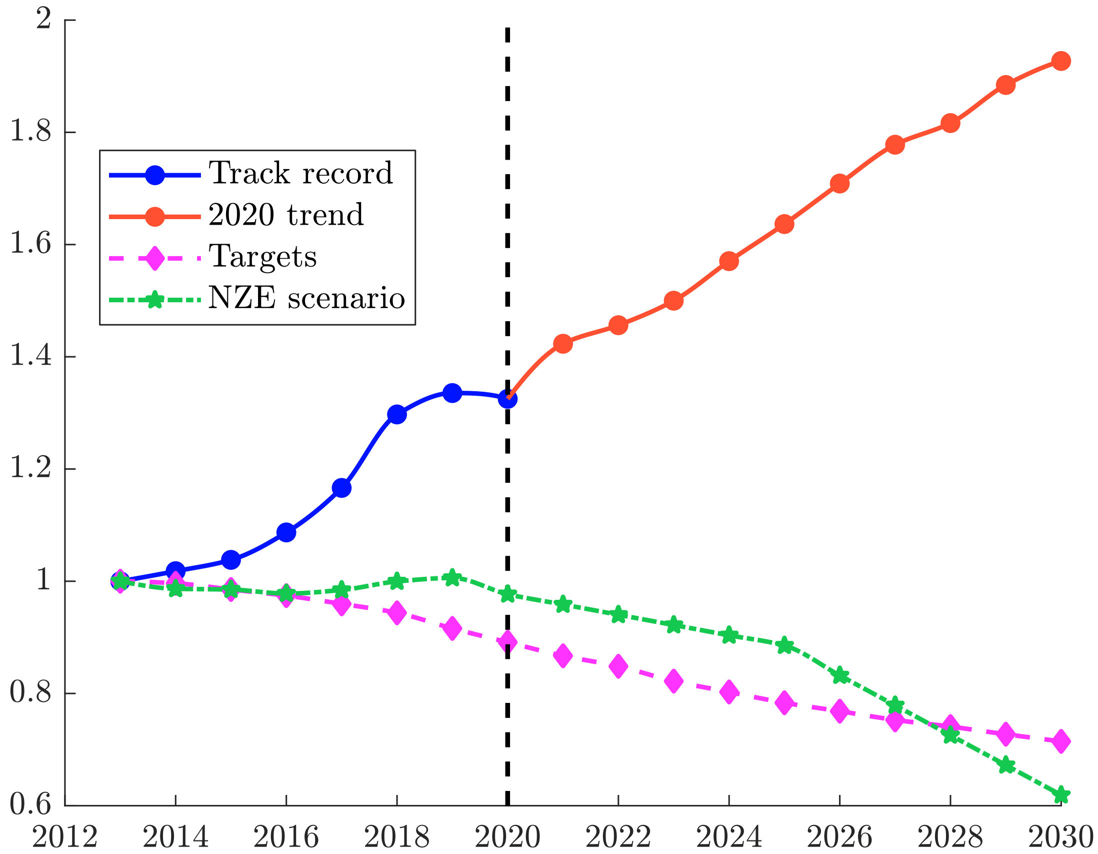
Figure 202: Carbon emissions, trend, targets and NZE scenario (Company *D*)



Source: CDP database (2021), IEA (2021) & Leguenedal *et al.* (2022)

Illustration

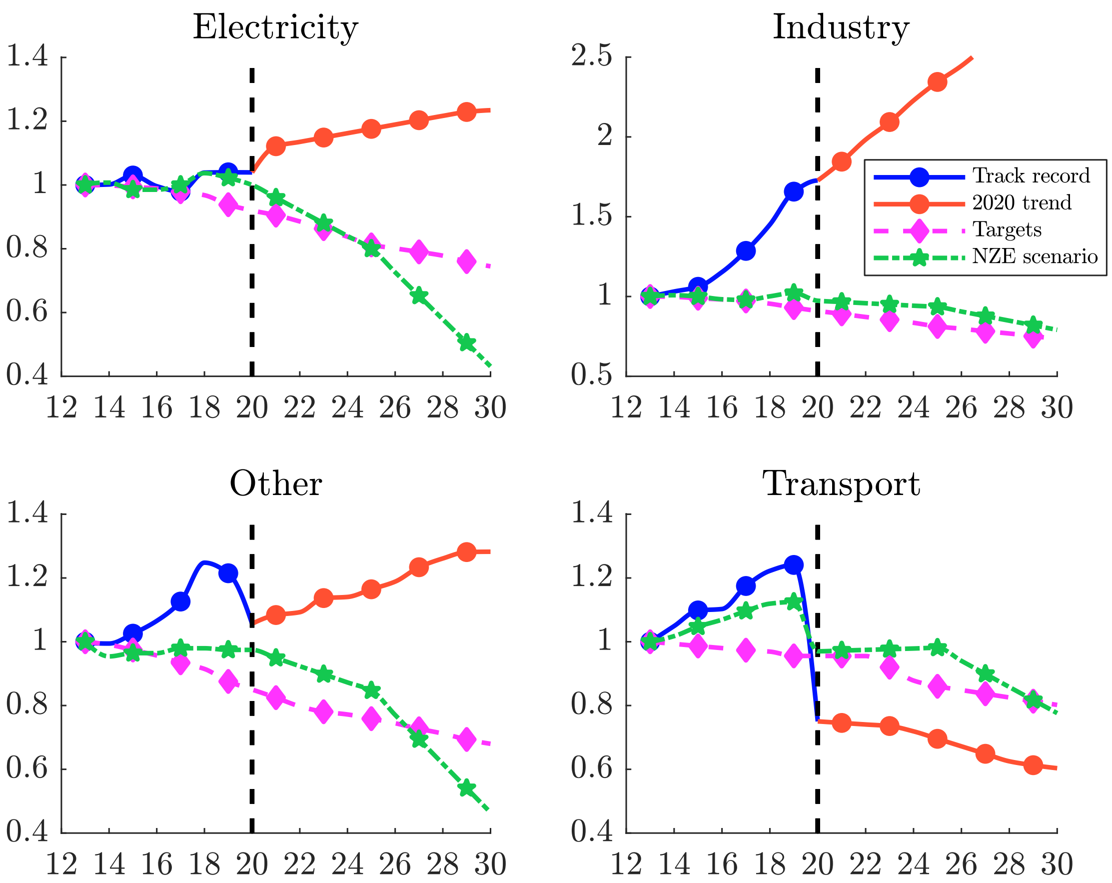
Figure 203: Carbon emissions, trend, targets and NZE scenario (median analysis, global universe)



Source: CDP database (2021), IEA (2021) & Leguenedal *et al.* (2022)

Illustration

Figure 204: Carbon emissions, trend, targets and NZE scenario (median analysis, sector universe)

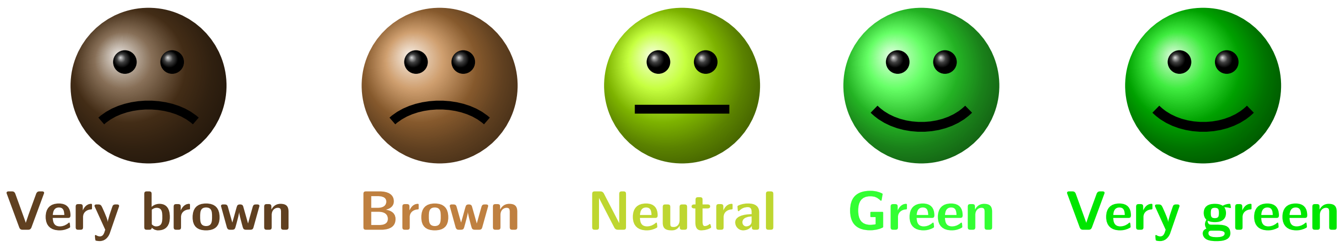


Source: CDP database (2021), IEA (2021) & Leguenedal *et al.* (2022)

Greenness measures

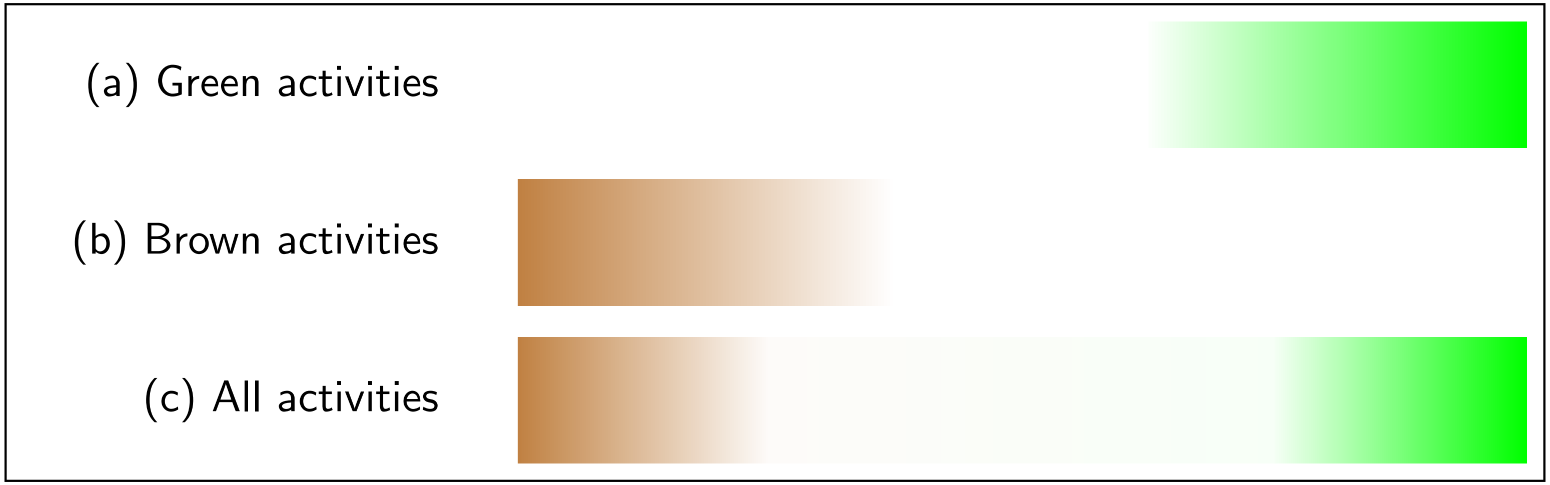
- Brown intensity: $\boldsymbol{\mathcal{BI}}$
- Green intensity: $\boldsymbol{\mathcal{GI}}$
- We have $\boldsymbol{\mathcal{BI}} \in [0,1]$, $\boldsymbol{\mathcal{GI}} \in [0,1]$ and $0 \leq \boldsymbol{\mathcal{BI}} + \boldsymbol{\mathcal{GI}} \leq 1$
- Most of the time, we have

$$\boldsymbol{\mathcal{BI}} + \boldsymbol{\mathcal{GI}} \neq 1$$



Greenness measures

Figure 205: Several taxonomies



Green taxonomy

Definition

The EU taxonomy for sustainable activities is "*a classification system, establishing a list of environmentally sustainable economic activities.*"

Green taxonomy

These economic activities must have a substantive contribution to at least one of the following six environmental objectives:

1. climate change mitigation
2. climate change adaptation
3. sustainable use and protection of water and marine resources
4. transition to a circular economy
5. pollution prevention and control
6. protection and restoration of biodiversity and ecosystem

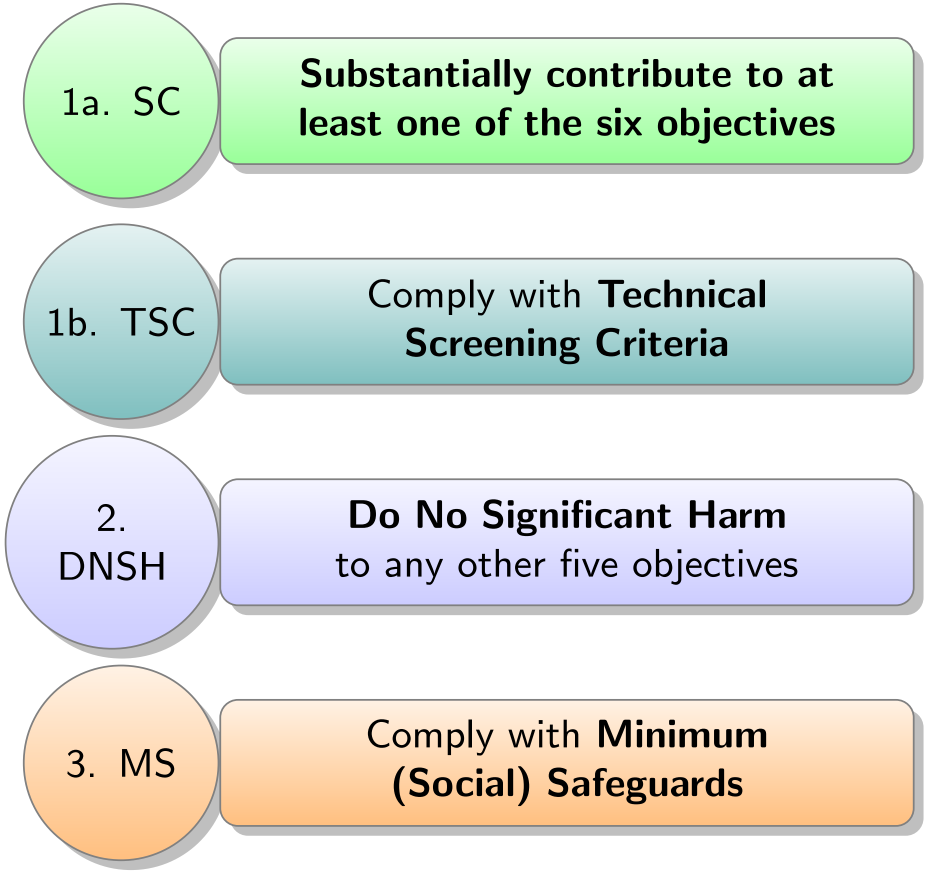
Green taxonomy

A business activity must also meet two other criteria to qualify as sustainable:

- The activity must do no significant harm to the other environmental objectives (**DNSH** constraint)
- It must comply with minimum social safeguards (**MS** constraint)

Green taxonomy

Figure 206: EU taxonomy for sustainable activities



Green revenue share

Relationship between the green intensity and the green revenue share

We have:

$$\boldsymbol{\mathcal{GI}} = \frac{\mathcal{GR}}{\mathcal{TR}} \cdot (1 - \mathcal{P}) \cdot \mathbb{1}\left\{\boldsymbol{\mathcal{S}} \geq \boldsymbol{\mathcal{S}}^{\star}\right\}$$

where:

- $\mathcal{GR}$ is the green revenue deduced from the six environmentally sustainable objectives
- $\mathcal{TR}$ is the total revenue
- $\mathcal{P}$ is the penalty coefficient reflecting the DNSH constraint
- $\boldsymbol{\mathcal{S}}$ is the minimum safeguard score
- $\boldsymbol{\mathcal{S}}^{\star}$ is the threshold

Green revenue share

- The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

$$\boldsymbol{\mathcal{GRS}} = \frac{\mathcal{GR}}{\mathcal{TR}}$$

- By construction, we have $0 \leq \boldsymbol{\mathcal{GRS}} \leq 1$
- This measure is then impacted by the DNSH coefficient
  - The two extreme cases are:

$$\left\{ \begin{array}{l} \mathcal{P} = 1 \Rightarrow \boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}} \\ \mathcal{P} = 0 \Rightarrow \boldsymbol{\mathcal{GI}} = 0 \end{array} \right.$$

  - We have $0 \leq \boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}} \cdot (1 - \mathcal{P}) \leq \boldsymbol{\mathcal{GRS}}$
- The indicator function $\mathbb{1}\left\{s \geq s^{\star}\right\}$ is a binary all-or-nothing variable:

$$\boldsymbol{\mathcal{S}} < \boldsymbol{\mathcal{S}}^{\star} \Rightarrow \boldsymbol{\mathcal{GI}} = 0$$

Green revenue share

Example #11

We consider a company in the hydropower sector which has five production sites. Below, we indicate the power density efficiency, the GHG emissions, the DNSH compliance with respect to the biodiversity and the corresponding revenue:

Site	#1	#2	#3	#4	#5
Efficiency (in Watt per m^2)	3.2	3.5	3.3	5.6	4.2
GHG emissions (in $\mathrm{gCO_2e}$ per kWh)	35	103	45	12	36
Biodiversity DNSH compliance	✓	✓	✓	✓	
Revenue (in $ mn)	103	256	89	174	218

Green revenue share

- The total revenue is equal to:

$$\mathcal{TR} = 103 + 256 + 89 + 174 + 218 = \$840 \text{ mn}$$

- The fourth site does not pass the technical screening, because the power density is above 5 Watt per m^2
- The second site does not also comply because it has a GHG emissions greater than 100 $\mathrm{gCO_2e}$ per kWh
- We deduce that the green revenue is equal to:

$$\mathcal{GR} = 103 + 89 + 218 = \$410 \text{ mn}$$

- We conclude that the green revenue share is equal to 48.8%
- According to the EU green taxonomy, the green intensity is lower because the last site is close to a biodiversity area and has a negative impact:

$$\boldsymbol{\mathcal{GI}} = \frac{103 + 89}{840} = 22.9\%$$

Statistics

Table 112: Statistics in % of green revenue share (MSCI ACWI IMI, June 2022)

Category	Frequency $\mathbf{F}(x)$				Quantile $\mathbf{Q}(\alpha)$				Mean	
	0	25%	50%	75%	75%	90%	95%	Max	Avg	Wgt
(1)	9.82	1.47	0.96	0.75	0.00	0.00	2.85	100.00	1.36	0.77
(2)	14.10	1.45	0.65	0.31	0.00	1.25	6.12	100.00	1.39	3.50
(3)	4.84	1.68	1.02	0.31	0.00	0.00	0.00	100.00	1.16	0.51
(4)	4.79	0.30	0.10	0.06	0.00	0.00	0.00	99.69	0.32	0.22
(5)	1.00	0.39	0.20	0.09	0.00	0.00	0.00	98.47	0.26	0.10
(6)	4.75	0.28	0.11	0.05	0.00	0.00	0.00	99.98	0.29	0.14
Total	27.85	5.82	3.17	1.68	0.42	11.82	30.36	100.00	4.78	5.24

Source: MSCI (2022) & Barahhou (2022)

$\mathbf{F}(x) = \Pr\{\mathcal{GRS} > x\}$, $\mathbf{Q}(\alpha) = \inf\{x : \Pr\{\mathcal{GRS} \leq x\} \geq \alpha\}$, arithmetic average $n^{-1}\sum_{i=1}^{n} \mathcal{GRS}_i$ and weighted mean $\mathcal{GRS}(b) = \sum_{i=1}^{n} b_i \mathcal{GRS}_i$

Statistics

- The green revenue share of the MSCI World index is equal to 5.24%
- The green revenue share of the Bloomberg Global Investment Grade Corporate Bond index is equal to 3.49%
- Alessi and Battiston (2022) estimated "*a greenness of about 2.8% for EU financial markets*"

Green capex

Green-to-brown ratio

Course 2022-2023 in Sustainable Finance
Lecture 9. Transition Risk Modeling

Thierry Roncalli*

*Amundi Asset Management[24]

*University of Paris-Saclay

March 2023

[24] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Climate transition risk

Definition

- Transition risks arise from the sudden shift towards a low-carbon economy
- Such transitions could mean that some sectors of the economy face big shifts in asset values or higher costs of doing business

" It's not that policies stemming from deals like the Paris Climate Agreement are bad for our economy — in fact, the risk of delaying action altogether would be far worse. Rather, it's about the speed of transition to a greener economy — and how this affects certain sectors and financial stability" (Bank of England, 2021)

Climate transition risk

The carbon footprint approach assumes that the **climate-related market risk** of a company is measured by its **current carbon intensity**

...But the **market perception** of the climate change may be different

Climate transition risk

Fundamental-based analysis

- Carbon footprint and pathway are measured by CO_2 emissions
- They are fundamental data

Market-based analysis

- Financial market's perception of the potentially reduced impact of climate policies' on securities issued by corporations
- These carbon risk metrics use market data
- How an increase in carbon prices and taxes influences the credit risk of the issuer?
- How sensitive the asset price is to a carbon market factor?

Carbon price

Two main pricing systems:

1. Carbon tax
2. Emissions trading system (ETS)

Underlying idea

- A high carbon tax impacts the creditworthiness of corporates
- This impact is different from one issuer to another one
- Identifying for each company the carbon price that would lead the default probability in the Merton model to exceed a certain threshold

Carbon price

Based on the assumptions that the enterprise value V is proportional to the earnings before interest, taxes, depreciation, and amortization (EBITDA) and that the debt D remains constant, we can define the carbon price margin as[25]:

$$\mathrm{CPM}_i = \left(1 - \exp\left(\sigma_i\sqrt{\tau}\Phi\left(-\theta\right) - \left(r + \frac{1}{2}\sigma_i^2\right)\tau\right)\frac{D_i}{V_i}\right)\frac{\mathrm{EBITDA_i}}{\boldsymbol{\mathcal{CE}}_{i,1}}$$

where σ_i is the volatility of the enterprise value, τ is the maturity and r is the risk-free rate

[25] The parameter θ is the threshold of default probability

Carbon tax

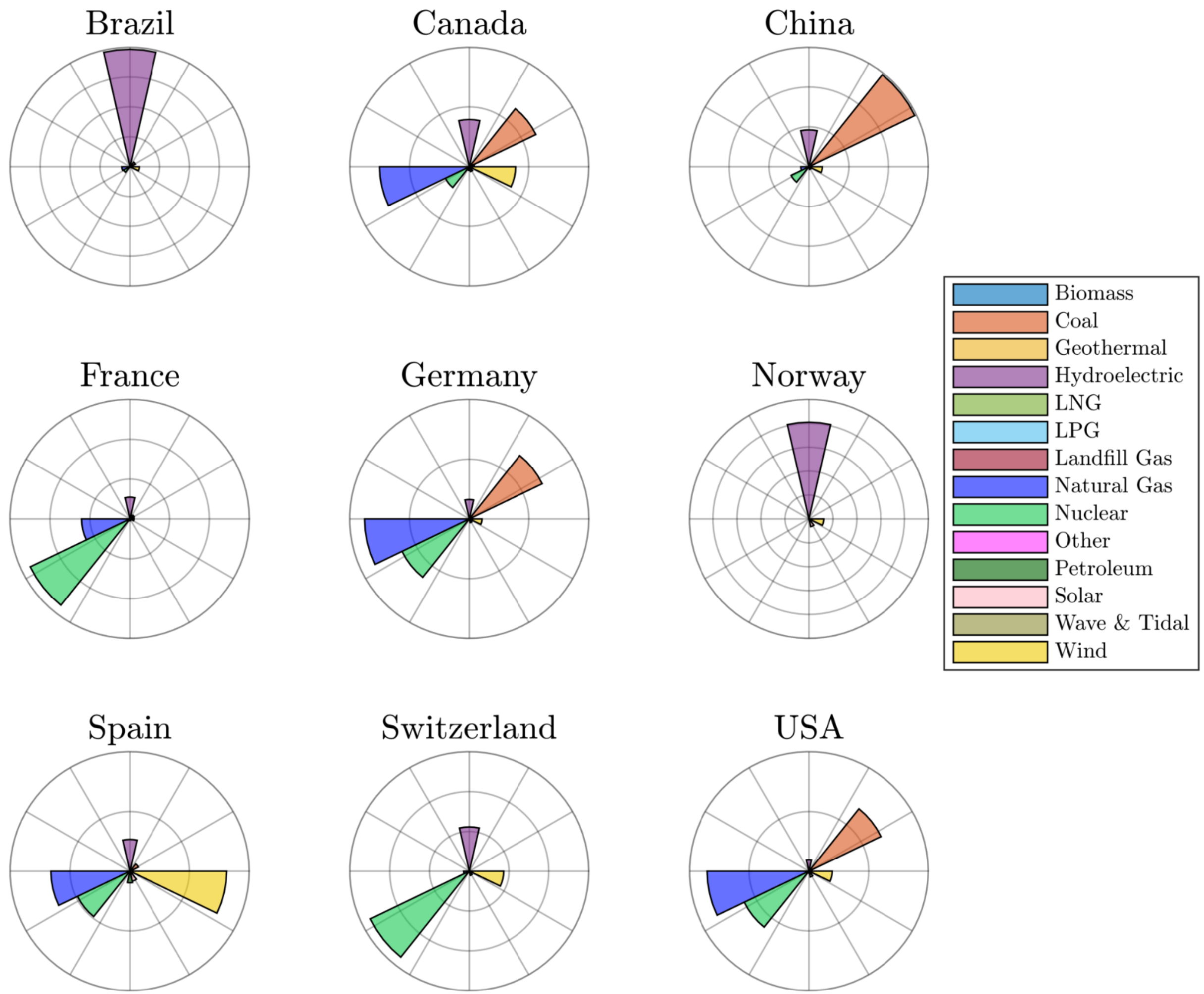
Stranded assets

Energy mix

- How to measure the environmental performance of an utility company?
- How to measure the environmental performance of a country?
- How to assess a company located in a country with a bad energy mix?

Bottom up energy mix$^{(*)}$ (in %)

This figure presents the energy generation breakdown for some countries. We can distinguish countries that rely on hydroelectric power (Brazil, Norway), nuclear (France, Switzerland) and mixed solutions (Canada, Germany, Spain, USA)



$^{(*)}$ Each grid circle represents 20% of energy generation. The scale of the radar chart is then 40% for Canada, Germany, Spain and USA, 60% for China, France and Switzerland, 80% for Brazil and 100% for Norway

Implied temperature rating

Carbon beta

- Introduced by Harris (2015) and Görgen *et al.* (2019)
- The underlying idea of the carbon beta is to estimate the sensitivity of the stock return with respect to a carbon/climate risk factor
- Climate risk is not only an idiosyncratic risk for the issuer, but also a systematic risk factor like the Fama-French-Carhart market factors

Carbon beta

Cross-section factor

- Long/short portfolio
- Long on stocks highly exposed to carbon risk
- Short on stocks lowly exposed to carbon risk
- The value of the factor is the return of the L/S portfolio
- High carbon beta = highly exposed to carbon risk

Time-series factor

- Synthetic index that represents the financial perception of climate risk
- Textual analysis of climate change-related news published by newspapers and media
- High carbon beta = highly exposed to carbon risk

Risk measure = carbon beta

Carbon beta

Let $R_i(t)$ be the return of stock i at time t. We assume that:

$$\begin{aligned} R_i(t) &= \alpha_i(t) + \beta_{i,\mathrm{mkt}}(t) R_{\mathrm{mkt}}(t) + \sum_{j=1}^{m} \beta_{i,\mathcal{F}_j}(t) R_{\mathcal{F}_j}(t) + \\ & \quad \boldsymbol{\beta_{i,\mathrm{Carbon}}(t) R_{\mathrm{Carbon}}(t)} + \varepsilon_i(t) \end{aligned}$$

where $R_{\mathrm{mkt}}(t)$ is the return of the market risk factor, $R_{\mathcal{F}_j}(t)$ is the return of the j^{th} alternative risk factor, $R_{\mathrm{Carbon}}(t)$ is the return of the carbon risk factor and $\varepsilon_i(t)$ is a white noise process

Remark

The carbon risk factor corresponds to a long/short portfolio between "green" and "brown" stocks

Climate beta

Engle *et al.* (2020) proposed a related approach where the carbon risk factor is replaced by a climate risk news index $\mathcal{I}_{\mathrm{Climate}}$:

$$
\begin{aligned}
R_i(t) &= \alpha_i(t) + \beta_{i,\mathrm{mkt}}(t) R_{\mathrm{mkt}}(t) + \sum_{j=1}^{m} \beta_{i,\mathcal{F}_j}(t) R_{\mathcal{F}_j}(t) + \\
&\quad \boldsymbol{\beta_{i,\mathrm{Climate}}(t) \mathcal{I}_{\mathrm{Climate}}(t)} + \varepsilon_i(t)
\end{aligned}
$$

Remark

The climate index $\mathcal{I}_{\mathrm{Climate}}$ *corresponds to a time series that measures the sentiment about the climate change. It is built using text mining and natural langage processing (NLP)*

Carbon beta

The carbon risk factor approach

Goal

The main objective is to define a **market** measure of carbon risk

Three-step approach

- Defining a brown green score (BGS) for each stock (scoring model)
- Building a brown minus green factor (Fama-French approach)
- Estimating the carbon beta of a stock with respect to the BMG factor (Multi-factor regression analysis)

Carbon beta = **market** measure of carbon risk

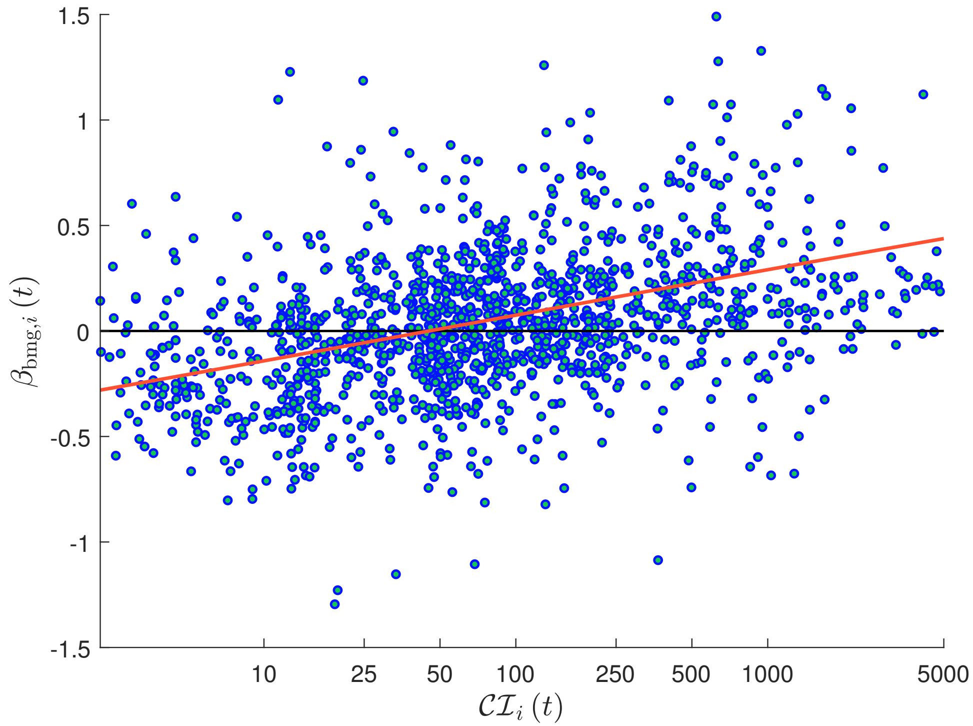
$\neq$

Carbon intensity = **fundamental** measure of carbon risk

Carbon beta

The carbon risk factor approach

Figure 207: Market-based vs fundamental-based measures of carbon risk



Source: Roncalli *et al.* (2021).

⇒ The market perception of a carbon risk measure depends on several dimensions: sector, country, etc.

Carbon beta

The carbon risk factor approach

Systematic carbon risk

- Common risk
- Carbon beta

Idiosyncratic carbon risk

- Specific risk
- Carbon intensity

Market measure ($\approx$ general carbon risk exposure, e.g. market repricing risk)

Fundamental measure ($\approx$ specific carbon risk exposure, e.g. reputational risk)

Carbon beta

The carbon risk factor approach

	Green	Neutral	Brown
Small	SG	SN	SB
Big	BG	BN	BB

The BMG factor return $R_{\mathrm{bmg}}(t)$ is derived from the Fama-French method:

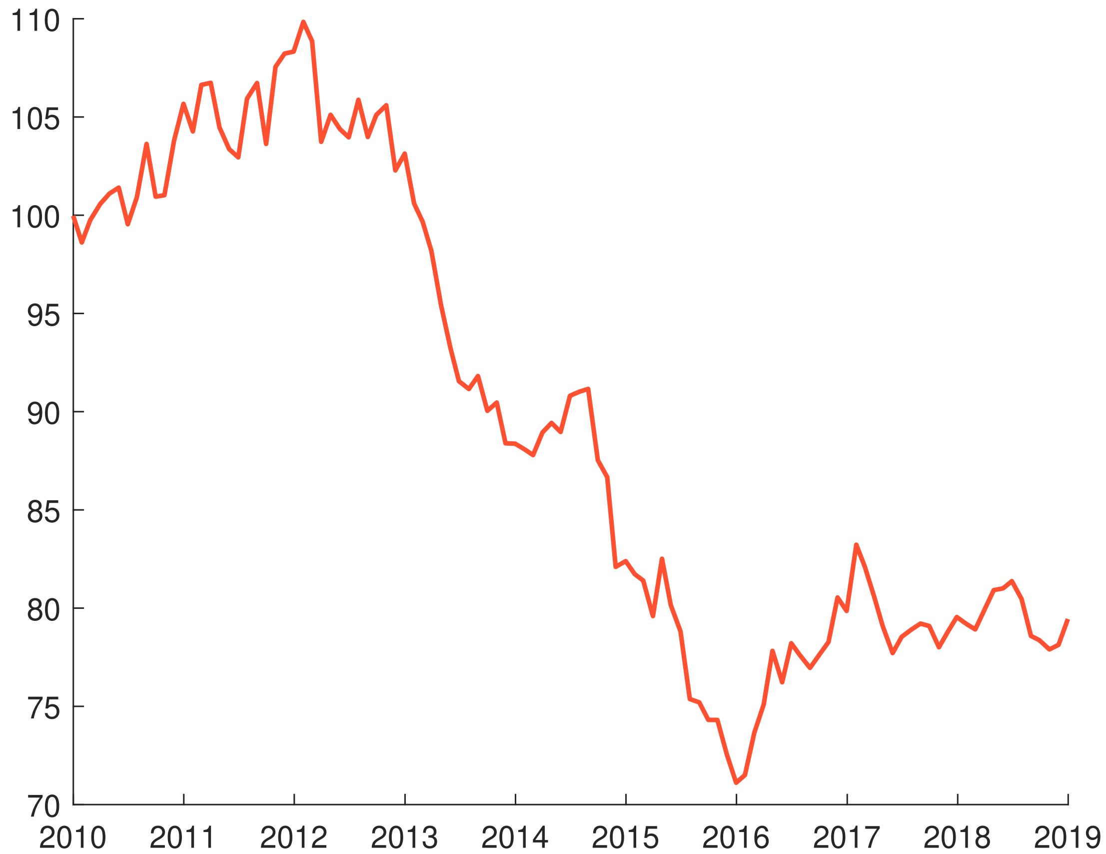
$$R_{\mathrm{bmg}}(t) = \frac{1}{2}\left(R_{\mathrm{SB}}(t) + R_{\mathrm{BB}}(t)\right) - \frac{1}{2}\left(R_{\mathrm{SG}}(t) + R_{\mathrm{BG}}(t)\right)$$

where the returns of each portfolio $R_j(t)$ (small green SG, big green BG, small brown SB, big brown BB) is value-weighted by the market capitalisation

$\Rightarrow$ The BMG factor is a Fama-French risk factor based on a scoring system (brown green score or BGS)

Carbon beta
The carbon risk factor approach

Figure 208: Cumulative performance of the BMG factor

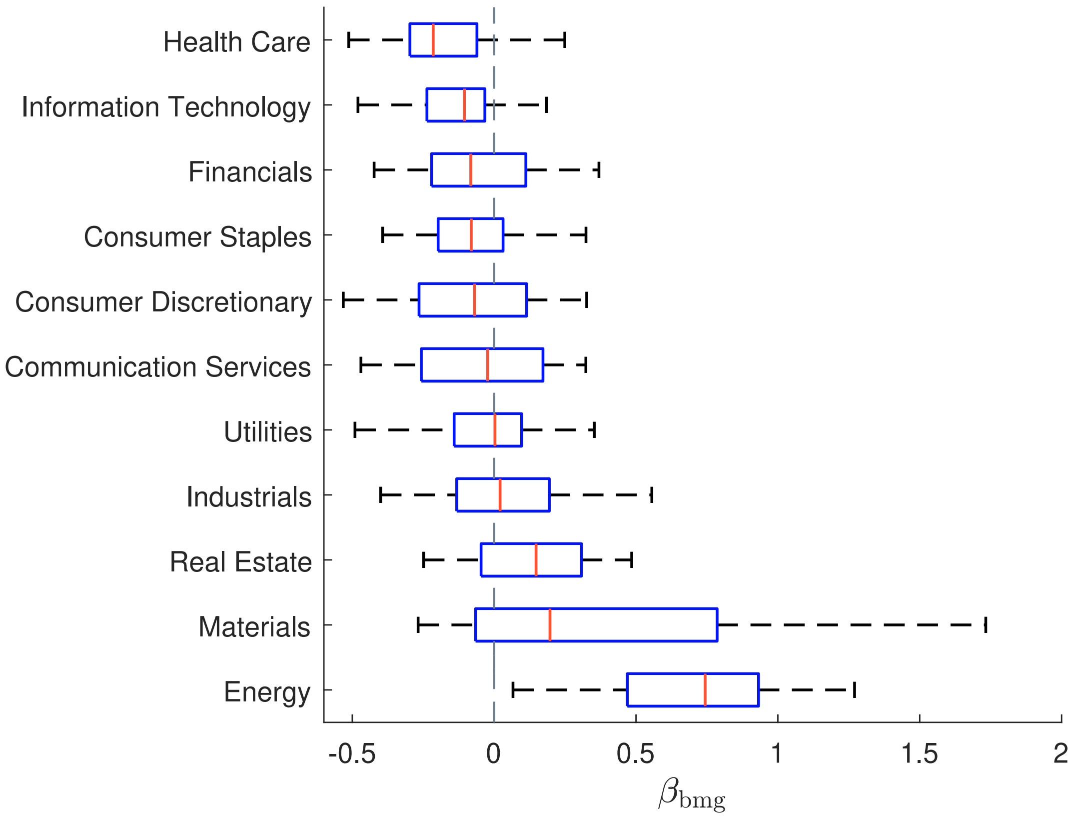


Source: Görgen *et al.* (2019).

Carbon beta

The carbon risk factor approach

Figure 209: Box plots of the carbon sensitivities[26]



Source: Roncalli *et al.* (2020).

[26] The box plots provide the median, the quartiles and the 5% and 95% quantiles

Carbon beta

The carbon risk factor approach

Relative carbon risk

- The right measure is β_{bmg}
- Sign matters
- **Negative exposure** is preferred

Absolute carbon risk

- The right measure is $|\beta_{\mathrm{bmg}}|$
- Sign doesn't matter
- **Zero exposure** is preferred

Two examples

1. We consider three portfolios with a carbon beta of -0.30, -0.05 and $+0.30$ respectively
2. We consider two portfolios with the following characteristics:
   - The value of the carbon beta is $+0.10$ and the stock dispersion of carbon beta is 0.20
   - The value of the carbon beta is -0.30 and the stock dispersion of carbon beta is 1.50

$\Rightarrow$ Impact of portfolio management and theory

Climate beta

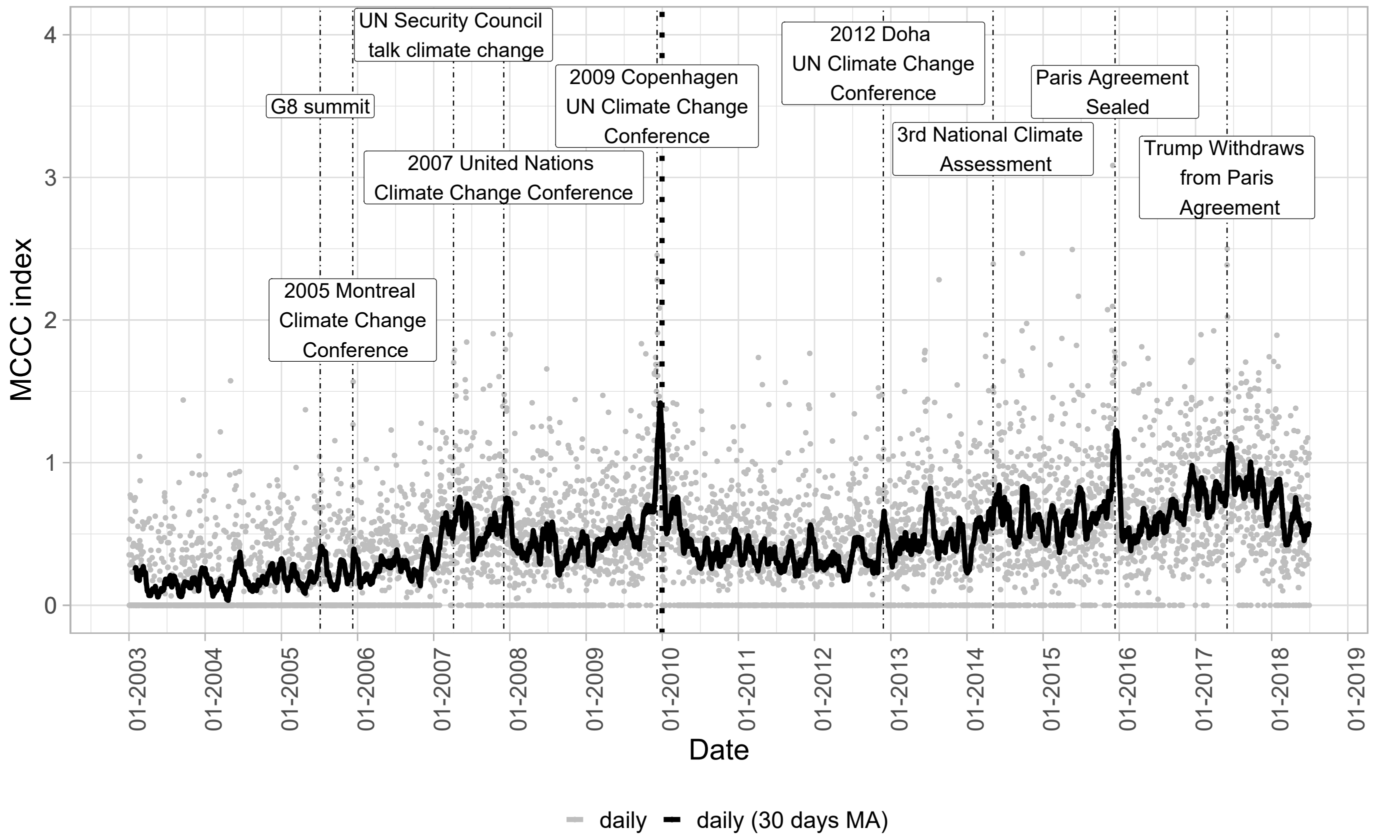
The climate index approach

- Two main references: Engle *et al.* (2020) & Ardia *et al.* (2021)
- We recall that brown assets must exhibit a positive risk premium
- Nevertheless, "*[...] If ESG concerns strengthen unexpectedly and sufficiently, green assets outperform brown ones despite having lower expected returns*" (Pástor *et al.*, 2021)
- Academics proxy concerns about climate change using climate indices based on news

Climate beta

The climate index approach

Figure 210: Media Climate Change Concerns (MCCC) index



Source: Ardia *et al.* (2021).

Course 2022-2023 in Sustainable Finance
Lecture 10. Climate Portfolio Construction

Thierry Roncalli*

*Amundi Asset Management[27]

*University of Paris-Saclay

March 2023

[27] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Quadratic programming

Definition

We have:

$$
\begin{aligned}
x^{\star} &= \arg\min \frac{1}{2} x^{\top} Q x - x^{\top} R \\
\text{s.t.} & \left\{ \begin{array}{l} Ax = B \\ Cx \leq D \\ x^{-} \leq x \leq x^{+} \end{array} \right.
\end{aligned}
$$

where x is a $n \times 1$ vector, Q is a $n \times n$ matrix, R is a $n \times 1$ vector, A is a $n_A \times n$ matrix, B is a $n_A \times 1$ vector, C is a $n_C \times n$ matrix, D is a $n_C \times 1$ vector, and x^{-} and x^{+} are two $n \times 1$ vectors

Quadratic form

A quadratic form is a polynomial with terms all of degree two

$$\mathcal{QF}(x_1, \ldots, x_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{i,j} x_i x_j = x^\top A x$$

Canonical form

$$\mathcal{QF}(x_1, \ldots, x_n) = \frac{1}{2}\left(x^\top A x + x^\top A^\top x\right) = \frac{1}{2} x^\top \left(A + A^\top\right) x = \frac{1}{2} x^\top Q x$$

Generalized quadratic form

$$\mathcal{QF}(x; Q, R, c) = \frac{1}{2} x^\top Q x - x^\top R + c$$

Quadratic form

Main properties

1. $\varphi \cdot \mathcal{QF}(w; Q, R, c) = \mathcal{QF}(w; \varphi Q, \varphi R, \varphi c)$
2. $\mathcal{QF}(x; Q_1, R_1, c_1) + \mathcal{QF}(x; Q_2, R_2, c_2) = \mathcal{QF}(x; Q_1 + Q_2, R_1 + R_2, c_1 + c_2)$
3. $\mathcal{QF}(x - y; Q, R, c) = \mathcal{QF}\left(x; Q, R + Qy, \frac{1}{2}y^\top Qy + y^\top R + c\right)$
4. $\mathcal{QF}(x - y; Q, R, c) = \mathcal{QF}\left(y; Q, Qx - R, \frac{1}{2}x^\top Qx - x^\top R + c\right)$
5. $\dfrac{1}{2}\sum_{i=1}^n q_i x_i^2 = \mathcal{QF}(x; \mathcal{D}(q), \mathbf{0}_n, 0)$ where $q = (q_1, \ldots, q_n)$ is a $n \times 1$ vector and $\mathcal{D}(q) = \mathrm{diag}(q)$
6. $\dfrac{1}{2}\sum_{i=1}^n q_i (x_i - y_i)^2 = \mathcal{QF}\left(x; \mathcal{D}(q), \mathcal{D}(q)y, \frac{1}{2}y^\top \mathcal{D}(q)y\right)$
7. $\dfrac{1}{2}\left(\sum_{i=1}^n q_i x_i\right)^2 = \mathcal{QF}(x; \mathcal{T}(q), \mathbf{0}_n, 0)$ where $\mathcal{T}(q) = qq^\top$
8. $\dfrac{1}{2}\left(\sum_{i=1}^n q_i (x_i - y_i)\right)^2 = \mathcal{QF}\left(x; \mathcal{T}(q), \mathcal{T}(q)y, \frac{1}{2}y^\top \mathcal{T}(q)y\right)$

Quadratic form

Main properties

We note $\omega = (\omega_1, \ldots, \omega_n)$ where $\omega_i = \mathbb{1}\{i \in \Omega\}$

1. $\frac{1}{2} \sum_{i \in \Omega} q_i x_i^2 = \mathcal{QF}\left(x; \mathcal{D}(\omega \circ q), \mathbf{0}_n, 0\right)$
2. $\frac{1}{2} \sum_{i \in \Omega} q_i (x_i - y_i)^2 =$
   $\mathcal{QF}\left(x; \mathcal{D}(\omega \circ q), \mathcal{D}(\omega \circ q) y, \frac{1}{2} y^\top \mathcal{D}(\omega \circ q) y\right)$
3. $\frac{1}{2} \left(\sum_{i \in \Omega} q_i x_i\right)^2 = \mathcal{QF}\left(x; \mathcal{T}(\omega \circ q), \mathbf{0}_n, 0\right)$
4. $\frac{1}{2} \left(\sum_{i \in \Omega} q_i (x_i - y_i)\right)^2 =$
   $\mathcal{QF}\left(x; \mathcal{T}(\omega \circ q), \mathcal{T}(\omega \circ q) y, \frac{1}{2} y^\top \mathcal{T}(\omega \circ q) y\right)$
5. $\mathcal{D}(\omega \circ q) = \operatorname{diag}(\omega \circ q) = \mathcal{D}(\omega) \mathcal{D}(q)$
6. $\mathcal{T}(\omega \circ q) = (\omega \circ q)(\omega \circ q)^\top = \left(\omega \omega^\top\right) \circ q q^\top = \mathcal{T}(\omega) \circ \mathcal{T}(q)$

Equity portfolio

Basic optimization problems

Mean-variance optimization

The long-only mean-variance optimization problem is given by:

$$
\begin{aligned}
w^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu \\
\text{s.t.} & \quad \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right.
\end{aligned}
$$

where:

- γ is the risk-tolerance coefficient
- the equality constraint is the budget constraint $\left(\sum_{i=1}^n w_i = 1\right)$
- the bounds correspond to the no short-selling restriction $(w_i \geq 0)$

QP form

$Q = \Sigma$, $R = \gamma\mu$, $A = \mathbf{1}_n^\top$, $B = 1$, $w^- = \mathbf{0}_n$ and $w^+ = \mathbf{1}$

Equity portfolio

Basic optimization problems

Tracking error optimization

The tracking error optimization problem is defined as:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - w^\top (\gamma\mu + \Sigma b) \\ &\text{s.t. } \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

QP form

$Q = \Sigma$, $\boxed{R = \gamma\mu + \Sigma b}$, $A = \mathbf{1}_n^\top$, $B = 1$, $w^- = \mathbf{0}_n$ and $w^+ = \mathbf{1}$

$\Rightarrow$ Portfolio replication: $\boxed{R = \Sigma b}$

Specification of the constraints

Sector weight constraint

- We have

$$s_j^- \leq \sum_{i \in \mathcal{S}ector_j} w_i \leq s_j^+$$

- $\boldsymbol{s}_j$ is the $n \times 1$ sector-mapping vector: $\boldsymbol{s}_{i,j} = \mathbb{1}\left\{i \in \mathcal{S}ector_j\right\}$
- We notice that:

$$\sum_{i \in \mathcal{S}ector_j} w_i = \boldsymbol{s}_j^\top w$$

- We deduce that:

$$s_j^- \leq \sum_{i \in \mathcal{S}ector_j} w_i \leq s_j^+ \Leftrightarrow \left\{ \begin{array}{l} s_j^- \leq \boldsymbol{s}_j^\top w \\ \boldsymbol{s}_j^\top w \leq s_j^+ \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} -\boldsymbol{s}_j^\top w \leq -s_j^- \\ \boldsymbol{s}_j^\top w \leq s_j^+ \end{array} \right.$$

QP form

$$\underbrace{\left(\begin{array}{c} -\boldsymbol{s}_j^\top \\ \boldsymbol{s}_j^\top \end{array} \right)}_{C} w \leq \underbrace{\left(\begin{array}{c} -s_j^- \\ s_j^+ \end{array} \right)}_{D}$$

Specification of the constraints
Score constraint

- General constraint:

$$\sum_{i=1}^{n} w_i \boldsymbol{\mathcal{S}}_i \geq \boldsymbol{\mathcal{S}}^\star \Leftrightarrow -\boldsymbol{\mathcal{S}}^\top w \leq -\boldsymbol{\mathcal{S}}^\star$$

QP form

- $C = -\boldsymbol{\mathcal{S}}^\top$
- $D = -\boldsymbol{\mathcal{S}}^\star$

Specification of the constraints

Score constraint

- Sector-specific constraint:

$$
\begin{aligned}
\sum_{i \in \mathcal{S}ector_j} w_i \mathcal{S}_i \geq \mathcal{S}_j^\star & \Leftrightarrow \sum_{i=1}^{n} \mathbb{1}\left\{i \in \mathcal{S}ector_j\right\} \cdot w_i \mathcal{S}_i \geq \mathcal{S}_j^\star \\
& \Leftrightarrow \sum_{i=1}^{n} \boldsymbol{s}_{i,j} w_i \mathcal{S}_i \geq \mathcal{S}_j^\star \\
& \Leftrightarrow \sum_{i=1}^{n} w_i \cdot \left(\boldsymbol{s}_{i,j} \mathcal{S}_i\right) \geq \mathcal{S}_j^\star \\
& \Leftrightarrow \left(\boldsymbol{s}_j \circ \mathcal{S}\right)^\top w \geq \mathcal{S}_j^\star
\end{aligned}
$$

QP form

- $C = -\left(\boldsymbol{s}_j \circ \mathcal{S}\right)^\top$
- $D = -\mathcal{S}_j^\star$

Equity portfolios

Example #1

- The capitalization-weighted equity index is composed of 8 stocks
- The weights are equal to 23%, 19%, 17%, 13%, 9%, 8%, 6% and 5%
- The ESG score, carbon intensity and sector of the eight stocks are the following:

Stock	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{S}$	−1.20	0.80	2.75	1.60	−2.75	−1.30	0.90	−1.70
$\mathcal{CI}$	125	75	254	822	109	17	341	741
$\mathcal{S}ector$	1	1	2	2	1	2	1	2

Equity portfolios

Example #1 (Cont'd)

- The stock volatilities are equal to 22%, 20%, 25%, 18%, 35%, 23%, 13% and 29%
- The correlation matrix is given by:

$$\mathbb{C} = \begin{pmatrix} 100\% & & & & & & & \\ 80\% & 100\% & & & & & & \\ 70\% & 75\% & 100\% & & & & & \\ 60\% & 65\% & 80\% & 100\% & & & & \\ 70\% & 50\% & 70\% & 85\% & 100\% & & & \\ 50\% & 60\% & 70\% & 80\% & 60\% & 100\% & & \\ 70\% & 50\% & 70\% & 75\% & 80\% & 50\% & 100\% & \\ 60\% & 65\% & 70\% & 75\% & 65\% & 70\% & 80\% & 100\% \end{pmatrix}$$

Equity portfolios

QP problem

- We have:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2} w^{\top} Q w - w^{\top} R \\
&\text{s.t.} \quad \left\{ \begin{array}{l} Aw = B \\ Cw \leq D \\ w^{-} \leq w \leq w^{+} \end{array} \right.
\end{aligned}
$$

Equity portfolios
Objective function

- Using $\Sigma_{i,j} = \mathbb{C}_{i,j}\sigma_i\sigma_j$, we obtain:

$$Q = \Sigma = 10^{-4} \times$$

$$\begin{pmatrix}
484.00 & 352.00 & 385.00 & 237.60 & 539.00 & 253.00 & 200.20 & 382.80 \\
352.00 & 400.00 & 375.00 & 234.00 & 350.00 & 276.00 & 130.00 & 377.00 \\
385.00 & 375.00 & 625.00 & 360.00 & 612.50 & 402.50 & 227.50 & 507.50 \\
237.60 & 234.00 & 360.00 & 324.00 & 535.50 & 331.20 & 175.50 & 391.50 \\
539.00 & 350.00 & 612.50 & 535.50 & 1225.00 & 483.00 & 364.00 & 659.75 \\
253.00 & 276.00 & 402.50 & 331.20 & 483.00 & 529.00 & 149.50 & 466.90 \\
200.20 & 130.00 & 227.50 & 175.50 & 364.00 & 149.50 & 169.00 & 301.60 \\
382.80 & 377.00 & 507.50 & 391.50 & 659.75 & 466.90 & 301.60 & 841.00
\end{pmatrix}$$

Equity portfolios
Objective function

- We have:

$$R = \Sigma b = \begin{pmatrix} 3.74 \\ 3.31 \\ 4.39 \\ 3.07 \\ 5.68 \\ 3.40 \\ 2.02 \\ 4.54 \end{pmatrix} \times 10^{-2}$$

Equity portfolios

Constraint specification (bounds)

- The portfolio is long-only

QP form

  - $w^{-} = \mathbf{0}_8$
  - $w^{+} = \mathbf{1}_8$

Equity portfolios
Constraint specification (equality)

- The budget constraint $\sum_{i=1}^{8} w_i = 1 \Rightarrow$ a first linear equation $A_0 w = B_0$

QP form

- $A_0 = \mathbf{1}_8^\top$
- $B_0 = 1$

Equity portfolios

Constraint specification (equality)

- We can impose the sector neutrality of the portfolio meaning that:

$$\sum_{i \in \mathcal{S}ector_j} w_i = \sum_{i \in \mathcal{S}ector_j} b_i$$

The sector neutrality constraint can be written as:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} w = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

QP form

- $A_1 = \boldsymbol{s}_1^\top = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$
- $A_2 = \boldsymbol{s}_2^\top = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
- $B_1 = \boldsymbol{s}_1^\top b = \sum_{i \in \mathcal{S}ector_1} b_i$
- $B_2 = \boldsymbol{s}_2^\top b = \sum_{i \in \mathcal{S}ector_2} b_i$

Equity portfolios

Constraint specification (inequality)

- We can impose a relative reduction of the benchmark carbon intensity:

$$\mathcal{CI}(w) \leq (1 - \mathcal{R})\,\mathcal{CI}(b) \Leftrightarrow C_1 w \leq D_1$$

QP form

- $C_1 = \mathcal{CI}^\top$ (because $\mathcal{CI}(w) = \mathcal{CI}^\top w$)
- $D_1 = (1 - \mathcal{R})\,\mathcal{CI}(b)$

- We can impose an absolute increase of the benchmark ESG score:

$$\mathcal{S}(w) \geq \mathcal{S}(b) + \Delta\mathcal{S}^\star$$

Since $\mathcal{S}(w) = \mathcal{S}^\top w$, we deduce that $C_2 w \leq D_2$

QP form

- $C_2 = -\mathcal{S}^\top$
- $D_2 = -\left(\mathcal{S}(b) + \Delta\mathcal{S}^\star\right)$

Equity portfolios

Combination of constraints

Set of constraints	Carbon intensity	ESG score	Sector neutrality	A	B	C	D
#1	✓			A_0	B_0	C_1	D_1
#2		✓		A_0	B_0	C_2	D_2
#3	✓	✓		A_0	B_0	$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$	$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$
#4	✓	✓	✓	$\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix}$	$\begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix}$	$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$	$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$

Equity portfolios

Results

Table 113: $\mathcal{R} = 30\%$ and $\Delta\mathcal{S}^\star = 0.50$ (Example #1)

		Benchmark	Set #1	Set #2	Set #3	Set #4
Weights (in %)	$w_1^\star$	23.00	18.17	25.03	8.64	12.04
	$w_2^\star$	19.00	24.25	14.25	29.27	23.76
	$w_3^\star$	17.00	16.92	21.95	26.80	30.55
	$w_4^\star$	13.00	2.70	27.30	1.48	2.25
	$w_5^\star$	9.00	12.31	3.72	10.63	8.51
	$w_6^\star$	8.00	11.23	1.34	6.30	10.20
	$w_7^\star$	6.00	11.28	1.68	16.87	12.69
	$w_8^\star$	5.00	3.15	4.74	0.00	0.00
Statistics	$\sigma(w^\star \mid b)$ (in %)	0.00	0.50	1.18	1.90	2.12
	$\mathcal{CI}(w^\star)$	261.72	183.20	367.25	183.20	183.20
	$\mathcal{R}(w^\star \mid b)$ (in %)		30.00	−40.32	30.00	30.00
	$\mathcal{S}(w^\star)$	0.17	0.05	0.67	0.67	0.67
	$\mathcal{S}(w^\star) - \mathcal{S}(b)$		−0.12	0.50	0.50	0.50
	$w^\star(\mathcal{S}ector_1)$ (in %)	57.00	66.00	44.67	65.41	57.00
	$w^\star(\mathcal{S}ector_2)$ (in %)	43.00	34.00	55.33	34.59	43.00

Equity portfolios

Dealing with constraints on relative weights

- The carbon intensity of the j^{th} sector within the portfolio w is:

$$\mathcal{CI}\left(w; \mathcal{S}ector_j\right) = \sum_{i \in \mathcal{S}ector_j} \tilde{w}_i \mathcal{CI}_i$$

where $\tilde{w}_i$ is the normalized weight in the sector bucket:

$$\tilde{w}_i = \frac{w_i}{\sum_{k \in \mathcal{S}ector_j} w_k}$$

- Another expression of $\mathcal{CI}\left(w; \mathcal{S}ector_j\right)$ is:

$$\mathcal{CI}\left(w; \mathcal{S}ector_j\right) = \frac{\sum_{i \in \mathcal{S}ector_j} w_i \mathcal{CI}_i}{\sum_{i \in \mathcal{S}ector_j} w_i} = \frac{\left(\boldsymbol{s}_j \circ \mathcal{CI}\right)^\top w}{\boldsymbol{s}_j^\top w}$$

Equity portfolios

Dealing with constraints on relative weights

- If we consider the constraint $\mathcal{CI}\left(w;\mathcal{S}ector_j\right) \leq \mathcal{CI}_j^{\star}$, we obtain:

$$
\begin{aligned}
(*) \quad & \Leftrightarrow \quad \mathcal{CI}\left(w;\mathcal{S}ector_j\right) \leq \mathcal{CI}_j^{\star} \\
& \Leftrightarrow \quad \left(\boldsymbol{s}_j \circ \mathcal{CI}\right)^\top w \leq \mathcal{CI}_j^{\star}\left(\boldsymbol{s}_j^\top w\right) \\
& \Leftrightarrow \quad \left(\left(\boldsymbol{s}_j \circ \mathcal{CI}\right) - \mathcal{CI}_j^{\star}\boldsymbol{s}_j\right)^\top w \leq 0 \\
& \Leftrightarrow \quad \left(\boldsymbol{s}_j \circ \left(\mathcal{CI} - \mathcal{CI}_j^{\star}\right)\right)^\top w \leq 0
\end{aligned}
$$

QP form

- $C = \left(\boldsymbol{s}_j \circ \left(\mathcal{CI} - \mathcal{CI}_j^{\star}\right)\right)^\top$
- $D = 0$

Equity portfolios

Dealing with constraints on relative weights

Example #2

- Example #1
- We would like to reduce the carbon footprint of the benchmark by 30%
- We impose the sector neutrality

Equity portfolios

Dealing with constraints on relative weights

QP form

- $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
- $B = \begin{pmatrix} 100\% \\ 57\% \\ 43\% \end{pmatrix}$
- $C = \begin{pmatrix} 125 & 75 & 254 & 822 & 109 & 17 & 341 & 741 \end{pmatrix}$
- $D = 183.2040$

Equity portfolios

Dealing with constraints on relative weights

- The optimal solution is:

$$w^\star = (21.54\%, 18.50\%, 21.15\%, 3.31\%, 10.02\%, 15.26\%, 6.94\%, 3.27\%)$$

- $\sigma(w^\star \mid b) = 112$ bps
- $\mathcal{CI}(w^\star) = 183.20$ vs. $\mathcal{CI}(b) = 261.72$

BUT

$$\left\{\begin{array}{l} \mathcal{CI}(w^\star; \mathcal{S}ector_1) = 132.25 \\ \mathcal{CI}(w^\star; \mathcal{S}ector_2) = 250.74 \end{array}\right. \quad \text{versus} \quad \left\{\begin{array}{l} \mathcal{CI}(b; \mathcal{S}ector_1) = 128.54 \\ \mathcal{CI}(b; \mathcal{S}ector_2) = 438.26 \end{array}\right.$$

The global reduction of 30% is explained by:

- an increase of 2.89% of the carbon footprint for the first sector
- a decrease of 42.79% of the carbon footprint for the second sector

Equity portfolios

Dealing with constraints on relative weights

- We impose $\mathcal{R}_1 = 20\%$

QP form

- $C = \begin{pmatrix} \mathcal{CI}^\top \\ \left(\boldsymbol{s}_1 \circ \left(\mathcal{CI} - (1 - \mathcal{R}_1)\,\mathcal{CI}\,(b; \mathcal{S}ector_1)\right)\right)^\top \end{pmatrix} =$
$\begin{pmatrix} 125 & 75 & 254 & 822 & 109 & 17 & 341 & 741 \\ 22.1649 & -27.8351 & 0 & 0 & 6.1649 & 0 & 238.1649 & 0 \end{pmatrix}$
- $D = \begin{pmatrix} 183.2040 \\ 0 \end{pmatrix}$

Equity portfolios

Dealing with constraints on relative weights

- Solving the new QP problem gives the following optimal portfolio:

$$w^{\star} = (22.70\%, 22.67\%, 19.23\%, 5.67\%, 11.39\%, 14.50\%, 0.24\%, 3.61\%)$$

- $\sigma\left(w^{\star} \mid b\right) = 144$ bps
- $\mathcal{CI}\left(w^{\star}\right) = 183.20$
  - $\mathcal{CI}\left(w^{\star}; \mathcal{S}ector_1\right) = 102.84$ (reduction of 20%)
  - $\mathcal{CI}\left(w^{\star}; \mathcal{S}ector_2\right) = 289.74$ (reduction of 33.89%)

Risk measure of a bond portfolio

- We consider a zero-coupon bond, whose price and maturity date are $B(t,T)$ and T:

$$B_t(t,T) = e^{-(r(t)+s(t))(T-t)+L(t)}$$

where $r(t)$, $s(t)$ and $L(t)$ are the interest rate, the credit spread and the liquidity premium

- We deduce that:

$$\begin{aligned}
\mathrm{d}\ln B(t,T) &= -(T-t)\,\mathrm{d}r(t) - (T-t)\,\mathrm{d}s(t) + \mathrm{d}L(t) \\
&= -D\,\mathrm{d}r(t) - (D\,s(t))\frac{\mathrm{d}s(t)}{s(t)} + \mathrm{d}L(t) \\
&= -D\,\mathrm{d}r(t) - \mathrm{DTS}(t)\frac{\mathrm{d}s(t)}{s(t)} + \mathrm{d}L(t)
\end{aligned}$$

where:

  - $D = T - t$ is the remaining maturity (or duration)
  - $\mathrm{DTS}(t)$ is the duration-times-spread factor

Risk measure of a bond portfolio

- If we assume that $r(t)$, $s(t)$ and $L(t)$ are independent, the risk of the defaultable bond is equal to:

$$\sigma^2 \left(\mathrm{d} \ln B(t,T)\right) = D^2 \sigma^2 \left(\mathrm{d} r(t)\right) + \mathrm{DTS}(t)^2 \sigma^2 \left(\frac{\mathrm{d} s(t)}{s(t)}\right) + \sigma^2 \left(\mathrm{d} L(t)\right)$$

- Three risk components

$$\sigma^2 \left(\mathrm{d} \ln B(t,T)\right) = D^2 \sigma_r^2 + \mathrm{DTS}(t)^2 \sigma_s^2 + \sigma_L^2$$

$\Longrightarrow$ **The historical volatility of a bond price is not a relevant risk measure**

Bond portfolio optimization

Without a benchmark

- Duration risk:

$$\mathrm{MD}(w) = \sum_{i=1}^{n} w_i \, \mathrm{MD}_i$$

- DTS risk:

$$\mathrm{DTS}(w) = \sum_{i=1}^{n} w_i \, \mathrm{DTS}_i$$

- Clustering approach = generalization of the sector approach, e.g. (EUR, Financials, AAA to A−, 1Y-3Y)
- We have:

$$\mathrm{MD}_j(w) = \sum_{i \in \mathcal{S}ector_j} w_i \, \mathrm{MD}_i$$

and:

$$\mathrm{DTS}_j(w) = \sum_{i \in \mathcal{S}ector_j} w_i \, \mathrm{DTS}_i$$

Bond portfolio optimization

Without a benchmark

Objective function without a benchmark

We have:

$$\begin{aligned} w^{\star} &= \arg\min \frac{\varphi_{\mathrm{MD}}}{2} \sum_{j=1}^{n_{\mathcal{S}ector}} \left(\mathrm{MD}_j(w) - \mathrm{MD}_j^{\star}\right)^2 + \\ & \frac{\varphi_{\mathrm{DTS}}}{2} \sum_{j=1}^{n_{\mathcal{S}ector}} \left(\mathrm{DTS}_j(w) - \mathrm{DTS}_j^{\star}\right)^2 - \gamma \sum_{i=1}^{n} w_i \mathcal{C}_i \end{aligned}$$

where:

- $\varphi_{\mathrm{MD}} \geq 0$ and $\varphi_{\mathrm{DTS}} \geq 0$ indicate the relative weight of each risk component
- $\mathcal{C}_i$ is the expected carry of bond i and γ is the risk-tolerance coefficient

Bond portfolio optimization

Without a benchmark

QP form

$$
\begin{aligned}
w^{\star} & = & \arg\min \mathcal{QF}\left(w; Q, R, c\right) \\
& \text{s.t.} & \left\{ \begin{array}{l} \mathbf{1}_n^{\top} w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right.
\end{aligned}
$$

where $\mathcal{QF}\left(w; Q, R, c\right)$ is the quadratic form of the objective function

Bond portfolio optimization
Without a benchmark

We have:

$$\begin{aligned}\frac{1}{2}\left(\mathrm{MD}_j(w)-\mathrm{MD}_j^\star\right)^2 &= \frac{1}{2}\left(\sum\nolimits_{i\in\mathcal{S}ector_j} w_i\,\mathrm{MD}_i-\mathrm{MD}_j^\star\right)^2 \\ &= \frac{1}{2}\left(\sum_{i=1}^n s_{i,j}w_i\,\mathrm{MD}_i-\mathrm{MD}_j^\star\right)^2 \\ &= \frac{1}{2}\left(\sum_{i=1}^n s_{i,j}\,\mathrm{MD}_i\,w_i\right)^2-w^\top\left(s_j\circ\mathrm{MD}\right)\mathrm{MD}_j^\star+\frac{1}{2}\mathrm{MD}_j^{\star 2} \\ &= \mathcal{QF}\left(w;\mathcal{T}\left(s_j\circ\mathrm{MD}\right),\left(s_j\circ\mathrm{MD}\right)\mathrm{MD}_j^\star,\frac{1}{2}\mathrm{MD}_j^{\star 2}\right)\end{aligned}$$

where $\mathrm{MD}=(\mathrm{MD}_1,\ldots,\mathrm{MD}_n)$ is the vector of modified durations and $\mathcal{T}(u)=uu^\top$

Bond portfolio optimization
Without a benchmark

We deduce that:

$$\frac{1}{2}\sum_{j=1}^{n_{\mathcal{S}ector}}\left(\mathrm{MD}_j\left(w\right)-\mathrm{MD}_j^{\star}\right)^2=\mathcal{QF}\left(w;Q_{\mathrm{MD}},R_{\mathrm{MD}},c_{\mathrm{MD}}\right)$$

where:

$$\left\{\begin{array}{l}Q_{\mathrm{MD}}=\sum_{j=1}^{n_{\mathcal{S}ector}}\mathcal{T}\left(\boldsymbol{s}_j\circ\mathrm{MD}\right)\\R_{\mathrm{MD}}=\sum_{j=1}^{n_{\mathcal{S}ector}}\left(\boldsymbol{s}_j\circ\mathrm{MD}\right)\mathrm{MD}_j^{\star}\\c_{\mathrm{MD}}=\frac{1}{2}\sum_{j=1}^{n_{\mathcal{S}ector}}\mathrm{MD}_j^{\star^2}\end{array}\right.$$

Bond portfolio optimization

Without a benchmark

In a similar way, we have:

$$\frac{1}{2}\sum_{j=1}^{n_{\mathcal{S}ector}} \left(\mathrm{DTS}_j(w) - \mathrm{DTS}_j^{\star}\right)^2 = \mathcal{QF}\left(w; Q_{\mathrm{DTS}}, R_{\mathrm{DTS}}, c_{\mathrm{DTS}}\right)$$

where:

$$\left\{\begin{array}{l} Q_{\mathrm{DTS}} = \sum_{j=1}^{n_{\mathcal{S}ector}} \mathcal{T}\left(\boldsymbol{s}_j \circ \mathrm{DTS}\right) \\ R_{\mathrm{MD}} = \sum_{j=1}^{n_{\mathcal{S}ector}} \left(\boldsymbol{s}_j \circ \mathrm{DTS}\right) \mathrm{DTS}_j^{\star} \\ c_{\mathrm{DTS}} = \frac{1}{2}\sum_{j=1}^{n_{\mathcal{S}ector}} \mathrm{DTS}_j^{\star^2} \end{array}\right.$$

Bond portfolio optimization

Without a benchmark

We have:

$$-\gamma \sum_{i=1}^{n} w_i \mathcal{C}_i = \gamma \mathcal{QF}\left(w; \mathbf{0}_{n,n}, \mathcal{C}, 0\right) = \mathcal{QF}\left(w; \mathbf{0}_{n,n}, \gamma \mathcal{C}, 0\right)$$

where $\mathcal{C} = (\mathcal{C}_1, \ldots, \mathcal{C}_n)$ is the vector of expected carry values

Bond portfolio optimization

Without a benchmark

Quadratic form of the objective function

The function to optimize is:

$$\begin{aligned} \mathcal{QF}\left(w; Q, R, c\right) &= \varphi_{\mathrm{MD}} \mathcal{QF}\left(w; Q_{\mathrm{MD}}, R_{\mathrm{MD}}, c_{\mathrm{MD}}\right) + \\ & \quad \varphi_{\mathrm{DTS}} \mathcal{QF}\left(w; Q_{\mathrm{DTS}}, R_{\mathrm{DTS}}, c_{\mathrm{DTS}}\right) + \\ & \quad \mathcal{QF}\left(w; \mathbf{0}_{n,n}, \gamma \mathcal{C}, 0\right) \end{aligned}$$

where:

$$\left\{ \begin{array}{l} Q = \varphi_{\mathrm{MD}} Q_{\mathrm{MD}} + \varphi_{\mathrm{DTS}} Q_{\mathrm{DTS}} \\ R = \gamma \mathcal{C} + \varphi_{\mathrm{MD}} R_{\mathrm{MD}} + \varphi_{\mathrm{DTS}} R_{\mathrm{DTS}} \\ c = \varphi_{\mathrm{MD}} c_{\mathrm{MD}} + \varphi_{\mathrm{DTS}} c_{\mathrm{DTS}} \end{array} \right.$$

Bond portfolio optimization

With a benchmark

- The MD- and DTS-based tracking error variances are equal to:

$$\mathcal{R}_{\mathrm{MD}}(w \mid b) = \sigma^2_{\mathrm{MD}}(w \mid b) = \sum_{j=1}^{n_{\mathcal{S}ector}} \left(\sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{MD}_i \right)^2$$

and:

$$\mathcal{R}_{\mathrm{DTS}}(w \mid b) = \sigma^2_{\mathrm{DTS}}(w \mid b) = \sum_{j=1}^{n_{\mathcal{S}ector}} \left(\sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{DTS}_i \right)^2$$

This means that $\mathrm{MD}_j^\star = \sum_{i \in \mathcal{S}ector_j} b_i \, \mathrm{MD}_i$ and $\mathrm{DTS}_j^\star = \sum_{i \in \mathcal{S}ector_j} b_i \, \mathrm{DTS}_i$.

- The active share risk is defined as:

$$\mathcal{R}_{\mathrm{AS}}(w \mid b) = \sigma^2_{\mathrm{AS}}(w \mid b) = \sum_{i=1}^{n} (w_i - b_i)^2$$

Bond portfolio optimization
With a benchmark

Objective function with a benchmark

The optimization problem becomes:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2}\mathcal{R}\left(w \mid b\right) - \gamma \sum_{i=1}^{n} \left(w_i - b_i\right) \mathcal{C}_i \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

where the synthetic risk measure is equal to:

$$\mathcal{R}\left(w \mid b\right) = \varphi_{\mathrm{AS}} \mathcal{R}_{\mathrm{AS}}\left(w \mid b\right) + \varphi_{\mathrm{MD}} \mathcal{R}_{\mathrm{MD}}\left(w \mid b\right) + \varphi_{\mathrm{DTS}} \mathcal{R}_{\mathrm{DTS}}\left(w \mid b\right)$$

Bond portfolio optimization

With a benchmark

We can show that

$$\begin{aligned} w^{\star} &= \arg\min \mathcal{QF}\left(w; Q\left(b\right), R\left(b\right), c\left(b\right)\right) \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

where:

$$\left\{ \begin{array}{l} Q\left(b\right) = \varphi_{\mathrm{AS}} Q_{\mathrm{AS}}\left(b\right) + \varphi_{\mathrm{MD}} Q_{\mathrm{MD}}\left(b\right) + \varphi_{\mathrm{DTS}} Q_{\mathrm{DTS}}\left(b\right) \\ R\left(b\right) = \gamma\mathcal{C} + \varphi_{\mathrm{AS}} R_{\mathrm{AS}}\left(b\right) + \varphi_{\mathrm{MD}} R_{\mathrm{MD}}\left(b\right) + \varphi_{\mathrm{DTS}} R_{\mathrm{DTS}}\left(b\right) \\ c\left(b\right) = \gamma b^\top\mathcal{C} + \varphi_{\mathrm{AS}} c_{\mathrm{AS}}\left(b\right) + \varphi_{\mathrm{MD}} c_{\mathrm{MD}}\left(b\right) + \varphi_{\mathrm{DTS}} c_{\mathrm{DTS}}\left(b\right) \end{array} \right.$$

$Q_{\mathrm{AS}}\left(b\right) = I_n$, $R_{\mathrm{AS}}\left(b\right) = b$, $c_{\mathrm{AS}}\left(b\right) = \frac{1}{2} b^\top b$, $Q_{\mathrm{MD}}\left(b\right) = Q_{\mathrm{MD}}$, $R_{\mathrm{MD}}\left(b\right) = Q_{\mathrm{MD}} b = R_{\mathrm{MD}}$, $c_{\mathrm{MD}}\left(b\right) = \frac{1}{2} b^\top Q_{\mathrm{MD}} b = c_{\mathrm{MD}}$, $Q_{\mathrm{DTS}}\left(b\right) = Q_{\mathrm{DTS}}$, $R_{\mathrm{DTS}}\left(b\right) = Q_{\mathrm{DTS}} b = R_{\mathrm{DTS}}$, and $c_{\mathrm{DTS}}\left(b\right) = \frac{1}{2} b^\top Q_{\mathrm{DTS}} b = c_{\mathrm{DTS}}$

Bond portfolio optimization
With a benchmark

Example #3

We consider an investment universe of 9 corporate bonds with the following characteristics[a]:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8	#9
b_i	21	19	16	12	11	8	6	4	3
$\mathcal{CI}_i$	111	52	369	157	18	415	17	253	900
MD_i	3.16	6.48	3.54	9.23	6.40	2.30	8.12	7.96	5.48
DTS_i	107	255	75	996	289	45	620	285	125
$\mathcal{S}ector$	1	1	1	2	2	2	3	3	3

We impose that $0.25 \times b_i \leq w_i \leq 4 \times b_i$. We have $\varphi_{\mathrm{AS}} = 100$, $\varphi_{\mathrm{MD}} = 25$ and $\varphi_{\mathrm{DTS}} = 0.001$.

[a] The units are: b_i in %, $\mathcal{CI}_i$ in $\mathrm{tCO_2e/\$\ mn}$, MD_i in years and DTS_i in bps

Bond portfolio optimization

With a benchmark

The optimization problem is defined as:

$$\begin{array}{rcl} w^{\star}\left(\mathcal{R}\right) & = & \arg\min \dfrac{1}{2} w^{\top} Q\left(b\right) w - w^{\top} R\left(b\right) \\ & & \text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_9^{\top} w = 1 \\ \mathcal{CI}^{\top} w \leq \left(1 - \mathcal{R}\right) \mathcal{CI}\left(b\right) \\ \dfrac{b}{4} \leq w \leq 4b \end{array} \right. \end{array}$$

where $\mathcal{R}$ is the reduction rate

Bond portfolio optimization

With a benchmark

Since the bonds are ordering by sectors, $Q(b)$ is a block diagonal matrix:

$$Q(b) = \begin{pmatrix} Q_1 & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & Q_2 & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & Q_3 \end{pmatrix} \times 10^3$$

where:

$$Q_1 = \begin{pmatrix} 0.3611 & 0.5392 & 0.2877 \\ 0.5392 & 1.2148 & 0.5926 \\ 0.2877 & 0.5926 & 0.4189 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 3.2218 & 1.7646 & 0.5755 \\ 1.7646 & 1.2075 & 0.3810 \\ 0.5755 & 0.3810 & 0.2343 \end{pmatrix}$$

and:

$$Q_3 = \begin{pmatrix} 2.1328 & 1.7926 & 1.1899 \\ 1.7926 & 1.7653 & 1.1261 \\ 1.1899 & 1.1261 & 0.8664 \end{pmatrix}$$

$R(b) = (2.243, 4.389, 2.400, 6.268, 3.751, 1.297, 2.354, 2.120, 1.424) \times 10^2$

Bond portfolio optimization

With a benchmark

Table 114: Weights in % of optimized bond portfolios (Example #3)

Portfolio	#1	#2	#3	#4	#5	#6	#7	#8	#9
b	21.00	19.00	16.00	12.00	11.00	8.00	6.00	4.00	3.00
$w^\star$ (10%)	21.92	19.01	15.53	11.72	11.68	7.82	6.68	4.71	0.94
$w^\star$ (30%)	26.29	20.24	10.90	10.24	16.13	3.74	9.21	2.50	0.75
$w^\star$ (50%)	27.48	23.97	4.00	6.94	22.70	2.00	11.15	1.00	0.75

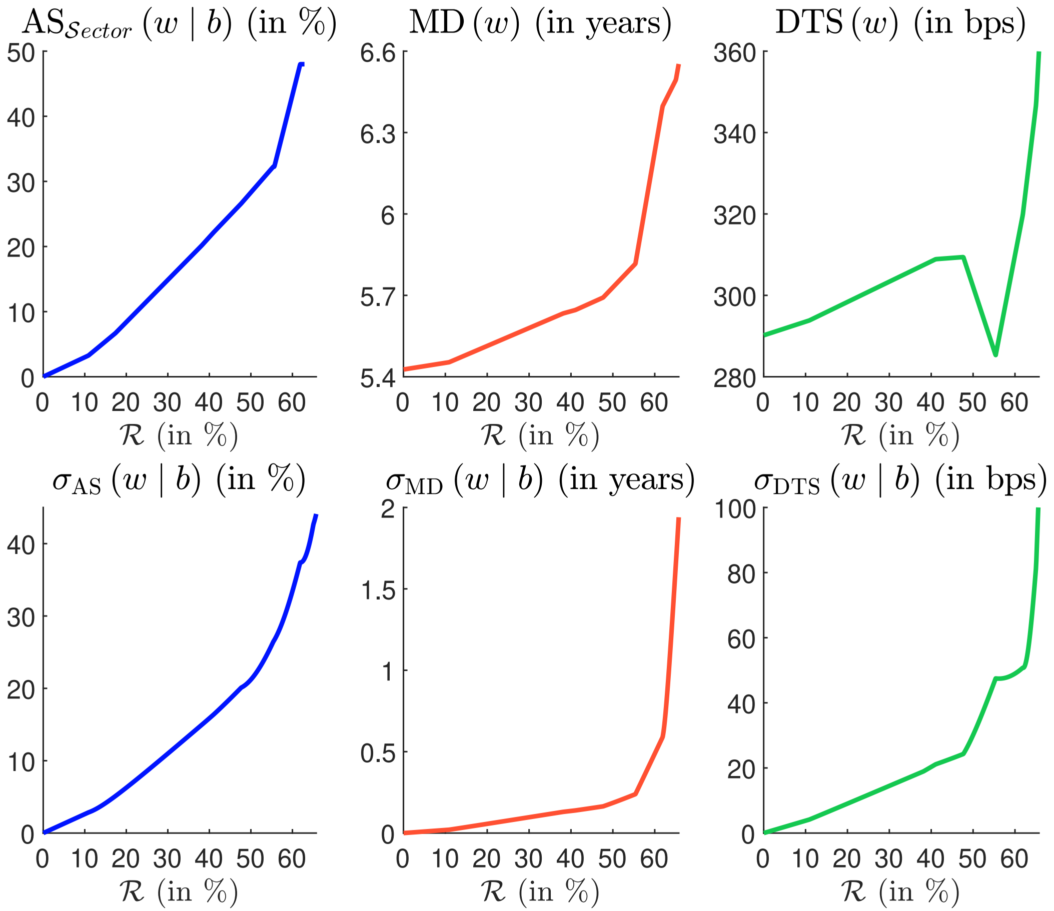
Table 115: Risk statistics of optimized bond portfolios (Example #3)

Portfolio	$\mathrm{AS}_{\mathcal{S}ector}$ (in %)	$\mathrm{MD}(w)$ (in years)	$\mathrm{DTS}(w)$ (in bps)	$\sigma_{\mathrm{AS}}(w \mid b)$ (in %)	$\sigma_{\mathrm{MD}}(w \mid b)$ (in years)	$\sigma_{\mathrm{DTS}}(w \mid b)$ (in bps)	$\mathcal{CI}(w)$ $\mathrm{gCO_2e}/\$$
b	0.00	5.43	290.18	0.00	0.00	0.00	184.39
$w^\star$ (10%)	3.00	5.45	293.53	2.62	0.02	3.80	165.95
$w^\star$ (30%)	14.87	5.58	303.36	10.98	0.10	14.49	129.07
$w^\star$ (50%)	28.31	5.73	302.14	21.21	0.19	30.11	92.19

Bond portfolio optimization

With a benchmark

Figure 211: Relationship between the reduction rate and the tracking risk (Example #3)



Advanced optimization problems
Large bond universe

- QP: $\mathbf{n \leq 5\,000}$ (the dimension of Q is $n \times n$)
- LP: $\mathbf{n \gg 10^6}$

- Some figures as of 31/01/2023
  - MSCI World Index (DM): $n = 1\,508$ stocks
  - MSCI World IMI (DM): $n = 5\,942$ stocks
  - MSCI World AC (DM + EM): $n = 2\,882$ stocks
  - MSCI World AC IMI (DM + EM): $n = 7\,928$ stocks

  - Bloomberg Global Aggregate Total Return Index: $n = 28\,799$ securities
  - ICE BOFA Global Broad Market Index: $n = 33\,575$ securities

- Trick: $\mathcal{L}_2$-norm risk measures $\Rightarrow$ $\mathcal{L}_1$-norm risk measures

Advanced optimization problems
Large bond universe

We replace the synthetic risk measure by:

$$\mathcal{D}(w \mid b) = \varphi'_{\mathrm{AS}} \mathcal{D}_{\mathrm{AS}}(w \mid b) + \varphi'_{\mathrm{MD}} \mathcal{D}_{\mathrm{MD}}(w \mid b) + \varphi'_{\mathrm{DTS}} \mathcal{D}_{\mathrm{DTS}}(w \mid b)$$

where:

$$\begin{aligned}
\mathcal{D}_{\mathrm{AS}}(w \mid b) &= \frac{1}{2} \sum_{i=1}^{n} \left| w_i - b_i \right| \\
\mathcal{D}_{\mathrm{MD}}(w \mid b) &= \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum\nolimits_{i \in \mathcal{S}ector_j} (w_i - b_i) \, \mathrm{MD}_i \right| \\
\mathcal{D}_{\mathrm{DTS}}(w \mid b) &= \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum\nolimits_{i \in \mathcal{S}ector_j} (w_i - b_i) \, \mathrm{DTS}_i \right|
\end{aligned}$$

Advanced optimization problems
Large bond universe

The optimization problem becomes:

$$\begin{aligned} w^{\star} &= \arg\min \mathcal{D}\left(w \mid b\right) - \gamma \sum_{i=1}^{n} \left(w_i - b_i\right) \mathcal{C}_i \\ \text{s.t.} & \left\{ \begin{array}{l} \mathbf{1}_n^{\top} w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

Advanced optimization problems

Large bond universe

Absolute value trick

If $c_i \geq 0$, then:

$$\min \sum_{i=1}^{n} c_i \left| f_i\left(x\right) \right| + g\left(x\right) \Leftrightarrow \left\{ \begin{array}{ll} \min & \displaystyle\sum_{i=1}^{n} c_i \tau_i + g\left(x\right) \\ \text{s.t.} & \left\{ \begin{array}{l} \left| f_i\left(x\right) \right| \leq \tau_i \\ \tau_i \geq 0 \end{array} \right. \end{array} \right.$$

The problem becomes linear:

$$\left| f_i\left(x\right) \right| \leq \tau_i \Leftrightarrow -\tau_i \leq f_i\left(x\right) \wedge f_i\left(x\right) \leq \tau_i$$

Advanced optimization problems

Large bond universe

Linear programming

The standard formulation of a linear programming problem is:

$$
\begin{aligned}
x^{\star} &= \arg\min c^{\top} x \\
\text{s.t.} & \left\{ \begin{array}{l} Ax = b \\ Cx \leq D \\ x^{-} \leq x \leq x^{+} \end{array} \right.
\end{aligned}
$$

where x is a $n \times 1$ vector, c is a $n \times 1$ vector, A is a $n_A \times n$ matrix, B is a $n_A \times 1$ vector, C is a $n_C \times n$ matrix, D is a $n_C \times 1$ vector, and x^{-} and x^{+} are two $n \times 1$ vectors.

Advanced optimization problems

Large bond universe

We have:

$$
\begin{aligned}
w^{\star} = & \arg\min \frac{1}{2}\varphi'_{\mathrm{AS}} \sum_{i=1}^{n} \tau_{i,w} + \varphi'_{\mathrm{MD}} \sum_{j=1}^{n_{\mathcal{S}ector}} \tau_{j,\mathrm{MD}} + \varphi'_{\mathrm{DTS}} \sum_{j=1}^{n_{\mathcal{S}ector}} \tau_{j,\mathrm{DTS}} - \\
& \gamma \sum_{i=1}^{n} (w_i - b_i)\, \mathcal{C}_i \\
\text{s.t.} & \left\{
\begin{array}{l}
\mathbf{1}_n^\top w = 1 \\
\mathbf{0}_n \leq w \leq \mathbf{1}_n \\
\left| w_i - b_i \right| \leq \tau_{i,w} \\
\left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{MD}_i \right| \leq \tau_{j,\mathrm{MD}} \\
\left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{DTS}_i \right| \leq \tau_{j,\mathrm{DTS}} \\
\tau_{i,w} \geq 0, \tau_{j,\mathrm{MD}} \geq 0, \tau_{j,\mathrm{DTS}} \geq 0
\end{array}
\right.
\end{aligned}
$$

Advanced optimization problems
Large bond universe

$$\left|w_i - b_i\right| \leq \tau_{i,w} \Leftrightarrow \left\{ \begin{array}{l} w_i - \tau_{i,w} \leq b_i \\ -w_i - \tau_{i,w} \leq -b_i \end{array} \right.$$

Advanced optimization problems

Large bond universe

$$
\begin{aligned}
(*) \quad & \Leftrightarrow \quad \left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i) \,\mathrm{MD}_i \right| \leq \tau_{j,\mathrm{MD}} \\
& \Leftrightarrow \quad -\tau_{j,\mathrm{MD}} \leq \sum_{i \in \mathcal{S}ector_j} (w_i - b_i) \,\mathrm{MD}_i \leq \tau_{j,\mathrm{MD}} \\
& \Leftrightarrow \quad -\tau_{j,\mathrm{MD}} + \sum_{i \in \mathcal{S}ector_j} b_i \,\mathrm{MD}_i \leq \sum_{i \in \mathcal{S}ector_j} w_i \,\mathrm{MD}_i \leq \tau_{j,\mathrm{MD}} + \\
& \qquad \sum_{i \in \mathcal{S}ector_j} b_i \,\mathrm{MD}_i \\
& \Leftrightarrow \quad -\tau_{j,\mathrm{MD}} + \mathrm{MD}_j^\star \leq (\boldsymbol{s}_j \circ \mathrm{MD})^\top w \leq \tau_{j,\mathrm{MD}} + \mathrm{MD}_j^\star \\
& \Leftrightarrow \quad \left\{ \begin{array}{l} (\boldsymbol{s}_j \circ \mathrm{MD})^\top w - \tau_{j,\mathrm{MD}} \leq \mathrm{MD}_j^\star \\ -(\boldsymbol{s}_j \circ \mathrm{MD})^\top w - \tau_{j,\mathrm{MD}} \leq -\mathrm{MD}_j^\star \end{array} \right.
\end{aligned}
$$

Advanced optimization problems

Large bond universe

$$\left|\sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\,\mathrm{DTS}_i\right| \leq \tau_{j,\mathrm{DTS}} \Leftrightarrow \left\{ \begin{array}{l} (\boldsymbol{s}_j \circ \mathrm{DTS})^\top w - \tau_{j,\mathrm{DTS}} \leq \mathrm{DTS}_j^\star \\ -(\boldsymbol{s}_j \circ \mathrm{DTS})^\top w - \tau_{j,\mathrm{DTS}} \leq -\mathrm{DTS}_j^\star \end{array} \right.$$

Advanced optimization problems
LP formulation

- x is a vector of dimension $n_x = 2 \times (n + n_{\mathcal{S}ector})$:

$$x = \begin{pmatrix} w \\ \tau_w \\ \tau_{\mathrm{MD}} \\ \tau_{\mathrm{DTS}} \end{pmatrix}$$

Advanced optimization problems

LP formulation

- The vector c is equal to:

$$c = \begin{pmatrix} -\gamma \mathcal{C} \\ \frac{1}{2}\varphi'_{\mathrm{AS}} \mathbf{1}_n \\ \varphi'_{\mathrm{MD}} \mathbf{1}_{n_{\mathcal{S}ector}} \\ \varphi'_{\mathrm{DTS}} \mathbf{1}_{n_{\mathcal{S}ector}} \end{pmatrix}$$

Advanced optimization problems
LP formulation

- The linear equality constraint $Ax = B$ is defined by:

$$A = \left(\begin{array}{cccc} \mathbf{1}_n^\top & \mathbf{0}_n^\top & \mathbf{0}_{n_{\mathcal{S}ector}}^\top & \mathbf{0}_{n_{\mathcal{S}ector}}^\top \end{array} \right)$$

and:

$$B = 1$$

Advanced optimization problems

LP formulation

- The linear inequality constraint $Cx \leq D$ is defined by:

$$C = \begin{pmatrix} I_n & -I_n & \mathbf{0}_{n,n_{\mathcal{S}ector}} & \mathbf{0}_{n,n_{\mathcal{S}ector}} \\ -I_n & -I_n & \mathbf{0}_{n,n_{\mathcal{S}ector}} & \mathbf{0}_{n,n_{\mathcal{S}ector}} \\ C_{\mathrm{MD}} & \mathbf{0}_{n_{\mathcal{S}ector},n} & -I_{n_{\mathcal{S}ector}} & \mathbf{0}_{n_{\mathcal{S}ector},n_{\mathcal{S}ector}} \\ -C_{\mathrm{MD}} & \mathbf{0}_{n_{\mathcal{S}ector},n} & -I_{n_{\mathcal{S}ector}} & \mathbf{0}_{n_{\mathcal{S}ector},n_{\mathcal{S}ector}} \\ C_{\mathrm{DTS}} & \mathbf{0}_{n_{\mathcal{S}ector},n} & \mathbf{0}_{n_{\mathcal{S}ector},n_{\mathcal{S}ector}} & -I_{n_{\mathcal{S}ector}} \\ -C_{\mathrm{DTS}} & \mathbf{0}_{n_{\mathcal{S}ector},n} & \mathbf{0}_{n_{\mathcal{S}ector},n_{\mathcal{S}ector}} & -I_{n_{\mathcal{S}ector}} \end{pmatrix}$$

end:

$$D = \begin{pmatrix} b \\ -b \\ \mathrm{MD}^{\star} \\ -\mathrm{MD}^{\star} \\ \mathrm{DTS}^{\star} \\ -\mathrm{DTS}^{\star} \end{pmatrix}$$

Advanced optimization problems
LP formulation

- C_{MD} and C_{DTS} are two $n_{\mathcal{S}ector} \times n$ matrices, whose elements are:

$$\left(C_{\mathrm{MD}}\right)_{j,i} = \boldsymbol{s}_{i,j}\,\mathrm{MD}_i$$

and:

$$\left(C_{\mathrm{DTS}}\right)_{j,i} = \boldsymbol{s}_{i,j}\,\mathrm{DTS}_i$$

- We have:

$$\mathrm{MD}^{\star} = \left(\mathrm{MD}_1^{\star}, \ldots, \mathrm{MD}_{n_{\mathcal{S}ector}}^{\star}\right)$$

and

$$\mathrm{DTS}^{\star} = \left(\mathrm{DTS}_1^{\star}, \ldots, \mathrm{DTS}_{n_{\mathcal{S}ector}}^{\star}\right)$$

Advanced optimization problems
LP formulation

- The bounds are:
$$x^- = \mathbf{0}_{n_x}$$
and:
$$x^+ = \infty \cdot \mathbf{1}_{n_x}$$

Advanced optimization problems

LP formulation

- Additional constraints:

$$\left\{ \begin{array}{l} A'w = B' \\ C'w \leq D' \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \left(\begin{array}{cc} A' & \mathbf{0}_{n_A, n_x - n} \end{array} \right) x = B' \\ \left(\begin{array}{cc} C' & \mathbf{0}_{n_A, n_x - n} \end{array} \right) x \leq D' \end{array} \right.$$

Advanced optimization problems

Large bond universe

Toy example

We consider a toy example with four corporate bonds:

Issuer	#1	#2	#3	#4
b_i (in %)	35	15	20	30
$\mathcal{CI}_i$ (in $\mathrm{tCO_2e/\$ mn}$)	117	284	162.5	359
MD_i (in years)	3.0	5.0	2.0	6.0
DTS_i (in bps)	100	150	200	250
$\mathcal{S}ector$	1	1	2	2

We would like to reduce the carbon footprint by 20%, and we set $\varphi'_{\mathrm{AS}} = 100$, $\varphi'_{\mathrm{MD}} = 25$ and $\varphi'_{\mathrm{DTS}} = 1$

Advanced optimization problems

Large bond universe

We have $n = 4$, $n_{\mathcal{S}ector} = 2$ and:

$$x = (\underbrace{w_1, w_2, w_3, w_4}_{w}, \underbrace{\tau_{w_1}, \tau_{w_2}, \tau_{w_3}, \tau_{w_4}}_{\tau_w}, \underbrace{\tau_{\mathrm{MD}_1}, \tau_{\mathrm{MD}_2}}_{\tau_{\mathrm{MD}}}, \underbrace{\tau_{\mathrm{DTS}_1}, \tau_{\mathrm{DTS}_2}}_{\tau_{\mathrm{DTS}}})$$

Since the vector $\mathcal{C}$ is equal to $\mathbf{0}_4$, we obtain:

$$c = (0, 0, 0, 0, 50, 50, 50, 50, 25, 25, 1, 1)$$

Advanced optimization problems

Large bond universe

The equality system $Ax = B$ is defined by:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and:

$$B = 1$$

Advanced optimization problems

Large bond universe

The inequality system $Cx \leq D$ is given by:

$$C = \left(\begin{array}{cccc|c|cccc}
\multicolumn{4}{c|}{I_4} & -I_4 & \multicolumn{4}{c}{\mathbf{0}_{4,4}} \\ \hline
\multicolumn{4}{c|}{-I_4} & -I_4 & \multicolumn{4}{c}{\mathbf{0}_{4,4}} \\ \hline
3 & 5 & 0 & 0 & & -1 & 0 & 0 & 0 \\
0 & 0 & 2 & 6 & \mathbf{0}_{4,4} & 0 & -1 & 0 & 0 \\
-3 & -5 & 0 & 0 & & -1 & 0 & 0 & 0 \\
0 & 0 & -2 & -6 & & 0 & -1 & 0 & 0 \\ \hline
100 & 150 & 0 & 0 & & 0 & 0 & -1 & 0 \\
0 & 0 & 200 & 250 & \mathbf{0}_{4,4} & 0 & 0 & 0 & -1 \\
-100 & -150 & 0 & 0 & & 0 & 0 & -1 & 0 \\
0 & 0 & -200 & -250 & & 0 & 0 & 0 & -1 \\ \hline
117 & 284 & 162.5 & 359 & \mathbf{0}_{1,4} & 0 & 0 & 0 & 0
\end{array}\right)$$

and:

$$\begin{array}{rcl}
D & = & (0.35, 0.15, 0.2, 0.3, \quad -0.35, -0.15, -0.2, -0.3, \ldots \\
& & 1.8, 2.2, -1.8, -2.2, \quad 57.5, 115, -57.5, -115, \quad 179)
\end{array}$$

Advanced optimization problems

Large bond universe

- The last row of $Cx \leq D$ corresponds to the carbon footprint constraint
- We have:

$$\mathcal{CI}(b) = 223.75 \text{ tCO}_2\text{e/\$ mn}$$

and:

$$(1 - \mathcal{R})\,\mathcal{CI}(b) = 0.80 \times 223.75 = 179.00 \text{ tCO}_2\text{e/\$ mn}$$

Advanced optimization problems

Large bond universe

We solve the LP program, and we obtain the following solution:

$$\begin{aligned} w^{\star} &= (47.34\%, 0\%, 33.3\%, 19.36\%) \\ \tau_{w}^{\star} &= (12.34\%, 15\%, 13.3\%, 10.64\%) \\ \tau_{\mathrm{MD}}^{\star} &= (0.3798, 0.3725) \\ \tau_{\mathrm{DTS}}^{\star} &= (10.1604, 0) \end{aligned}$$

Advanced optimization problems

Large bond universe

- Interpretation of $\tau_w^\star$:

$$w^\star \pm \tau_w^\star = \begin{pmatrix} 47.34\% \\ 0.00\% \\ 33.30\% \\ 19.36\% \end{pmatrix} \begin{pmatrix} - \\ + \\ - \\ + \end{pmatrix} \begin{pmatrix} 12.34\% \\ 15.00\% \\ 13.30\% \\ 10.64\% \end{pmatrix} = \begin{pmatrix} 35\% \\ 15\% \\ 20\% \\ 30\% \end{pmatrix} = b$$

- Interpretation of $\tau_{\mathrm{MD}}^\star$:

$$\begin{pmatrix} \sum_{i \in \mathcal{S}ector_1} w_i^\star \,\mathrm{MD}_i \\ \sum_{i \in \mathcal{S}ector_2} w_i^\star \,\mathrm{MD}_i \end{pmatrix} \pm \tau_{\mathrm{MD}}^\star = \begin{pmatrix} 1.42 \\ 1.83 \end{pmatrix} \begin{pmatrix} + \\ + \end{pmatrix} \begin{pmatrix} 0.38 \\ 0.37 \end{pmatrix} = \begin{pmatrix} 1.80 \\ 2.20 \end{pmatrix} = \begin{pmatrix} \mathrm{MD}_1^\star \\ \mathrm{MD}_2^\star \end{pmatrix}$$

- Interpretation of $\tau_{\mathrm{DTS}}^\star$:

$$\begin{pmatrix} \sum_{i \in \mathcal{S}ector_1} w_i^\star \,\mathrm{DTS}_i \\ \sum_{i \in \mathcal{S}ector_2} w_i^\star \,\mathrm{DTS}_i \end{pmatrix} \pm \tau_{\mathrm{DTS}}^\star = \begin{pmatrix} 47.34 \\ 115.00 \end{pmatrix} \begin{pmatrix} + \\ + \end{pmatrix} \begin{pmatrix} 10.16 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 57.50 \\ 115.00 \end{pmatrix} = \begin{pmatrix} \mathrm{DTS}_1^\star \\ \mathrm{DTS}_2^\star \end{pmatrix}$$

Advanced optimization problems

Large bond universe

Example #4 (Example #3 again)

We consider an investment universe of 9 corporate bonds with the following characteristics[a]:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8	#9
b_i	21	19	16	12	11	8	6	4	3
$\mathcal{CI}_i$	111	52	369	157	18	415	17	253	900
MD_i	3.16	6.48	3.54	9.23	6.40	2.30	8.12	7.96	5.48
DTS_i	107	255	75	996	289	45	620	285	125
$\mathcal{S}ector$	1	1	1	2	2	2	3	3	3

We impose that $0.25 \times b_i \leq w_i \leq 4 \times b_i$ and assume that $\varphi'_{\mathrm{AS}} = \varphi_{\mathrm{AS}} = 100$, $\varphi'_{\mathrm{MD}} = \varphi_{\mathrm{MD}} = 25$ and $\varphi'_{\mathrm{DTS}} = \varphi'_{\mathrm{DTS}} = 0.001$

[a] The units are: b_i in %, $\mathcal{CI}_i$ in $\mathrm{tCO_2e}/\$\ \mathrm{mn}$, MD_i in years and DTS_i in bps

Advanced optimization problems

Large bond universe

Table 116: Weights in % of optimized bond portfolios (Example #4)

Portfolio	#1	#2	#3	#4	#5	#6	#7	#8	#9
b	21.00	19.00	16.00	12.00	11.00	8.00	6.00	4.00	3.00
$w^\star$ (10%)	21.70	19.00	16.00	12.00	11.00	8.00	7.46	4.00	0.84
$w^\star$ (30%)	34.44	19.00	4.00	11.65	11.98	6.65	7.52	4.00	0.75
$w^\star$ (50%)	33.69	19.37	4.00	3.91	24.82	2.00	10.46	1.00	0.75

Table 117: Risk statistics of optimized bond portfolios (Example #4)

Portfolio	$\mathrm{AS}_{\mathcal{S}ector}$ (in %)	$\mathrm{MD}(w)$ (in years)	$\mathrm{DTS}(w)$ (in bps)	$\sigma_{\mathrm{AS}}(w \mid b)$ (in %)	$\sigma_{\mathrm{MD}}(w \mid b)$ (in years)	$\sigma_{\mathrm{DTS}}(w \mid b)$ (in bps)	$\mathcal{CI}(w)$ $\mathrm{gCO_2e/\$}$
b	0.00	5.43	290.18	0.00	0.00	0.00	184.39
$w^\star$ (10%)	2.16	5.45	297.28	2.16	0.02	7.10	165.95
$w^\star$ (30%)	15.95	5.43	300.96	15.95	0.00	13.20	129.07
$w^\star$ (50%)	31.34	5.43	268.66	31.34	0.00	65.12	92.19

Equity portfolios

Threshold approach

The optimization problem is:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2} (w - b)^{\top} \Sigma (w - b) \\ & \text{s.t.} \left\{ \begin{array}{l} \mathbf{1}_n^{\top} w = 1 \\ w \in \Omega \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \\ \mathcal{CI}(w) \leq (1 - \mathcal{R}) \mathcal{CI}(b) \end{array} \right. \end{aligned}$$

Equity portfolios

Order-statistic approach

- $\mathcal{CI}_{i:n}$ is the order statistics of $(\mathcal{CI}_1, \ldots, \mathcal{CI}_n)$:

$$\min \mathcal{CI}_i = \mathcal{CI}_{1:n} \leq \mathcal{CI}_{2:n} \leq \cdots \leq \mathcal{CI}_{i:n} \leq \cdots \leq \mathcal{CI}_{n:n} = \max \mathcal{CI}_i$$

- The carbon intensity bound $\mathcal{CI}^{(m,n)}$ is defined as:

$$\mathcal{CI}^{(m,n)} = \mathcal{CI}_{n-m+1:n}$$

where $\mathcal{CI}_{n-m+1:n}$ is the $(n-m+1)$-th order statistic of $(\mathcal{CI}_1, \ldots, \mathcal{CI}_n)$

- Exclusion process:

$$\mathcal{CI}_i \geq \mathcal{CI}^{(m,n)} \Rightarrow w_i = 0$$

Equity portfolios

Order-statistic approach (Cont'd)

The optimization problem is:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2}(w-b)^{\top}\Sigma(w-b) \\ \text{s.t.} & \left\{ \begin{array}{l} \mathbf{1}_n^{\top} w = 1 \\ w \in \Omega \\ \mathbf{0}_n \leq w \leq \mathbb{1}\left\{\mathcal{CI} < \mathcal{CI}^{(m,n)}\right\} \end{array} \right. \end{aligned}$$

Equity portfolios

Naive approach

We re-weight the remaining assets:

$$w_i^\star = \frac{\mathbb{1}\left\{\mathcal{CI}_i < \mathcal{CI}^{(m,n)}\right\} \cdot b_i}{\sum_{k=1}^{n} \mathbb{1}\left\{\mathcal{CI}_k < \mathcal{CI}^{(m,n)}\right\} \cdot b_k}$$

Equity portfolios

Example #5

We consider a capitalization-weighted equity index, which is composed of eight stocks. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in $\mathrm{tCO_2e}/\$ \mathrm{mn}$) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the market one-factor model: the beta β_i of each stock is equal to 0.30, 1.80, 0.85, 0.83, 1.47, 0.94, 1.67 and 1.08, the idiosyncratic volatilities $\tilde{\sigma}_i$ are respectively equal to 10%, 5%, 6%, 12%, 15%, 4%, 8% and 7%, and the estimated market volatility σ_m is 18%.

Equity portfolios

The covariance matrix is:

$$\Sigma = \beta\beta^\top \sigma_m^2 + D$$

where:

1. β is the vector of beta coefficients
2. σ_m^2 is the variance of the market portfolio
3. $D = diag\left(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_n^2\right)$ is the diagonal matrix of idiosyncratic variances

Equity portfolios

Table 118: Optimal decarbonization portfolios (Example #5, threshold approach)

$\mathcal{R}$	0	10	20	30	40	50	$\mathcal{CI}_i$
$w_1^\star$	20.00	20.54	21.14	21.86	22.58	22.96	100.5
$w_2^\star$	19.00	19.33	19.29	18.70	18.11	17.23	97.2
$w_3^\star$	17.00	15.67	12.91	8.06	3.22	0.00	250.4
$w_4^\star$	13.00	12.28	10.95	8.74	6.53	3.36	352.3
$w_5^\star$	12.00	12.26	12.60	13.07	13.53	14.08	27.1
$w_6^\star$	8.00	11.71	16.42	22.57	28.73	34.77	54.2
$w_7^\star$	6.00	6.36	6.69	7.00	7.30	7.59	78.6
$w_8^\star$	5.00	1.86	0.00	0.00	0.00	0.00	426.7
$\sigma(w^\star \mid b)$	0.00	30.01	61.90	104.10	149.65	196.87	
$\mathcal{CI}(w)$	160.57	144.52	128.46	112.40	96.34	80.29	
$\mathcal{R}(w \mid b)$	0.00	10.00	20.00	30.00	40.00	50.00	

The reduction rate and the weights are expressed in % whereas the tracking error volatility is measured in bps

Equity portfolios

Table 119: Optimal decarbonization portfolios (Example #5, order-statistic approach)

m	0	1	2	3	4	5	6	7	$\mathcal{CI}_i$
$w_1^\star$	20.00	20.40	22.35	26.46	0.00	0.00	0.00	0.00	100.5
$w_2^\star$	19.00	19.90	20.07	20.83	7.57	0.00	0.00	0.00	97.2
$w_3^\star$	17.00	17.94	21.41	0.00	0.00	0.00	0.00	0.00	250.4
$w_4^\star$	13.00	13.24	0.00	0.00	0.00	0.00	0.00	0.00	352.3
$w_5^\star$	12.00	12.12	12.32	12.79	13.04	14.26	18.78	100.00	27.1
$w_6^\star$	8.00	10.04	17.14	32.38	74.66	75.12	81.22	0.00	54.2
$w_7^\star$	6.00	6.37	6.70	7.53	4.73	10.62	0.00	0.00	78.6
$w_8^\star$	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	426.7
$\sigma(w^\star \mid b)$	0.00	0.37	1.68	2.25	3.98	4.04	4.30	15.41	
$\mathcal{CI}(w)$	160.57	145.12	113.48	73.78	55.08	52.93	49.11	27.10	
$\mathcal{R}(w \mid b)$	0.00	9.62	29.33	54.05	65.70	67.04	69.42	83.12	

The reduction rate, the weights and the tracking error volatility are expressed in %

Equity portfolios

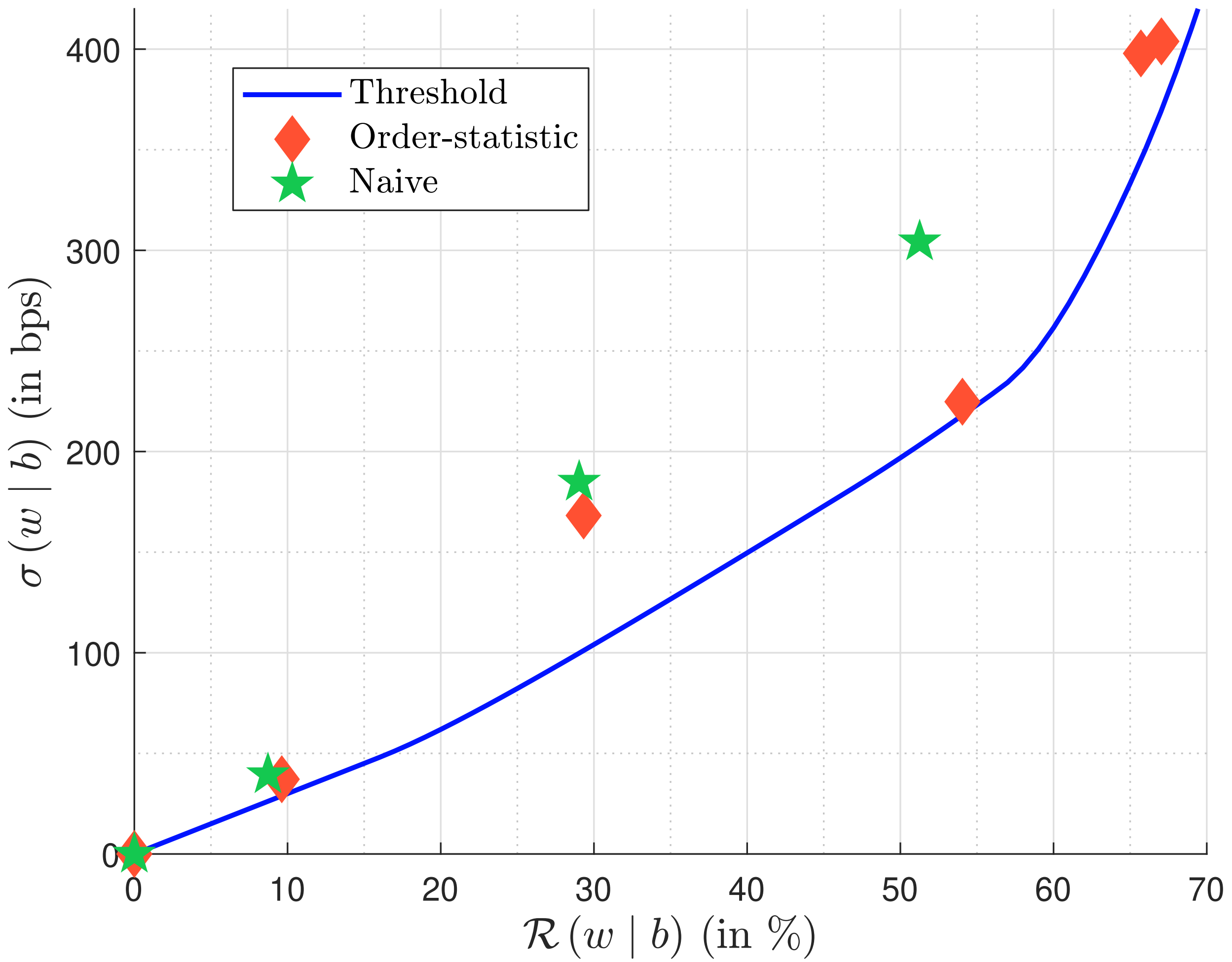
Table 120: Optimal decarbonization portfolios (Example #5, naive approach)

m	0	1	2	3	4	5	6	7	$\mathcal{CI}_i$
$w_1^\star$	20.00	21.05	24.39	30.77	0.00	0.00	0.00	0.00	100.5
$w_2^\star$	19.00	20.00	23.17	29.23	42.22	0.00	0.00	0.00	97.2
$w_3^\star$	17.00	17.89	20.73	0.00	0.00	0.00	0.00	0.00	250.4
$w_4^\star$	13.00	13.68	0.00	0.00	0.00	0.00	0.00	0.00	352.3
$w_5^\star$	12.00	12.63	14.63	18.46	26.67	46.15	60.00	100.00	27.1
$w_6^\star$	8.00	8.42	9.76	12.31	17.78	30.77	40.00	0.00	54.2
$w_7^\star$	6.00	6.32	7.32	9.23	13.33	23.08	0.00	0.00	78.6
$w_8^\star$	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	426.7
$\sigma(w^\star \mid b)$	0.00	0.39	1.85	3.04	9.46	8.08	8.65	15.41	
$\mathcal{CI}(w)$	160.57	146.57	113.95	78.26	68.38	47.32	37.94	27.10	
$\mathcal{R}(w \mid b)$	0.00	8.72	29.04	51.26	57.41	70.53	76.37	83.12	

The reduction rate, the weights and the tracking error volatility are expressed in %.

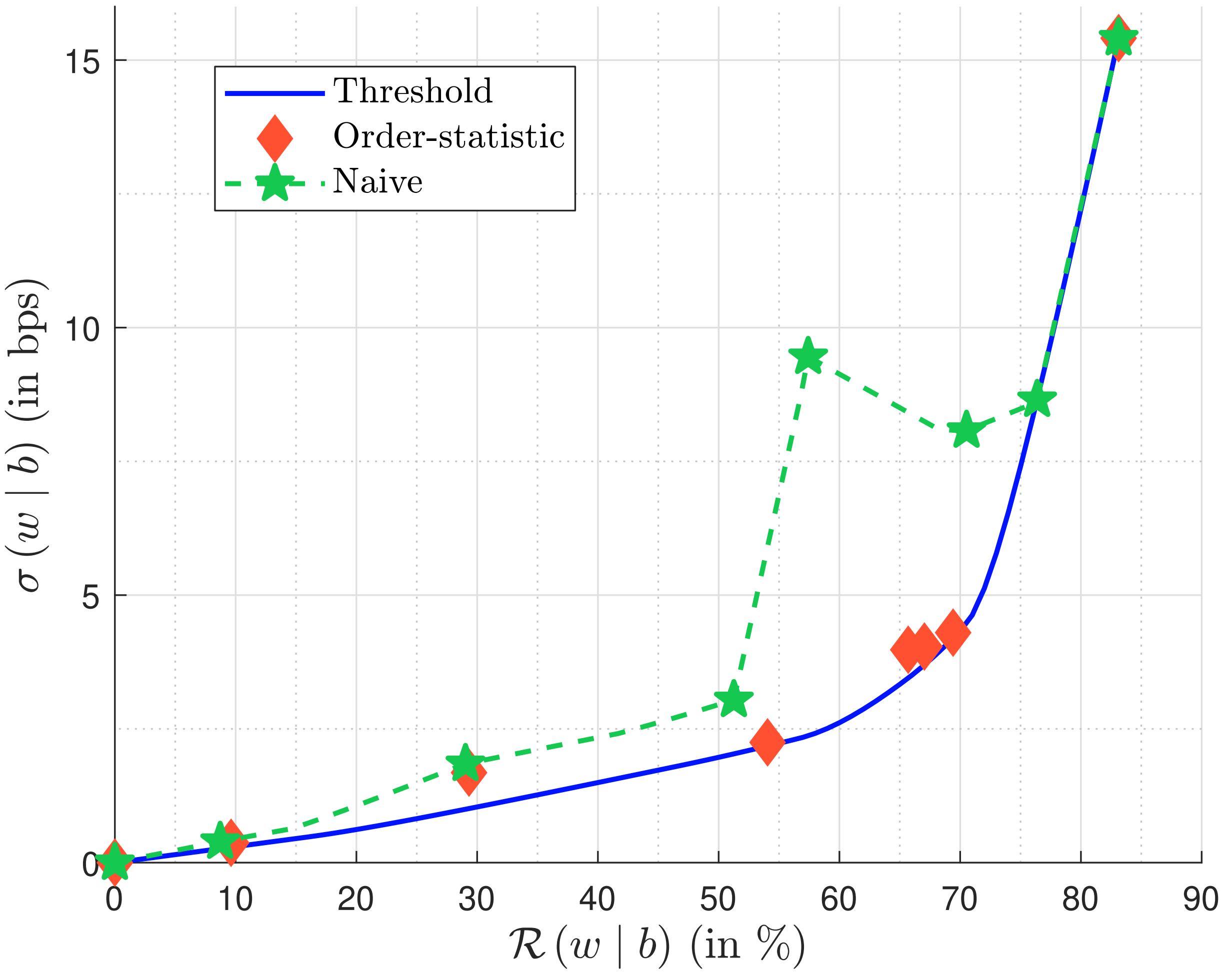
Equity portfolios

Figure 212: Efficient decarbonization frontier (Example #5)



Equity portfolios

Figure 213: Efficient decarbonization frontier of the interpolated naive approach (Example #5)



Bond portfolios

Example #6

We consider a debt-weighted bond index, which is composed of eight bonds. Their weights are equal to 20%, 19%, 17%, 13%, 12%, 8%, 6% and 5%. The carbon intensities (expressed in $\mathrm{tCO_2e/\$\ mn}$) are respectively equal to 100.5, 97.2, 250.4, 352.3, 27.1, 54.2, 78.6 and 426.7. To evaluate the risk of the portfolio, we use the modified duration which is respectively equal to 3.1, 6.6, 7.2, 5, 4.7, 2.1, 8.1 and 2.6 years, and the duration-times-spread factor, which is respectively equal to 100, 155, 575, 436, 159, 145, 804 and 365 bps. There are two sectors. Bonds #1, #3, #4 and #8 belong to $\mathcal{S}ector_1$ while Bonds #2, #5, #6 and #7 belong to $\mathcal{S}ector_2$

Bond portfolios

Table 121: Optimal decarbonization portfolios (Example #6, threshold approach)

$\mathcal{R}$	0	10	20	30	40	50	$\mathcal{CI}_i$
$w_1^\star$	20.00	21.62	23.93	26.72	30.08	33.44	100.5
$w_2^\star$	19.00	18.18	16.98	14.18	7.88	1.58	97.2
$w_3^\star$	17.00	18.92	21.94	22.65	16.82	11.00	250.4
$w_4^\star$	13.00	11.34	5.35	0.00	0.00	0.00	352.3
$w_5^\star$	12.00	13.72	16.14	21.63	33.89	46.14	27.1
$w_6^\star$	8.00	9.60	10.47	10.06	7.21	4.36	54.2
$w_7^\star$	6.00	5.56	5.19	4.75	4.11	3.48	78.6
$w_8^\star$	5.00	1.05	0.00	0.00	0.00	0.00	426.7
$\mathrm{AS}_{\mathcal{S}ector}$	0.00	6.87	15.49	24.07	31.97	47.58	
$\mathrm{MD}(w)$	5.48	5.49	5.45	5.29	4.90	4.51	
$\mathrm{DTS}(w)$	301.05	292.34	282.28	266.12	236.45	206.78	
$\sigma_{\mathrm{AS}}(w \mid b)$	0.00	5.57	12.31	19.82	30.04	43.58	
$\sigma_{\mathrm{MD}}(w \mid b)$	0.00	0.01	0.04	0.17	0.49	0.81	
$\sigma_{\mathrm{DTS}}(w \mid b)$	0.00	8.99	19.29	35.74	65.88	96.01	
$\mathcal{CI}(w)$	160.57	144.52	128.46	112.40	96.34	80.29	
$\mathcal{R}(w \mid b)$	0.00	10.00	20.00	30.00	40.00	50.00	

Bond portfolios

Table 122: Optimal decarbonization portfolios (Example #6, order-statistic approach)

m	0	1	2	3	4	5	6	7	$\mathcal{CI}_i$
$w_1^\star$	20.00	20.83	24.62	64.64	0.00	0.00	0.00	0.00	100.5
$w_2^\star$	19.00	18.60	18.13	21.32	3.32	0.00	0.00	0.00	97.2
$w_3^\star$	17.00	17.79	26.30	0.00	0.00	0.00	0.00	0.00	250.4
$w_4^\star$	13.00	14.53	0.00	0.00	0.00	0.00	0.00	0.00	352.3
$w_5^\star$	12.00	12.89	13.96	6.00	36.57	41.27	41.27	100.00	27.1
$w_6^\star$	8.00	9.74	11.85	0.00	60.11	58.73	58.73	0.00	54.2
$w_7^\star$	6.00	5.62	5.15	8.03	0.00	0.00	0.00	0.00	78.6
$w_8^\star$	5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	426.7
$\mathrm{AS}_{\mathcal{S}ector}$	0.00	5.78	19.72	49.00	76.68	80.00	80.00	88.00	
$\mathrm{MD}(w)$	5.48	5.52	5.54	4.77	3.27	3.17	3.17	4.70	
$\mathrm{DTS}(w)$	301.05	295.08	284.71	171.82	150.45	150.78	150.78	159.00	
$\sigma_{\mathrm{AS}}(w \mid b)$	0.00	5.73	17.94	50.85	66.96	68.63	68.63	95.33	
$\sigma_{\mathrm{MD}}(w \mid b)$	0.00	0.03	0.04	0.63	2.66	2.64	2.64	3.21	
$\sigma_{\mathrm{DTS}}(w \mid b)$	0.00	6.21	16.87	128.04	197.22	197.29	197.29	199.22	
$\mathcal{CI}(w)$	160.57	147.94	122.46	93.63	45.72	43.02	43.02	27.10	
$\mathcal{R}(w \mid b)$	0.00	7.87	23.74	41.69	71.53	73.21	73.21	83.12	

Sector-specific constraints

Sector scenario

- Decarbonization scenario per sector:

$$\mathcal{CI}\left(w;\mathcal{S}ector_j\right) \leq \left(1-\mathcal{R}_j\right)\mathcal{CI}\left(b;\mathcal{S}ector_j\right)$$

- We have:

$$\left(\boldsymbol{s}_j \circ \left(\boldsymbol{\mathcal{CI}} - \boldsymbol{\mathcal{CI}}_j^{\star}\right)\right)^{\top} w \leq 0$$

where $\mathcal{CI}_j^{\star} = \left(1-\mathcal{R}_j\right)\mathcal{CI}\left(b;\mathcal{S}ector_j\right)$

Sector-specific constraints

Sector scenario

QP form

$$C = \begin{pmatrix} \left(\boldsymbol{s}_1 \circ \left(\mathcal{CI} - \mathcal{CI}_1^\star\right)\right)^\top \\ \vdots \\ \left(\boldsymbol{s}_j \circ \left(\mathcal{CI} - \mathcal{CI}_j^\star\right)\right)^\top \\ \vdots \\ \left(\boldsymbol{s}_{n_{\mathcal{S}ector}} \circ \left(\mathcal{CI} - \mathcal{CI}_{n_{\mathcal{S}ector}}^\star\right)\right)^\top \end{pmatrix}$$

$$D = \begin{pmatrix} \left(1 - \mathcal{R}_1\right) \mathcal{CI}\left(b; \mathcal{S}ector_1\right) \\ \vdots \\ \left(1 - \mathcal{R}_j\right) \mathcal{CI}\left(b; \mathcal{S}ector_j\right) \\ \vdots \\ \left(1 - \mathcal{R}_{n_{\mathcal{S}ector}}\right) \mathcal{CI}\left(b; \mathcal{S}ector_{n_{\mathcal{S}ector}}\right) \end{pmatrix}$$

Sector-specific constraints

Sector scenario

Table 123: Carbon intensity and threshold in $\mathrm{tCO_2e}/\$\ \mathrm{mn}$ per GICS sector (MSCI World, 2030)

Sector	$\mathcal{CI}(b;\mathcal{S}ector_j)$ $\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_{1-3}$	$\mathcal{R}_j$ (in %)	$\mathcal{CI}_j^\star$ $\mathcal{SC}_1$	$\mathcal{SC}_{1-2}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$	$\mathcal{SC}_{1-3}$
Communication Services	2	28	134	172	52.4	1	13	64	82
Consumer Discretionary	23	65	206	590	52.4	11	31	98	281
Consumer Staples	28	55	401	929	52.4	13	26	191	442
Energy	632	698	1 006	6 823	56.9	272	301	434	2 941
Financials	13	19	52	244	52.4	6	9	25	116
Health Care	10	22	120	146	52.4	5	10	57	70
Industrials	111	130	298	1 662	18.8	90	106	242	1 350
Information Technology	7	23	112	239	52.4	3	11	53	114
Materials	478	702	1 113	2 957	36.7	303	445	704	1 872
Real Estate	22	101	167	571	36.7	14	64	106	361
Utilities	1 744	1 794	2 053	2 840	56.9	752	773	885	1 224
MSCI World	130	163	310	992	36.6	82	103	196	629

Sector-specific constraints

Sector and weight deviation constraints (equity portfolio)

1. Asset weight deviation constraint:

$$\Omega := \mathcal{C}_1\left(m_w^-, m_w^+\right) = \left\{w : m_w^- b \leq w \leq m_w^+ b\right\}$$

2. Sector weight deviation constraint:

$$\Omega := \mathcal{C}_2\left(m_s^-, m_s^+\right) == \left\{\forall j : m_s^- \sum_{i \in \mathcal{S}ector_j} b_i \leq \sum_{i \in \mathcal{S}ector_j} w_i \leq m_s^+ \sum_{i \in \mathcal{S}ector_j} b_i\right\}$$

3. $\mathcal{C}_2\left(m_s\right) = \mathcal{C}_2\left(1/m_s, m_s\right)$
4. $\mathcal{C}_3\left(m_w^-, m_w^+, m_s\right) = \mathcal{C}_1\left(m_w^-, m_w^+\right) \cap \mathcal{C}_2\left(m_s\right)$

Sector-specific constraints

Sector and weight deviation constraints (bond portfolio)

1. Modified duration constraint:

$$\Omega := \mathcal{C}_1' = \{w : \mathrm{MD}(w) = \mathrm{MD}(b)\} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i)\, \mathrm{MD}_i = 0 \right\}$$

2. DTS constraint

$$\Omega := \mathcal{C}_2' = \{w : \mathrm{DTS}(w) = \mathrm{DTS}(b)\} = \left\{ w : \sum_{i=1}^{n} (x_i - b_i)\, \mathrm{DTS}_i = 0 \right\}$$

3. Maturity/rating buckets:

$$\Omega := \left\{ w : \sum_{i \in \mathcal{B}ucket_j} (x_i - b_i) = 0 \right\}$$

   1. $\mathcal{C}_3'$: $\mathcal{B}ucket_j$ is the j^{th} maturity bucket, e.g., 0–1, 1–3, 3–5, 5–7, 7–10 and 10+
   2. $\mathcal{C}_4'$: $\mathcal{B}ucket_j$ is the j^{th} rating category, e.g., AAA–AA (AAA, AA+, AA and AA−), A (A+, A and A−) and BBB (BBB+, BBB, BBB−)

Sector-specific constraints

HCIS constraint

Two types of sectors:

1. High climate impact sectors (HCIS):

   "**sectors that are key to the low-carbon transition**" (TEG, 2019)
2. Low climate impact sectors (LCIS)

Let $\mathcal{HCIS}(w) = \sum_{i \in \text{HCIS}} w_i$ be the HCIS weight of portfolio w:

$$\mathcal{HCIS}(w) \geq \mathcal{HCIS}(b)$$

Sector-specific constraints

HCIS constraint

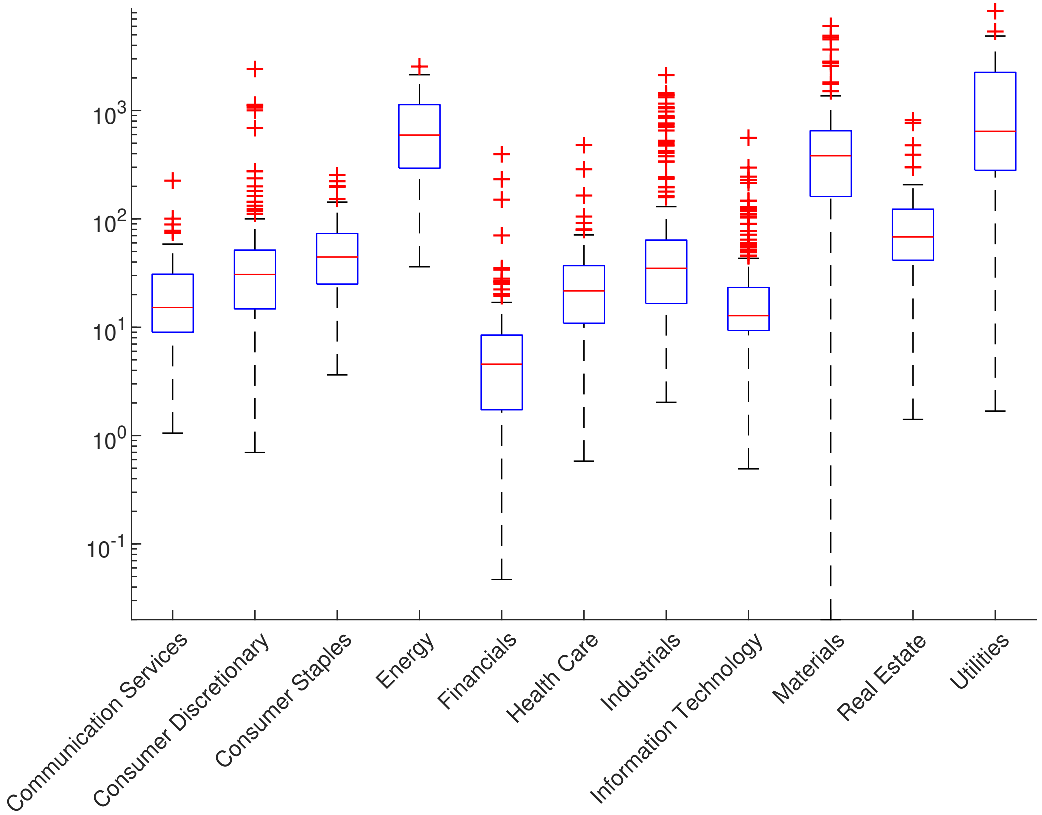
Table 124: Weight and carbon intensity when applying the HCIS filter (MSCI World, June 2022)

Sector	Index b_j	HCIS b'_j	$\mathcal{SC}_1$ $\mathcal{CI}$	$\mathcal{SC}_1$ $\mathcal{CI}'$	$\mathcal{SC}_{1-2}$ $\mathcal{CI}$	$\mathcal{SC}_{1-2}$ $\mathcal{CI}'$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$ $\mathcal{CI}$	$\mathcal{SC}_{1-3}^{\mathrm{up}}$ $\mathcal{CI}'$	$\mathcal{SC}_{1-3}$ $\mathcal{CI}$	$\mathcal{SC}_{1-3}$ $\mathcal{CI}'$
Communication Services	7.58	0.00	2		28		134		172	
Consumer Discretionary	10.56	8.01	23	14	65	31	206	189	590	462
Consumer Staples	7.80	7.80	28	28	55	55	401	401	929	929
Energy	4.99	4.99	632	632	698	698	1 006	1 006	6 823	6 823
Financials	13.56	0.00	13		19		52		244	
Health Care	14.15	9.98	10	13	22	26	120	141	146	177
Industrials	9.90	7.96	111	132	130	151	298	332	1 662	1 921
Information Technology	21.08	10.67	7	12	23	30	112	165	239	390
Materials	4.28	4.28	478	478	702	702	1 113	1 113	2 957	2 957
Real Estate	2.90	2.90	22	22	101	101	167	167	571	571
Utilities	3.21	3.21	1 744	1 744	1 794	1 794	2 053	2 053	2 840	2 840
MSCI World	100.00	59.79	130	210	163	252	310	458	992	1 498

Source: MSCI (2022), Trucost (2022) & Author's calculations

Empirical results (equity portfolios)

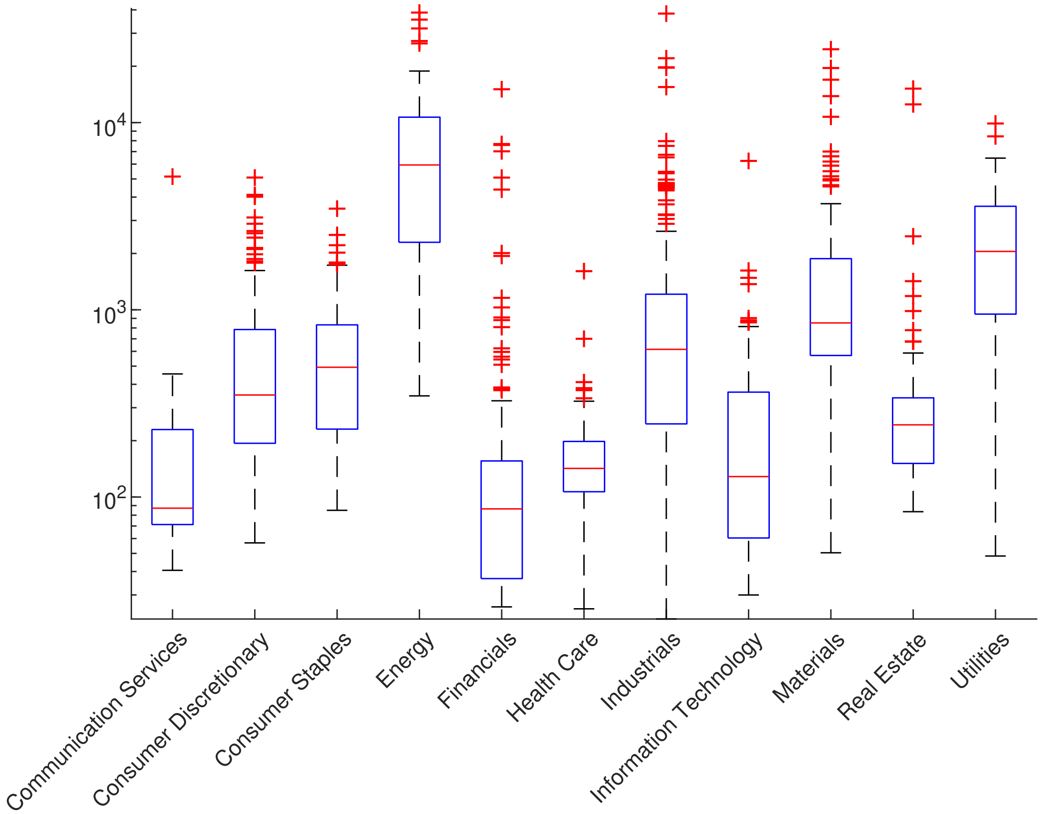
Figure 214: Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $\mathcal{SC}_{1-2}$)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Empirical results (equity portfolios)

Figure 215: Boxplot of carbon intensity per sector (MSCI World, June 2022, scope $\mathcal{SC}_{1-3}$)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Equity portfolios

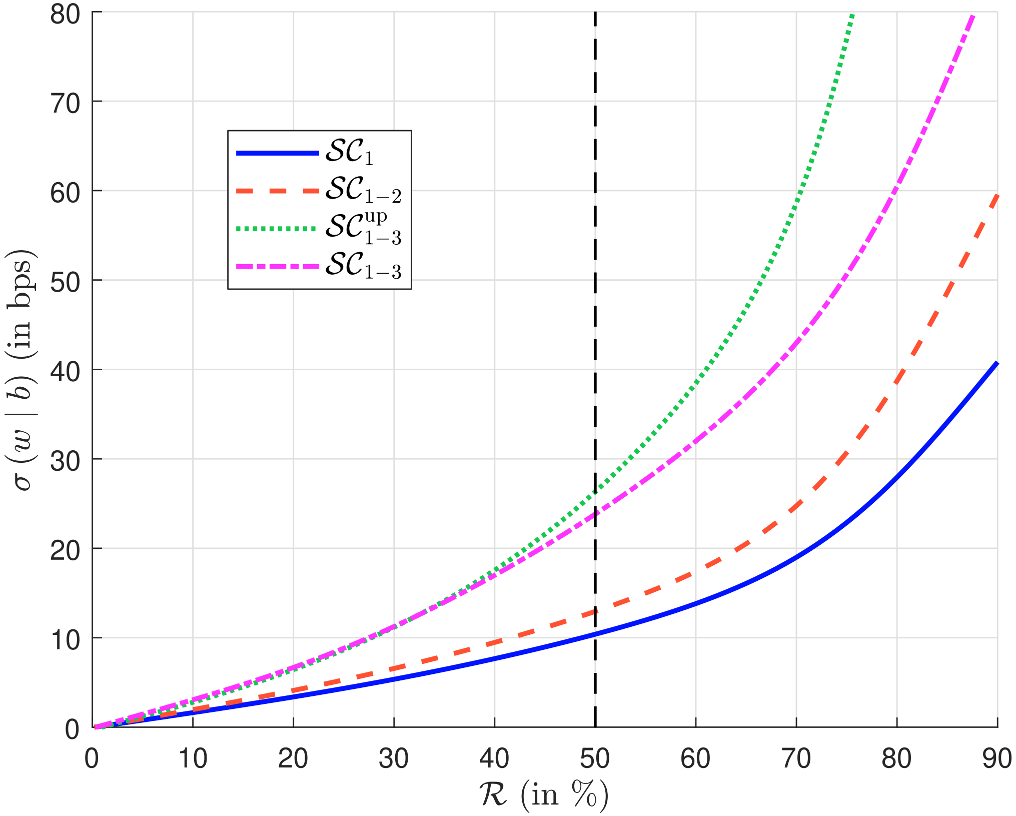
Barahhou *et al.* (2022) consider the basic optimization problem:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2} (w-b)^{\top} \Sigma (w-b) \\ \text{s.t.} & \quad \left\{ \begin{array}{l} \mathcal{CI}(w) \leq (1-\mathcal{R})\, \mathcal{CI}(b) \\ w \in \Omega_0 \cap \Omega \end{array} \right. \end{aligned}$$

What is the impact of constraints $\Omega_0 \cap \Omega$?

Equity portfolios

Figure 216: Impact of the carbon scope on the tracking error volatility (MSCI World, June 2022, $\mathcal{C}_0$ constraint)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Equity portfolios

Table 125: Sector allocation in % (MSCI World, June 2022, scope $\mathcal{SC}_{1-3}$)

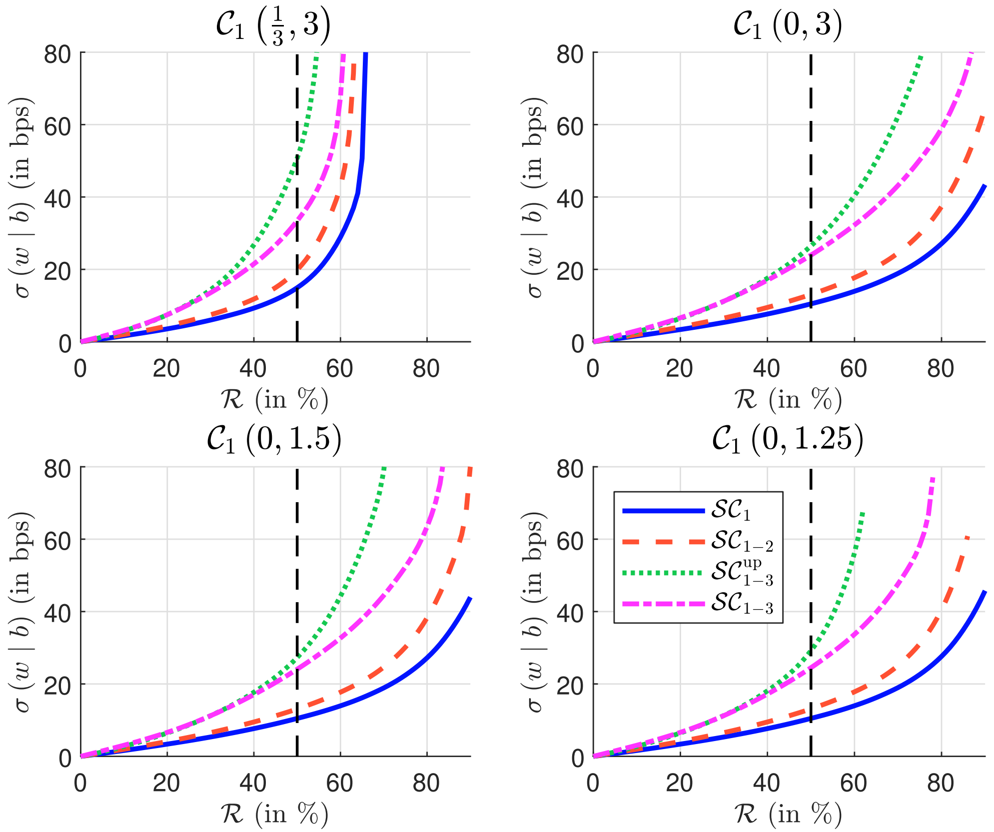
Sector	Index	Reduction rate $\mathcal{R}$ 30%	40%	50%	60%	70%	80%	90%
Communication Services	7.58	7.95	8.15	8.42	8.78	9.34	10.13	12.27
Consumer Discretionary	10.56	10.69	10.69	10.65	10.52	10.23	9.62	6.74
Consumer Staples	7.80	7.80	7.69	7.48	7.11	6.35	5.03	1.77
Energy	**4.99**	4.14	3.65	3.10	2.45	**1.50**	**0.49**	**0.00**
Financials	**13.56**	14.53	15.17	15.94	16.90	**18.39**	**20.55**	**28.62**
Health Care	14.15	14.74	15.09	15.50	16.00	16.78	17.77	17.69
Industrials	9.90	9.28	9.01	8.71	8.36	7.79	7.21	6.03
Information Technology	21.08	21.68	22.03	22.39	22.88	23.51	24.12	24.02
Materials	**4.28**	3.78	3.46	3.06	2.56	**1.85**	**1.14**	**0.24**
Real Estate	2.90	3.12	3.27	3.41	3.57	3.72	3.71	2.51
Utilities	**3.21**	2.28	1.79	1.36	0.90	**0.54**	**0.24**	**0.12**

Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Portfolio decarbonization = strategy **long on Financials** and **short on Energy, Materials and Utilities**

Equity portfolios

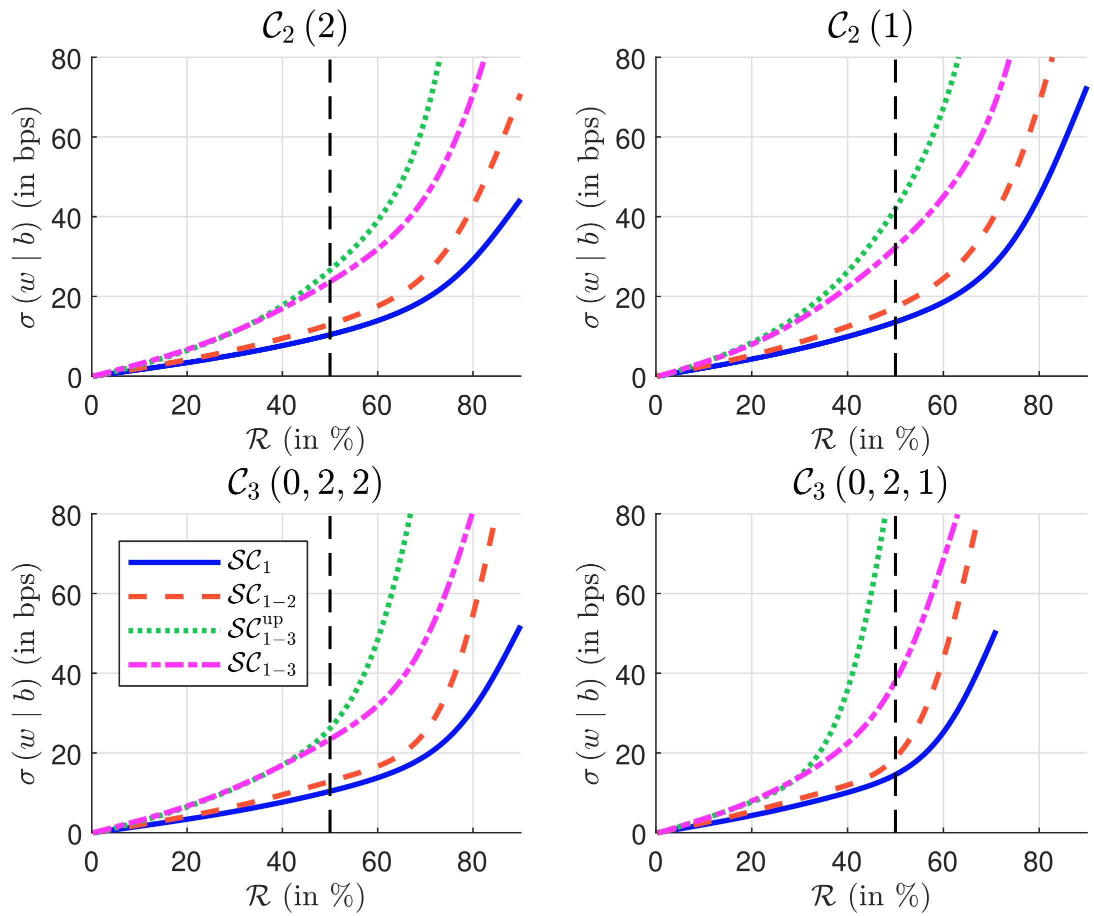
Figure 217: Impact of $\mathcal{C}_1$ constraint on the tracking error volatility (MSCI World, June 2022)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Equity portfolios

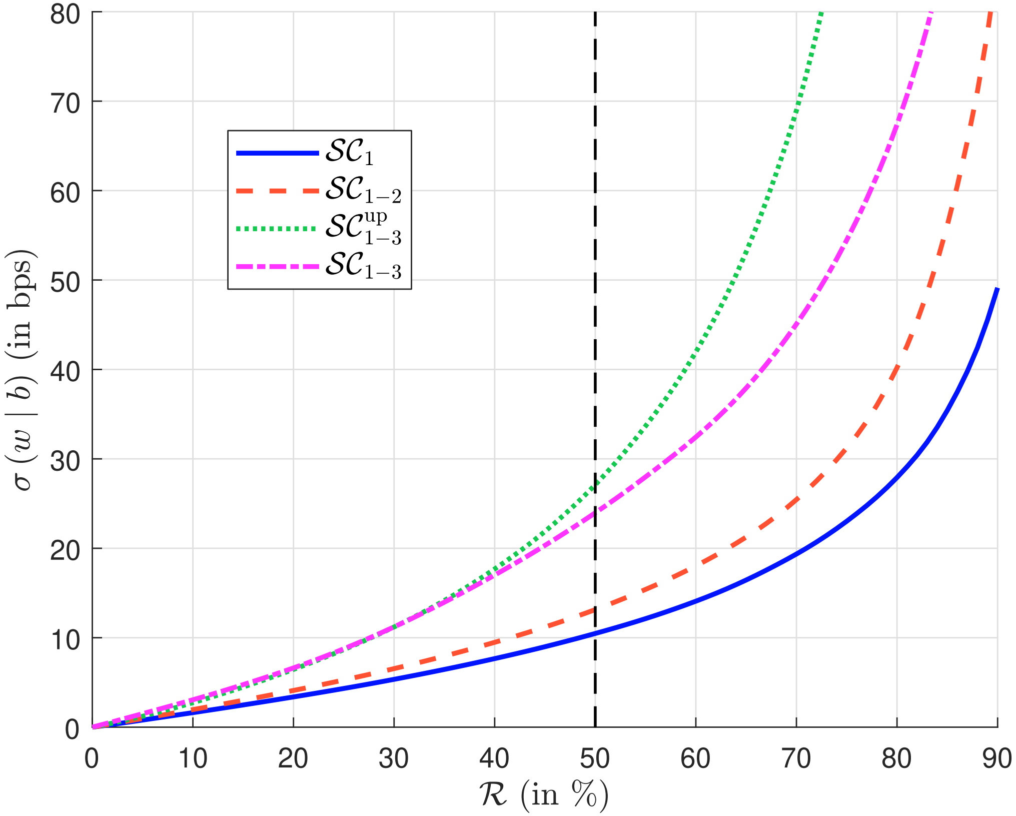
Figure 218: Impact of $\mathcal{C}_2$ and $\mathcal{C}_3$ constraints (MSCI World, June 2022)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Equity portfolios

Figure 219: Tracking error volatility with $\mathcal{C}_3(0, 10, 2)$ constraint (MSCI World, June 2022)



Source: MSCI (2022), Trucost (2022) & Barahhou *et al.* (2022)

Equity portfolios

First approach

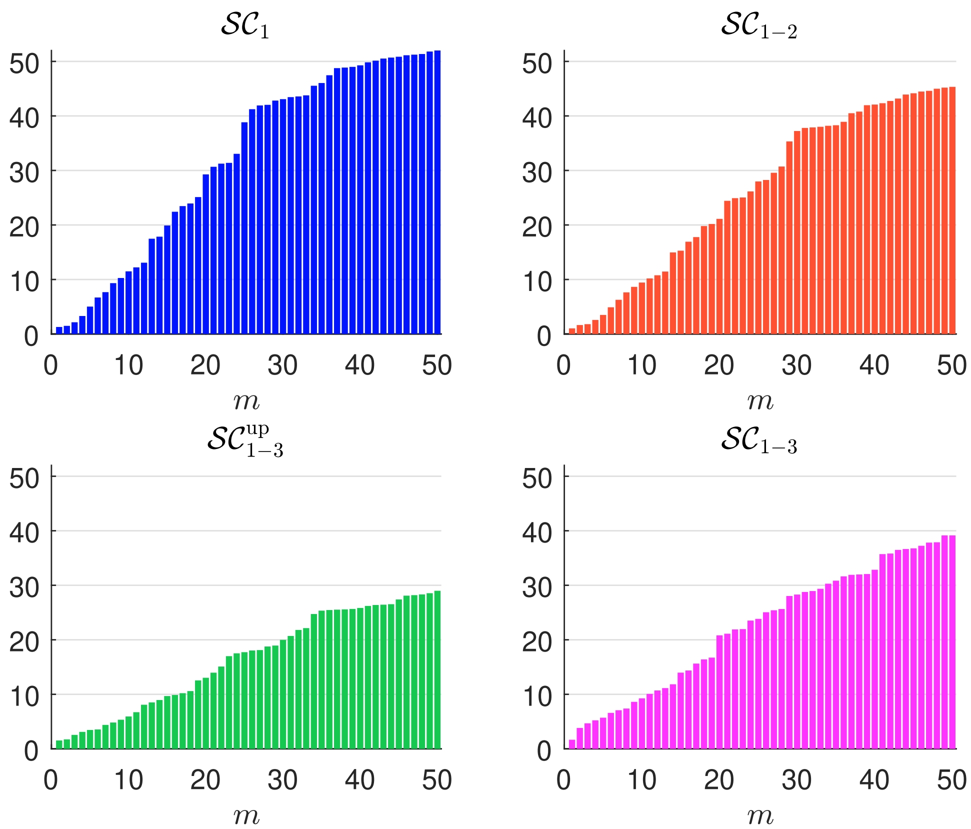
- The carbon footprint contribution of the m worst performing assets is:

$$\mathcal{CFC}^{(m,n)} = \frac{\sum_{i=1}^{n} \mathbb{1}\left\{\mathcal{CI}_i \geq \mathcal{CI}^{(m,n)}\right\} \cdot b_i \mathcal{CI}_i}{CI\left(b\right)}$$

where $\mathcal{CI}^{(m,n)} = \mathcal{CI}_{n-m+1:n}$ is the $(n-m+1)$-th order statistic

Equity portfolios

Figure 220: Carbon footprint contribution $\mathcal{CFC}^{(m,n)}$ in % (MSCI World, June 2022, first approach)



Source: MSCI (2022), Trucost (2022) & Author's calculations

Equity portfolios

Second approach

- Another definition:

$$\mathcal{CFC}^{(m,n)} = \frac{\sum_{i=1}^{n} \mathbb{1}\left\{\mathcal{CIC}_i \geq \mathcal{CIC}^{(m,n)}\right\} \cdot b_i \boldsymbol{\mathcal{CI}}_i}{CI(b)}$$

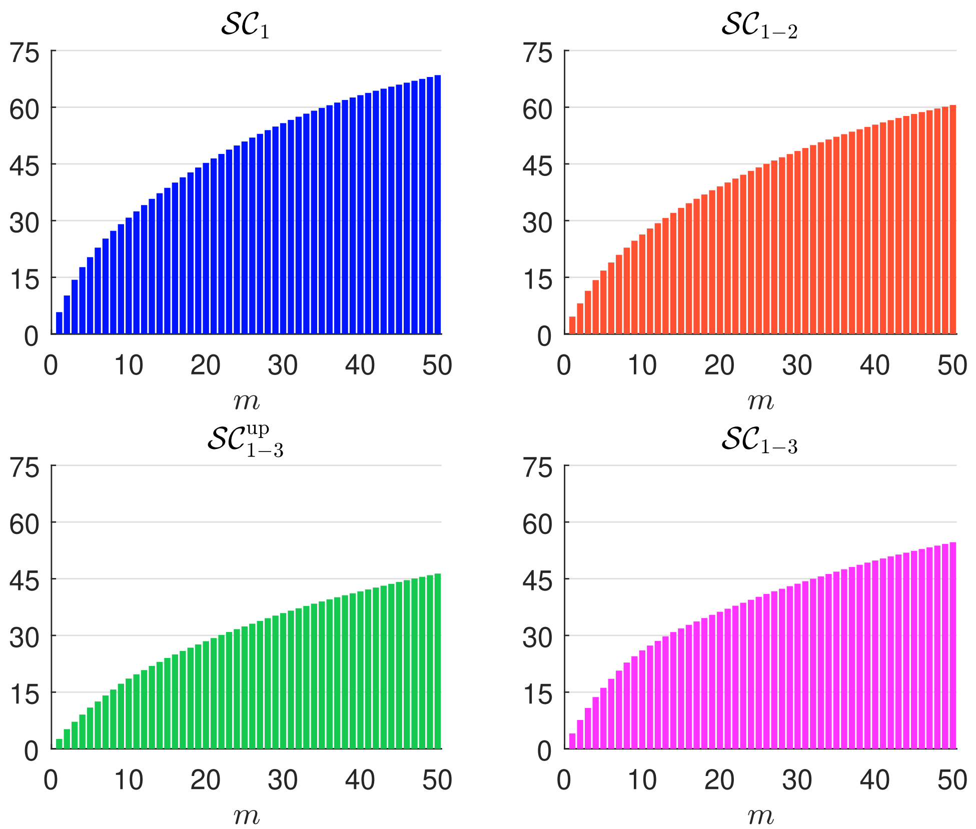
  where $\mathcal{CIC}_i = b_i \boldsymbol{\mathcal{CI}}_i$ and $\mathcal{CIC}^{(m,n)} = \mathcal{CIC}_{n-m+1:n}$

- Weight contribution:

$$\mathcal{WC}^{(m,n)} = \sum_{i=1}^{n} \mathbb{1}\left\{\mathcal{CIC}_i \geq \mathcal{CIC}^{(m,n)}\right\} \cdot b_i$$

Equity portfolios

Figure 221: Carbon footprint contribution $\mathcal{CFC}^{(m,n)}$ in % (MSCI World, June 2022, second approach)



Source: MSCI (2022), Trucost (2022) & Author's calculations

Equity portfolios

Table 126: Carbon footprint contribution $\mathcal{CFC}^{(m,n)}$ in % (MSCI World, June 2022, second approach, $\mathcal{SC}_{1-3}$)

Sector	m							
	1	5	10	25	50	75	100	200
Communication Services						0.44	0.44	0.73
Consumer Discretionary				0.78	1.37	2.44	2.93	4.28
Consumer Staples		2.46	2.46	2.46	3.75	4.44	4.92	5.62
Energy		9.61	17.35	23.78	29.56	31.78	33.02	33.89
Financials						0.72	1.53	1.88
Health Care							0.21	0.37
Industrials			2.16	5.59	7.13	8.70	9.48	13.05
Information Technology				0.98	1.58	1.94	2.15	3.30
Materials	4.08	4.08	4.08	5.81	7.31	8.81	9.59	10.75
Real Estate					0.77	0.77	0.77	0.85
Utilities				0.81	3.20	3.89	5.24	7.98
Total	4.08	16.15	26.06	40.21	54.66	63.94	70.29	82.70

Source: MSCI (2022), Trucost (2022) & Author's calculations

Equity portfolios

Table 127: Weight contribution $\mathcal{WC}^{(m,n)}$ in % (MSCI World, June 2022, second approach, $\boldsymbol{\mathcal{SC}}_{1-3}$)

Sector	b_j (in %)	m = 1	5	10	25	50	75	100	200
Communication Services	7.58						0.08	0.08	3.03
Consumer Discretionary	10.56				0.58	1.79	2.44	4.51	5.89
Consumer Staples	7.80		0.70	0.70	0.70	1.90	2.50	2.84	3.84
Energy	4.99		1.71	2.25	2.96	3.62	3.99	4.33	4.65
Financials	13.56						0.74	1.17	2.33
Health Care	14.15							0.95	1.34
Industrials	9.90			0.06	0.32	0.70	0.96	1.20	4.12
Information Technology	21.08				0.16	4.70	8.42	8.78	11.62
Materials	4.28	0.29	0.29	0.29	0.47	0.88	1.10	1.40	1.87
Real Estate	2.90					0.05	0.05	0.05	0.23
Utilities	3.21				0.31	0.86	1.04	1.31	2.33
Total		0.29	2.71	3.30	5.49	14.50	21.32	26.63	41.24

Source: MSCI (2022), Trucost (2022) & Author's calculations

Equity portfolios

- The order-statistic optimization problem is:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2}(w-b)^{\top}\Sigma(w-b) \\
\text{s.t.} & \left\{\begin{array}{l} \mathbf{1}_n^{\top} w = 1 \\ \mathbf{0}_n \leq w \leq w^{(m,n)} \end{array}\right.
\end{aligned}
$$

  where the upper bound $w^{(m,n)}$ is equal to $\mathbb{1}\left\{\boldsymbol{\mathcal{CI}} < \boldsymbol{\mathcal{CI}}^{(m,n)}\right\}$ for the first ordering approach and $\mathbb{1}\left\{\mathcal{CIC} < \mathcal{CIC}^{(m,n)}\right\}$ for the second ordering approach

Equity portfolios

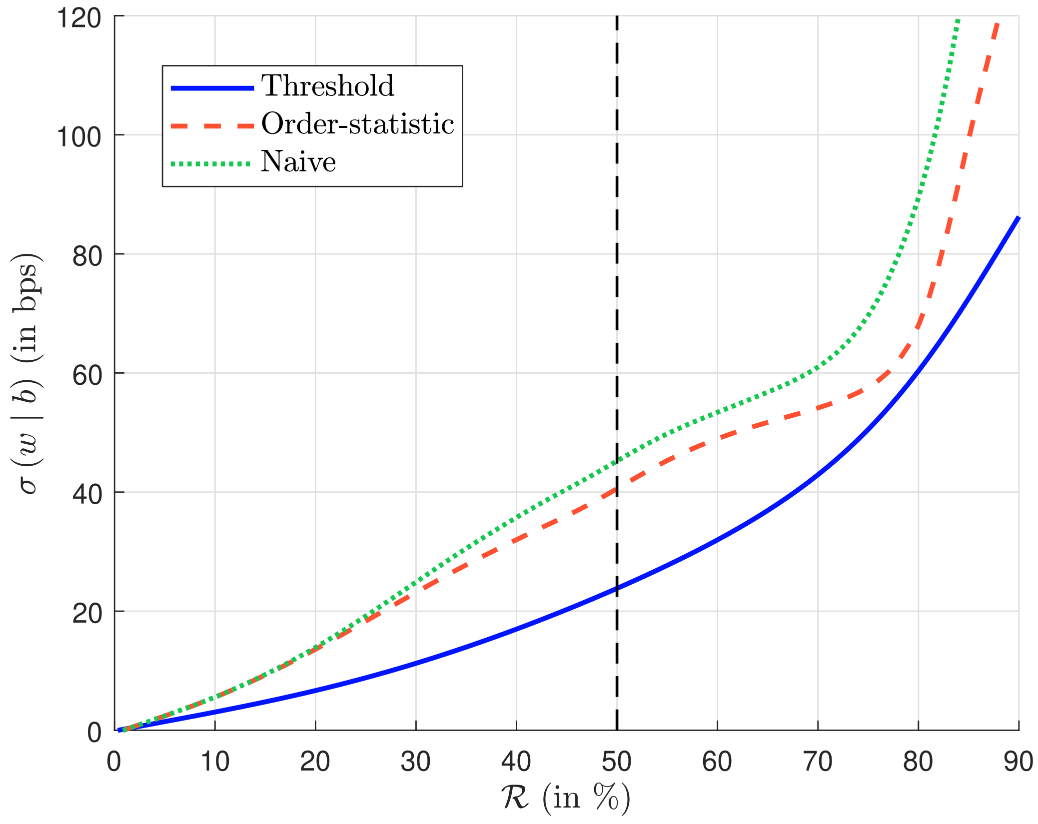
- The naive method is:

$$w_i^\star = \frac{e_i b_i}{\sum_{k=1}^n e_k b_k}$$

where e_i is defined as $\mathbb{1}\left\{\mathcal{CI}_i < \mathcal{CI}^{(m,n)}\right\}$ for the first ordering approach and $\mathbb{1}\left\{\mathcal{CIC}_i < \mathcal{CIC}^{(m,n)}\right\}$ for the second ordering approach

Equity portfolios

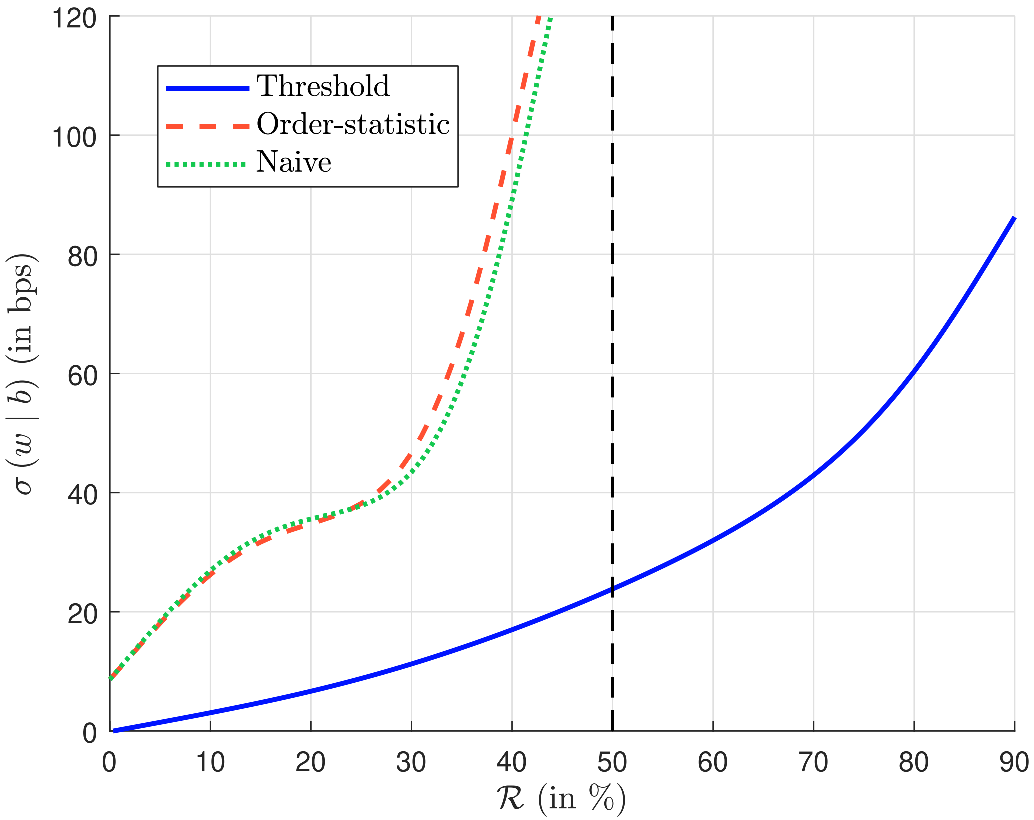
Figure 222: Tracking error volatility (MSCI World, June 2022, $\mathcal{SC}_{1-3}$, first ordering method)



Source: MSCI (2022), Trucost (2022) & Author's calculations

Equity portfolios

Figure 223: Tracking error volatility (MSCI World, June 2022, $\mathcal{SC}_{1-3}$, second ordering method)



Source: MSCI (2022), Trucost (2022) & Author's calculations

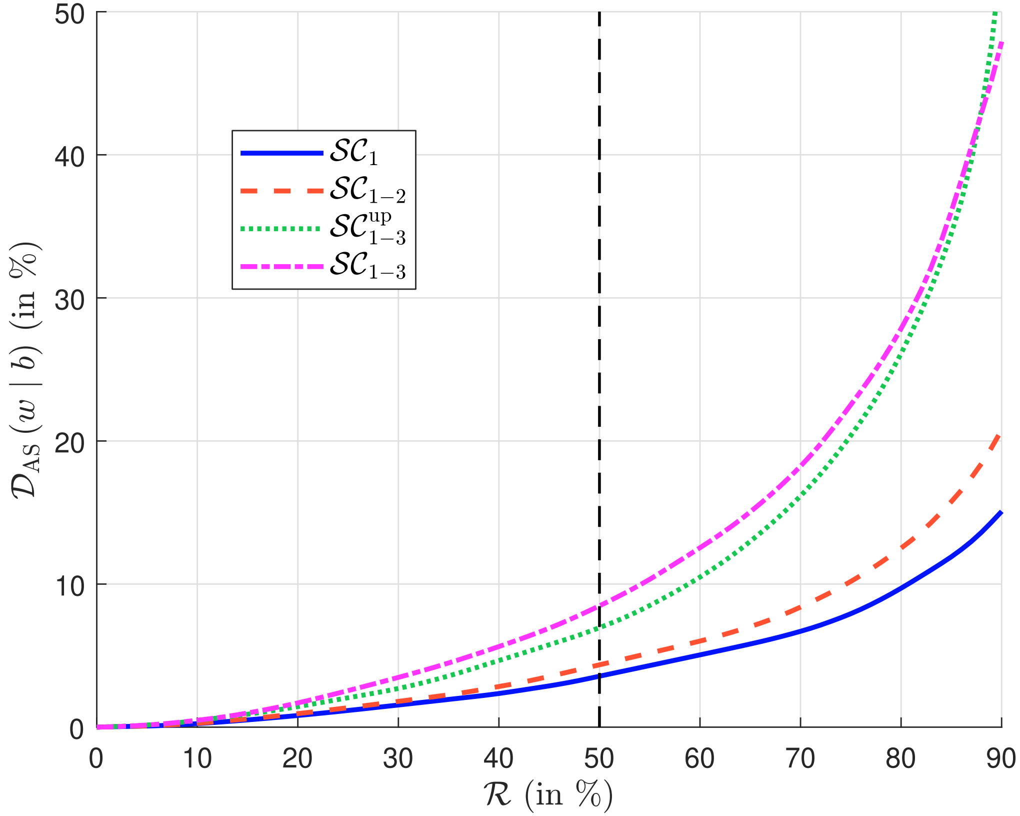
Bond portfolios

The optimization problem is:

$$
\begin{aligned}
w^{\star} &= \arg\min \frac{1}{2}\sum_{i=1}^{n} \left| w_i - b_i \right| + 50 \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum_{i \in \mathcal{S}ector_j} \left(w_i - b_i \right) \mathrm{DTS}_i \right| \\
& \text{s.t.} \quad \left\{ \begin{array}{l} \mathcal{CI}\left(w\right) \leq \left(1 - \mathcal{R}\right) \mathcal{CI}\left(b\right) \\ w \in \mathcal{C}_0 \cap \mathcal{C}_1' \cap \mathcal{C}_3' \cap \mathcal{C}_4' \end{array} \right.
\end{aligned}
$$

Bond portfolios

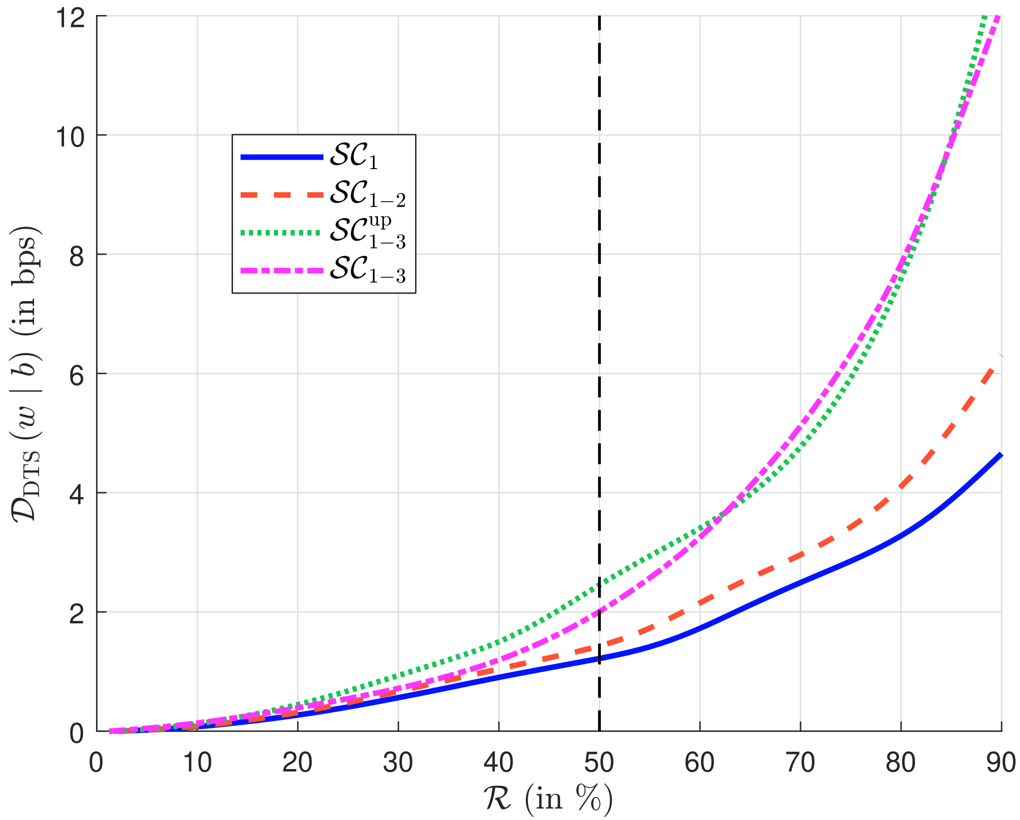
Figure 224: Impact of the carbon scope on the active share in % (ICE Global Corp., June 2022)



Source: ICE (2022), Trucost (2022) & Barahhou *et al.* (2022)

Bond portfolios

Figure 225: Impact of the carbon scope on the DTS risk in bps (ICE Global Corp., June 2022)



Source: ICE (2022), Trucost (2022) & Barahhou *et al.* (2022)

Bond portfolios

Table 128: Sector allocation in % (ICE Global Corp., June 2022, scope $\mathcal{SC}_{1-3}$)

Sector	Index	Reduction rate $\mathcal{R}$						
		30%	40%	50%	60%	70%	80%	90%
Communication Services	7.34	7.35	7.34	7.37	7.43	7.43	7.31	7.30
Consumer Discretionary	5.97	5.97	5.96	5.94	5.93	5.46	4.48	3.55
Consumer Staples	6.04	6.04	6.04	6.04	6.04	6.02	5.39	4.06
Energy	6.49	5.49	4.42	3.84	3.69	3.23	2.58	2.52
Financials	33.91	34.64	35.66	35.96	36.09	37.36	38.86	39.00
Health Care	7.50	7.50	7.50	7.50	7.50	7.50	7.52	7.48
Industrials	8.92	9.38	9.62	10.19	11.34	12.07	13.55	18.13
Information Technology	5.57	5.57	5.59	5.59	5.60	5.60	5.52	5.27
Materials	3.44	3.43	3.31	3.18	3.12	2.64	2.25	1.86
Real Estate	4.76	4.74	4.74	4.74	4.74	4.66	4.61	3.93
Utilities	10.06	9.89	9.82	9.64	8.52	8.04	7.92	6.88

Source: ICE (2022), Trucost (2022) & Barahhou *et al.* (2022)

Course 2022-2023 in Sustainable Finance
Lecture 11. Exercise — Equity and Bond Portfolio Optimization with Green Preferences

Thierry Roncalli*

*Amundi Asset Management[28]

*University of Paris-Saclay

March 2023

[28]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

We consider an investment universe of 8 issuers. In the table below, we report the carbon emissions $\mathcal{CE}_{i,j}$ (in $\mathrm{ktCO_2e}$) of these companies and their revenues Y_i (in \$ bn), and we indicate in the last row whether the company belongs to sector $\mathcal{S}ector_1$ or $\mathcal{S}ector_2$:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{CE}_{i,1}$	75	5 000	720	50	2 500	25	30 000	5
$\mathcal{CE}_{i,2}$	75	5 000	1 030	350	4 500	5	2000	64
$\mathcal{CE}_{i,3}$	24 000	15 000	1 210	550	500	187	30 000	199
Y_i	300	328	125	100	200	102	107	25
$\mathcal{S}ector$	1	2	1	1	2	1	2	2

The benchmark b of this investment universe is defined as:

$$b = (22\%, 19\%, 17\%, 13\%, 11\%, 8\%, 6\%, 4\%)$$

In what follows, we consider long-only portfolios.

Question 1

We want to compute the carbon intensity of the benchmark.

Question (a)

Compute the carbon intensities $\mathcal{CI}_{i,j}$ of each company i for the scopes 1, 2 and 3.

We have:

$$\mathcal{CI}_{i,j} = \frac{\mathcal{CE}_{i,j}}{Y_i}$$

For instance, if we consider the 8^{th} issuer, we have[29]:

$$\begin{aligned}
\mathcal{CI}_{8,1} &= \frac{\mathcal{CE}_{8,1}}{Y_8} = \frac{5}{25} = 0.20 \text{ tCO}_2\text{e}/\$ \text{ mn} \\
\mathcal{CI}_{8,2} &= \frac{\mathcal{CE}_{8,2}}{Y_8} = \frac{64}{25} = 2.56 \text{ tCO}_2\text{e}/\$ \text{ mn} \\
\mathcal{CI}_{8,3} &= \frac{\mathcal{CE}_{8,3}}{Y_8} = \frac{199}{25} = 7.96 \text{ tCO}_2\text{e}/\$ \text{ mn}
\end{aligned}$$

[29] Because 1 $\text{ktCO}_2\text{e}/\$$ bn $= 1$ $\text{tCO}_2\text{e}/\$ \text{ mn}$.

Since we have:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{CE}_{i,1}$	75	5 000	720	50	2 500	25	30 000	5
$\mathcal{CE}_{i,2}$	75	5 000	1 030	350	4 500	5	2000	64
$\mathcal{CE}_{i,3}$	24 000	15 000	1 210	550	500	187	30 000	199
Y_i	300	328	125	100	200	102	107	25

we obtain:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{CI}_{i,1}$	0.25	15.24	5.76	0.50	12.50	0.25	280.37	0.20
$\mathcal{CI}_{i,2}$	0.25	15.24	8.24	3.50	22.50	0.05	18.69	2.56
$\mathcal{CI}_{i,3}$	80.00	45.73	9.68	5.50	2.50	1.83	280.37	7.96

Question (b)

Deduce the carbon intensities $\mathcal{CI}_{i,j}$ of each company i for the scopes $1+2$ and $1+2+3$.

We have:

$$\mathcal{CI}_{i,1-2} = \frac{\mathcal{CE}_{i,1} + \mathcal{CE}_{i,2}}{Y_i} = \mathcal{CI}_{i,1} + \mathcal{CI}_{i,2}$$

and:

$$\mathcal{CI}_{i,1-3} = \mathcal{CI}_{i,1} + \mathcal{CI}_{i,2} + \mathcal{CI}_{i,3}$$

We deduce that:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{CI}_{i,1}$	0.25	15.24	5.76	0.50	12.50	0.25	280.37	0.20
$\mathcal{CI}_{i,1-2}$	0.50	30.49	14.00	4.00	35.00	0.29	299.07	2.76
$\mathcal{CI}_{i,1-3}$	80.50	76.22	23.68	9.50	37.50	2.12	579.44	10.72

Question (c)

Deduce the weighted average carbon intensity (WACI) of the benchmark if we consider the scope $1 + 2 + 3$.

We have:

$$\begin{aligned}\mathcal{CI}(b) &= \sum_{i=1}^{8} b_i \mathcal{CI}_i \\ &= 0.22 \times 80.50 + 0.19 \times 76.2195 + 0.17 \times 23.68 + 0.13 \times 9.50 + \\ &\quad 0.11 \times 37.50 + 0.08 \times 2.1275 + 0.06 \times 579.4393 + 0.04 \times 10.72 \\ &= 76.9427 \text{ tCO}_2\text{e}/\$ \text{ mn}\end{aligned}$$

Question (d)

We assume that the market capitalization of the benchmark portfolio is equal to \$10 tn and we invest \$1 bn.

Question (d).i

Deduce the market capitalization of each company (expressed in $ bn).

We have:

$$b_i = \frac{MC_i}{\sum_{k=1}^{8} \mathrm{MC}_k}$$

and $\sum_{k=1}^{8} \mathrm{MC}_k = \$10$ tn. We deduce that:

$$\mathrm{MC}_i = 10 \times b_i$$

We obtain the following values of market capitalization expressed in \$ bn:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
MC_i	2 200	1 900	1 700	1 300	1 100	800	600	400

Question (d).ii

Compute the ownership ratio for each asset (expressed in bps).

Let W be the wealth invested in the benchmark portfolio b. The wealth invested in asset i is equal to $b_i W$. We deduce that the ownership ratio is equal to:

$$\varpi_i = \frac{b_i W}{\mathrm{MC}_i} = \frac{b_i W}{b_i \sum_{k=1}^{n} \mathrm{MC}_k} = \frac{W}{\sum_{k=1}^{n} \mathrm{MC}_k}$$

When we invest in a capitalization-weighted portfolio, the ownership ratio is the same for all the assets. In our case, we have:

$$\varpi_i = \frac{1}{10 \times 1000} = 0.01\%$$

The ownership ratio is equal to 1 basis point.

Question (d).iii

Compute the carbon emissions of the benchmark portfolio[a] if we invest \$1 bn and we consider the scope $1 + 2 + 3$.

[a] We assume that the float percentage is equal to 100% for all the 8 companies.

Using the financed emissions approach, the carbon emissions of our investment is equal to:

$$\begin{aligned}
\mathcal{CE}\,(\$1\text{ bn}) &= 0.01\% \times (75 + 75 + 24\,000) + \\
&\quad 0.01\% \times (5\,000 + 5\,000 + 15\,000) + \\
&\quad \ldots + \\
&\quad 0.01\% \times (5 + 64 + 199) \\
&= 12.3045 \text{ ktCO}_2\text{e}
\end{aligned}$$

Question (d).iv

Compare the (exact) carbon intensity of the benchmark portfolio with the WACI value obtained in Question 1.(c).

We compute the revenues of our investment:

$$Y(\$1 \text{ bn}) = 0.01\% \sum_{i=1}^{8} Y_i = \$0.1287 \text{ bn}$$

We deduce that the exact carbon intensity is equal to:

$$\mathcal{CI}(\$1 \text{ bn}) = \frac{\mathcal{CE}(\$1 \text{ bn})}{Y(\$1 \text{ bn})} = \frac{12.3045}{0.1287} = 95.6061 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

We notice that the WACI of the benchmark underestimates the exact carbon intensity of our investment by 19.5%:

$$76.9427 < 95.6061$$

Question 2

We want to manage an equity portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $\mathcal{R}$. We assume that the volatility of the stocks is respectively equal to 22%, 20%, 25%, 18%, 40%, 23%, 13% and 29%. The correlation matrix between these stocks is given by:

$$\rho = \begin{pmatrix} 100\% & & & & & & & \\ 80\% & 100\% & & & & & & \\ 70\% & 75\% & 100\% & & & & & \\ 60\% & 65\% & 80\% & 100\% & & & & \\ 70\% & 50\% & 70\% & 85\% & 100\% & & & \\ 50\% & 60\% & 70\% & 80\% & 60\% & 100\% & & \\ 70\% & 50\% & 70\% & 75\% & 80\% & 50\% & 100\% & \\ 60\% & 65\% & 70\% & 75\% & 65\% & 70\% & 60\% & 100\% \end{pmatrix}$$

Question (a)

Compute the covariance matrix Σ.

The covariance matrix $\Sigma = (\Sigma_{i,j})$ is defined by:

$$\Sigma_{i,j} = \rho_{i,j}\sigma_i\sigma_j$$

We obtain the following numerical values (expressed in bps):

$$\Sigma = \begin{pmatrix} 484.0 & 352.0 & 385.0 & 237.6 & 616.0 & 253.0 & 200.2 & 382.8 \\ 352.0 & 400.0 & 375.0 & 234.0 & 400.0 & 276.0 & 130.0 & 377.0 \\ 385.0 & 375.0 & 625.0 & 360.0 & 700.0 & 402.5 & 227.5 & 507.5 \\ 237.6 & 234.0 & 360.0 & 324.0 & 612.0 & 331.2 & 175.5 & 391.5 \\ 616.0 & 400.0 & 700.0 & 612.0 & 1600.0 & 552.0 & 416.0 & 754.0 \\ 253.0 & 276.0 & 402.5 & 331.2 & 552.0 & 529.0 & 149.5 & 466.9 \\ 200.2 & 130.0 & 227.5 & 175.5 & 416.0 & 149.5 & 169.0 & 226.2 \\ 382.8 & 377.0 & 507.5 & 391.5 & 754.0 & 466.9 & 226.2 & 841.0 \end{pmatrix}$$

Question (b)

Write the optimization problem if the objective function is to minimize the tracking error risk under the constraint of carbon intensity reduction.

The tracking error variance of portfolio w with respect to benchmark b is equal to:

$$\sigma^2 (w \mid b) = (w - b)^\top \Sigma (w - b)$$

The carbon intensity constraint has the following expression:

$$\sum_{i=1}^{8} w_i \mathcal{CI}_i \leq (1 - \mathcal{R}) \mathcal{CI}(b)$$

where $\mathcal{R}$ is the reduction rate and $\mathcal{CI}(b)$ is the carbon intensity of the benchmark. Let $\mathcal{CI}^\star = (1 - \mathcal{R}) \mathcal{CI}(b)$ be the target value of the carbon footprint. The optimization problem is then:

$$\begin{array}{rcl} w^\star & = & \arg\min \frac{1}{2} \sigma^2 (w \mid b) \\ & \text{s.t.} & \left\{ \begin{array}{l} \sum_{i=1}^{8} w_i \mathcal{CI}_i \leq \mathcal{CI}^\star \\ \sum_{i=1}^{8} w_i = 1 \\ 0 \leq w_i \leq 1 \end{array} \right. \end{array}$$

We add the second and third constraints in order to obtain a long-only portfolio.

Question (c)

Give the QP formulation of the optimization problem.

The objective function is equal to:

$$f(w) = \frac{1}{2}\sigma^2(w \mid b) = \frac{1}{2}(w-b)^\top \Sigma (w-b) = \frac{1}{2}w^\top \Sigma w - w^\top \Sigma b + \frac{1}{2}b^\top \Sigma b$$

while the matrix form of the carbon intensity constraint is:

$$\boldsymbol{\mathcal{CI}}^\top w \leq \boldsymbol{\mathcal{CI}}^\star$$

where $\boldsymbol{\mathcal{CI}} = (\mathcal{CI}_1, \ldots, \mathcal{CI}_8)$ is the column vector of carbon intensities. Since $b^\top \Sigma b$ is a constant and does not depend on w, we can cast the previous optimization problem into a QP problem:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2}w^\top Q w - w^\top R \\ & \text{s.t.} \quad \left\{ \begin{array}{l} Aw = B \\ Cw \leq D \\ w^- \leq w \leq w^+ \end{array} \right. \end{aligned}$$

We have $Q = \Sigma$, $R = \Sigma b$, $A = \mathbf{1}_8^\top$, $B = 1$, $C = \boldsymbol{\mathcal{CI}}^\top$, $D = \boldsymbol{\mathcal{CI}}^\star$, $w^- = \mathbf{0}_8$ and $w^+ = \mathbf{1}_8$.

Question (d)

$\mathcal{R}$ is equal to 20%. Find the optimal portfolio if we target scope $1+2$. What is the value of the tracking error volatility?

We have:

$$\begin{aligned}\mathcal{CI}(b) &= 0.22 \times 0.50 + 0.19 \times 30.4878 + \ldots + 0.04 \times 2.76 \\ &= 30.7305 \text{ tCO}_2\text{e}/\$ \text{ mn}\end{aligned}$$

We deduce that:

$$\mathcal{CI}^{\star} = (1 - \mathcal{R})\,\mathcal{CI}(b) = 0.80 \times 30.7305 = 24.5844 \text{ tCO}_2\text{e}/\$ \text{ mn}$$

Therefore, the inequality constraint of the QP problem is:

$$\begin{pmatrix} 0.50 & 30.49 & 14.00 & 4.00 & 35.00 & 0.29 & 299.07 & 2.76 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_7 \\ w_8 \end{pmatrix} \leq 24.5844$$

We obtain the following optimal solution:

$$w^{\star} = \begin{pmatrix} 23.4961\% \\ 17.8129\% \\ 17.1278\% \\ 15.4643\% \\ 10.4037\% \\ 7.5903\% \\ 4.0946\% \\ 4.0104\% \end{pmatrix}$$

The minimum tracking error volatility $\sigma(w^{\star} \mid b)$ is equal to 15.37 bps.

Question (e)

Same question if $\mathcal{R}$ is equal to 30%, 50%, and 70%.

Table 129: Solution of the equity optimization problem (scope $\mathcal{SC}_{1-2}$)

$\mathcal{R}$	0%	20%	30%	50%	70%
w_1	22.0000	23.4961	24.2441	25.7402	30.4117
w_2	19.0000	17.8129	17.2194	16.0323	9.8310
w_3	17.0000	17.1278	17.1917	17.3194	17.8348
w_4	13.0000	15.4643	16.6964	19.1606	23.3934
w_5	11.0000	10.4037	10.1055	9.5091	7.1088
w_6	8.0000	7.5903	7.3854	6.9757	6.7329
w_7	6.0000	4.0946	3.1418	1.2364	0.0000
w_8	4.0000	4.0104	4.0157	4.0261	4.6874
$\mathcal{CI}(w)$	30.7305	24.5844	21.5114	15.3653	9.2192
$\sigma(w \mid b)$	0.00	15.37	23.05	38.42	72.45

In Table 129, we report the optimal solution $w^{\star}$ (expressed in %) of the optimization problem for different values of $\mathcal{R}$. We also indicate the carbon intensity of the portfolio (in $\mathrm{tCO_2e/\$ mn}$) and the tracking error volatility (in bps). For instance, if $\mathcal{R}$ is set to 50%, the weights of assets #1, #3, #4 and #8 increase whereas the weights of assets #2, #5, #6 and #7 decrease. The carbon intensity of this portfolio is equal to 15.3653 $\mathrm{tCO_2e/\$ mn}$. The tracking error volatility is below 40 bps, which is relatively low.

Question (f)

We target scope $1+2+3$. Find the optimal portfolio if $\mathcal{R}$ is equal to $20\%, 30\%$, 50% and 70%. Give the value of the tracking error volatility for each optimized portfolio.

In this case, the inequality constraint $Cw \leq D$ is defined by:

$$C = \mathcal{CI}_{1-3}^{\top} = \begin{pmatrix} 80.5000 \\ 76.2195 \\ 23.6800 \\ 9.5000 \\ 37.5000 \\ 2.1275 \\ 579.4393 \\ 10.7200 \end{pmatrix}^{\top}$$

and:

$$D = (1 - \mathcal{R}) \times 76.9427$$

We obtain the results given in Table 130.

Table 130: Solution of the equity optimization problem (scope $\mathcal{SC}_{1-3}$)

$\mathcal{R}$	0%	20%	30%	50%	70%
w_1	22.0000	23.9666	24.9499	26.4870	13.6749
w_2	19.0000	17.4410	16.6615	8.8001	0.0000
w_3	17.0000	17.1988	17.2981	19.4253	24.1464
w_4	13.0000	16.5034	18.2552	25.8926	41.0535
w_5	11.0000	10.2049	9.8073	7.1330	3.5676
w_6	8.0000	7.4169	7.1254	7.0659	8.8851
w_7	6.0000	3.2641	1.8961	0.0000	0.0000
w_8	4.0000	4.0043	4.0065	5.1961	8.6725
$\mathcal{CI}(w)$	76.9427	61.5541	53.8599	38.4713	23.0828
$\sigma(w \mid b)$	0.00	21.99	32.99	104.81	414.48

Question (g)

Compare the optimal solutions obtained in Questions 2.(e) and 2.(f).

Figure 226: Impact of the scope on the tracking error volatility

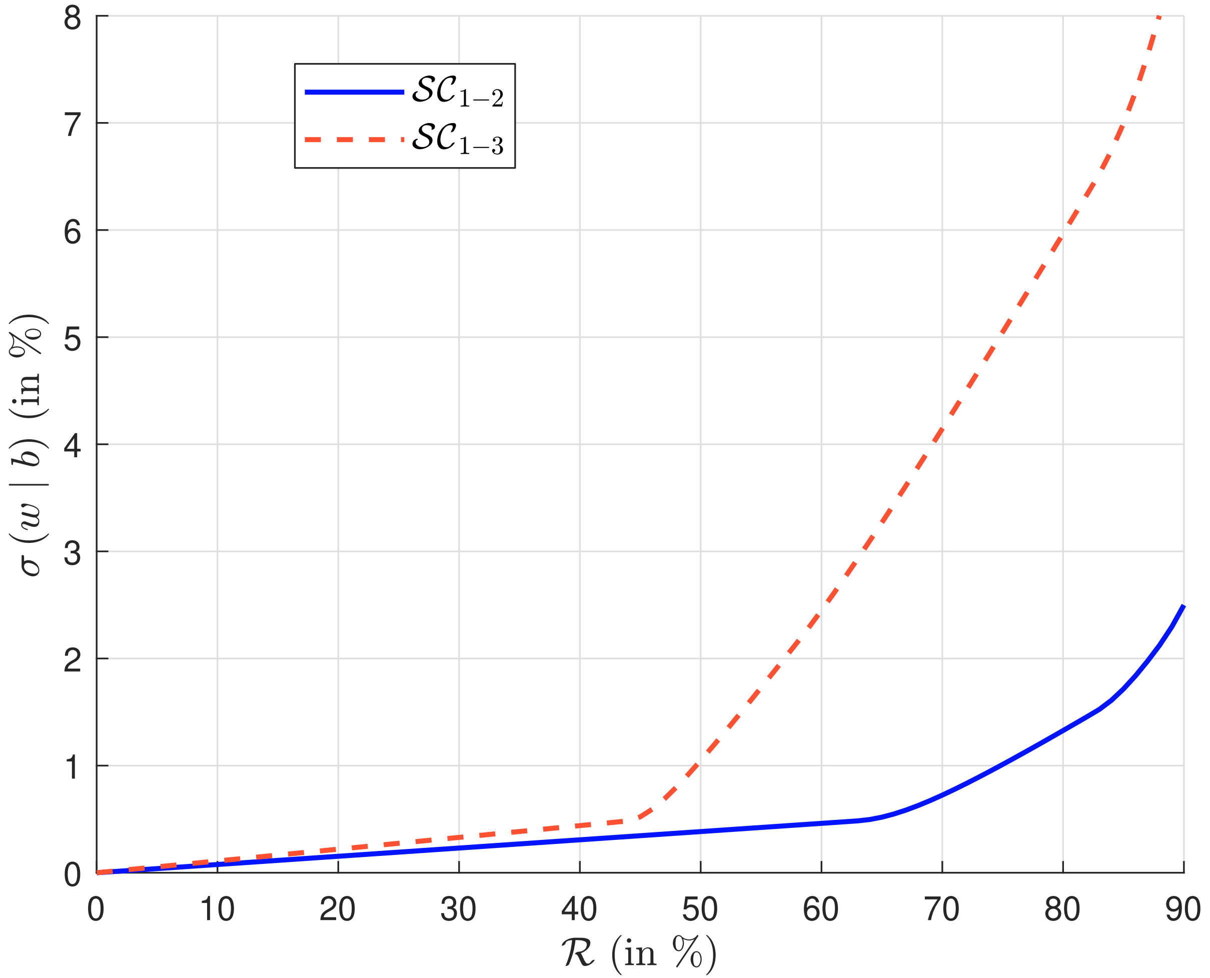
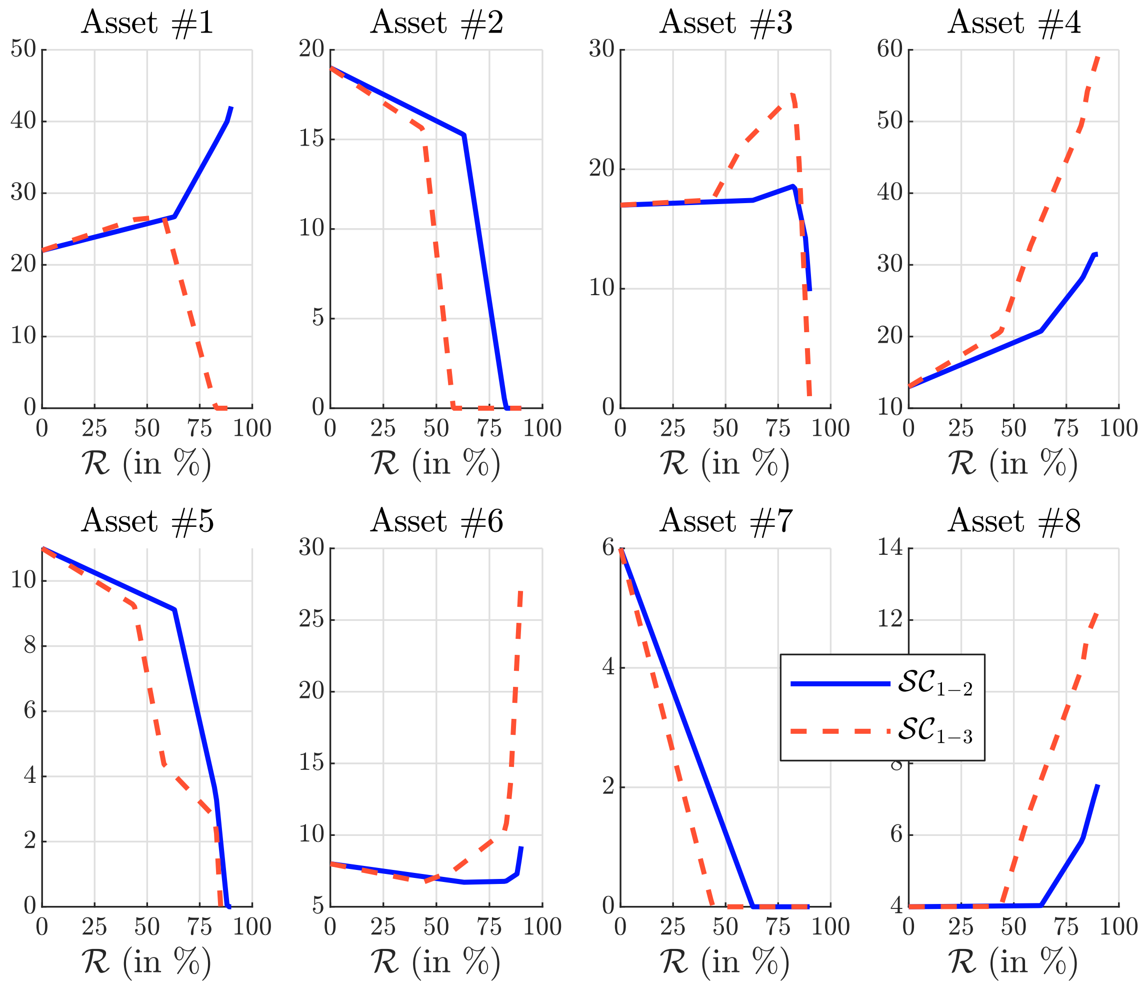


Figure 227: Impact of the scope on the portfolio allocation (in %)



In Figure 226, we report the relationship between the reduction rate $\mathcal{R}$ and the tracking error volatility $\sigma(w \mid b)$. The choice of the scope has little impact when $\mathcal{R} \leq 45\%$. Then, we notice a high increase when we consider the scope $1+2+3$. The portfolio's weights are given in Figure 227. For assets $\#1$ and $\#3$, the behavior is divergent when we compare scopes $1+2$ and $1+2+3$.

Question 3

We want to manage a bond portfolio with respect to the previous investment universe and reduce the weighted average carbon intensity of the benchmark by the rate $\mathcal{R}$. We use the scope $1+2+3$. In the table below, we report the modified duration MD_i and the duration-times-spread factor DTS_i of each corporate bond i:

Asset	#1	#2	#3	#4	#5	#6	#7	#8
MD_i (in years)	3.56	7.48	6.54	10.23	2.40	2.30	9.12	7.96
DTS_i (in bps)	103	155	75	796	89	45	320	245
$\mathcal{S}$ector	1	2	1	1	2	1	2	2

Question 3 (Cont'd)

We remind that the active risk can be calculated using three functions. For the active share, we have:

$$\mathcal{R}_{\mathrm{AS}}\left(w \mid b\right) = \sigma_{\mathrm{AS}}^2\left(w \mid b\right) = \sum_{i=1}^{n} \left(w_i - b_i\right)^2$$

We also consider the MD-based tracking error risk:

$$\mathcal{R}_{\mathrm{MD}}\left(w \mid b\right) = \sigma_{\mathrm{MD}}^2\left(w \mid b\right) = \sum_{j=1}^{n_{\boldsymbol{\mathcal{S}}ector}} \left(\sum\nolimits_{i \in \boldsymbol{\mathcal{S}}ector_j} \left(w_i - b_i\right) \mathrm{MD}_i\right)^2$$

and the DTS-based tracking error risk:

$$\mathcal{R}_{\mathrm{DTS}}\left(w \mid b\right) = \sigma_{\mathrm{DTS}}^2\left(w \mid b\right) = \sum_{j=1}^{n_{\boldsymbol{\mathcal{S}}ector}} \left(\sum\nolimits_{i \in \boldsymbol{\mathcal{S}}ector_j} \left(w_i - b_i\right) \mathrm{DTS}_i\right)^2$$

Question 3 (Cont'd)

Finally, we define the synthetic risk measure as a combination of AS, MD and DTS active risks:

$$\mathcal{R}(w \mid b) = \varphi_{\mathrm{AS}} \mathcal{R}_{\mathrm{AS}}(w \mid b) + \varphi_{\mathrm{MD}} \mathcal{R}_{\mathrm{MD}}(w \mid b) + \varphi_{\mathrm{DTS}} \mathcal{R}_{\mathrm{DTS}}(w \mid b)$$

where $\varphi_{\mathrm{AS}} \geq 0$, $\varphi_{\mathrm{MD}} \geq 0$ and $\varphi_{\mathrm{DTS}} \geq 0$ indicate the weight of each risk. In what follows, we use the following numerical values: $\varphi_{\mathrm{AS}} = 100$, $\varphi_{\mathrm{MD}} = 25$ and $\varphi_{\mathrm{DTS}} = 1$. The reduction rate $\boldsymbol{\mathcal{R}}$ of the weighted average carbon intensity is set to 50% for the scope $1 + 2 + 3$.

Question (a)

Compute the modified duration $\mathrm{MD}(b)$ and the duration-times-spread factor $\mathrm{DTS}(b)$ of the benchmark.

We have:

$$\begin{aligned}\mathrm{MD}(b) &= \sum_{i=1}^{n} b_i \,\mathrm{MD}_i \\ &= 0.22 \times 3.56 + 0.19 \times 7.48 + \ldots + 0.04 \times 7.96 \\ &= 5.96 \text{ years}\end{aligned}$$

and:

$$\begin{aligned}\mathrm{DTS}(b) &= \sum_{i=1}^{n} b_i \,\mathrm{DTS}_i \\ &= 0.22 \times 103 + 0.19 \times 155 + \ldots + 0.04 \times 155 \\ &= 210.73 \text{ bps}\end{aligned}$$

Question (b)

Let w_{ew} be the equally-weighted portfolio. Compute[a] $\mathrm{MD}\left(w_{\mathrm{ew}}\right)$, $\mathrm{DTS}\left(w_{\mathrm{ew}}\right)$, $\sigma_{\mathrm{AS}}\left(w_{\mathrm{ew}} \mid b\right)$, $\sigma_{\mathrm{MD}}\left(w_{\mathrm{ew}} \mid b\right)$ and $\sigma_{\mathrm{DTS}}\left(w_{\mathrm{ew}} \mid b\right)$.

[a] Precise the corresponding unit (years, bps or %) for each metric.

We have:

$$\left\{\begin{array}{l} \mathrm{MD}\left(w_{\mathrm{ew}}\right) = 6.20 \text{ years} \\ \mathrm{DTS}\left(w_{\mathrm{ew}}\right) = 228.50 \text{ bps} \\ \sigma_{\mathrm{AS}}\left(w_{\mathrm{ew}} \mid b\right) = 17.03\% \\ \sigma_{\mathrm{MD}}\left(w_{\mathrm{ew}} \mid b\right) = 1.00 \text{ years} \\ \sigma_{\mathrm{DTS}}\left(w_{\mathrm{ew}} \mid b\right) = 36.19 \text{ bps} \end{array}\right.$$

Question (c)

We consider the following optimization problem:

$$\begin{aligned} w^{\star} &= \arg\min \frac{1}{2}\mathcal{R}_{\mathrm{AS}}\left(w \mid b\right) \\ &\text{s.t. } \left\{ \begin{array}{l} \sum_{i=1}^{n} w_i = 1 \\ \mathrm{MD}\left(w\right) = \mathrm{MD}\left(b\right) \\ \mathrm{DTS}\left(w\right) = \mathrm{DTS}\left(b\right) \\ \mathcal{CI}\left(w\right) \leq \left(1 - \mathcal{R}\right)\mathcal{CI}\left(b\right) \\ 0 \leq w_i \leq 1 \end{array} \right. \end{aligned}$$

Give the analytical value of the objective function. Find the optimal portfolio $w^{\star}$. Compute $\mathrm{MD}\left(w^{\star}\right)$, $\mathrm{DTS}\left(w^{\star}\right)$, $\sigma_{\mathrm{AS}}\left(w^{\star} \mid b\right)$, $\sigma_{\mathrm{MD}}\left(w^{\star} \mid b\right)$ and $\sigma_{\mathrm{DTS}}\left(w^{\star} \mid b\right)$.

We have:

$$\begin{aligned}\mathcal{R}_{\mathrm{AS}}(w \mid b) &= (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 + \\ &\quad (w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2\end{aligned}$$

The objective function is then:

$$f(w) = \frac{1}{2}\mathcal{R}_{\mathrm{AS}}(w \mid b)$$

The optimal solution is equal to:

$$w^{\star} = (17.30\%, 17.41\%, 20.95\%, 14.41\%, 10.02\%, 11.09\%, 0\%, 8.81\%)$$

The risk metrics are:

$$\left\{\begin{array}{l}\mathrm{MD}(w^{\star}) = 5.96 \text{ years} \\ \mathrm{DTS}(w^{\star}) = 210.73 \text{ bps} \\ \sigma_{\mathrm{AS}}(w^{\star} \mid b) = 10.57\% \\ \sigma_{\mathrm{MD}}(w^{\star} \mid b) = 0.43 \text{ years} \\ \sigma_{\mathrm{DTS}}(w^{\star} \mid b) = 15.21 \text{ bps}\end{array}\right.$$

Question (d)

We consider the following optimization problem:

$$
\begin{aligned}
w^{\star} \quad &= \quad \arg\min \frac{\varphi_{\mathrm{AS}}}{2}\mathcal{R}_{\mathrm{AS}}\left(w \mid b\right) + \frac{\varphi_{\mathrm{MD}}}{2}\mathcal{R}_{\mathrm{MD}}\left(w \mid b\right) \\
&\text{s.t.} \quad \left\{
\begin{array}{l}
\sum_{i=1}^{n} w_i = 1 \\
\mathrm{DTS}\left(w\right) = \mathrm{DTS}\left(b\right) \\
\boldsymbol{\mathcal{CI}}\left(w\right) \leq \left(1 - \boldsymbol{\mathcal{R}}\right)\boldsymbol{\mathcal{CI}}\left(b\right) \\
0 \leq w_i \leq 1
\end{array}
\right.
\end{aligned}
$$

Give the analytical value of the objective function. Find the optimal portfolio $w^{\star}$. Compute $\mathrm{MD}\left(w^{\star}\right)$, $\mathrm{DTS}\left(w^{\star}\right)$, $\sigma_{\mathrm{AS}}\left(w^{\star} \mid b\right)$, $\sigma_{\mathrm{MD}}\left(w^{\star} \mid b\right)$ and $\sigma_{\mathrm{DTS}}\left(w^{\star} \mid b\right)$.

We have[30]:

$$\begin{aligned}\mathcal{R}_{\mathrm{MD}}(w \mid b) &= \left(\sum_{i=1,3,4,6}(w_i - b_i)\,\mathrm{MD}_i\right)^2 + \left(\sum_{i=2,5,7,8}(w_i - b_i)\,\mathrm{MD}_i\right)^2 \\ &= \left(\sum_{i=1,3,4,6} w_i\,\mathrm{MD}_i - \sum_{i=1,3,4,6} b_i\,\mathrm{MD}_i\right)^2 + \\ &\quad \left(\sum_{i=2,5,7,8} w_i\,\mathrm{MD}_i - \sum_{i=2,5,7,8} b_i\,\mathrm{MD}_i\right)^2 \\ &= (3.56w_1 + 6.54w_3 + 10.23w_4 + 2.30w_6 - 3.4089)^2 + \\ &\quad (7.48w_2 + 2.40w_5 + 9.12w_7 + 7.96w_8 - 2.5508)^2\end{aligned}$$

The objective function is then:

$$f(w) = \frac{\varphi_{\mathrm{AS}}}{2}\mathcal{R}_{\mathrm{AS}}(w \mid b) + \frac{\varphi_{\mathrm{MD}}}{2}\mathcal{R}_{\mathrm{MD}}(w \mid b)$$

[30] We verify that $3.4089 + 2.5508 = 5.9597$ years.

The optimal solution is equal to:

$$w^{\star} = (16.31\%, 18.44\%, 17.70\%, 13.82\%, 11.67\%, 11.18\%, 0\%, 10.88\%)$$

The risk metrics are:

$$\begin{cases} \mathrm{MD}\left(w^{\star}\right) = 5.93 \text{ years} \\ \mathrm{DTS}\left(w^{\star}\right) = 210.73 \text{ bps} \\ \sigma_{\mathrm{AS}}\left(w^{\star} \mid b\right) = 11.30\% \\ \sigma_{\mathrm{MD}}\left(w^{\star} \mid b\right) = 0.03 \text{ years} \\ \sigma_{\mathrm{DTS}}\left(w^{\star} \mid b\right) = 3.70 \text{ bps} \end{cases}$$

Question (e)

We consider the following optimization problem:

$$w^{\star} = \arg\min \frac{1}{2}\mathcal{R}\left(w \mid b\right)$$
$$\text{s.t.} \quad \left\{\begin{array}{l} \sum_{i=1}^{n} w_i = 1 \\ \mathcal{CI}\left(w\right) \leq \left(1 - \boldsymbol{\mathcal{R}}\right)\mathcal{CI}\left(b\right) \\ 0 \leq w_i \leq 1 \end{array}\right.$$

Give the analytical value of the objective function. Find the optimal portfolio $w^{\star}$. Compute $\mathrm{MD}\left(w^{\star}\right)$, $\mathrm{DTS}\left(w^{\star}\right)$, $\sigma_{\mathrm{AS}}\left(w^{\star} \mid b\right)$, $\sigma_{\mathrm{MD}}\left(w^{\star} \mid b\right)$ and $\sigma_{\mathrm{DTS}}\left(w^{\star} \mid b\right)$.

We have[31]:

$$\begin{aligned}
\mathcal{R}_{\mathrm{DTS}}(w \mid b) &= \left(\sum_{i=1,3,4,6}(w_i - b_i)\,\mathrm{DTS}_i\right)^2 + \left(\sum_{i=2,5,7,8}(w_i - b_i)\,\mathrm{DTS}_i\right)^2 \\
&= (103w_1 + 75w_3 + 796w_4 + 45w_6 - 142.49)^2 + \\
&\quad (155w_2 + 89w_5 + 320w_7 + 245w_8 - 68.24)^2
\end{aligned}$$

The objective function is then:

$$f(w) = \frac{\varphi_{\mathrm{AS}}}{2}\mathcal{R}_{\mathrm{AS}}(w \mid b) + \frac{\varphi_{\mathrm{MD}}}{2}\mathcal{R}_{\mathrm{MD}}(w \mid b) + \frac{\varphi_{\mathrm{DTS}}}{2}\mathcal{R}_{\mathrm{DTS}}(w \mid b)$$

[31] We verify that $142.49 + 68.24 = 210.73$ bps.

The optimal solution is equal to:

$$w^{\star} = (16.98\%, 17.21\%, 18.26\%, 13.45\%, 12.10\%, 9.46\%, 0\%, 12.55\%)$$

The risk metrics are:

$$\left\{ \begin{array}{l} \mathrm{MD}\left(w^{\star}\right) = 5.97 \text{ years} \\ \mathrm{DTS}\left(w^{\star}\right) = 210.68 \text{ bps} \\ \sigma_{\mathrm{AS}}\left(w^{\star} \mid b\right) = 11.94\% \\ \sigma_{\mathrm{MD}}\left(w^{\star} \mid b\right) = 0.03 \text{ years} \\ \sigma_{\mathrm{DTS}}\left(w^{\star} \mid b\right) = 0.06 \text{ bps} \end{array} \right.$$

Question (f)

Comment on the results obtained in Questions 3.(c), 3.(d) and 3.(e).

Table 131: Solution of the bond optimization problem (scope $\mathcal{SC}_{1-3}$)

Problem	Benchmark	3.(c)	3.(d)	3.(e)
w_1	22.0000	17.3049	16.3102	16.9797
w_2	19.0000	17.4119	18.4420	17.2101
w_3	17.0000	20.9523	17.6993	18.2582
w_4	13.0000	14.4113	13.8195	13.4494
w_5	11.0000	10.0239	11.6729	12.1008
w_6	8.0000	11.0881	11.1792	9.4553
w_7	6.0000	0.0000	0.0000	0.0000
w_8	4.0000	8.8075	10.8769	12.5464
$\mathrm{MD}(w)$	5.9597	5.9597	5.9344	5.9683
$\mathrm{DTS}(w)$	210.7300	210.7300	210.7300	210.6791
$\sigma_{\mathrm{AS}}(w \mid b)$	0.0000	10.5726	11.3004	11.9400
$\sigma_{\mathrm{MD}}(w \mid b)$	0.0000	0.4338	0.0254	0.0308
$\sigma_{\mathrm{DTS}}(w \mid b)$	0.0000	15.2056	3.7018	0.0561
$\mathcal{CI}(w)$	76.9427	38.4713	38.4713	38.4713

Question (g)

How to find the previous solution of Question 3.(e) using a QP solver?

The goal is to write the objective function into a quadratic function:

$$\begin{aligned} f(w) &= \frac{\varphi_{\mathrm{AS}}}{2}\mathcal{R}_{\mathrm{AS}}(w \mid b) + \frac{\varphi_{\mathrm{MD}}}{2}\mathcal{R}_{\mathrm{MD}}(w \mid b) + \frac{\varphi_{\mathrm{DTS}}}{2}\mathcal{R}_{\mathrm{DTS}}(w \mid b) \\ &= \frac{1}{2}w^\top Q(b)\, w - w^\top R(b) + c(b) \end{aligned}$$

where:

$$\begin{aligned} \mathcal{R}_{\mathrm{AS}}(w \mid b) &= (w_1 - 0.22)^2 + (w_2 - 0.19)^2 + (w_3 - 0.17)^2 + (w_4 - 0.13)^2 + \\ &\quad (w_5 - 0.11)^2 + (w_6 - 0.08)^2 + (w_7 - 0.06)^2 + (w_8 - 0.04)^2 \\ \mathcal{R}_{\mathrm{MD}}(w \mid b) &= (3.56 w_1 + 6.54 w_3 + 10.23 w_4 + 2.30 w_6 - 3.4089)^2 + \\ &\quad (7.48 w_2 + 2.40 w_5 + 9.12 w_7 + 7.96 w_8 - 2.5508)^2 \\ \mathcal{R}_{\mathrm{DTS}}(w \mid b) &= (103 w_1 + 75 w_3 + 796 w_4 + 45 w_6 - 142.49)^2 + \\ &\quad (155 w_2 + 89 w_5 + 320 w_7 + 245 w_8 - 68.24)^2 \end{aligned}$$

We use the analytical approach which is described in Section 11.1.2 on pages 332-339. Moreover, we rearrange the universe such that the first fourth assets belong to the first sector and the last fourth assets belong to the second sector. In this case, we have:

$$w = \left(\underbrace{w_1, w_3, w_4, w_6}_{\mathcal{S}ector_1}, \underbrace{w_2, w_5, w_7, w_8}_{\mathcal{S}ector_2} \right)$$

The matrix $Q(b)$ is block-diagonal:

$$Q(b) = \begin{pmatrix} Q_1 & \mathbf{0}_{4,4} \\ \mathbf{0}_{4,4} & Q_2 \end{pmatrix}$$

where the matrices Q_1 and Q_2 are equal to:

$$Q_1 = \begin{pmatrix} 11\,025.8400 & 8\,307.0600 & 82\,898.4700 & 4\,839.7000 \\ 8\,307.0600 & 6\,794.2900 & 61\,372.6050 & 3\,751.0500 \\ 82\,898.4700 & 61\,372.6050 & 636\,332.3225 & 36\,408.2250 \\ 4\,839.7000 & 3\,751.0500 & 36\,408.2250 & 2\,257.2500 \end{pmatrix}$$

and:

$$Q_2 = \begin{pmatrix} 25\,523.7600 & 14\,243.8000 & 51\,305.4400 & 39\,463.5200 \\ 14\,243.8000 & 8\,165.0000 & 29\,027.2000 & 22\,282.6000 \\ 51\,305.4400 & 29\,027.2000 & 104\,579.3600 & 80\,214.8800 \\ 39\,463.5200 & 22\,282.6000 & 80\,214.8800 & 61\,709.0400 \end{pmatrix}$$

The vector $R(b)$ is defined as follows:

$$R(b) = \begin{pmatrix} 15\,001.8621 \\ 11\,261.1051 \\ 114\,306.8662 \\ 6\,616.0617 \\ 11\,073.1996 \\ 6\,237.4080 \\ 22\,424.3824 \\ 17\,230.4092 \end{pmatrix}$$

Finally, the value of $c(b)$ is equal to:

$$c(b) = 12\,714.3386$$

Using a QP solver, we obtain the following numerical solution:

$$\begin{pmatrix} w_1 \\ w_3 \\ w_4 \\ w_6 \\ w_2 \\ w_5 \\ w_7 \\ w_8 \end{pmatrix} = \begin{pmatrix} 16.9796 \\ 18.2582 \\ 13.4494 \\ 9.4553 \\ 17.2102 \\ 12.1009 \\ 0.0000 \\ 12.5464 \end{pmatrix} \times 10^{-2}$$

We observe some small differences (after the fifth digit) because the QP solver is more efficient than a traditional nonlinear solver.

Question 4

We consider a variant of Question 3 and assume that the synthetic risk measure is:

$$\mathcal{D}(w \mid b) = \varphi_{\mathrm{AS}} \mathcal{D}_{\mathrm{AS}}(w \mid b) + \varphi_{\mathrm{MD}} \mathcal{D}_{\mathrm{MD}}(w \mid b) + \varphi_{\mathrm{DTS}} \mathcal{D}_{\mathrm{DTS}}(w \mid b)$$

where:

$$\begin{aligned}
\mathcal{D}_{\mathrm{AS}}(w \mid b) &= \frac{1}{2} \sum_{i=1}^{n} |w_i - b_i| \\
\mathcal{D}_{\mathrm{MD}}(w \mid b) &= \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum\nolimits_{i \in \mathcal{S}ector_j} (w_i - b_i) \, \mathrm{MD}_i \right| \\
\mathcal{D}_{\mathrm{DTS}}(w \mid b) &= \sum_{j=1}^{n_{\mathcal{S}ector}} \left| \sum\nolimits_{i \in \mathcal{S}ector_j} (w_i - b_i) \, \mathrm{DTS}_i \right|
\end{aligned}$$

Question (a)

Define the corresponding optimization problem when the objective is to minimize the active risk and reduce the carbon intensity of the benchmark by $\mathcal{R}$.

The optimization problem is:

$$
\begin{aligned}
w^{\star} & = \arg\min \mathcal{D}\left(w \mid b\right) \\
& \text{s.t.} \quad \left\{
\begin{array}{l}
\mathbf{1}_8^\top w = 1 \\
\boldsymbol{\mathcal{CI}}^\top w \leq (1 - \boldsymbol{\mathcal{R}})\, \boldsymbol{\mathcal{CI}}(b) \\
\mathbf{0}_8 \leq w \leq \mathbf{1}_8
\end{array}
\right.
\end{aligned}
$$

Question (b)

Give the LP formulation of the optimization problem.

We use the absolute value trick and obtain the following optimization problem:

$$
\begin{aligned}
w^{\star} & = & \arg\min \frac{1}{2}\varphi_{\mathrm{AS}} \sum_{i=1}^{8} \tau_{i,w} + \varphi_{\mathrm{MD}} \sum_{j=1}^{2} \tau_{j,\mathrm{MD}} + \varphi_{\mathrm{DTS}} \sum_{j=1}^{2} \tau_{j,\mathrm{DTS}} \\
& \text{s.t.} & \left\{
\begin{array}{l}
\mathbf{1}_8^\top w = 1 \\
\mathbf{0}_8 \leq w \leq \mathbf{1}_8 \\
\mathcal{CI}^\top w \leq (1 - \mathcal{R})\, \mathcal{CI}(b) \\
\left| w_i - b_i \right| \leq \tau_{i,w} \\
\left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{MD}_i \right| \leq \tau_{j,\mathrm{MD}} \\
\left| \sum_{i \in \mathcal{S}ector_j} (w_i - b_i)\, \mathrm{DTS}_i \right| \leq \tau_{j,\mathrm{DTS}} \\
\tau_{i,w} \geq 0, \tau_{j,\mathrm{MD}} \geq 0, \tau_{j,\mathrm{DTS}} \geq 0
\end{array}
\right.
\end{aligned}
$$

We can now formulate this problem as a standard LP problem:

$$\begin{aligned} x^\star &= \arg\min c^\top x \\ \text{s.t.} & \left\{ \begin{array}{l} Ax = B \\ Cx \leq D \\ x^- \leq x \leq x^+ \end{array} \right. \end{aligned}$$

where x is the 20×1 vector defined as follows:

$$x = \begin{pmatrix} w \\ \tau_w \\ \tau_{\mathrm{MD}} \\ \tau_{\mathrm{DTS}} \end{pmatrix}$$

The 20×1 vector c is equal to:

$$c = \begin{pmatrix} \mathbf{0}_8 \\ \frac{1}{2}\varphi_{\mathrm{AS}}\mathbf{1}_8 \\ \varphi_{\mathrm{MD}}\mathbf{1}_2 \\ \varphi_{\mathrm{DTS}}\mathbf{1}_2 \end{pmatrix}$$

The equality constraint is defined by $A = \begin{pmatrix} \mathbf{1}_8^\top & \mathbf{0}_8^\top & \mathbf{0}_2^\top & \mathbf{0}_2^\top \end{pmatrix}$ and $B = 1$. The bounds are $x^- = \mathbf{0}_{20}$ and $x^+ = \infty \cdot \mathbf{1}_{20}$.

For the inequality constraint, we have[32]:

$$Cx \leq D \Leftrightarrow \left(\begin{array}{cccc} I_8 & -I_8 & \mathbf{0}_{8,2} & \mathbf{0}_{8,2} \\ -I_8 & -I_8 & \mathbf{0}_{8,2} & \mathbf{0}_{8,2} \\ C_{\mathrm{MD}} & \mathbf{0}_{2,8} & -I_2 & \mathbf{0}_{2,2} \\ -C_{\mathrm{MD}} & \mathbf{0}_{2,8} & -I_2 & \mathbf{0}_{2,2} \\ C_{\mathrm{DTS}} & \mathbf{0}_{2,8} & \mathbf{0}_{2,2} & -I_2 \\ -C_{\mathrm{DTS}} & \mathbf{0}_{2,8} & \mathbf{0}_{2,2} & -I_2 \\ \mathcal{CI}^\top & \mathbf{0}_{1,8} & 0 & 0 \end{array}\right) x \leq \left(\begin{array}{c} b \\ -b \\ \mathrm{MD}^\star \\ -\mathrm{MD}^\star \\ \mathrm{DTS}^\star \\ -\mathrm{DTS}^\star \\ (1-\mathcal{R})\,\mathcal{CI}(b) \end{array}\right)$$

where:

$$C_{\mathrm{MD}} = \left(\begin{array}{cccccccc} 3.56 & 0.00 & 6.54 & 10.23 & 0.00 & 2.30 & 0.00 & 0.00 \\ 0.00 & 7.48 & 0.00 & 0.00 & 2.40 & 0.00 & 9.12 & 7.96 \end{array}\right)$$

and:

$$C_{\mathrm{DTS}} = \left(\begin{array}{cccccccc} 103 & 0 & 75 & 796 & 0 & 45 & 0 & 0 \\ 0 & 155 & 0 & 0 & 89 & 0 & 320 & 245 \end{array}\right)$$

The 2×1 vectors $\mathrm{MD}^\star$ and $\mathrm{DTS}^\star$ are respectively equal to (3.4089, 2.5508) and (142.49, 68.24).

[32] C is a 25×8 matrix and D is a 25×1 vector.

Question (c)

Find the optimal portfolio when $\mathcal{R}$ is set to 50%. Compare the solution with this obtained in Question 3.(e).

We obtain the following solution:

$$
\begin{aligned}
w^{\star} &= (18.7360, 15.8657, 17.8575, 13.2589, 11, 9.4622, 0, 13.8196) \times 10^{-2} \\
\tau_w^{\star} &= (3.2640, 3.1343, 0.8575, 0.2589, 0, 1.4622, 6, 9.8196) \times 10^{-2} \\
\tau_{\mathrm{MD}} &= (0, 0) \\
\tau_{\mathrm{DTS}} &= (0, 0)
\end{aligned}
$$

Table 132: Solution of the bond optimization problem (scope $\mathcal{SC}_{1-3}$)

Problem	Benchmark	3.(e)	4.(c)
w_1	22.0000	16.9796	18.7360
w_2	19.0000	17.2102	15.8657
w_3	17.0000	18.2582	17.8575
w_4	13.0000	13.4494	13.2589
w_5	11.0000	12.1009	11.0000
w_6	8.0000	9.4553	9.4622
w_7	6.0000	0.0000	0.0000
w_8	4.0000	12.5464	13.8196
$\mathrm{MD}(w)$	5.9597	5.9683	5.9597
$\mathrm{DTS}(w)$	210.7300	210.6791	210.7300
$\sigma_{\mathrm{AS}}(w \mid b)$	0.0000	11.9400	12.4837
$\sigma_{\mathrm{MD}}(w \mid b)$	0.0000	0.0308	0.0000
$\sigma_{\mathrm{DTS}}(w \mid b)$	0.0000	0.0561	0.0000
$\mathcal{D}_{\mathrm{AS}}(w \mid b)$	0.0000	25.6203	24.7964
$\mathcal{D}_{\mathrm{MD}}(w \mid b)$	0.0000	0.0426	0.0000
$\mathcal{D}_{\mathrm{DTS}}(w \mid b)$	0.0000	0.0608	0.0000
$\mathcal{CI}(w)$	76.9427	38.4713	38.4713

In Table 132, we compare the two solutions[33]. They are very close. In fact, we notice that the LP solution matches perfectly the MD and DTS constraints, but has a higher AS risk $\sigma_{\mathrm{AS}}(w \mid b)$. If we note the two solutions $w^\star(\mathcal{L}_1)$ and $w^\star(\mathcal{L}_2)$, we have:

$$\left\{ \begin{array}{l} \mathcal{R}(w^\star(\mathcal{L}_2) \mid b) = 1.4524 < \mathcal{R}(w^\star(\mathcal{L}_1) \mid b) = 1.5584 \\ \mathcal{D}(w^\star(\mathcal{L}_2) \mid b) = 13.9366 > \mathcal{D}(w^\star(\mathcal{L}_1) \mid b) = 12.3982 \end{array} \right.$$

There is a trade-off between the $\mathcal{L}_1$- and $\mathcal{L}_2$-norm risk measures. This is why we cannot say that one solution dominates the other.

[33] The units are the following: % for the weights w_i, and the active share metrics $\sigma_{\mathrm{AS}}(w \mid b)$ and $\mathcal{D}_{\mathrm{AS}}(w \mid b)$; years for the modified duration metrics $\mathrm{MD}(w)$, $\sigma_{\mathrm{MD}}(w \mid b)$ and $\mathcal{D}_{\mathrm{MD}}(w \mid b)$; bps for the duration-times-spread metrics $\mathrm{DTS}(w)$, $\sigma_{\mathrm{DTS}}(w \mid b)$ and $\mathcal{D}_{\mathrm{DTS}}(w \mid b)$; $\mathrm{tCO_2e/\$\ mn}$ for the carbon intensity $\mathrm{DTS}(w)$.

Course 2022-2023 in Sustainable Finance
Lecture 12. Physical Risk Modeling

Thierry Roncalli*

*Amundi Asset Management[34]

*University of Paris-Saclay

March 2023

[34]The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

Physical risk and investors

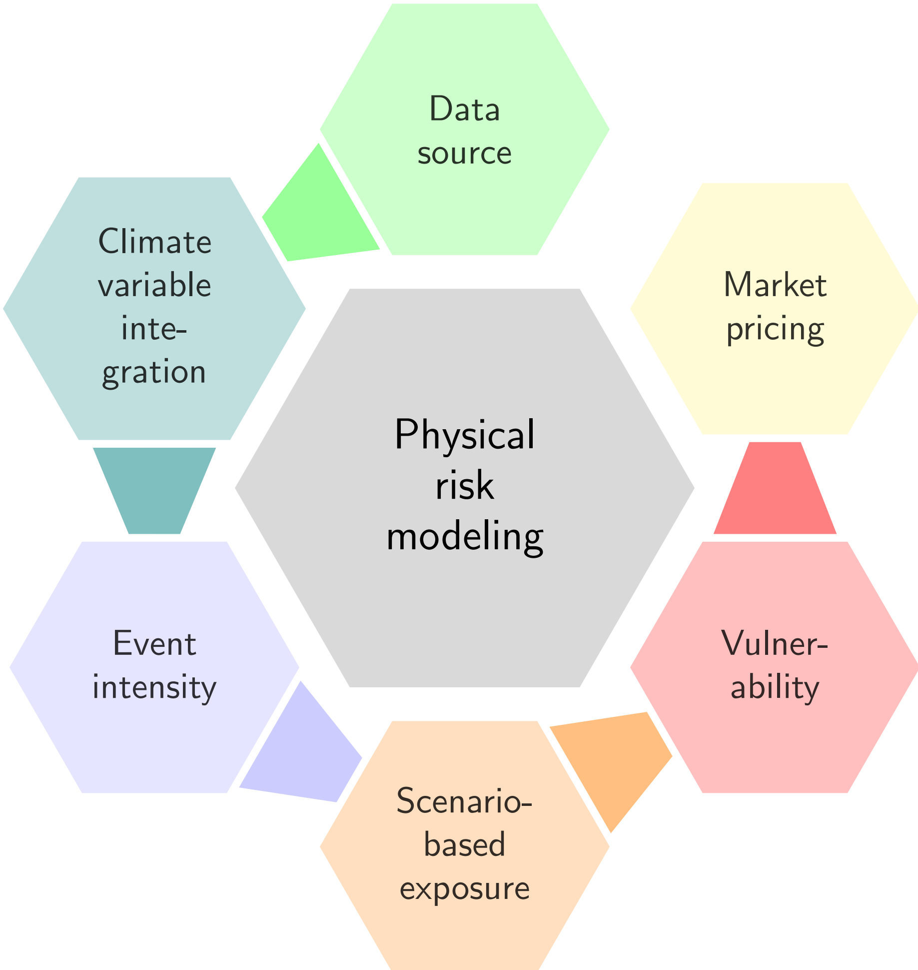
"Responsible investors have paid more attention to the transition risk than to the physical risk. However, recent events show that physical risk is also a big concern. It corresponds to the financial losses that really come from climate change, and not from the adaptation of the economy to prevent them. It includes droughts, floods, storms, etc." (Le Guenedal and Roncalli, 2022).

Chronic risk

Acute risk

Statistical modeling of physical risk

Figure 228: Physical risk modeling



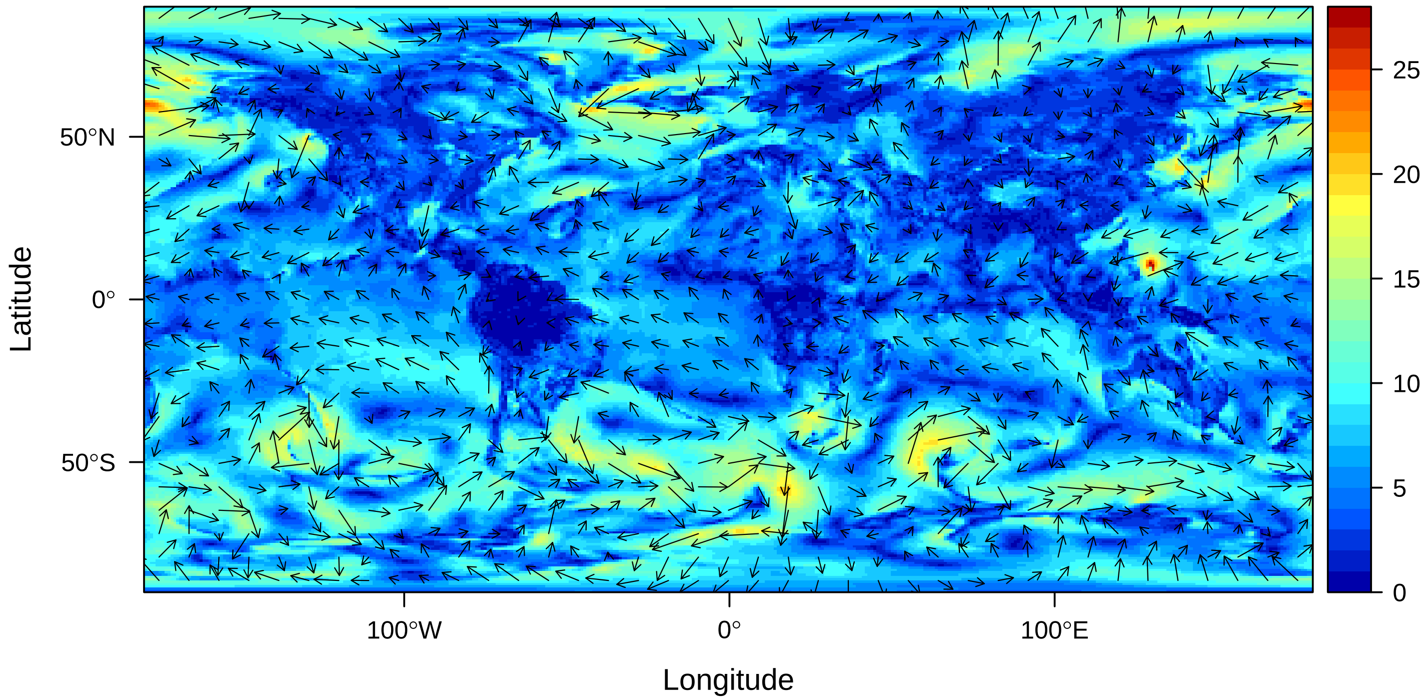
Statistical modeling of physical risk

Climate variable and data source

- The climate data source is the set $\Theta_s = \{\theta(\lambda, \varphi, z, t)\}$
- $\theta = (\theta_1, \ldots, \theta_k)$ is a vector of k climate variables such as temperature, pressure or wind speed
- Each variable θ_k has four coordinates:
  1. Latitude λ
  2. Longitude φ
  3. Height (or altitude) z
  4. Time t
- Three types of sources:
  1. Meteorological records
  2. Reanalysis
  3. Historical simulations by a climate model

Statistical modeling of physical risk

Figure 229: Slice* of wind speed (07/11/2013, tropical cyclone Haiyan)



Source: Modern-Era Retrospective analysis for Research and Applications, Version 2, Global Modeling and Assimilation Office, NASA.

*This is a slice of the MERRA-2 reanalysis at a height of 10 meters on 7^{th} November 2013. The red dot is the location of the eye of the tropical cyclone Haiyan, which affected more than 10 million people in the Philippines

Statistical modeling of physical risk

Event intensity sensitivity

- We first have define the sensitivity of the intensity of extreme events to climate change
- Let $\mathbb{E}\left[I\left(\Theta_s\left(C\right)\right)\right]$ be the expected intensity of the event in the scenario associated with the GHG concentration C
- The sensitivity of the event is equal to:

$$\Delta I\left(C\right)=\mathbb{E}\left[I\left(\Theta_s\left(C\right)\right)\right]-I\left(\Theta_s\left(C_0\right)\right)$$

  where $I\left(\Theta_s\left(C_0\right)\right)$ is the current intensity or the reference intensity in a scenario where climate objectives are met
- For instance, we know that the maximum wind of tropical cyclones increases by more than 10% in scenarios with a high GHG concentration

Statistical modeling of physical risk

Asset exposure

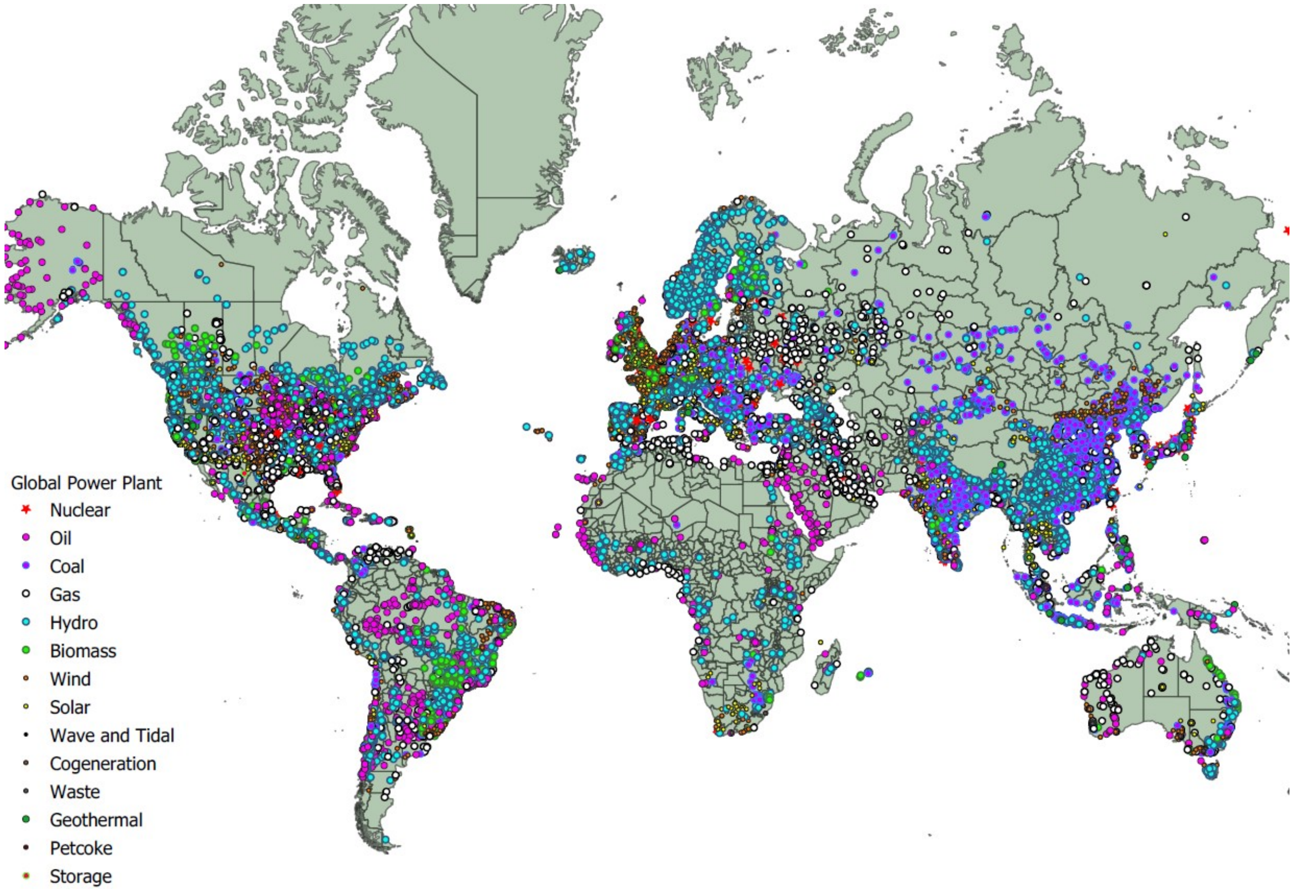
- The asset value of the portfolio can then be written as:

$$\Psi(t) = \sum_{j=1}^{n} x_j \Psi_j(\lambda, \varphi, t)$$

  where $\Psi_j(\lambda, \varphi, t)$ is the geolocated asset value estimated at time t and x_j is the weight of asset j in the portfolio
- This requires the geolocation of the portfolio

Statistical modeling of physical risk

Figure 230: Geolocation of world power plants by energy source



Source: Global Power Database version 1.3 (June 2021).

Statistical modeling of physical risk

Vulnerability

- The damage function $\Omega_j(I) \in [0,1]$ is the fraction of property loss with respect to the intensity
- It is generally calibrated on past damages (insurance claims, economic loss, etc.) and disasters

Statistical modeling of physical risk

Market pricing

- The physical risk implied by the concentration scenario C is equal to:

$$\Delta\mathcal{L}oss\left(t, C\right) = \beta \cdot \mathcal{DD}\left(t, C\right) = \beta \sum_{j=1}^{n} x_j \Psi_j\left(\lambda, \varphi, t\right) \Omega_j\left(\Delta I\left(t, C\right)\right)$$

- $\Delta\mathcal{L}oss\left(t, C\right)$ is the relative loss due to the events on the portfolio
- β is the transmission factor of the direct damage $\mathcal{DD}\left(t, C\right)$ on the underlying to the loss of financial value in the investment portfolio
- For example, if the facilities of an energy producer are damaged at 50%, the securities issued by this company will be impacted at $50\% \times \beta$

Climate hazard location

Asset location

Applications
Tropical cyclone damage modeling

Le Guenedal, Drobinski, and Tankov (2021), Measuring and Pricing Cyclone-Related Physical Risk under Changing Climate, *Amundi Working Paper*, `www.ssrn.com/abstract=3850673`

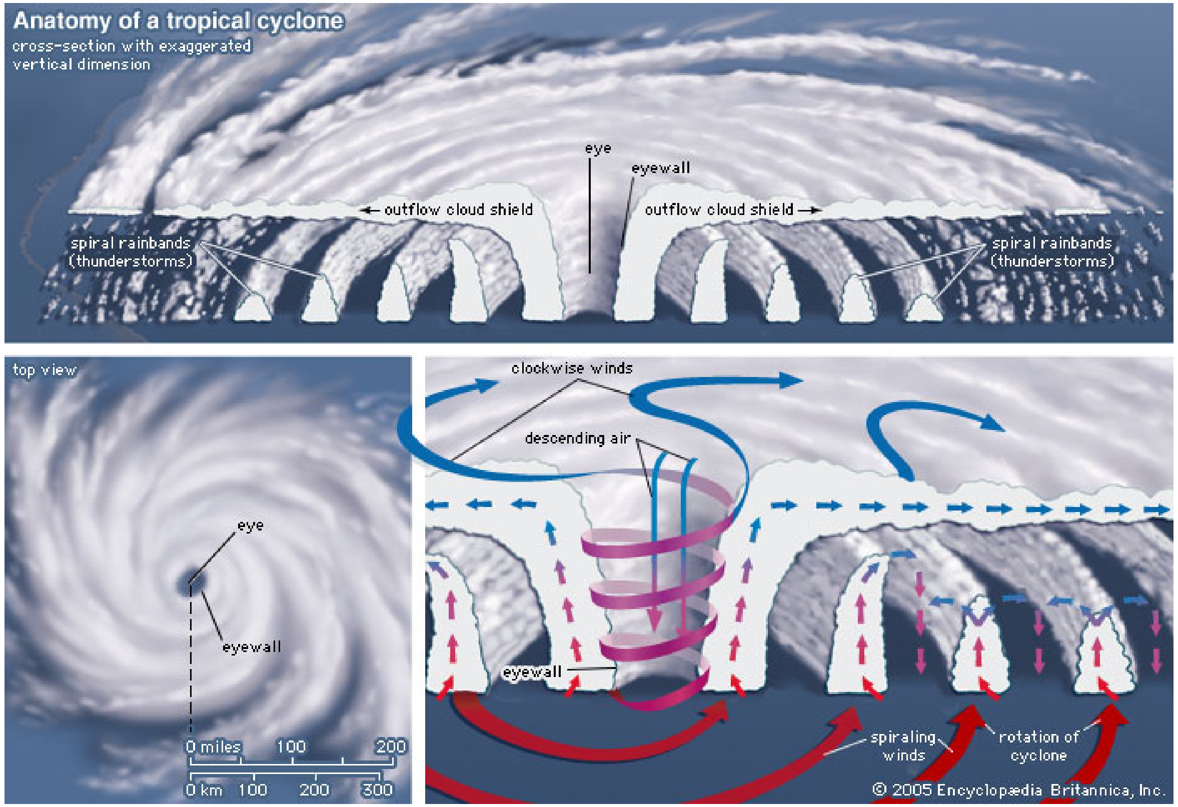
Two main modules:

- Simulation and generation of tropical cyclones under a given climate change scenario
- Geolocation of assets, damage modeling and loss estimation

Applications

Tropical cyclone damage modeling

Figure 231: What is a cyclone?

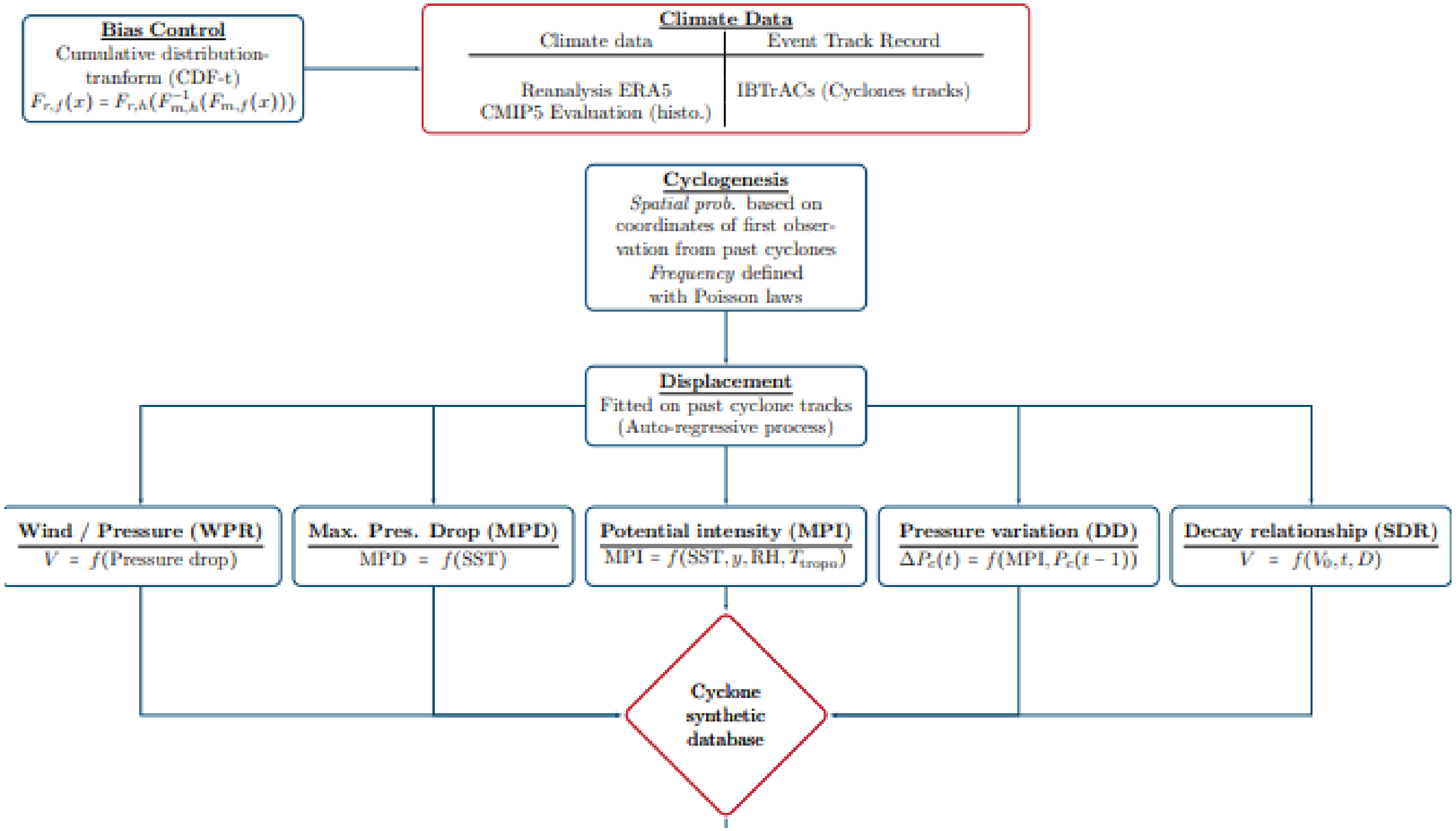


Source: www.geosci.usyd.edu.au/users/prey/teaching/geos-2111gis/cyclone/cln006.html

Applications

Tropical cyclone damage modeling

Figure 232: Modeling framework (Module 1)

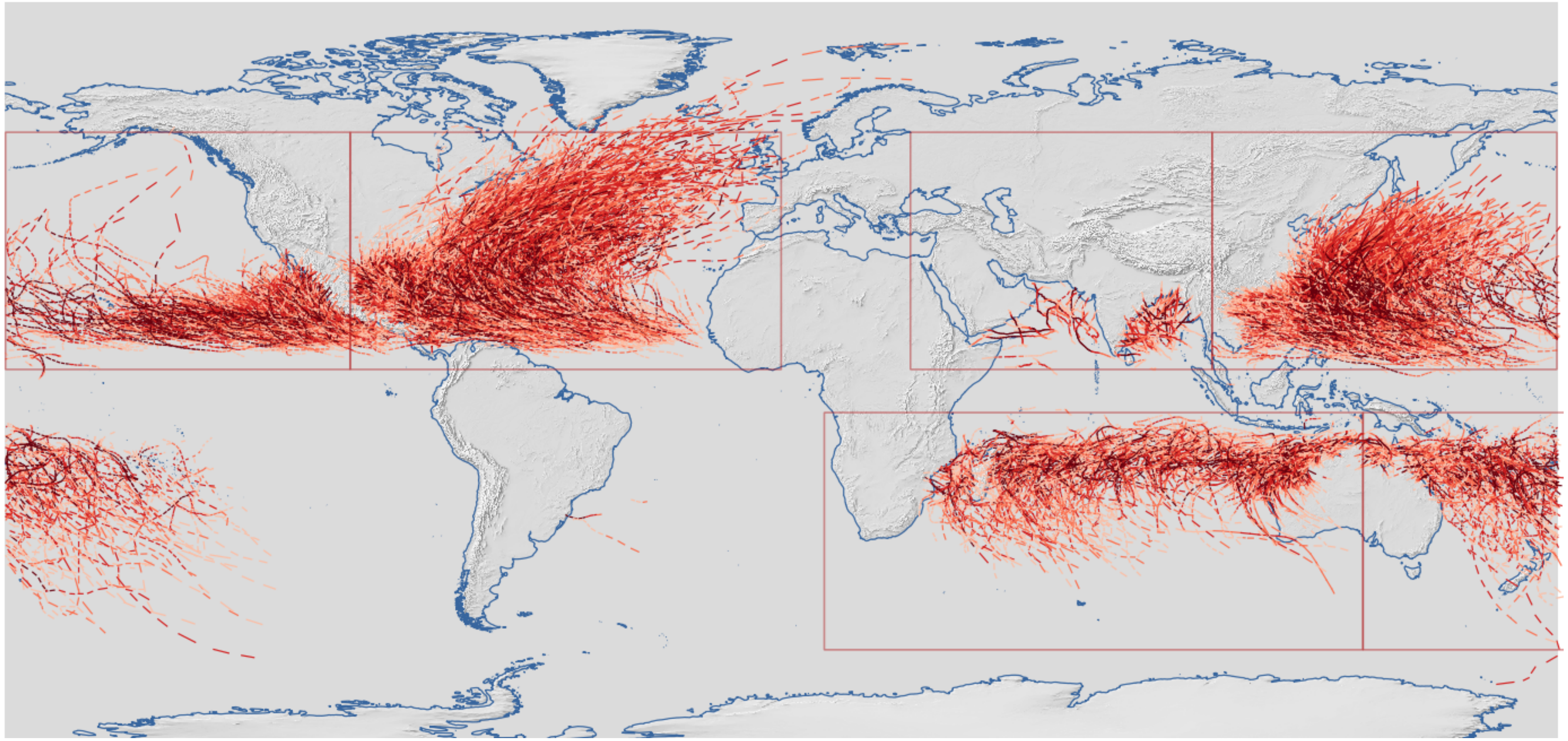


Source: Le Guenedal *et al.* (2021).

Applications

Tropical cyclone damage modeling

Figure 233: Sample of storms (ERA-5 climate data)



Source: Le Guenedal *et al.* (2021).

Applications

Tropical cyclone damage modeling

Physics of cyclones

1. Wind pressure relationship (Bloemendaal *et al.*, 2020):

$$V = a\left(P_{\mathrm{env}} - P_c\right)^b$$

2. Maximum potential intensity (Holland, 1997; Emanuel, 1999):

$$MPI = f\left(y, SST, T_{tropo}, MSLP, RH, P_c\right)$$

3. Maximum pressure drop (Bloemendaal *et al.*, 2020):

$$MPD \sim P_{\mathrm{env}} - P_c = A + Be^{C(\mathrm{SST}-T_0)} \qquad T_0 = 30^o\mathrm{C}$$

4. Pressure incremental variation (James and Mason, 2005):

$$\begin{array}{rcl} \Delta_t P_c(t) & = & c_0 + c_1 \Delta_t P_c(t-1) + c_2 e^{-c_3(P_c(t) - MPI(x,y,t))} + \varepsilon(P_c, t) \\ \varepsilon(P_c, t) & \sim & \mathcal{N}\left(0, \sigma^2_{P_c}\right) \end{array}$$

5. Decay function (Kaplan and DeMaria, 1995):

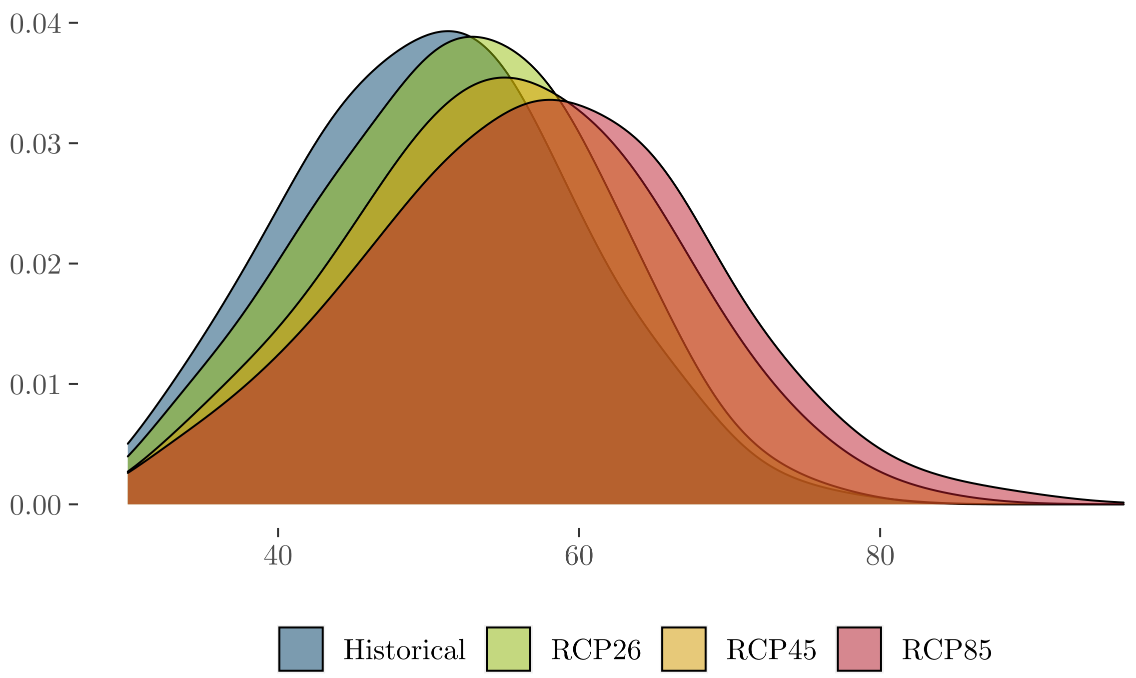
$$V(t_L) = V_b + (R \cdot V_0 - V_b)e^{-\alpha t} - C$$

where $C = m\left(\ln \frac{D}{D_0}\right) + b$, $m = \tilde{c}_1 t_L \left(t_{0,L} - t_L\right)$ and $b = d_1 t_L \left(t_{0,L} - t_L\right)$

Applications

Tropical cyclone damage modeling

Figure 234: Maximum wind speed in m/s (2070-2100)

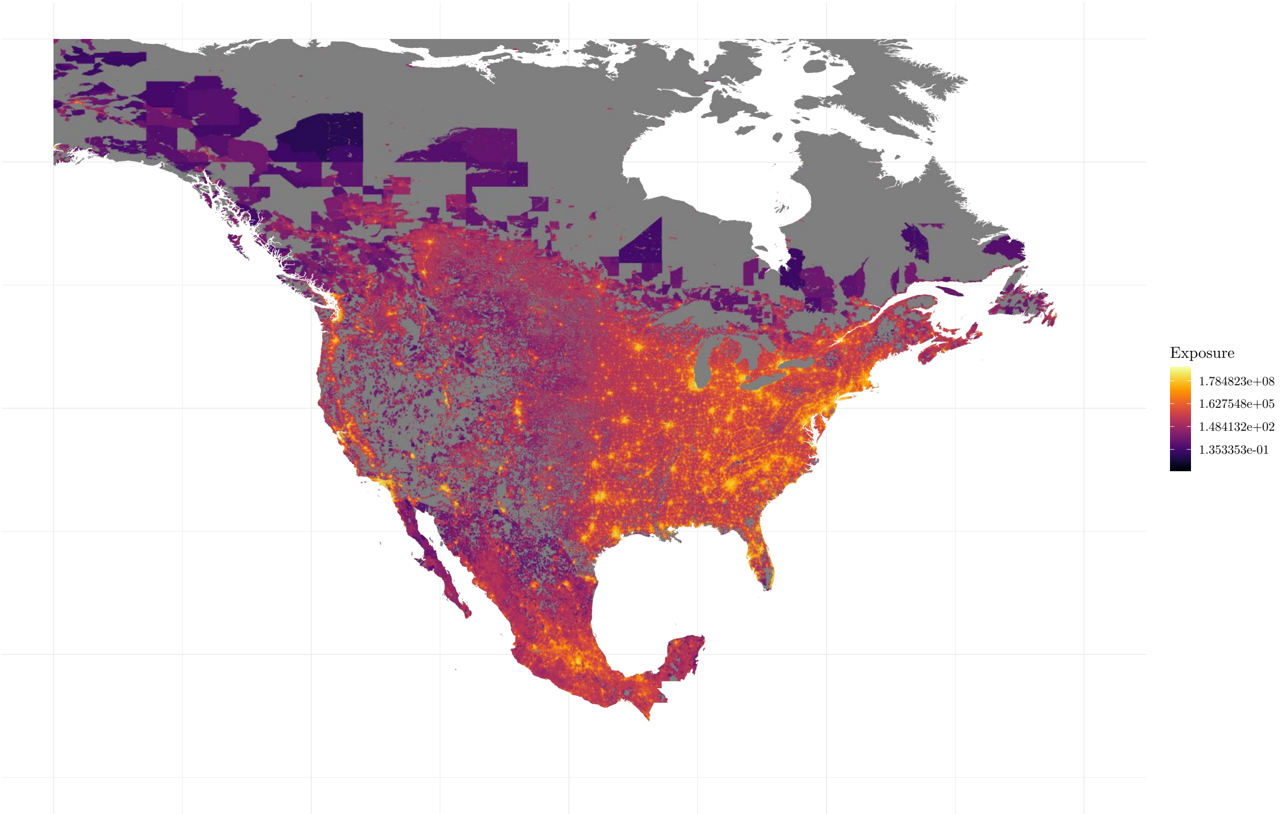


Source: Le Guenedal *et al.* (2021).

The cyclone simulation database must be sensitive to the climate change scenario

Applications
Tropical cyclone damage modeling

Figure 235: GDP decomposition of North America (or physical asset values) (Litpop database)

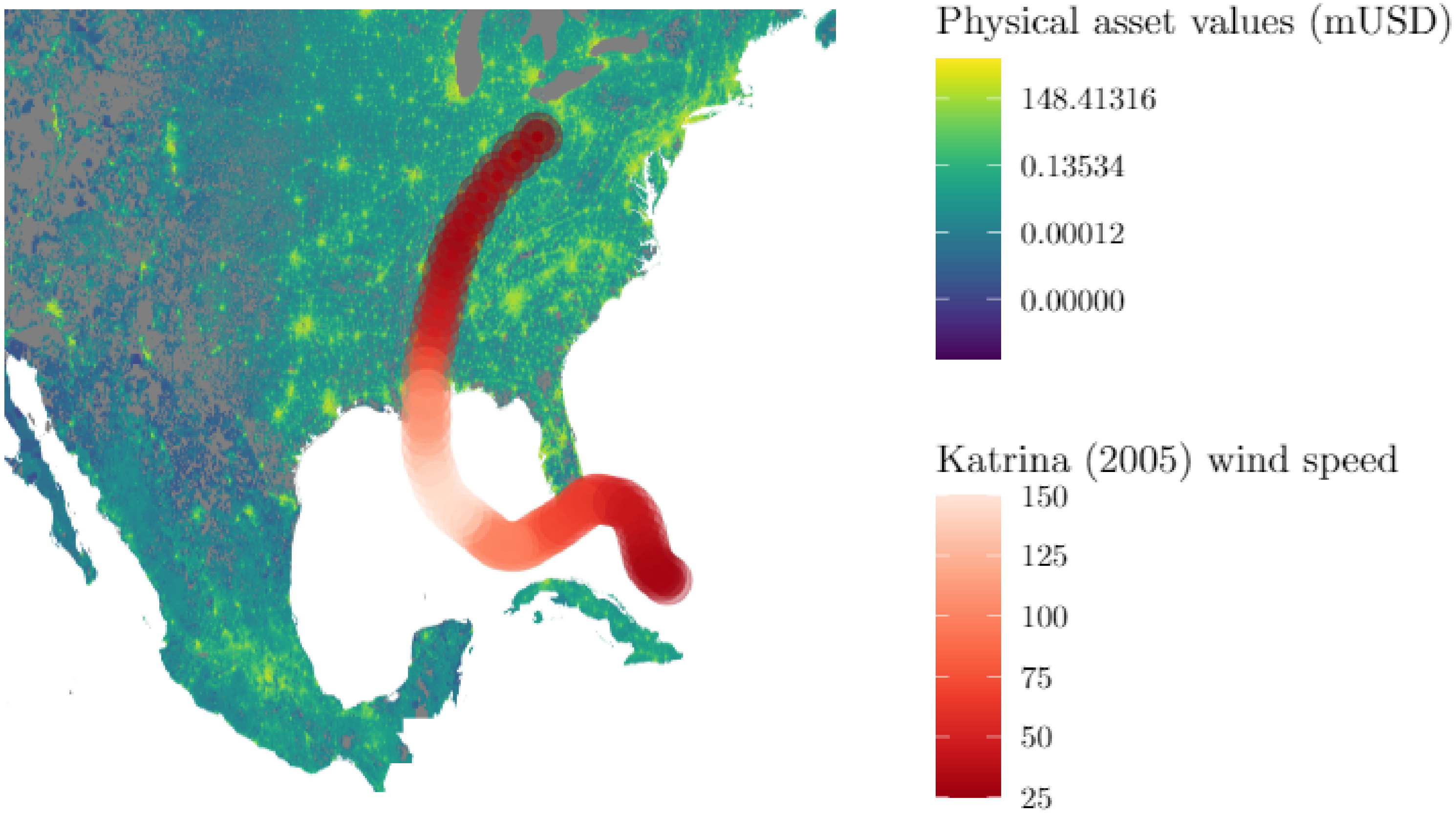


Source: Le Guenedal *et al.* (2021).

Applications

Tropical cyclone damage modeling

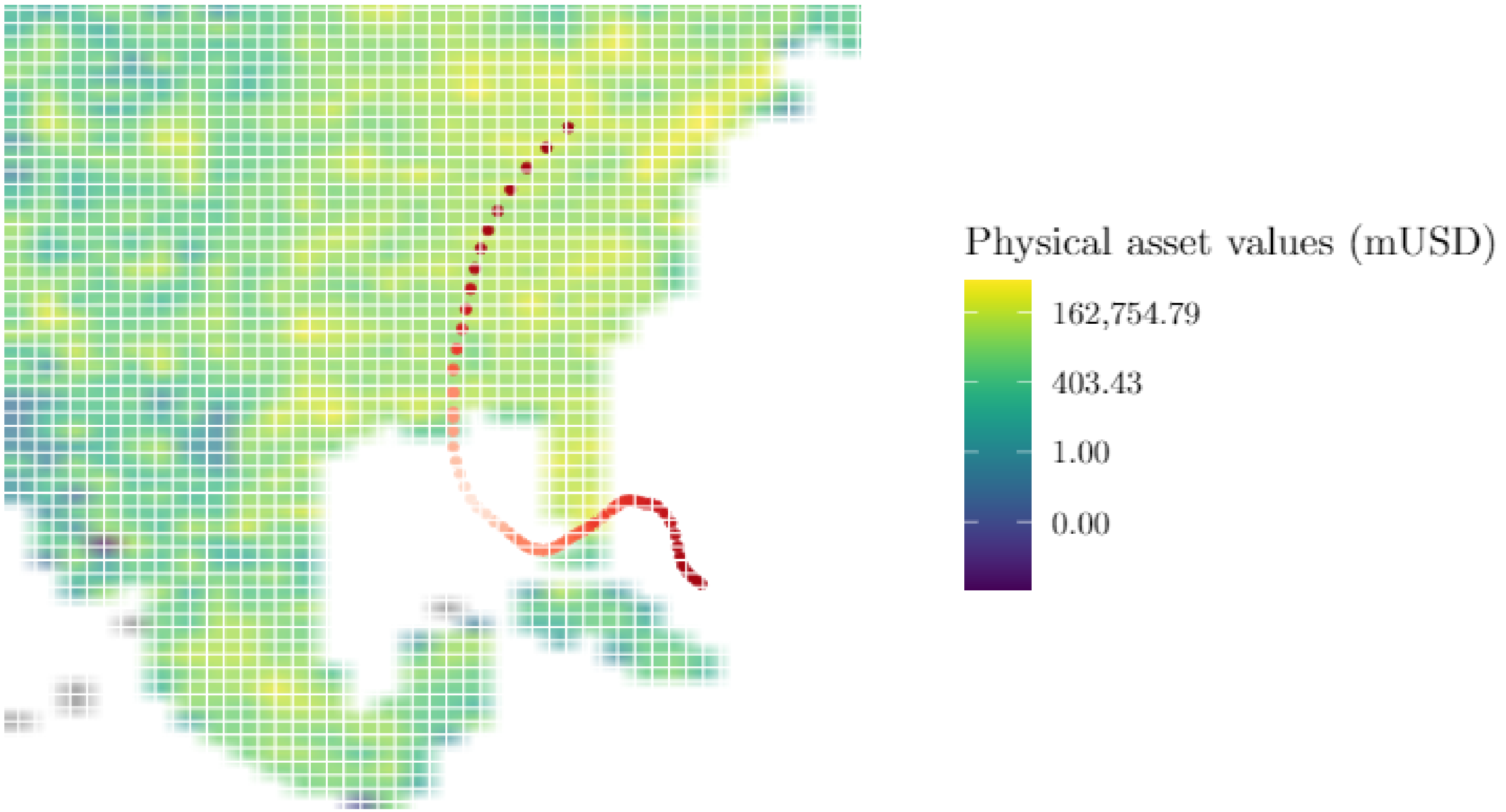
Figure 236: The case of Katrina (2005)



Source: Le Guenedal *et al.* (2021).

Applications
Tropical cyclone damage modeling

Figure 237: The grid approach

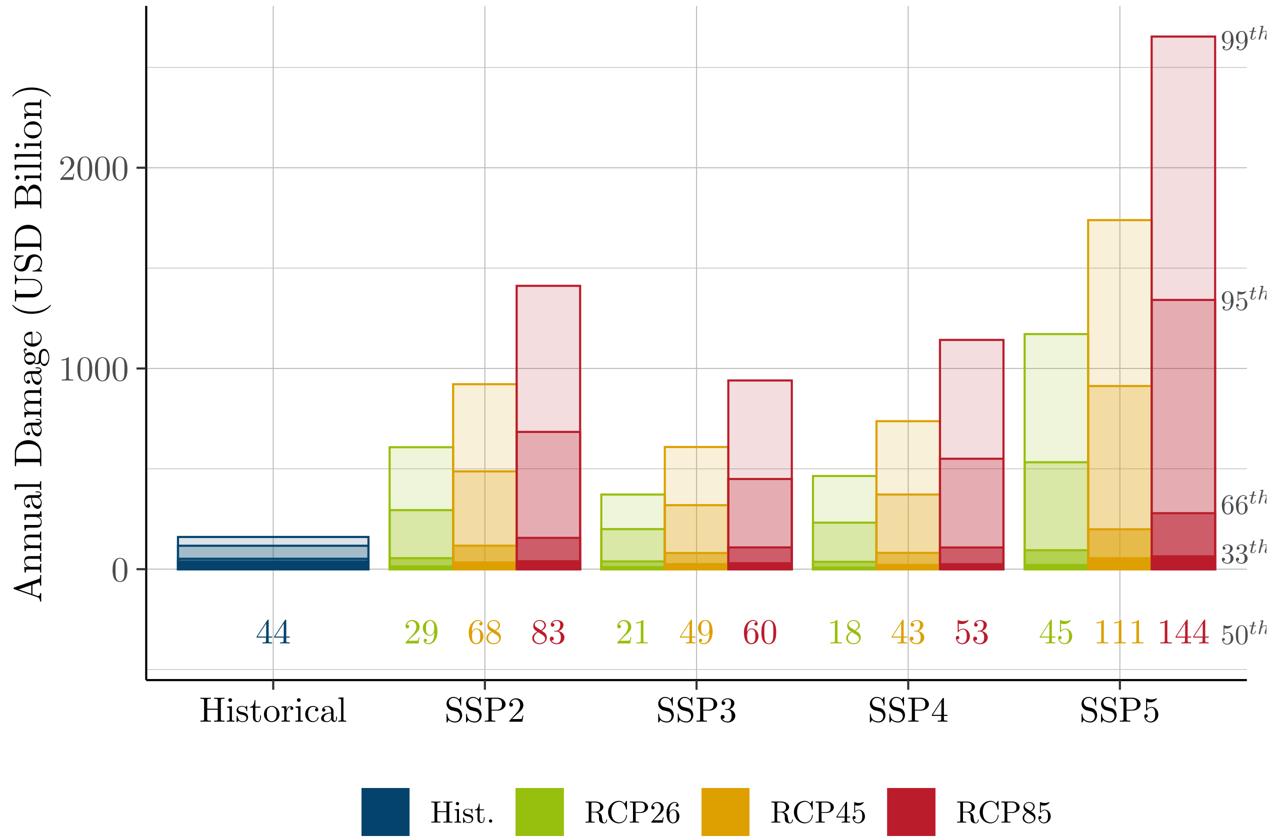


Source: Le Guenedal *et al.* (2021).

Applications

Tropical cyclone damage modeling

Figure 238: Average global losses



Source: Le Guenedal *et al.* (2021).

Applications

Tropical cyclone damage modeling

Table 133: Average increase of financial losses per year

SSP	RCP 2.6	RCP 4.5	RCP 8.5
SSP2	+43%	+153%	+247%
SSP5	+157%	+360%	+543%

Source: Le Guenedal *et al.* (2021).

Remark

- There are simulations that lead to annual losses that easily exceed 2 or 3 trillion dollars per year
- 1 Katrina = $180 billion in 2005

Floods

Drought

Water stress

Extreme heat

Wildfire

Course 2022-2023 in Sustainable Finance
Lecture 13. Climate Stress Testing and Risk Management

Thierry Roncalli*

*Amundi Asset Management[35]

*University of Paris-Saclay

March 2023

[35] The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.

forthcoming