Portfolio Construction with Climate Risk Measures*

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Abstract

Because of the 2015 Paris Agreement, the development of ESG investing and the emergence of net zero emission policies, climate risk is certainly the most important topic and challenge for asset owners and managers now and will remain so over the next five years. It considerably changes portfolio allocation and the investment framework of both passive and active investors. The goal of this paper is to conduct a survey of the various climate risk measures that are available in the asset management industry and the practices of portfolio construction that use these metrics. Therefore, the first part of this paper lists the different climate risk metrics — e.g., carbon footprint, carbon transition pathway, carbon transition and physical risks. The second part is dedicated to portfolio optimization, in particular portfolio decarbonization and portfolio alignment (Paris-based benchmarks and net zero carbon objective). Among the different findings, two are of great importance for investors. First, portfolio decarbonization is more difficult when we include scope 3 carbon emissions. Indeed, optimizing using the sum of scopes 1, 2 and 3 emissions leads to a portfolio with more tracking error risk than using direct plus first tier indirect carbon emissions. Second, portfolio alignment is more complex than portfolio decarbonization. Since aligning portfolios with scope 3 is becoming the standard approach to climate portfolio construction, the impact on portfolio management may be substantial, and the divergence between carbon investing and traditional investing will increase.

Keywords: Climate change, risk measure, carbon emissions, reduction scenario, carbon trajectory, net zero emission, optimized portfolio, decarbonization, portfolio alignment, index portfolio.

JEL classification: C61, G11, Q54.

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1 Introduction

Climate risk is the biggest challenge to humanity of the 21st century. Indeed, climate change implies rises in temperature that increase the likelihood of extreme weather events and impact living patterns. Similarly, the latest IPCC assessment report paints a grim picture of the future of the world (IPCC, 2021). Beyond the direct effect on natural hazards, biodiversity, health or migration of human and animal populations, climate change may also result in a new economic order because of the transition to a low-carbon economy. In this context, the resilience of the financial system to climate-related risks is being called into question. This is why climate change has become one of the largest systemic risks (FSB, 2020) and is the top priority on the agenda of financial institutions, supervisors and policymakers. Since all stakeholders play a key role in tackling climate risk, it also concerns the asset management industry. Therefore, portfolio decarbonization, temperature alignment, net zero carbon investment, Paris-aligned benchmark, etc. are the everyday reality of both asset owners and managers.

Since 2014, interest in climate-related financial risks has been boosted by the development of ESG investing in Europe (Bennani *et al.*, 2018; Drei *et al.*, 2019). While environmental issues have been lagging behind social issues during the Covid-19 crisis, the net zero carbon race and the Glasgow COP 26 event have recently changed the equation, and climate risk is now the hottest topic in asset management. Climate risk is definitively back on the agenda of investors and goes beyond ESG ratings. However, climate risk assessment methodologies have not reached maturity. We are in an earlier stage and there are many initiatives for proposing climate risk measures that are different from traditional scope $1+2$ carbon emissions. In particular, taking into account scope 3 changes the way we measure the risk to portfolio construction. Indeed, the dispersion of carbon intensities with scope 3 is dramatically reduced, implying that it is more difficult to decarbonize a portfolio. Similarly, investors are increasingly using carbon trajectories rather than just current carbon measures. Therefore, the introduction of trend or target dynamics means a new approach to portfolio allocation. More generally, the development of climate risk measures and their sophistication have an impact on portfolio optimization. We notice that these approaches were first developed on the side of passive management and low-carbon equity indices but are now gaining a lot of traction in active management. As such, the goal of our research is to conduct a survey of the current practices in terms of climate risk measures and portfolio construction, which are now widely used by asset owners and managers.

This paper is organized as follows. In Section Two, we review the different climate risk measures. First, we consider the carbon footprint and explain the concept of carbon intensity. Then, we focus on carbon transition pathways that are a dynamic extension of carbon emissions. These metrics are particularly important for defining a net zero carbon emission policy. We also study carbon transition and physical risks and show their differences from an investment point of view. Finally, this section ends with several key performance indicators that are derived from ESG scoring, green revenues, etc. Section Three is dedicated to portfolio optimization when imposing climate risk constraints. First, we present the several portfolio decarbonization methodologies. In particular, we show how using carbon emissions differs from using carbon intensity. Then, we investigate the portfolio alignment approach. In particular, we focus on two methods. The first one corresponds to the approach of Paris-aligned benchmarks. We highlight the impact of using scope 3 instead of scopes 1 and 2. The second method defines the net zero objective. We show how the dynamics of carbon intensity change the portfolio allocation with respect to a pure decarbonization approach. Finally, Section Four offers some concluding remarks.

2 Climate risk measures

Traditional portfolio optimization requires input data such as the vector μ of expected returns, the vector σ of asset volatilities and the correlation matrix ρ of asset returns[1] (Roncalli, 2013). Let $x = (x_1, \ldots, x_n)$ be the vector of portfolio weights. We can then compute the first and second moments of the stochastic portfolio return[2]. In particular, the portfolio risk corresponds to the portfolio volatility $\sigma(x)$. When introducing climate-related risks, we have to define another measure that assesses the risk of climate change. For that, we generally consider a climate metric $\boldsymbol{\mathcal{C}}_i$ associated with asset i and assume that the climate risk measure is linear:

$$\boldsymbol{\mathcal{C}}(x) = \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{C}}_i \tag{1}$$

The nature of the climate risk measure is then different from the volatility risk measure since the latter satisfies the subadditivity property:

$$\sigma(x) \leq \sum_{i=1}^{n} x_i \sigma_i \tag{2}$$

Even if $\boldsymbol{\mathcal{C}}(x)$ is a convex risk measure, it is an abuse of language because there is no way to diversify the climate risk:

$$\boldsymbol{\mathcal{C}}(x) \not< \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{C}}_i \tag{3}$$

Therefore, $\boldsymbol{\mathcal{C}}(x)$ plays more the role of an expected loss than a risk measure.

2.1 Carbon footprint

Carbon footprint is a generic term used to define the total greenhouse gas (GHG) emissions caused by a given system, activity, company, country, or region. Greenhouse gases are made up of water vapor (H_2O), carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), Ozone (O_3), etc. They absorb and emit radiation energy, causing the greenhouse effect. The greenhouse effect was a crucial factor for the development of human life on Earth. Indeed, without the greenhouse effect, the average temperature of Earth's surface would be about $-18°$C. With the greenhouse effect, the current temperature of Earth's surface is about $+15°$C. Nevertheless, the increasing concentration of some GHGs is an issue because it is a factor in global warming. It mainly concerns carbon dioxide, and to a lesser extent, methane and nitrous oxide.

Carbon footprint is generally measured in carbon dioxide equivalent (CO_2e), which is a term for describing different GHGs in a common unit. In this framework, a quantity of GHG is expressed as CO_2e by multiplying the GHG amount by its global warming potential (GWP). The GWP of a gas is the amount of CO_2 that would warm the earth equally. For instance, the IPCC's 4$^{\text{th}}$ Assessment Report has used the following rules[3]: 1 kg of methane corresponds to 25 kg of CO_2 and 1 kg of nitrous oxide corresponds to 298 kg of CO_2. The definition of a common unit allows two companies to be compared properly. However, carbon emissions cannot be compared fairly if the size of the two companies differs. Therefore, it can be useful to transform carbon emissions into normalized metrics, which are called carbon intensities.

[1] Using the parameters σ and ρ, we can form the covariance matrix Σ, which is defined by $\Sigma_{i,j} = \rho_{i,j}\sigma_i\sigma_j$.

[2] The first moment is equal to the expected return of the portfolio $\mu(x) = x^\top \mu$ while the second moment is equal to the variance of the portfolio return $\sigma^2(x) = x^\top \Sigma x$.

[3] GWP estimates depend on several factors such as the atmospheric lifetime of the gas. This explains that estimates are different from one study to another (Derwent, 2020).

2.1.1 Carbon emissions

It is now possible to access the carbon emissions of an increasing number of companies. However, since GHG are not physically material, even this raw indicator is subject to uncertainty at the company level. To provide a common accounting basis that can be used by states and companies, and to limit the measurement gap, the Greenhouse Gas Protocol (GHGP) classifies a company's greenhouse gas emissions in three scopes[4]:

- Scope 1 denotes direct GHG emissions occurring from sources that are owned or controlled by the company.
- Scope 2 corresponds to the indirect greenhouse gas emissions from consumption of purchased electricity, heat or steam. Scope 2 emissions can be computed using the energy mix of the country (location-based) or the energy mix of the utility company supplying the electricity.
- Finally, Scope 3 are other indirect emissions[5], such as the extraction and production of purchased materials and fuels, transport-related activities in vehicles not owned or controlled by the reporting entity, electricity-related activities not covered in Scope 2. Scope 3 upstream emissions include the indirect emissions that come from the supply side, while scope 3 downstream emissions are mostly associated with the product sold by the entity.

In what follows, we distinguish these three carbon emission measures by introducing the notations $\mathcal{CE}_1$, $\mathcal{CE}_2$ and $\mathcal{CE}_3$. In carbon emission databases, we also observe that carbon scopes can be self-reported or estimated. Moreover, they are generally expressed in tons of carbon dioxide equivalent or tCO_2e.

Data on GHG emissions are easily and freely available when they concern countries and regions[6]. For corporations, two main sources of GHG emissions are generally used: the CDP database and the S&P Trucost data[7]. In what follows, we use the latter since it seems to be more comprehensive. Indeed, it contains the self-reported values of companies from CDP. When they are not available, estimated values are provided. For the year 2019, we have about 15 700 corporations. The carbon emission histograms for the three scopes are reported in Figure 22 on page 47. If we sum up all the observations, we obtain the following figures (expressed in GtCO_2e): $\mathcal{CE}_1 = 15.57$, $\mathcal{CE}_2 = 2.45$ and $\mathcal{CE}_3 = 10.17$. It follows that the total emissions of these corporate firms are about 28.2 GtCO_2e, which represents more than 75% of the 36 GtCO_2e global emissions.

Figure 1 shows the breakdown of these GHG emissions by sector. We notice that the direct emissions are concentrated in a few sectors: Utilities, Materials, Energy and to some extent Industrials. Scope 2 emissions are more equally distributed. On the other hand, every sector's scope 2 emissions are already accounted for in the scope 1 emissions of the Utilities sector. This *double-counting* issue becomes particularly problematic when trying to build the carbon footprint of a portfolio. Finally, scope 3 emissions are discriminant for Information Technology, Health Care, Consumer Staples, Consumer Discretionary and Financials. However, estimating scope 3 emissions is still subject to uncertainty. The distribution of the absolute emissions within GICS sectors is also an issue. Contrary to

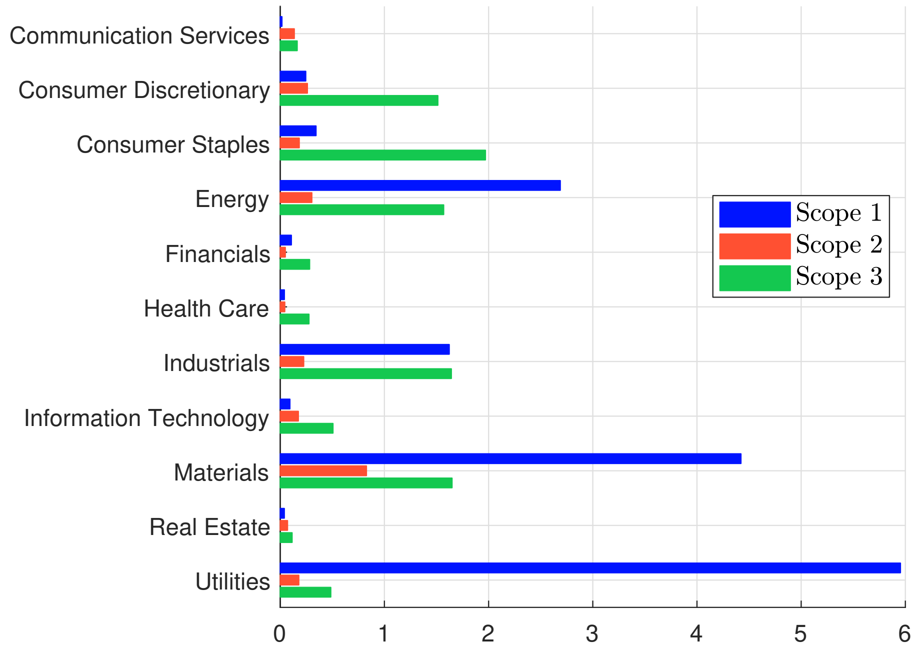
[4] The standards can be found at `www.ghgprotocol.org`.

[5] See Appendix A.3 on page 44 about the issues of scope 3 emissions measurement.

[6] They can be retrieved from the World Bank (`data.worldbank.org/topic/climate-change`), Climate Watch Data (`www.climatewatchdata.org/ghg-emissions`), Global Carbon Project (`www.globalcarbonproject.org`), etc.

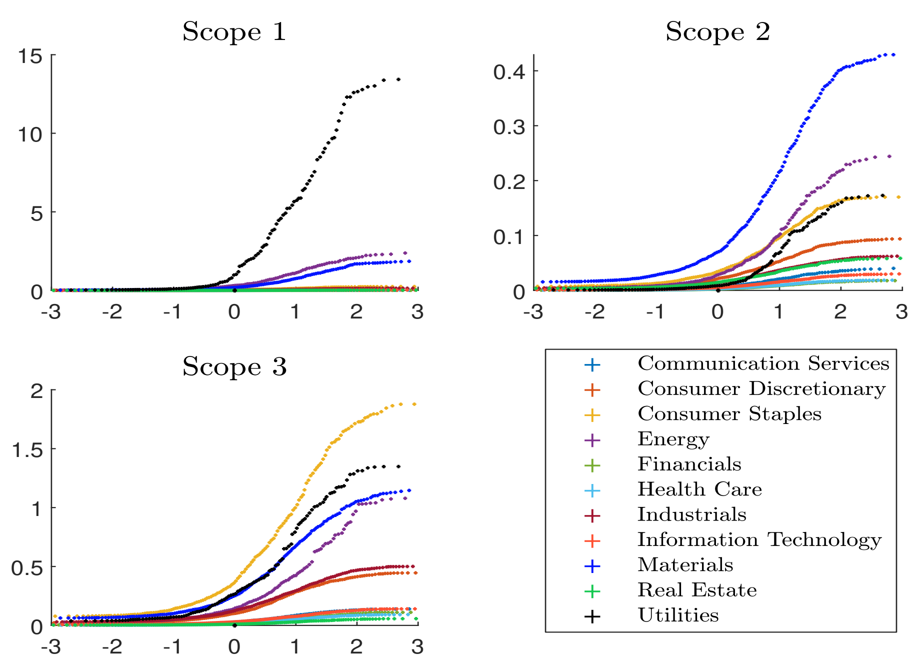
[7] A description of these two databases can be found at `www.cdp.net/en/data` and `www.spglobal.com/esg/trucost`.

Figure 1: Total absolute scopes per GICS sector in $GtCO_2e$



Source: Trucost reporting year 2019 & Authors' calculations.

Figure 2: QQ-plot of carbon scopes per GICS sector in $MtCO_2e$



Source: Trucost reporting year 2019 & Authors' calculations.

commonly used centered scores, the emissions are concentrated on very few actors. Filtering the outliers[8], we obtain the distribution of the values given in Figure 2. Although the filtration allows us to visualize the distribution, we are faced with multiple scaling issues (sector, size, country), making absolute emissions difficult to use in portfolio construction.

2.1.2 Carbon metrics

Definition The carbon intensity of company i with respect to scope j is a normalization of the carbon emissions:

$$\mathcal{CI}_{i,j} = \frac{\mathcal{CE}_{i,j}}{Y_i} \tag{4}$$

where $\mathcal{CE}_{i,j}$ is the company's absolute scope j emissions and Y_i is an output indicator measuring its activity. In general, revenues (expressed in \$) are used to compute carbon intensities. For some major sectors, it is possible to find intensities per production unit. For instance, for a company from the Utilities sector, a carbon intensity in CO_2e/kWh is more informative in terms of carbon efficiency than a carbon intensity in CO_2e/\$. In what follows, we use revenue as the normalization variable when one is not defined. Some examples are provided in Table 1. Comparing the carbon emissions of a football club like Juventus with a petroleum company like BP does not make sense, because their economic size is not comparable. This explains why the carbon intensity measure is more popular than the carbon emission measure in portfolio management. These examples also illustrate how interconnected the three scopes are. Thus, if a company extensively outsources the manufacturing of its products, it reduces its scope 1 but increases its scope 3 emissions. It is also possible to deal with more disaggregated data, for instance by making the distinction between upstream and downstream emissions resulting from the entire supply chain. However, it is not straightforward since the accuracy of scope 3 is not obvious[9].

Table 1: Examples of carbon emissions and intensity

Company	Emission (in tCO_2e)			Revenue	Intensity (in tCO_2e/\$ mn)		
	Scope 1	Scope 2	Scope 3	(in \$ mn)	Scope 1	Scope 2	Scope 3
Alphabet	74 462	5 116 949	7 166 240	161 857	0.460	31.614	44.275
Amazon	5 760 000	5 500 000	20 054 722	280 522	20.533	19.606	71.491
Apple	50 463	862 127	27 618 943	260 174	0.194	3.314	106.156
BP	49 199 999	5 200 000	103 840 194	276 850	177.714	18.783	375.077
Danone	722 122	944 877	28 969 780	28 308	25.509	33.378	1023.365
Enel	69 981,891	5 365 386	8 726 973	86 610	808.016	61.949	100.762
Juventus	6 665	15 739	35 842	709	9.401	22.198	50.553
LVMH	67 613	262 609	11 853 749	60 083	1.125	4.371	197.291
Microsoft	113 414	3 556 553	5 977 488	125 843	0.901	28.262	47.500
Nestle	3 291 303	3 206 495	61 262 078	93 153	35.332	34.422	657.647
Netflix	38 481	145 443	1 900 283	20 156	1.909	7.216	94.277
Total	40 909 135	3 596 127	49 831 487	200 316	204.223	17.952	248.764
Volkswagen	4 494 066	5 973 894	65 335 372	282 817	15.890	21.123	231.016

Source: Trucost reporting year 2019.

Another advantage of carbon intensity is reducing the skewness of the distribution compared to absolute emissions. In Figure 23 on page 48, we show the carbon intensity histograms when the normalization variable is revenues in dollars. In Table 2, we report some

[8]We exclude the companies below the 20th percentile and above the 80th percentile.

[9]For example, the figures of BP, Enel and Total are very different and may be difficult to understand.

statistics (average, median, 95% percentile and maximum). We verify the skewness reduction. For instance, if we compute the ratio between $Q(95\%)$ and the median, it takes the value 233, 43 and 37 for the three scopes when we consider carbon emissions. These figures become 69, 9 and 5 when the ratio is computed with the carbon intensity.

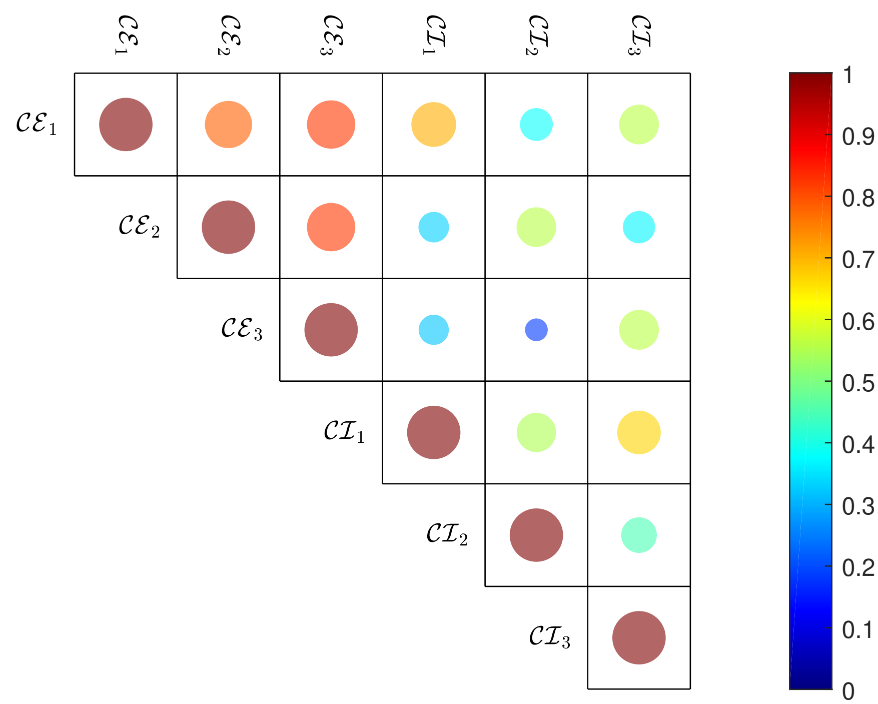
Table 2: Statistics of carbon emissions and intensity

Scope	Emission (in $10^6 \cdot$ tCO$_2$e)				Intensity (in $10^3 \cdot$ tCO$_2$e/\$ mn)			
	Avg.	Med.	$Q(95\%)$	Max.	Avg.	Med.	$Q(95\%)$	Max.
1	0.992	0.010	2.28	587.1	0.277	0.016	1.14	207.4
2	0.156	0.012	0.53	99.1	0.053	0.021	0.19	11.9
3	0.648	0.067	2.50	137.5	0.170	0.099	0.51	2.0

Source: Trucost reporting year 2019 & Authors' calculations.

In Figure 3, we show the Spearman correlations between the several carbon metrics[10]. Because of the economic size effect, we verify that carbon emissions are more correlated than carbon intensities (80% vs. 55% on average). Regarding these latter measures, we observe a high correlation of 66% between $\mathcal{CI}_1$ and $\mathcal{CI}_3$. Nevertheless, this figure is mainly explained by the sector effect. Indeed, if we compute rank correlations by sector[11], they are lower except for some specific sectors such as Utilities.

Figure 3: Rank correlation matrix of carbon metrics



Source: Trucost reporting year 2019 & Authors' calculations.

[10]The values are reported in Table 14 on page 47.
[11]See Table 15 on page 48.

Additivity property Carbon intensity is additive when we consider a given issuer. For instance, the carbon intensity of scopes 1, 2 and 3 is the sum of the different scopes:

$$\begin{aligned}\mathcal{CI}_{i,1+2+3} &= \frac{\mathcal{CE}_{i,1}+\mathcal{CE}_{i,2}+\mathcal{CE}_{i,3}}{Y_i} \\ &= \mathcal{CI}_{i,1}+\mathcal{CI}_{i,2}+\mathcal{CI}_{i,3}\end{aligned} \tag{5}$$

Nevertheless, the additivity property is not satisfied when we consider a set of issuers. Let us consider a portfolio x investing in n assets (stocks or bonds). Its carbon emissions are equal to:

$$\mathcal{CE}_j(x) = \sum_{i=1}^{n} \frac{W_i}{\mathcal{MV}_i}\mathcal{CE}_{i,j} = \sum_{i=1}^{n} \varpi_i \mathcal{CE}_{i,j} \tag{6}$$

where $\mathcal{MV}_i$ is the market value of the company i, W_i is the dollar value invested in the company i and ϖ_i is the ownership ratio:

$$\varpi_i = \frac{W_i}{\mathcal{MV}_i} \tag{7}$$

Let x_i be the weight of the asset i in the portfolio x. By definition, the wealth invested in this asset is equal to $W_i = x_i W$ where $W = \sum_{i=1}^{n} W_i$ is the dollar value of the portfolio. We deduce that:

$$\varpi_i = x_i \frac{W}{\mathcal{MV}_i} \tag{8}$$

and:

$$\mathcal{CE}_j(x) = \sum_{i=1}^{n} \left(x_i \frac{W}{\mathcal{MV}_i} \right) \mathcal{CE}_{i,j} \tag{9}$$

If we consider the weighted-average carbon intensity (WACI), we obtain:

$$\begin{aligned}\mathcal{CI}_j(x) &= \sum_{i=1}^{n} x_i \mathcal{CI}_{i,j} \\ &= \sum_{i=1}^{n} x_i \frac{\mathcal{CE}_{i,j}}{Y_i}\end{aligned} \tag{10}$$

It is obvious that the Equations (9) and (10) are mutually satisfied if and only if the normalization metric Y_i of the carbon intensity is proportional to the market value $\mathcal{MV}_i$. Indeed, we can interpret the carbon emissions of a portfolio as a weighted-average carbon intensity:

$$\mathcal{CE}_j(x) = W \left(\sum_{i=1}^{n} x_i \mathcal{CI}_{i,j}^{\mathcal{MV}} \right) \tag{11}$$

where $\mathcal{CI}_{i,j}^{\mathcal{MV}}$ is a carbon intensity measure, where the normalization factor is the market value of the company:

$$\mathcal{CI}_{i,j}^{\mathcal{MV}} = \frac{\mathcal{CE}_{i,j}}{\mathcal{MV}_i} \tag{12}$$

Let us assume that Equation (6) holds. In Appendix A.4.1 on page 45, we show that:

$$\mathcal{CI}_j(x) = \sum_{i=1}^{n} \omega_i \mathcal{CI}_{i,j} \tag{13}$$

where the weights ω_i are equal to:

$$\omega_i = \frac{x_i \dfrac{Y_i}{\mathcal{MV}_i}}{\sum_{k=1}^n x_k \dfrac{Y_k}{\mathcal{MV}_k}} = \frac{x_i \mathcal{SR}_i}{\sum_{k=1}^n x_k \mathcal{SR}_k} \tag{14}$$

and $\mathcal{SR}_i = Y_i / \mathcal{MV}_i$ is the revenue-to-market value (or sales-to-price) ratio. Contrary to common belief, analyzing the carbon risk of a portfolio using carbon intensity or carbon emissions is then not equivalent.

Remark 1. *$\mathcal{CE}_j(x)$ is generally expressed in tCO2e per 1\$ mn invested ($W = \$10^6$)*

Let us illustrate the discrepancy between the two approaches. We consider two issuers. We assume that the first issuer's carbon emissions, revenue and market value are equal to $\mathcal{CE}_{1,j} = 5 \times 10^6$, $Y_1 = 2 \times 10^5$ and $\mathcal{MV}_1 = 10^7$. It follows that its carbon intensity is equal to $\mathcal{CI}_{1,j} = 25$. For the second issuer, we have $\mathcal{CE}_{2,j} = 5 \times 10^7$, $Y_2 = 4 \times 10^6$ and $\mathcal{MV}_2 = 10^7$. We deduce that $\mathcal{CI}_{2,j} = 12.5$. In Table 3, we report the carbon intensity of the portfolio when it is made up of these two issuers ($W = \$10$ mn). We observe that the intensity-based approach overestimates the carbon intensity in our case, because the first issuer presents a higher sales-to-price ratio. In fact, the discrepancy error made by the intensity-based approach is highly dependent on the distribution of $\mathcal{CI}_{i,j}$ and $\mathcal{SR}_i$. For instance, it may be a huge problem when we mix large cap and small cap stocks.

Table 3: Comparison between the two approaches for computing the carbon intensity

x_1	x_2	$\mathcal{CE}_j(x)$ $(\times 10^6)$	$Y(x)$ $(\times 10^6)$	$\frac{\mathcal{CE}_j(x)}{Y(x)}$	$\mathcal{CI}_j(x)$
0%	100%	50.00	4.00	12.50	12.50
10%	90%	45.50	3.62	12.57	13.75
20%	80%	41.00	3.24	12.65	15.00
30%	70%	36.50	2.86	12.76	16.25
50%	50%	27.50	2.10	13.10	18.75
70%	30%	18.50	1.34	13.81	21.25
80%	20%	14.00	0.96	14.58	22.50
90%	10%	9.50	0.58	16.38	23.75
100%	0%	5.00	0.20	25.00	25.00

2.2 Carbon transition pathway

2.2.1 Carbon reduction scenario

The most recent IPCC report urges action to drastically reduce carbon emissions to net zero by 2050 (IPCC, 2021). The carbon reduction trajectory imposes that we need to reduce total emissions by at least 7% every year between 2019 and 2050. IEA (2021) has also published its net zero emission (NZE) scenario (see Table 4). It implies a 40.11% reduction of carbon emissions in 2030 and 61.84% in 2035. In 2050, gross emissions would be 1.94 $GtCO_2e$ offset by the carbon capture and storage (CCS) technology. These two scenarios are reported in Figure 4.

Besides net zero carbon scenarios, we can find other carbon scenarios, whose goal is to limit global warming. For instance, the European Union has framed new climate benchmarks

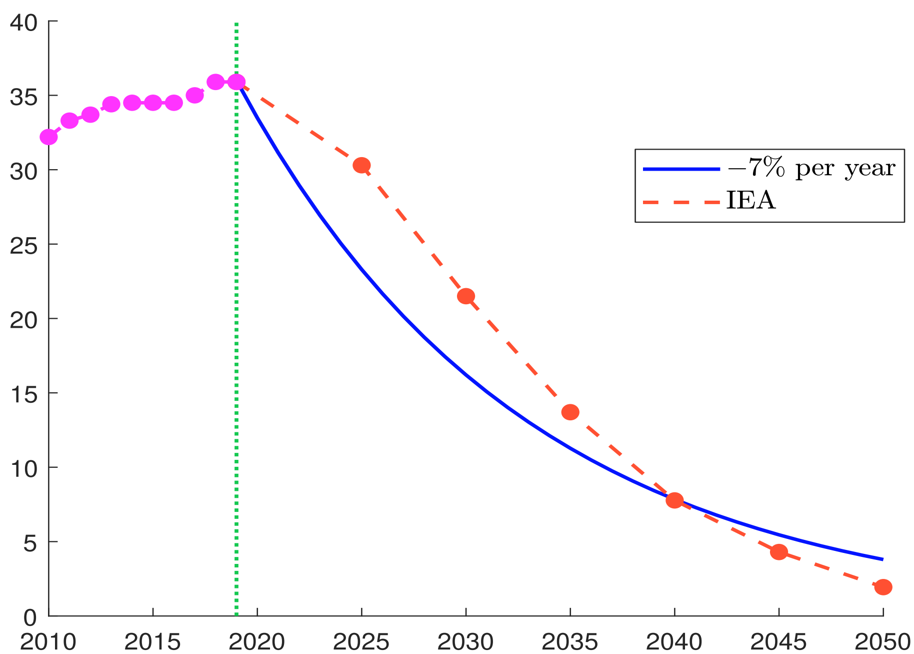
Table 4: IEA NZE scenario (in $GtCO_2e$)

Year	2019	2025	2030	2035	2040	2045	2050
Gross emission	35.90	30.30	21.50	13.70	7.77	4.30	1.94
CCS	0.00	−0.06	−0.32	−0.96	−1.46	−1.80	−1.94
Net emission	35.90	30.24	21.18	12.74	6.31	2.50	0.00
Reduction (in %)	0.00	15.60	40.11	61.84	78.36	88.02	94.60

Source: IEA (2021, Chapter 2, Figure 2.3, page 55).

designed to align investors' portfolios with the Paris agreement. For instance, the EU Paris aligned benchmark (PAB) label implies a year-on-year self-decarbonization of 7% and an initial carbon intensity reduction of 50% with respect to the investable universe in 2021. In the case of the EU climate transition benchmark (CTB) label, this last figure is equal to 30%. We notice that these two labels use the carbon intensity[12] instead of the carbon emission measure.

Figure 4: Two net zero emission scenarios



Source: IEA (2021) & Authors' calculations.

Remark 2. *Carbon reduction scenarios can be applied to regions, countries, sectors, etc. For example, IEA (2021, Chapter 3, pages 99-150) proposes sectoral pathways to net zero emissions for electricity, transport, buildings, etc. Moreover, we observe the development of physical carbon intensity by production unit (CO_2/km, CO_2/ton, CO_2/km^2, CO_2/kWh, etc.). The issue is then how to compare and aggregate these different intensity measures.*

[12]The normalization variable is then the enterprise value including cash and not the revenue.

2.2.2 Carbon trajectory

Trend Defining indicators describing individual transitions is not trivial. For some issuers, we can retrieve declared emissions since 2005, but the coverage starts to become more widespread starting in 2010. Using the historical records, it is possible to estimate the trend and make some carbon budget projections. For instance, we can consider a linear regression model:

$$\mathcal{CE}_{i,j}(t) = \beta_0 + \beta_1 t + u(t) \qquad \text{for } t \leq t_0 \tag{15}$$

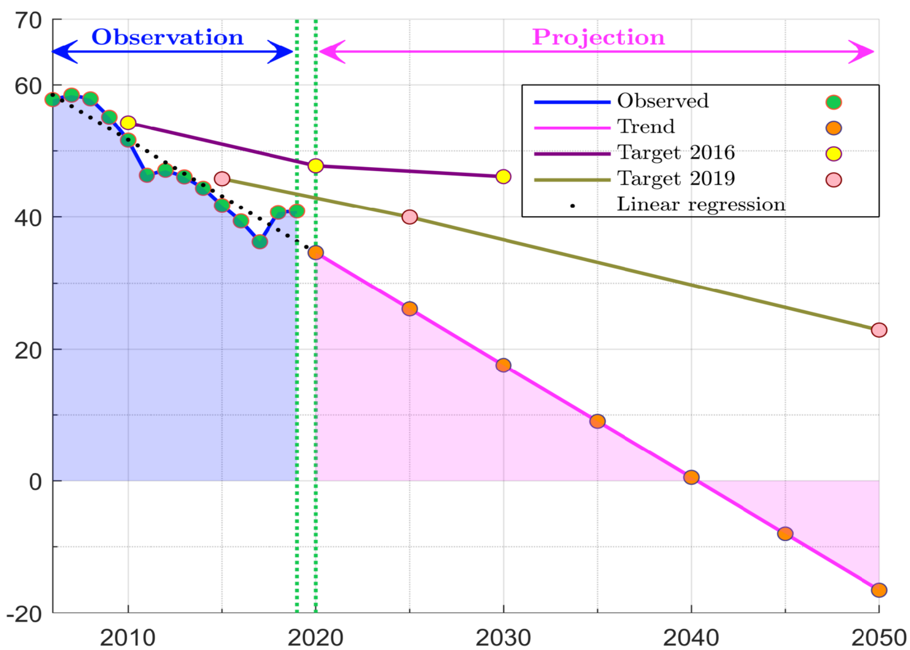
where t_0 is the current date. Once the model is estimated, we can project the future carbon emissions:

$$\mathcal{CE}_{i,j}^{\text{trend}}(t_0 + h) = \hat{\beta}_0 + \hat{\beta}_1 (t_0 + h) \qquad \text{for } h = 1, 2, \ldots \tag{16}$$

Figure 5 illustrates this approach when we consider the company $\mathbf{\Lambda}$ and the direct carbon emissions (scope 1). Computational details are given in Appendix A.5.2 on page 51. We notice that the projected value of direct carbon emissions is equal to zero in 2040. Using these trends, we can define the annual contribution of company i to reducing emissions:

$$\mathcal{R}_i^{\text{trend}} = \frac{1}{t - t_0} \frac{\sum_{j=1}^{3} \mathcal{CE}_{i,j}^{\text{trend}}(t_0) - \sum_{j=1}^{3} \mathcal{CE}_{i,j}^{\text{trend}}(t)}{\sum_{j=1}^{3} \mathcal{CE}_{i,j}^{\text{trend}}(t_0)} \tag{17}$$

Figure 5: Scope 1 trajectory of the company $\mathbf{\Lambda}$ (in $MtCO_2e$)



Source: Carbon Disclosure Project reporting year 2019 & Authors' calculations.

Target In addition, companies can directly disclose carbon targets $\mathcal{CE}_{i,j}^{\text{target}}(t)$ at a given horizon date t for a given scope j. The Carbon Disclosure Project gathers information about reduction targets (base year, target year, target value, scope, percentage of the scope concerned, etc.) in its climate change emissions management dataset. For example, we have reported the company $\mathbf{\Lambda}$'s targets announced in 2016 and 2019. We observe that the

2019 targets are more ambitious than the 2016 targets, and the targets are greater than the projections. As previously, we can compute the annual reduction using Equation (17) and replacing $\mathcal{CE}_{i,j}^{\text{trend}}(t)$ with $\mathcal{CE}_{i,j}^{\text{target}}(t)$. Therefore, the synthetic trajectory of the company can be derived from both projected and reported annual reductions.

2.2.3 Temperature

Temperature tags are indicators assigned to companies by mixing various metrics. The main ingredient is the carbon trajectory. Based on the carbon reduction required to achieve a climate scenario, and individual trajectories, it is possible to interpolate a temperature for each issuer. For instance, Table 5 provides some figures at the global level. Indeed, if we would like to achieve a 1.8°C scenario, this implies reducing absolute carbon emissions with respect to year 2005 by 51.5%. For each company, we can consider a linear projection of the carbon trajectory, and perform some corrections using additional information such as reduction targets, changes in the share of green revenues, investment in research and development associated with transition and green patents, etc.

Table 5: Global temperature changes in % (wrt year 2005)

Scenario	2010	2020	2030	2040	2050
1.5°C (SSP2, RCP 1.9W/m^2)	+7.5%	17.1%	−28.6%	−63.9%	−87.3%
1.8°C (SSP2, RCP 2.6W/m^2)	+8.1%	17.4%	−14.5%	−34.5%	−51.5%
2.2°C (SSP2, RCP 3.4W/m^2)	+8.1%	19.9%	+8.2%	−5.8%	−20.6%
3.0°C (SSP2, RCP 4.5W/m^2)	+8.1%	21.5%	+24.4%	+22.2%	+11.3%
4.0°C (SSP2, RCP 6.0W/m^2)	+8.1%	22.2%	+33.6%	+42.7%	+45.9%
5.0°C (SSP2, Baseline)	+8.1%	26.8%	+43.7%	+58.7%	+70.0%

Source: SSP Database (Shared Socioeconomic Pathways), Version 2.0.

However, there are multiple methodologies that do not always provide the same output in terms of aggregated temperature (Raynaud *et al.*, 2021a,b). One of the main pitfalls is the aggregation of individual trajectories, or in other terms, the definition of the portfolio trajectory based on the trajectories of issuers (Le Guenedal *et al.*, 2020). For example, the Carbon Disclosure Project (CDP) provides temperature ratings that mix the trajectories with issuer targets[13]. Among the 8 500 companies in the database, about 3 300 have a CDP temperature rating. When carbon targets are used, they are based on the Science Based Target initiative (SBT), however it only represents 12% of the sample for scope $1+2$, compared to 1% for scope 3. In Table 6, we report the frequencies of the temperature ratings provided by CDP and Iceberg. For CDP data, we notice that most of companies have a rating corresponding to a temperature of 3.2°C.

2.3 Carbon transition risk

While carbon footprint and pathway are measured by CO_2 emissions, which are based on fundamentals and physical data, we now consider carbon risk metrics, which are by nature based on market data. In other words, they are related to the financial market's perception of the potentially reduced impact of climate policies' on securities issued by corporations. Recently, two main approaches have been developed. The first one assesses how an increase in carbon prices and taxes influences the credit risk of the issuer whereas the second method measures how sensitive the asset price is to a carbon market factor.

[13] The methodology is available at `www.cdp.net/en/investor/temperature-ratings`.

Table 6: Frequency of temperature ratings (in %)

Range	CDP				Iceberg
	Scope $1+2+3$	Short-term	Mid-term	Long-term	
$\mathcal{T} \leq 1.0°\mathrm{C}$	0.00	0.00	0.00	0.00	1.01
$1.0°\mathrm{C} < \mathcal{T} \leq 1.5°\mathrm{C}$	1.44	2.92	10.68	2.71	2.60
$1.5°\mathrm{C} < \mathcal{T} \leq 2.0°\mathrm{C}$	6.20	1.26	13.03	3.94	3.14
$2.0°\mathrm{C} < \mathcal{T} \leq 2.5°\mathrm{C}$	6.86	3.07	7.46	2.68	21.76
$2.5°\mathrm{C} < \mathcal{T} \leq 3.0°\mathrm{C}$	7.64	1.99	4.21	0.48	30.87
$3.0°\mathrm{C} < \mathcal{T} \leq 3.5°\mathrm{C}$	76.95	89.77	62.80	90.07	32.30
$3.5°\mathrm{C} < \mathcal{T} \leq 4.0°\mathrm{C}$	0.78	0.81	1.44	0.09	2.23
$4.0°\mathrm{C} < \mathcal{T} \leq 4.5°\mathrm{C}$	0.12	0.18	0.36	0.03	3.31
$4.5°\mathrm{C} < \mathcal{T} \leq 5.0°\mathrm{C}$	0.00	0.00	0.00	0.00	0.77
$\mathcal{T} = 3.2°\mathrm{C}$	52.09	88.50	61.39	89.95	0.01

Source: CDP Temperature Ratings Dataset, version 1.1, February 2021 & Iceberg Data Lab (2021).

2.3.1 Carbon price

The Paris agreement defines reduction paths that are determined at the national level. These targets are often translated into pricing mechanisms for GHG emissions whose requirements increase over time. In practice, there are two main pricing systems: the carbon tax and the emissions trading system (ETS). Thus, emitting companies must pay the tax corresponding to their emissions if they are subject to a carbon tax, or buy carbon allowance on a cap-and-trade market if they are subject to an ETS. The coverage of these mechanisms is rapidly increasing[14]. Rapid fluctuations in the price of carbon can represent a risk for companies' businesses. Therefore, Bouchet and Le Guenedal (2020) suggested identifying for each company the carbon price that would lead the default probability in the Merton model to exceed a certain threshold. Based on the assumptions that the enterprise value V is proportional to the earnings before interest, taxes, depreciation, and amortization (EBITDA) and that the debt D remains constant, we can define the carbon price margin as:

$$\mathcal{CPM}_i = \left(1 - \exp\left(\sigma_i\sqrt{\tau}\Phi\left(-\theta\right) - \left(r + \frac{1}{2}\sigma_i^2\right)\tau\right)\frac{D_i}{V_i}\right)\frac{\mathrm{EBITDA_i}}{\mathcal{CE}_{i,1}}$$

where σ_i is the volatility of the enterprise value, τ is the maturity and r is the risk-free rate. The parameter θ is a threshold of default probability, for instance 50%. A low value of $\mathcal{CPM}$ means that company's capital structure leaves it vulnerable to a minor change in the carbon price. Inversely, using this methodology, it would be possible to assess the default risk over a portfolio subject to changes in local carbon prices. However, data availability on the breakdown of regional emissions and the lack of diffusion effect, from intensive sectors to the rest of the economy, limits the interest of this exercise on a diversified portfolio.

[14] The state of the art pricing mechanism can be found on https://carbonpricingdashboard.worldbank.org.

2.3.2 Carbon beta

Introduced by Harris (2015) and Görgen *et al.* (2019), the underlying idea of the carbon beta is to estimate the sensitivity of the stock return with respect to a carbon risk factor. Therefore, this framework adopts the viewpoint of the arbitrage pricing theory by considering that carbon is not only an idiosyncratic risk for an issuer, but also a systematic risk factor like the Fama-French-Carhart market factors (Roncalli *et al.*, 2021). Let $R_i(t)$ be the return of stock i at time t. We assume that:

$$R_i(t) = \alpha_i(t) + \beta_{i,\text{mkt}}(t) R_{\text{mkt}}(t) + \sum_{j=1}^{m} \beta_{i,\mathcal{F}_j}(t) R_{\mathcal{F}_j}(t) + \beta_{i,\text{Carbon}}(t) R_{\text{Carbon}}(t) + \varepsilon_i(t)$$

where $R_{\text{mkt}}(t)$ is the return of the market risk factor, $R_{\mathcal{F}_j}(t)$ is the return of the j^{th} alternative risk factor, $R_{\text{Carbon}}(t)$ is the return of the carbon risk factor and $\varepsilon_i(t)$ is a white noise process. The estimation $\hat{\beta}_{i,\text{Carbon}}(t)$ of the carbon beta $\boldsymbol{\mathcal{CB}}_i$ can be done using ordinary least squares (Görgen *et al.*, 2019) or Kalman filtering (Roncalli *et al.*, 2020). One of the difficulties is building the carbon risk factor, which systematically corresponds to a long/short portfolio between "*green*" and "*brown*" stocks. For instance, Görgen *et al.* (2019) defined a brown-minus-green factor by using many carbon metrics. Nevertheless, Roncalli *et al.* (2020) showed that this risk factor is highly correlated to a simpler factor based on small number of carbon indicators[15].

Engle *et al.* (2020) proposed a related approach where the carbon risk factor is replaced by a climate risk news index $\mathcal{I}_{\text{Climate}}$:

$$R_i(t) = \alpha_i(t) + \beta_{i,\text{mkt}}(t) R_{\text{mkt}}(t) + \sum_{j=1}^{m} \beta_{i,\mathcal{F}_j}(t) R_{\mathcal{F}_j}(t) + \beta_{i,\text{Climate}}(t) \mathcal{I}_{\text{Climate}}(t) + \varepsilon_i(t)$$

In this case, $\mathcal{I}_{\text{Climate}}$ corresponds to a time series that measures the sentiment about the climate change. The big challenge is to apply the appropriate text mining tools on newspapers and media data (Apel *et al.*, 2021; Ardia *et al.*, 2021; Engle *et al.*, 2020).

2.4 Climate physical risk

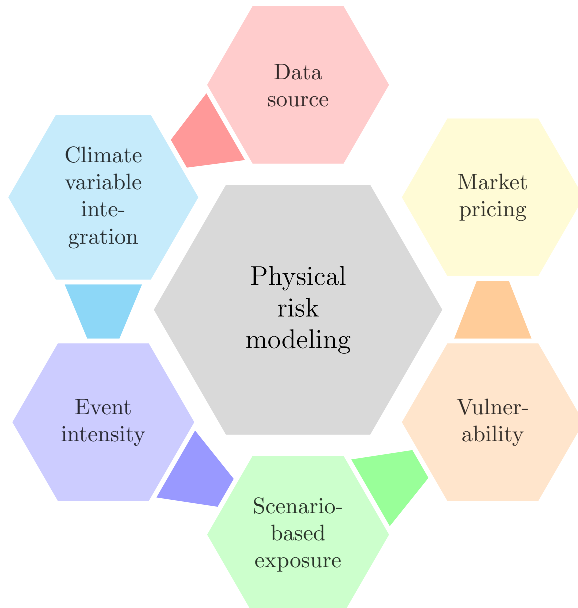
Responsible investors have paid more attention to the transition risk than to the physical risk. However, recent events show that physical risk is also a big concern. It corresponds to the financial losses that really come from climate change, and not from the adaptation of the economy to prevent them. It includes droughts, floods, storms, etc. This risk is more difficult to quantify, and its evaluation requires multidisciplinary methodologies: climate modeling, physical asset geolocation, financial loss estimation, etc. The general steps for developing an integrated approach are illustrated in Figure 6. For example, we refer to Le Guenedal *et al.* (2021) for the description of a full integrated methodology to measure cyclone-related physical risk.

2.4.1 Climate variable and data source

We define the climate data of source s as $\Theta_s = \left\{\theta(\lambda, \varphi, z, t)\right\}$ where $\theta = (\theta_1, \ldots, \theta_k)$ is a vector of k climate variables such as temperature, pressure or wind speed. Each variable θ_k has four coordinates: the latitude λ, the longitude φ, the height (or altitude) z and the time t. We can consider three types of sources: meteorological records, reanalysis and historical simulations by a climate model.

[15] See also Gurvich and Creamer (2021) for alternative definitions of the carbon risk factor.

Figure 6: Physical risk modeling



Essentially, the first source corresponds to physical measures made in space and time, but these measures do not necessarily cover the entire grid. The second source collects the outputs of a model that has been calibrated with meteorological records and gives the best estimates of past climate. For example, Figure 7 is a slice of the MERRA-2 reanalysis at a height of 10 meters on 7^{th} November 2013. The red dot is the location of the eye of the tropical cyclone Haiyan, which affected more than 10 million people in the Philippines. For instance, these data can be used to fit natural disaster sensitivity to climate change and calibrate damage functions. The last possible source of climate data is a climate model. It can be run in the past and in the future for different representative concentration pathway (RCP) scenarios.

Climate models are constantly improved and the different phases of the coupled model intercomparison project (CMIP) provide outputs of climate variables. For instance, the latest results used by the intergovernmental panel on climate change (IPCC) are based on the sixth phase of the project (CMIP6). From a financial point of view, it is important to take into account the data of this third source. For instance, in the case of wind, the output of the projection can provide information about the future value of wind farms in different representative concentration pathways (see Figure 7).

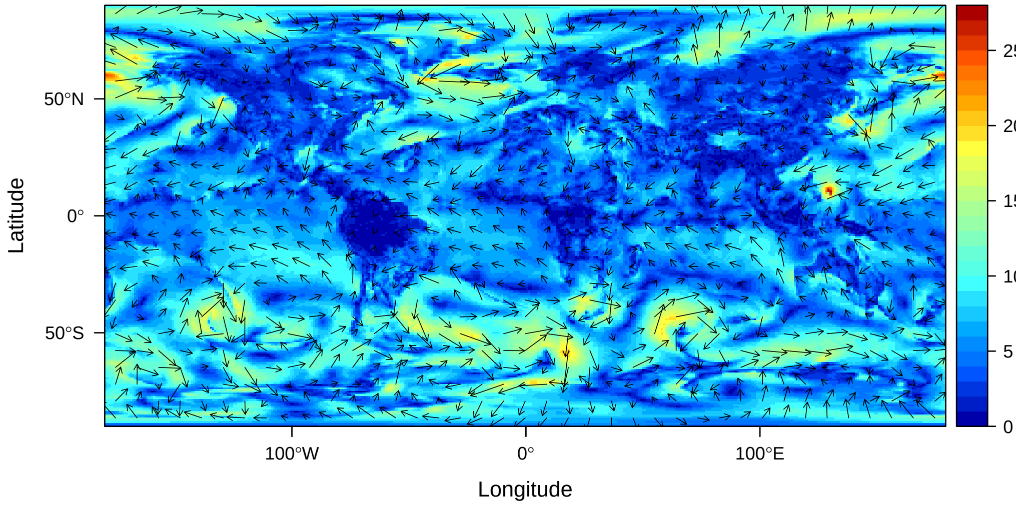
2.4.2 Event intensity sensitivity

To measure physical risks from these raw climate variables, we first define the sensitivity of the intensity of extreme events to climate change[16]. Let $\mathbb{E}\left[I\left(\Theta_s\left(C\right)\right)\right]$ be the expected intensity of the event in the scenario associated with the GHG concentration C. The sensitivity of the event is equal to:

$$\Delta I\left(C\right) = \mathbb{E}\left[I\left(\Theta_s\left(C\right)\right)\right] - I\left(\Theta_s\left(C_0\right)\right) \tag{18}$$

[16] More precisely, one can simulate events using Monte-Carlo methods in such a way that these events are sensitive to raw climate data.

Figure 7: Slice of wind speed at 10 meters (07/11/2013, tropical cyclone Haiyan)



Source: Modern-Era Retrospective analysis for Research and Applications, Version 2 (MERRA-2), Global Modeling and Assimilation Office, NASA.

where $I\left(\Theta_s\left(C_0\right)\right)$ is the current intensity or the reference intensity in a scenario where climate objectives are met. For instance, we know that the maximum wind of tropical cyclones increases by more than 10% in scenarios with a high GHG concentration.

2.4.3 Asset exposure

Estimating the future asset value $\Psi(t)$ of the portfolio can be puzzling. For instance, Le Guenedal *et al.* (2021) used the shared-socioeconomic pathways (SSP) to project GDP per capita and local values of physical assets in the long term. The asset value of the portfolio can then be written as:

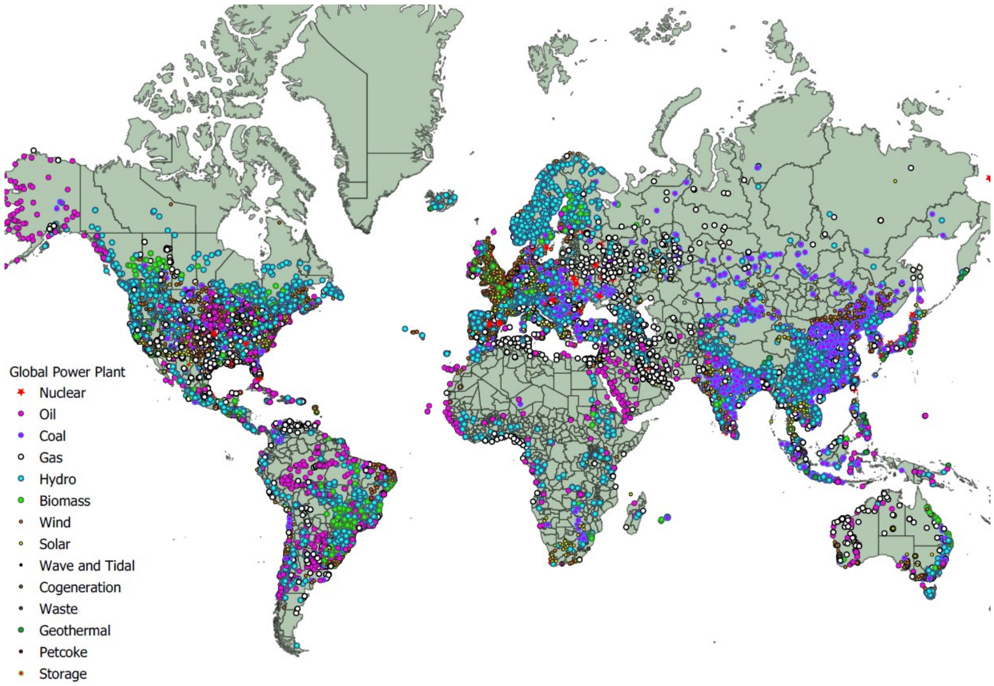
$$\Psi(t)=\sum_{j=1}^{n} x_j \Psi_j(\lambda, \varphi, t) \tag{19}$$

where $\Psi_j(\lambda, \varphi, t)$ is the geolocated asset value estimated at time t and x_j is the weight of asset j in the portfolio. Therefore, geolocation of the portfolio is another important feature in the context of physical risks. Some open-source initiatives allow us to estimate the risk at the country level. For instance, for the utilities sector, one could map issuer production (see Figure 11 on page 20) and production sites given in Figure 8 to estimate the physical risk exposure of this sector.

2.4.4 Vulnerability

Based on historical records, we define the damage function $\Omega_j(I) \in [0,1]$ as the fraction of property loss with respect to the intensity. It is generally calibrated on past damages (insurance claims, economic loss, etc.) and disasters. For example, the global disaster database (EM-DAT) can be used in this context. This function should be defined in a bottom-up fashion and specific to each asset (or asset class) to better translate their idiosyncratic vulnerability.

Figure 8: Geolocation of world power plants by energy source



Source: Global Power Database version 1.3 (June 2021).

2.4.5 Market pricing

We can define the physical risk implied by the concentration scenario C as:

$$\Delta\mathcal{L}oss\left(t,C\right)=\beta\cdot\mathcal{DD}\left(t,C\right)=\beta\sum_{j=1}^{n}x_j\Psi_j\left(\lambda,\varphi,t\right)\Omega_j\left(\Delta I\left(t,C\right)\right) \tag{20}$$

where $\Delta\mathcal{L}oss\left(t,C\right)$ is the relative loss due to the events on the portfolio and β is the transmission factor of the direct damage $\mathcal{DD}\left(t,C\right)$ on the underlying to the loss of financial value in the investment portfolio. For example, if the facilities of an energy producer are damaged at 50%, the securities issued by this company will be impacted at 50% $\times\,\beta$.

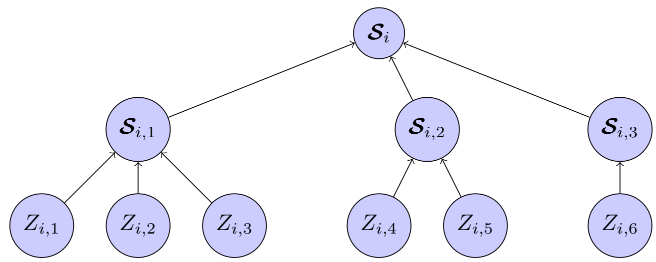
2.5 Other metrics

2.5.1 Scoring

The success of ESG scoring systems is one factor that explains the development of ESG investing. Most of them are based on scoring trees. Let us consider the construction of a two-level scoring tree. Raw data $\left(X_{i,1},\ldots,X_{i,m_1}\right)$ are normalized in order to obtain features $\left(Z_{i,1},\ldots,Z_{i,m_1}\right)$. These features are then aggregated to obtain sub-scores $\left(\boldsymbol{\mathcal{S}}_{i,1},\ldots,\boldsymbol{\mathcal{S}}_{i,m_2}\right)$ generally by using a linear function:

$$\boldsymbol{\mathcal{S}}_{i,j}=\sum_{k=1}^{m_1}\omega_{j,k}^{(1)}Z_{i,k} \tag{21}$$

Figure 9: An example of a two-level scoring tree



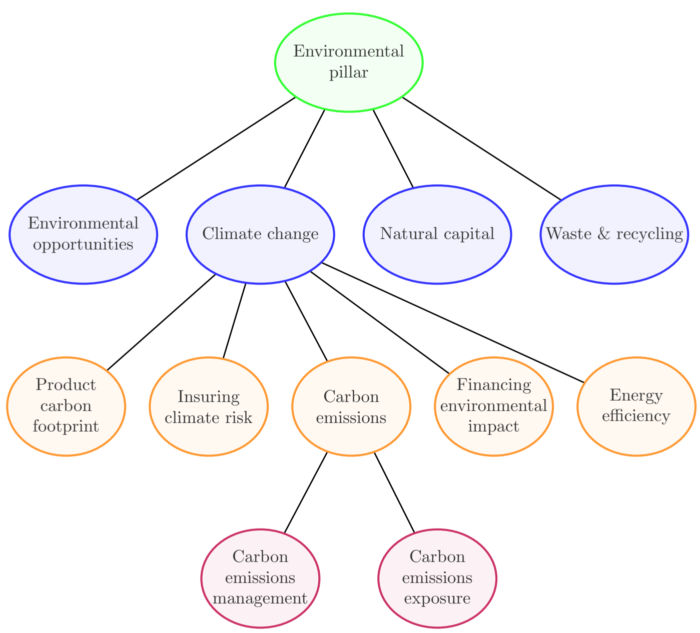
For the level 1, we assume that $\omega_{1,1}^{(1)} = 50\%$, $\omega_{1,2}^{(1)} = 25\%$ and $\omega_{1,3}^{(1)} = 25\%$ (sub-score $\mathcal{S}_{i,1}$), $\omega_{2,4}^{(1)} = 50\%$ and $\omega_{2,5}^{(1)} = 50\%$ (sub-score $\mathcal{S}_{i,2}$), and $\omega_{3,6}^{(1)} = 100\%$ (sub-score $\mathcal{S}_{i,3}$). For the level 2, we have $\omega_1^{(2)} = \omega_2^{(2)} = \omega_3^{(2)} = 33.33\%$ (final score $\mathcal{S}_i$).

Finally, we combine sub-scores in order to obtain the final score:

$$\mathcal{S}_i = \sum_{j=1}^{m_2} \omega_j^{(2)} \mathcal{S}_{i,j} \tag{22}$$

In the example provided in Figure 9, we have six raw variables and three sub-scores. The first sub-score depends on the first three variables, the second sub-score is related to the next two variables whereas the last variable is already a sub-score. In the case of the first sub-score, we notice that the weight of the first variable is twice the weight of the second or third variables, whereas the second sub-score is an equally-weighted score between the fourth and fifth variables. Finally, the final score is an average of the three sub-scores. In practice, ESG rating agencies use multi-level tree structures, which is a straightforward extension of the two-level tree structure. An example is given in Figure 10.

Figure 10: An example of a multi-level scoring tree (MSCI methodology)



Source: MSCI (2020).

Remark 3. *Generally, ESG rating providers consider two approaches for normalizing raw data. They can use the z-scores method:*

$$Z_{i,j} = \frac{X_{i,j} - \hat{m}_j}{\hat{\sigma}_j} \tag{23}$$

where $\hat{m}_j$ and $\hat{\sigma}_j$ are the empirical mean and standard deviation of the j^{th} variable. In this case, the range of $Z_{i,j}$ is between -3 and $+3$ most of the time, except when there are some outliers or extreme observations. The second approach consists in applying a transformation function on a given range $\left[z^-, z^+\right]$:

$$Z_{i,j} = \mathbf{T}_j \left(X_{i,j}\right) \tag{24}$$

For instance, a typical range is the $0-10$ normalization. Among the different transformation functions, the most popular is the p-scores method:

$$Z_{i,j} = z^- + \left(z^+ - z^-\right) \hat{\mathbf{F}}_j \left(X_{i,j}\right) \tag{25}$$

where $\hat{\mathbf{F}}_j$ is the empirical probability distribution the j^{th} variable[17]*.*

2.5.2 KPI

Below, we list other metrics that can be used for the construction of an investment portfolio in a climate change framework. We focus on three metrics: avoided emissions, green revenues and energy mix. However, the fund manager can use many other metrics, and we have observed a proliferation of indicators in recent years. For example, we can create reserves-based indicators. Indeed, from a stranding risk perspective, it is important to assess and price available reserves. Both the reserves of coal, natural gas, etc. and infrastructure are included in companies' balance sheets. A change in the price of carbon or other regulations could cause these assets to suddenly depreciate. This phenomenon is called capital stranding. Another example of recent indicators concerns green patent, R&D and capital expenditure, because the transition to a low-carbon economy requires high levels of investment and research. For instance, the most ambitious scenarios introduce carbon capture and storage technology, which are not currently available.

Avoided emissions Another metric to measure the environmental impact of a portfolio is the avoided emissions. This metric aims to compare the carbon emissions of a product to a reference or benchmark. For example, a hybrid car emits CO_2, especially if we take into account the life cycle of batteries. Nevertheless, by increasing their offer of hybrid cars, car manufacturers still avoid the emissions that would have been emitted otherwise[18]. Assuming that each company proposes n_p products or services, the avoided emissions is defined as:

$$\boldsymbol{\mathcal{AE}}_i = \sum_{k=1}^{n_p} \sum_{j=1}^{3} \left(\boldsymbol{\mathcal{CE}}_j \left(k; \text{reference}\right) - \boldsymbol{\mathcal{CE}}_j \left(k; \text{green}\right)\right)$$

where $\boldsymbol{\mathcal{CE}}_j \left(k; \text{green}\right)$ and $\boldsymbol{\mathcal{CE}}_j \left(k; \text{reference}\right)$ are the scope j emissions of the green and reference products k. The major issue is that this metric is subject to multiple accounting issues. Therefore, defining this metric for a portfolio is particularly complex.

[17] A related approach is the Gaussian probability integral transform:

$$Z_{i,j} = z^- + \left(z^+ - z^-\right) \Phi \left(\frac{X_{i,j} - \hat{m}_j}{\hat{\sigma}_j}\right)$$

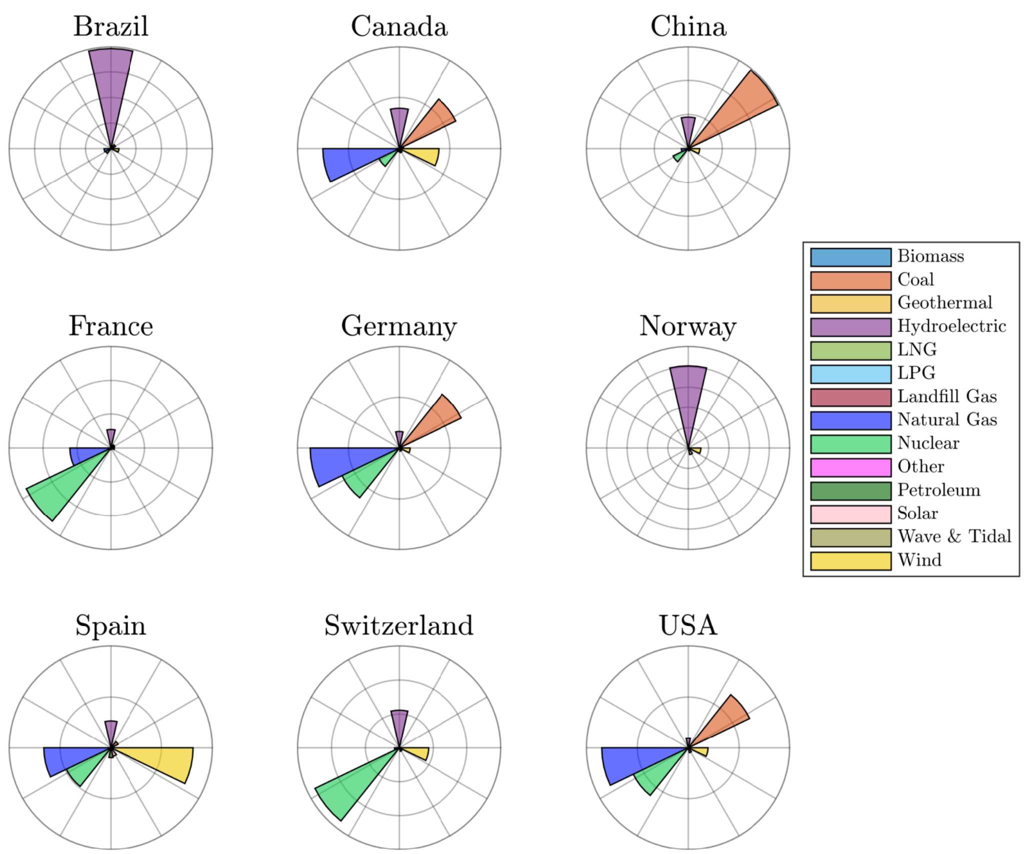
where $\Phi(x)$ is the normal probability distribution. This method is a special case of the p-scores method by assuming the normality of the observations.

[18] Indeed, the direct emissions of these cars are at first reported in scope 2 of the electric producers.

Green revenues An increasing number of companies operate in many sectors with a diversified range of products and services. The green revenues measure the share of the company's business in sustainable activities. An issuer with 100% of green revenues is called a *pure player*. For instance, revenues from electric cars can be accounted as green revenues for car manufacturers. For utility companies, we can classify renewable production (wind, solar, etc.) as green and other sources as brown. We note that even this simple distinction can be controversial when it comes to nuclear. It becomes all the more elusive to make the distinction between green and non-green revenues when looking at information technology or health care sectors.

Energy mix For utility companies the energy mix can provide information on the environmental performance of the company. This indicator is also particularly relevant when assessing the risk at the sovereign level. Figure 11 presents the energy generation breakdown for some countries[19]. We can distinguish countries that rely on hydroelectric power (Brazil, Norway), nuclear (France, Switzerland) and mixed solutions (Canada, Germany, Spain, USA).

Figure 11: Bottom up energy mix (in %)



Source: Trucost reporting year 2019 & Authors' calculations.

[19]Each grid circle represents 20% of energy generation. The scale of the radar chart is then 40% for Canada, Germany, Spain and USA, 60% for China, France and Switzerland, 80% for Brazil and 100% for Norway.

3 Portfolio optimization

The development of sustainable and impact investing strategies related to climate change implies including climate risk metrics in modern portfolio optimization. Generally, the solution consists in adding some management constraints. From a theoretical and mathematical viewpoint, this does not fundamentally change the traditional approach to portfolio construction. However, this can lead to high portfolio distortion from a practical viewpoint, especially when constraints are tight, which can be the case with net zero carbon portfolios.

3.1 General framework

Traditional portfolio optimization problems can be written as:

$$\begin{aligned} x^{\star}(\gamma) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b)-\gamma(x-b)^{\top}\mu \\ &\text{s.t.} \quad \left\{\begin{array}{l} \mathbf{1}_n^{\top}x=1 \\ x\in\Omega \end{array}\right. \end{aligned} \tag{26}$$

where $x=(x_1,\ldots,x_n)$ is the active portfolio, $b=(b_1,\ldots,b_n)$ is the benchmark portfolio, Σ is the covariance matrix of asset returns, μ is the vector of expected returns (or alpha tilts/scores) and Ω is the set of constraints. For instance, we can impose a long-only constraint[20], maximum sector deviation[21], etc. If there is no benchmark ($b=\mathbf{0}_n$), we retrieve the Markowitz framework with the trade-off between the portfolio variance $\sigma^2(x)=x^{\top}\Sigma x$ and the average return $\mu(x)=x^{\top}\mu$. The efficient frontier is then obtained by considering different values of $\gamma\in\mathbb{R}$. If we consider a benchmark ($b\neq\mathbf{0}_n$), the trade-off concerns the tracking error variance $\sigma^2\left(x\mid b\right)=(x-b)^{\top}\Sigma(x-b)$ and the average excess return of the portfolio $\mu\left(x\mid b\right)=(x-b)^{\top}\mu$.

A climate risk measure can be introduced into the portfolio optimization by adding a new constraint[22]:

$$\Omega=\left\{x:\boldsymbol{\mathcal{C}}(x)=\sum_{i=1}^{n}x_i\boldsymbol{\mathcal{C}}_i\leq\boldsymbol{\mathcal{C}}^{+}\right\} \tag{27}$$

For instance, we may want to limit the portfolio's carbon emissions — $\boldsymbol{\mathcal{CE}}_j(x)\leq\boldsymbol{\mathcal{CE}}_j^{+}$ — or its carbon intensity — $\boldsymbol{\mathcal{CI}}_j(x)\leq\boldsymbol{\mathcal{CI}}_j^{+}$. We may also want to reduce the portfolio's carbon emissions or intensity with respect to a benchmark: $\boldsymbol{\mathcal{CE}}_j(x)\leq(1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CE}}_j(b)$ or $\boldsymbol{\mathcal{CI}}_j(x)\leq(1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CI}}_j(b)$ where $\boldsymbol{\mathcal{R}}>0$ is the reduction rate. An alternative approach to Problem (26-27) is to formulate the following objective function:

$$\begin{aligned} x^{\star}(\gamma,\delta) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b)-\gamma(x-b)^{\top}\mu+\gamma\boldsymbol{\mathcal{C}}(x) \\ &\text{s.t.} \quad \left\{\begin{array}{l} \mathbf{1}_n^{\top}x=1 \\ x\in\Omega \end{array}\right. \end{aligned} \tag{28}$$

In this case, we obtain a three-dimensional efficient frontier: mean return vs. variance risk vs. climate risk. As noticed by Pedersen *et al.* (2020), optimizing over three characteristics can

[20] In this case, we have $\Omega=\{x:0\leq x_i\leq 1\}$.

[21] Let $\omega^{-}(\mathcal{S}_j)$ and $\omega^{+}(\mathcal{S}_j)$ be the minimum and maximum weights for the sector $\mathcal{S}_j$. We can impose that $\Omega=\left\{x:\omega^{-}(\mathcal{S}_j)\leq\sum_{i\in\mathcal{S}_j}x_i\leq\omega^{+}(\mathcal{S}_j)\right\}$.

[22] We assume that the climate metric is an increasing function with respect to the climate risk. If we consider a decreasing function, we replace the constraint by $\boldsymbol{\mathcal{C}}(x)=\sum_{i=1}^{n}x_i\boldsymbol{\mathcal{C}}_i\geq\boldsymbol{\mathcal{C}}^{-}$. In this case, we would like to achieve a minimum value $\boldsymbol{\mathcal{C}}^{-}$ of the climate metric $\boldsymbol{\mathcal{C}}(x)$.

be challenging. However, these authors showed that the problem can be reduced to a trade-off between the Sharpe ratio and the climate risk measure[23]. In what follows, we consider another approach by fixing a minimum level of expected excess return — $\mu\left(x \mid b\right) \geq \mu^{-}$ — or a maximum level of tracking error volatility — $\sigma\left(x \mid b\right) \leq \sigma^{+}$. In this case, the efficient frontier is reduced to a two-dimensional trade-off: variance risk vs. climate risk or mean return vs. climate risk. This is the preferred approach of professionals when implementing active management. A third approach consists in defining the benchmark portfolio and finding the climate-optimal portfolio that minimizes the tracking error volatility. This is equivalent imposing $\gamma = 0$ (or $\mu = \mathbf{0}_n$) in Problem (28). This approach is extensively used in passive management (Andersson *et al.*, 2016).

3.2 Portfolio decarbonization

3.2.1 Using carbon intensity

One of the most important topics in climate finance is portfolio decarbonization. The underlying idea is to construct a portfolio x that tracks the benchmark portfolio b but with a lower carbon risk metric. From a mathematical point of view, the optimization problem is defined as[24]:

$$\begin{aligned} x^{\star}\left(\boldsymbol{\mathcal{R}}\right) &= \arg\min \frac{1}{2}\left(x-b\right)^{\top}\Sigma\left(x-b\right) \\ & \text{s.t.} \quad \left\{\begin{array}{l} \mathbf{1}_n^{\top}x = 1 \\ x \geq \mathbf{0}_n \\ \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{CI}}_i \leq \left(1-\boldsymbol{\mathcal{R}}\right)\boldsymbol{\mathcal{CI}}\left(b\right) \end{array}\right. \end{aligned} \tag{29}$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate. Therefore, we minimize the tracking error volatility of the portfolio x with respect to the benchmark by imposing a long-only constraint and reducing the weighted average carbon intensity (WACI) of the benchmark. We notice that the carbon intensity metric is used and not the carbon emissions metric, which may cause problems when considering net zero carbon portfolios[25]. Since we both impose a constraint and minimize the tracking error volatility, the portfolio x has fewer stocks than the benchmark b. In fact, the number of stocks depends on several parameters: the reduction rate $\boldsymbol{\mathcal{R}}$, the number n of stocks in the benchmark and the covariance matrix. This implies that the portfolio x is less diversified than the benchmark b. In order to explicitly control the number of removed stocks, Andersson *et al.* (2016) proposed a second portfolio decarbonization approach by eliminating the m worst performing issuers in terms of carbon intensity. Let $\boldsymbol{\mathcal{CI}}_{i:n}$ be the order statistics of $(\boldsymbol{\mathcal{CI}}_1, \ldots, \boldsymbol{\mathcal{CI}}_n)$ such that:

$$\min \boldsymbol{\mathcal{CI}}_i = \boldsymbol{\mathcal{CI}}_{1:n} \leq \boldsymbol{\mathcal{CI}}_{2:n} \leq \cdots \leq \boldsymbol{\mathcal{CI}}_{i:n} \leq \cdots \leq \boldsymbol{\mathcal{CI}}_{n:n} = \max \boldsymbol{\mathcal{CI}}_i \tag{30}$$

The carbon intensity bound $\boldsymbol{\mathcal{CI}}^{(m,n)}$ is defined as $\boldsymbol{\mathcal{CI}}^{(m,n)} = \boldsymbol{\mathcal{CI}}_{n-m+1:n}$ where $\boldsymbol{\mathcal{CI}}_{n-m+1:n}$ is the $(n-m+1)$-th order statistic of $(\boldsymbol{\mathcal{CI}}_1, \ldots, \boldsymbol{\mathcal{CI}}_n)$. Eliminating the m worst performing assets is equivalent to imposing the following constraint: $\boldsymbol{\mathcal{CI}}_i \geq \boldsymbol{\mathcal{CI}}^{(m,n)} \Rightarrow x_i = 0$. Finally,

[23]Although Pedersen *et al.* (2020) used ESG scores instead of climate scores, their analysis remains valid when considering climate risks and not ESG risks.
[24]In the sequel, we omit the subscript j that defines the scope to simplify the notations.
[25]This issue is developed in Section 3.3 on page 28.

we obtain the following optimization problem:

$$\begin{aligned} x^{\star}(m) &= \arg\min \frac{1}{2}(x-b)^{\top}\Sigma(x-b) \\ & \text{s.t.} \left\{ \begin{array}{l} \mathbf{1}_n^{\top} x = 1 \\ x \geq \mathbf{0}_n \\ \mathcal{CI}_i \geq \mathcal{CI}^{(m,n)} \Rightarrow x_i = 0 \end{array} \right. \end{aligned} \tag{31}$$

Problem (31) is called the "*order-statistic*" approach, whereas Problem (29) is known as "*max-threshold*". Finally, a "*naive*" solution consists in re-weighting the remaining assets:

$$x_i = \frac{\mathbb{1}\left\{\mathcal{CI}_i < \mathcal{CI}^{(m,n)}\right\} \cdot b_i}{\sum_{k=1}^{n} \mathbb{1}\left\{\mathcal{CI}_k < \mathcal{CI}^{(m,n)}\right\} \cdot b_k} \tag{32}$$

Remark 4. *Portfolio decarbonization requires the covariance matrix Σ to be estimated. A first approach is to use the empirical covariance matrix of stock returns. A second solution is to postulate a multivariate risk factor model:*

$$R_i(t) = \alpha_i + B\mathcal{F}(t) + \varepsilon_i(t) \tag{33}$$

where $\mathcal{F}(t)$ is the vector of risk factors and $\varepsilon_i(t)$ is the specific risk. By denoting $\Omega = \operatorname{var}\left(\mathcal{F}(t)\right)$ and $D = \operatorname{var}\left(\varepsilon_1(t), \ldots, \varepsilon_n(t)\right) = \operatorname{diag}\left(\tilde{\sigma}_1^2, \ldots, \tilde{\sigma}_n^2\right)$, we obtain $\Sigma = B\Omega B^{\top} + D$.

Let us illustrate the difference between the three approaches. The benchmark corresponds to the S&P 500 index at the end of October 2021. To estimate the covariance matrix, we use the single-factor CAPM model and the historical prices of the last three years. In Figure 12, we report the tracking error volatility $\sigma\left(x \mid b\right)$ with respect to the reduction rate $\mathcal{R}$ when we consider scope $1+2$. For example, we notice that reducing the carbon intensity of 50% leads to a tracking error volatility of 12% for the max-threshold solution. For the naive solution, the tracking error volatility is equal to 20%. We observe some discrepancies between the three approaches[26]. In particular, the order-statistic and naive solutions are satisfying only when the reduction is small, e.g. less than 30%. We also notice that the slope of the tracking error risk is steeper when the reduction is high (see Figure 13). For scope 1 or $1+2$, the slope changes when $\mathcal{R} \approx 50\%$. Today, most asset managers use scope $1+2$ to decarbonize their portfolios because the data are more robust than scope $1+2+3$. Nevertheless, there is pressure from ESG investors to increasingly consider scope $1+2+3$. In this case, the changing point is lower and equal to $\mathcal{R} \approx 30\%$. In Figure 14, we show the overlap[27] between optimal decarbonization portfolios. We notice that the portfolio composition is very different if we consider scope $1+2$ or scope $1+2+3$. Indeed, when the reduction is greater than 50%, the overlap is less than 90%. Considering the preference of ESG investors for scope $1+2+3$, we can anticipate large portfolio rebalancing in the future.

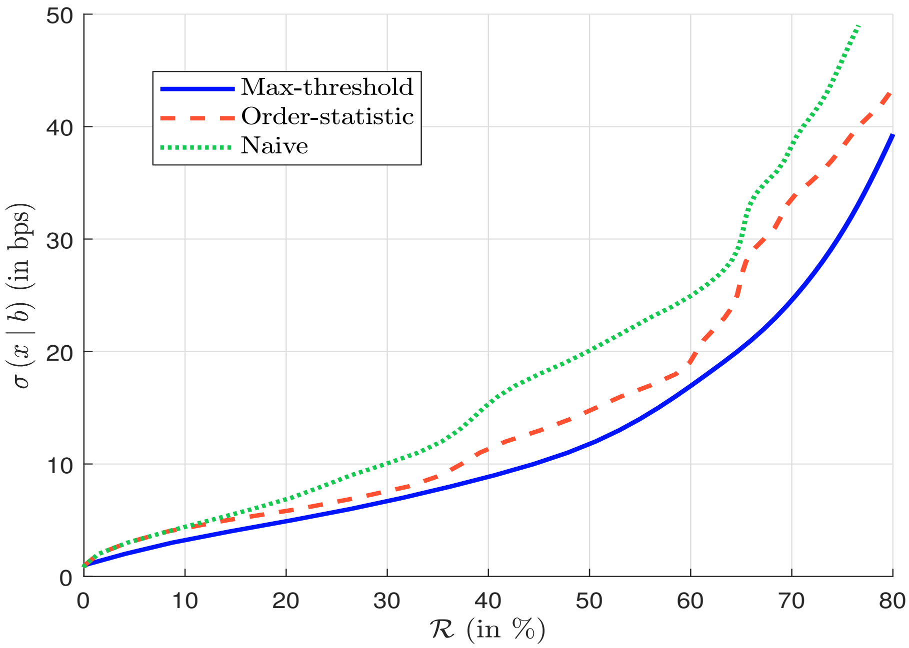
[26] This is particularly true when the carbon intensity corresponds to scope $1+2+3$ (see Figure 25 on page 49).

[27] The overlap measure between two portfolios x and y is defined as:

$$\operatorname{overlap}(x, y) = 1 - \frac{1}{2}\sum |x_i - y_i| \tag{34}$$

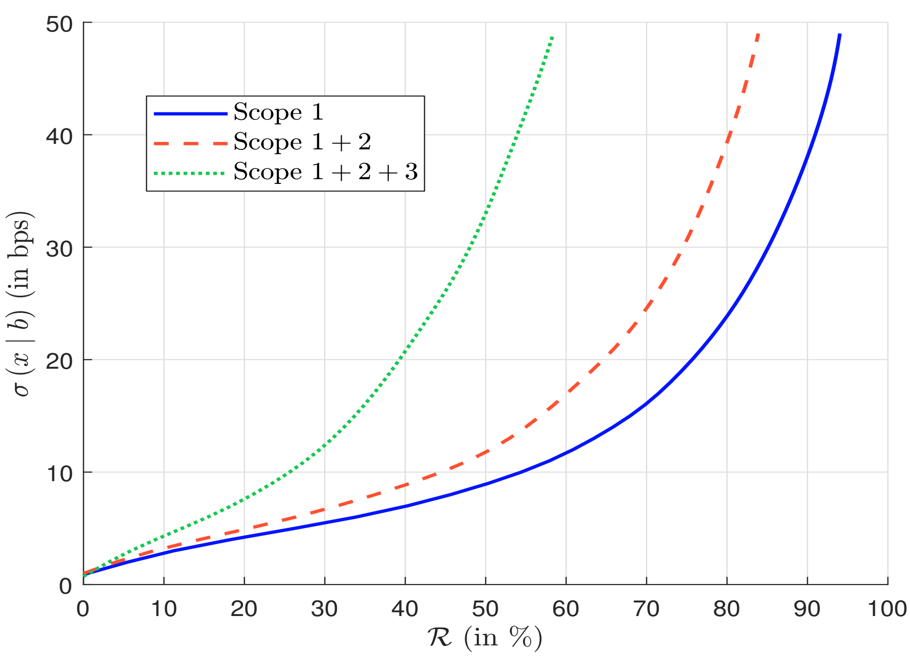
It is equal to 100% if the two portfolios are the same and 0% if the two portfolios have no common trading positions.

Figure 12: Efficient frontier of optimal decarbonization portfolios (S&P 500 index, October 2021, scope $1+2$)



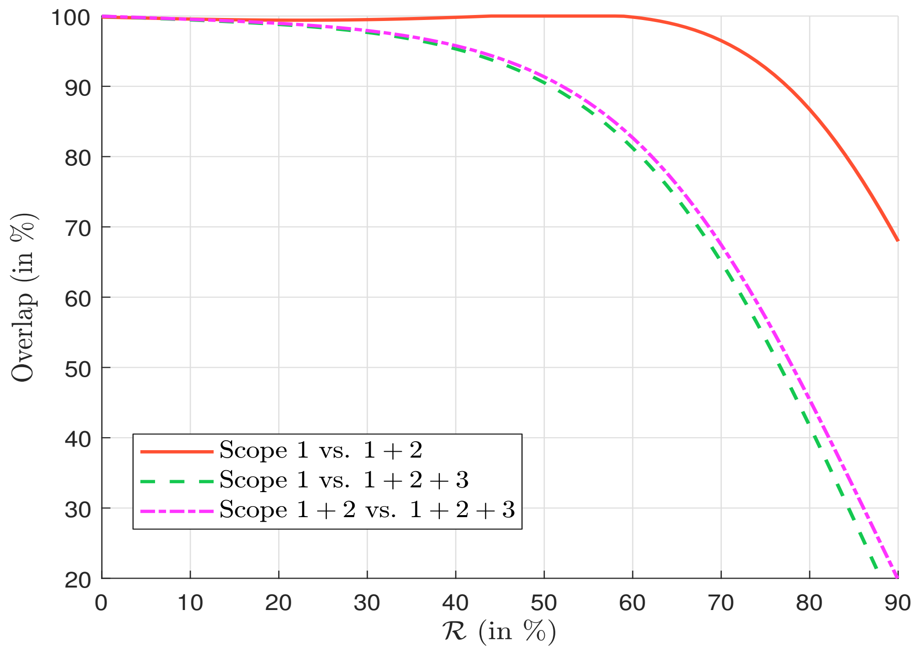
Source: Trucost reporting year 2020 & Authors' calculations.

Figure 13: Impact of the carbon scope on the tracking error volatility (S&P 500 index, October 2021)



Source: Trucost reporting year 2020 & Authors' calculations.

Figure 14: Overlap of optimal decarbonization portfolios (S&P 500 index, October 2021)



Source: Trucost reporting year 2020 & Authors' calculations.

3.2.2 Using carbon emissions

Let us consider an investor who has a nominal exposure W_i to the stock i. His carbon emissions contribution is equal to:

$$\boldsymbol{\mathcal{CEC}}_i\left(W_i\right) = \frac{W_i \mathcal{FP}_i}{\mathcal{MC}_i} \boldsymbol{\mathcal{CE}}_i \tag{35}$$

where $\boldsymbol{\mathcal{CE}}_i$ is the carbon emissions of the company i, $\mathcal{FP}_i$ is the float percentage associated with the stock i and $\mathcal{MC}_i$ is the free-float market capitalization. If we consider a portfolio $x = (x_1, \ldots, x_n)$, we cannot directly compute its carbon emissions contribution by averaging the carbon emissions:

$$\boldsymbol{\mathcal{CE}}\left(x\right) \neq \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{CE}}_i \tag{36}$$

Indeed, this formula does not depend on the nominal value W of the portfolio. Therefore, the correct formula is:

$$\begin{aligned} \boldsymbol{\mathcal{CE}}\left(x; W\right) &= \boldsymbol{\mathcal{CEC}}\left(x; W\right) \\ &= \sum_{i=1}^{n} \frac{W_i \mathcal{FP}_i}{\mathcal{MC}_i} \boldsymbol{\mathcal{CE}}_i \end{aligned} \tag{37}$$

where $W_i = x_i W$. We notice that the carbon emission $\boldsymbol{\mathcal{CE}}\left(x\right)$ of the portfolio is in fact the carbon emission contribution $\boldsymbol{\mathcal{CEC}}\left(x\right)$.

As explained on page 8, we can compute the carbon intensity of this investment as:

$$\boldsymbol{\mathcal{CI}}^{\text{exact}}\left(x\right) = \frac{\boldsymbol{\mathcal{CE}}\left(x; W\right)}{Y\left(x; W\right)} \tag{38}$$

where:

$$Y(x;W) = \sum_{i=1}^{n} \frac{W_i \mathcal{FP}_i}{\mathcal{MC}_i} Y_i \tag{39}$$

Finally, we obtain[28]:

$$\mathcal{CI}^{\text{exact}}(x) = \frac{\sum_{i=1}^{n} \omega_i \mathcal{CE}_i}{\sum_{i=1}^{n} \omega_i Y_i} \tag{40}$$

where:

$$\omega_i = \frac{x_i \mathcal{FP}_i}{\mathcal{MC}_i} \tag{41}$$

Formula (40) differs from the direct computation:

$$\mathcal{CI}^{\text{direct}}(x) = \sum_{i=1}^{n} x_i \mathcal{CI}_i \tag{42}$$

Remark 5. *If we assume that $\mathcal{FP}_i = 100\%$, ω_i is constant for a market-cap portfolio because $x_i \propto \mathcal{MC}_i^{-1}$. Therefore, the exact value of the carbon intensity reduces to:*

$$\mathcal{CI}^{\text{exact}}(x) = \frac{\sum_{i=1}^{n} \mathcal{CE}_i}{\sum_{i=1}^{n} Y_i} \tag{43}$$

Let us illustrate the discrepancy between the exact and direct computation. In Table 7, we report the $\mathcal{CE}(x;W)$ of three S&P 500 index portfolios. The first one corresponds to the total value of the S&P 500 index as if the investor buys the 500 companies that make up the index. Therefore, the carbon emissions of the S&P 500 index are respectively equal to 1.66 $GtCO_2e$, 2.01 $GtCO_2e$ and 3.75 $GtCO_2e$ for the three scopes. If we consider an investment of $1 bn, these figures become 10.5 $ktCO_2e$, 48.9 $ktCO_2e$ and 91.4 $ktCO_2e$. For an S&P 500 index portfolio whose value is $5 bn, we multiply these numbers by a factor of five. If we compute the carbon intensity, we notice that it does not depend on the portfolio notional even if we use the exact approach. We also remark that the direct computation of the carbon intensity is less than the exact computation. For instance, we obtain 161.7 vs. 129.8 tCO_2e/$ mn for scope $1+2$. As explained before, there is no ordering between these two measures. For instance, in Figure 15, we simulate portfolios of size n by randomly choosing n stocks from the S&P 500 index and drawing their weights from the uniform distribution. For each simulated portfolio x, we report the exact and direct values of $\mathcal{CI}(x)$. We verify that the direct value can be less than or greater than the exact value. However, when we fix n to 500, the direct value is systematically lower than the exact value.

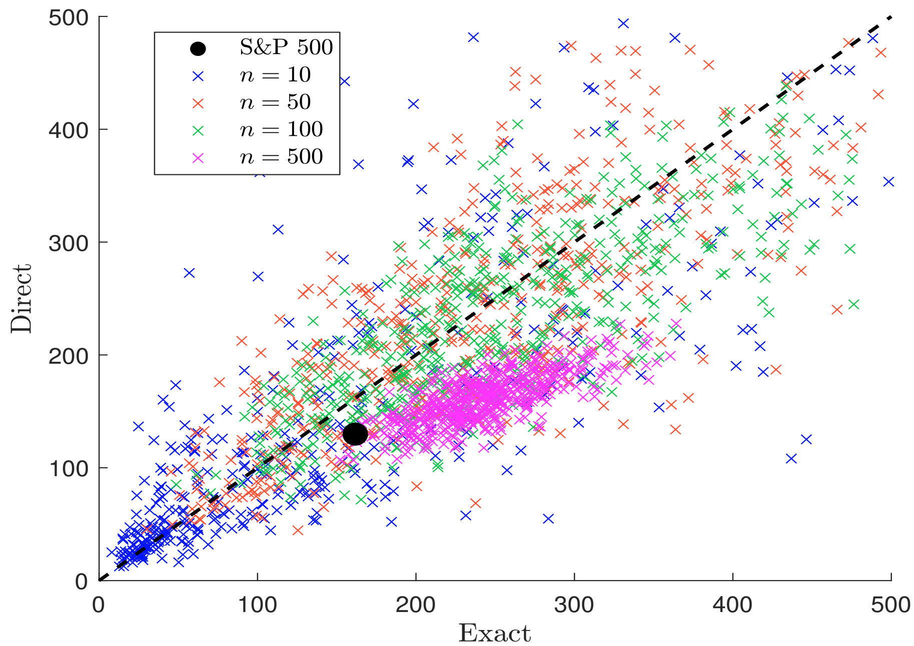
Table 7: Carbon emission (in tCO_2e) and intensity (tCO_2e/$ mn) of S&P 500 index portfolios

Scope	$\mathcal{CE}(x;W)$			$\mathcal{CI}(x)$	
	S&P 500	$1 bn	$5 bn	Exact	Direct
1	1.66×10^9	40.5×10^3	202.5×10^3	133.8	99.2
$1+2$	2.01×10^9	48.9×10^3	244.7×10^3	161.7	129.8
$1+2+3$	3.75×10^9	91.4×10^3	457.2×10^3	302.1	245.2

Source: Trucost reporting year 2020 & Authors' calculations.

[28] We notice that the variable W vanishes.

Figure 15: Exact vs. direct computation of scope 1 + 2 carbon intensity (S&P 500 index, October 2021)



Source: Trucost reporting year 2020 & Authors' calculations.

For the last year, we have observed a trend from some investors that prefer to decarbonize their portfolios using carbon emissions and not carbon intensities. This is equivalent to defining the carbon constraint as $\boldsymbol{\mathcal{CE}}(x;W) \leq \boldsymbol{\mathcal{CE}}^{+}$ (or $\boldsymbol{\mathcal{CE}}(x;W) \leq (1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CE}}(b;W)$). We have:

$$\begin{aligned}\boldsymbol{\mathcal{CE}}(x;W) \leq \boldsymbol{\mathcal{CE}}^{+} \quad &\Leftrightarrow \quad \sum_{i=1}^{n} \frac{W_i \mathcal{FP}_i}{\mathcal{MC}_i} \boldsymbol{\mathcal{CE}}_i \leq \boldsymbol{\mathcal{CE}}^{+} \\ &= \quad \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{CI}}_i^{\mathcal{MV}} \leq \frac{\boldsymbol{\mathcal{CE}}^{+}}{W} \end{aligned} \tag{44}$$

where $\boldsymbol{\mathcal{CI}}_i^{\mathcal{MV}}$ is a carbon intensity measure normalized by the market value $\mathcal{MV}_i$ of the issuer i:

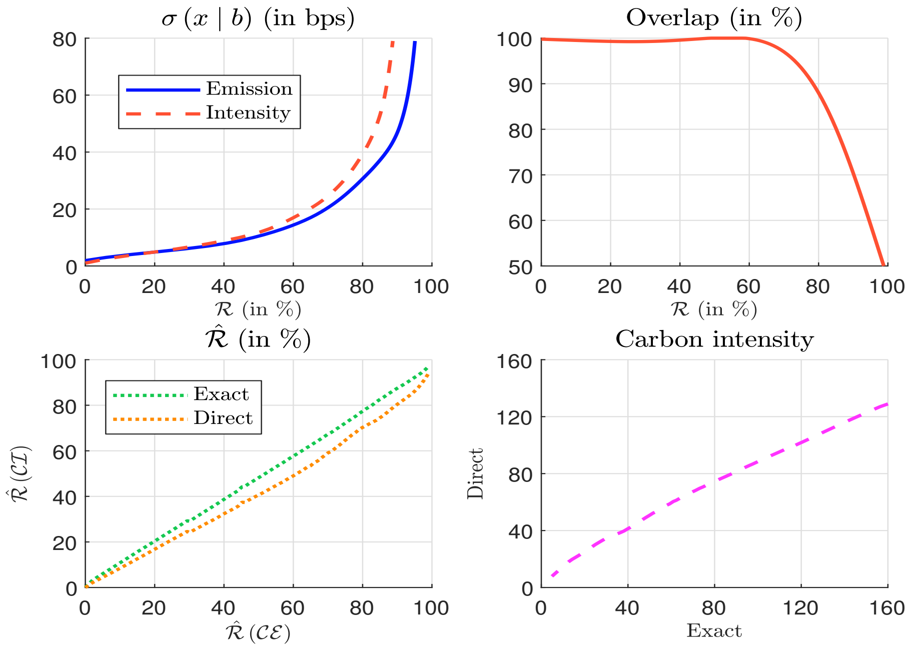
$$\boldsymbol{\mathcal{CI}}_i^{\mathcal{MV}} = \frac{\mathcal{FP}_i}{\mathcal{MC}_i} \boldsymbol{\mathcal{CE}}_i = \frac{\boldsymbol{\mathcal{CE}}_i}{\mathcal{MV}_i} \tag{45}$$

and $\mathcal{MV}_i = \mathcal{MC}_i / \mathcal{FP}_i$. Therefore, Problem (29) becomes:

$$\begin{aligned} x^{\star}(\boldsymbol{\mathcal{R}}) \quad &= \quad \arg\min \frac{1}{2} (x-b)^{\top} \Sigma (x-b) \\ &\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_n^{\top} x = 1 \\ x \geq \mathbf{0}_n \\ \sum_{i=1}^{n} x_i \boldsymbol{\mathcal{CI}}_i^{\mathcal{MV}} \leq (1-\boldsymbol{\mathcal{R}}) \sum_{i=1}^{n} b_i \boldsymbol{\mathcal{CI}}_i^{\mathcal{MV}} \end{array} \right. \end{aligned} \tag{46}$$

Again, we obtain a quadratic program, which is appealing because it is very efficient to solve from a computational point of view (Perrin and Roncalli, 2020).

Figure 16: Portfolio decarbonization with carbon emissions (S&P 500 index, October 2021, scope $1 + 2$)



Source: Trucost reporting year 2020 & Authors' calculations.

Figure 16 shows the results of this portfolio decarbonization program. The first top-left panel compares the tracking errors when targeting carbon emission or carbon intensity reduction. We obtain higher tracking errors in the last case. Indeed, since absolute emissions are concentrated on few issuers, it is possible to reduce carbon emissions by reducing the allocation on these issuers, whereas the distribution of carbon intensities is less skewed, implying that the constraint potentially impacts the weights of more issuers. The overlap of the two optimized portfolios is given in the second top-right panel. We notice that we obtain similar holdings, but the overlap decreases dramatically when the reduction is greater than 80%. In the bottom-left panel, we compute the emission and intensity reduction levels $\hat{\boldsymbol{\mathcal{R}}}\left(\boldsymbol{\mathcal{CE}}\right) = 1 - \boldsymbol{\mathcal{CE}}\left(x^{\star}\right) / \boldsymbol{\mathcal{CE}}\left(b\right)$ and $\hat{\boldsymbol{\mathcal{R}}}\left(\boldsymbol{\mathcal{CI}}\right) = 1 - \boldsymbol{\mathcal{CI}}\left(x^{\star}\right) / \boldsymbol{\mathcal{CI}}\left(b\right)$ using the direct and exact formulas. Finally, the fourth bottom-right panel shows that the carbon intensity is underestimated when we consider the direct formula.

3.3 Portfolio alignment

While portfolio decarbonization is a static problem, portfolio alignment implies a dynamic strategy in order to comply with a given climate policy. Therefore, the dynamic problem is trickier since it involves several rebalancing decisions and depends on the future behavior of corporate issuers. Two main climate policies are generally used: the Paris-aligned benchmarks (PAB) approach and the net zero emissions (NZE) scenario for 2050.

3.3.1 Paris-aligned benchmarks

The goal of the Paris agreement on climate change is to limit global warming to well below 2°C. In this context, the EU Technical Expert Group on sustainable finance (hereafter

TEG) has proposed to create two climate benchmark labels[29]: climate transition benchmark (CTB) and Paris-aligned benchmark (PAB). These labels are structured along the following common principles:

1. A year-on-year self-decarbonization of 7% on average per annum, based on scope 1, 2 and 3 emissions;
2. A minimum carbon intensity reduction $\boldsymbol{\mathcal{R}}^{-}$ compared to the investable universe;
3. A minimum exposure to sectors highly exposed to climate change.

For the CTB label, the minimum reduction $\boldsymbol{\mathcal{R}}^{-}$ is set to 30% whereas it is equal to 50% for the PAB label. Other constraints are also imposed such as issuer exclusions (controversial weapons and societal norms violators), a minimum green share revenue or some activity exclusions. Nevertheless, in what follows, we focus on the three principles that correspond to the main constraints.

Decarbonization pathway Let t_0 be the base date of the climate benchmark. The initial carbon intensity reduction must be equal to $\boldsymbol{\mathcal{R}}^{-}$ (30% for CTB and 50% for PAB). Then, the carbon intensity must decrease by 7% every year. The minimum reduction $\boldsymbol{\mathcal{R}}(t_0, t)$ of the carbon intensity between the current date t and the base date t_0 is then equal to:

$$\boldsymbol{\mathcal{R}}(t_0, t) = 1 - \left(1 - 7\%\right)^{t-t_0} \cdot \left(1 - \boldsymbol{\mathcal{R}}^{-}\right) \tag{47}$$

Figure 17 shows this decarbonization pathway for CTB and PAB indexes when the base year is 2021. We observe that the difference between the two trajectories mainly concerns the start of the period, because the two reductions converge when t is sufficiently large ($t > 2040$).

At date t, the CTB and PAB labels impose the following inequality constraint for the portfolio $x(t)$:

$$\boldsymbol{\mathcal{CI}}\left(x(t)\right) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(b(t_0)\right) \tag{48}$$

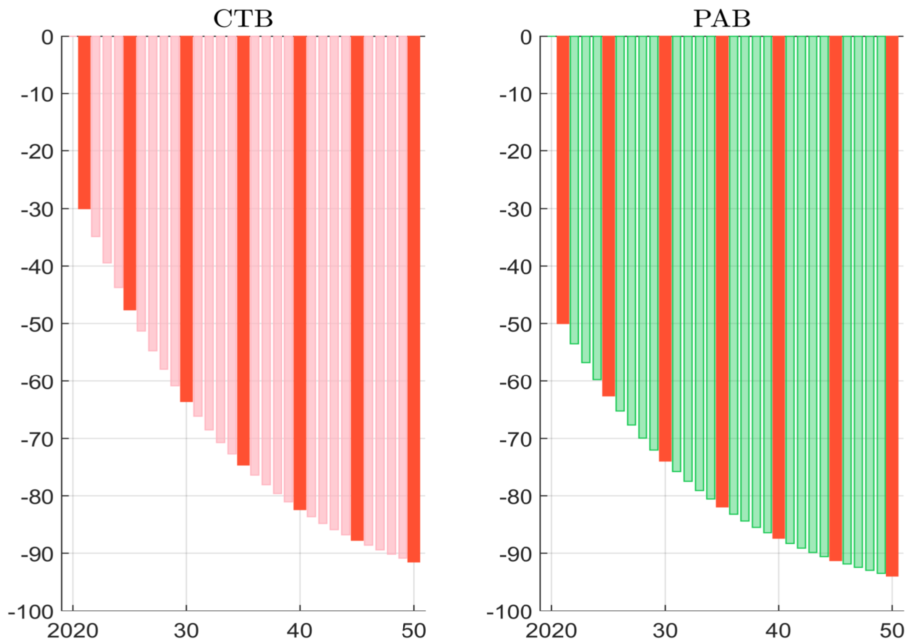
We notice the importance of the base year t_0 since it defines the reference level of the carbon intensity. Indeed, the reference level is equal to $\boldsymbol{\mathcal{CI}}\left(b(t_0)\right)$ and not $\boldsymbol{\mathcal{CI}}\left(b(t)\right)$. If we collectively and significantly reduce the carbon emissions and the corporations make effort to fight climate change, we can assume that $\boldsymbol{\mathcal{CI}}\left(b(t)\right) \leq \boldsymbol{\mathcal{CI}}\left(b(t_0)\right)$, meaning that the carbon intensity of the traditional benchmark will be reduced. Let T be a reference date for the climate scenario (T may be equal to 2030 or 2050), we must observe that $\boldsymbol{\mathcal{CI}}\left(b(T)\right) \ll \boldsymbol{\mathcal{CI}}\left(b(t_0)\right)$ and even $\boldsymbol{\mathcal{CI}}\left(x(T)\right) \approx \boldsymbol{\mathcal{CI}}\left(b(T)\right)$. Otherwise, this implies that we will be unsuccessful in fighting climate change, reducing GHG emissions and respecting the Paris agreement.

Climate impact sector The CTB and PAB labels require that the exposure to sectors highly exposed to climate change is at least equal to the exposure in the investment universe. TEG (2019a) distinguishes two types of sectors: (1) High climate impact sectors (HCIS or $\mathcal{CIS}_{\mathcal{High}}$), (2) Low climate impact sectors (LCIS or $\mathcal{CIS}_{\mathcal{Low}}$). The first category is made up of sectors that are key to the low-carbon transition. They correspond to the following NACE classes[30]: A. Agriculture, Forestry, and Fishing; B. Mining and Quarrying;

[29] According to the TEG (2019a), "*a climate benchmark is defined as an investment benchmark that incorporates — next to financial investment objectives — specific objectives related to greenhouse gas (GHG) emission reductions and the transition to a low-carbon economy through the selection and weighting of underlying benchmark constituents*".

[30] NACE is the European Union's classification of economic activities. It is made up of 21 classes.

Figure 17: Decarbonization pathway of CTB and PAB labels



C. Manufacturing; D. Electricity, Gas, Steam, and Air Conditioning Supply; E. Water Supply; Sewerage, Waste Management, and Remediation Activities; F. Construction; G. Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles; H. Transportation and Storage; L. Real Estate Activities. Let $\mathcal{CIS}_{\mathcal{H}igh}(x) = \sum_{i \in \mathcal{CIS}_{\mathcal{H}igh}} x_i$ be the HCIS weight of Portfolio x. At each rebalancing date t, we must verify that:

$$\mathcal{CIS}_{\mathcal{H}igh}\left(x(t)\right) \geq \varphi_{\mathcal{CIS}} \cdot \mathcal{CIS}_{\mathcal{H}igh}\left(b(t)\right) \tag{49}$$

where $\varphi_{\mathcal{CIS}} = 1$.

TEG (2019b, Appendix B, pages 26-170) has published a mapping between the NACE classes and several sector classification structures: BICS (Bloomberg), GICS (MSCI and S&P), ICB (FTSE) and TRBC (Refinitiv). In the case of the GICS, there are 129 sub-industries out of a total of 185 that are classified as high climate impact sectors. This represents 69.73% of the sub-industries. This classification has been criticized because this figure is very high[31]. This would mean that almost all activities are critical for building a low-carbon economy. Therefore, only two sectors are classified in low climate impact sectors (Communication Services and Financials), but more than half of the Health Care and Information Technology sub-industries are viewed as high climate impact sectors. The original idea of the CIS constraint was to continue financing the sectors that are essential for reaching a low-carbon economy (e.g. Energy and Utilities) and at the same time promoting investments in green issuers instead of brown issuers in these sectors. Nevertheless, the constraint (49) is not very restrictive with this broad HCIS measure. Moreover, this constraint encourages substitutions between sectors or industries and not substitutions between issuers within a same sector. Therefore, the trade-off is not necessarily between green electricity and brown electricity, but for example between electricity generation and health care equipment.

[31] Certainly too high to be credible.

Besides the broad HCIS measure, we propose a narrow measure, which is more restrictive and focuses on a small number of sub-industries. The mapping between the NACE classes and the GICS sectors is given in Table 13 on page 44. We notice that the narrow HCIS measure concerns four main sectors: Energy, Industrials, Utilities and Real Estate. This scope is complemented by one industry group (Transportation) and four industries (Food Products, Metals & Mining, Construction Materials, Food & Staples Retailing). To compare the narrow and broad measures, we report in Table 8 the weights and the carbon intensity (scope 1+2+3) of high climate impact sectors when the investment universe is the S&P 500 index. Since the carbon intensity of the S&P 500 is equal to 245 tCO_2e/\$ mn, it is equal to 380 tCO_2e/\$ mn for the sectors of the broad HCIS classification. We verify that the broad HCIS universe has a higher carbon intensity than the reference universe. Nevertheless, the broad HCIS universe represents 54.59% of the S&P 500 index, which is a very high figure. This means that more than half of the S&P 500 index has a high climate impact. If we consider the narrow HCIS universe, the weighted average carbon intensity is equal to 681 tCO_2e/\$ mn, which is 80% larger than the carbon intensity of the broad HCIS universe. Moreover, the narrow HCIS universe represents less than 20% of the S&P 500 index. These figures are more consistent than those obtained for the broad measures. Indeed, it is difficult to justify that 69% of the Health Care sector, 48% of the Information Technology but 0% of the Financial sector are classified as high climate impact sectors when we consider the broad HCIS measure.

Table 8: Weights and carbon intensity of high climate impact sectors

Sector	S&P 500		Narrow HCIS		Broad HCIS	
	b_s	$\mathcal{CI}_s$	b_s	$\mathcal{CI}_s$	b_s	$\mathcal{CI}_s$
Communication Services	10.89%	80				
Consumer Discretionary	13.57%	190			10.22%	185
Consumer Staples	6.10%	355	2.73%	348	6.10%	355
Energy	2.81%	790	2.81%	790	2.81%	790
Financials	11.13%	67				
Health Care	12.74%	126			8.56%	152
Industrials	7.97%	330	7.97%	330	6.32%	368
Information Technology	27.50%	99			13.30%	139
Materials	2.45%	966	0.44%	850	2.45%	966
Real Estate	2.55%	198	2.55%	198	2.55%	198
Utilities	2.30%	2 669	2.30%	2 669	2.30%	2 669
Total	100.00%	245	18.79%	681	54.59%	380

One-period optimization problem The optimization program is an extension of Problem (29) where we introduce the two inequality constraints (48) and (49):

$$x^{\star}(t) = \arg\min_{x(t)} \frac{1}{2}\sigma^2\left(x(t) \mid b(t)\right) + \lambda\tau\left(x(t) \mid x^{\star}(t-1)\right) \tag{50}$$

$$\text{s.t.} \quad \begin{cases} \mathbf{1}_n^\top x(t) = 1 \\ x(t) \geq \mathbf{0}_n \\ \boldsymbol{\mathcal{CI}}\left(x(t)\right) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}\left(b(t_0)\right) \\ \mathcal{CIS}_{\mathcal{H}igh}\left(x(t)\right) \geq \varphi_{\mathcal{CIS}} \cdot \mathcal{CIS}_{\mathcal{H}igh}\left(b(t)\right) \\ \left|\sum_{i \in \mathcal{S}ector_j} x_i(t) - \sum_{i \in \mathcal{S}ector_j} b_i(t)\right| \leq \delta_j \end{cases}$$

where $\lambda \geq 0$. The objective function depends on the tracking error risk:

$$\sigma\left(x(t) \mid b(t)\right) = \sqrt{\left(x(t) - b(t)\right)^\top \Sigma(t) \left(x(t) - b(t)\right)} \tag{51}$$

Since we face a dynamic process, it is useful to control the one-way turnover of the portfolio between $t-1$ and t defined as:

$$\tau\left(x(t) \mid x^\star(t-1)\right) = \frac{1}{2} \left\|x(t) - x^\star(t-1)\right\|_1 \tag{52}$$

We can impose a maximum turnover constraint $\tau\left(x(t) \mid x^\star(t-1)\right) \leq \tau^+$. Nevertheless, the convergence toward an optimal solution is not guaranteed. Therefore, we prefer to introduce the turnover in the objective function and control the turnover aversion with the coefficient λ. For instance, we minimize the tracking error risk if λ is set to 0 and the turnover risk if λ is set to $+\infty$. Otherwise, λ measures the trade-off between tracking error and turnover risks. We can also impose some limits concerning sector deviation. This is the purpose of the fifth constraint.

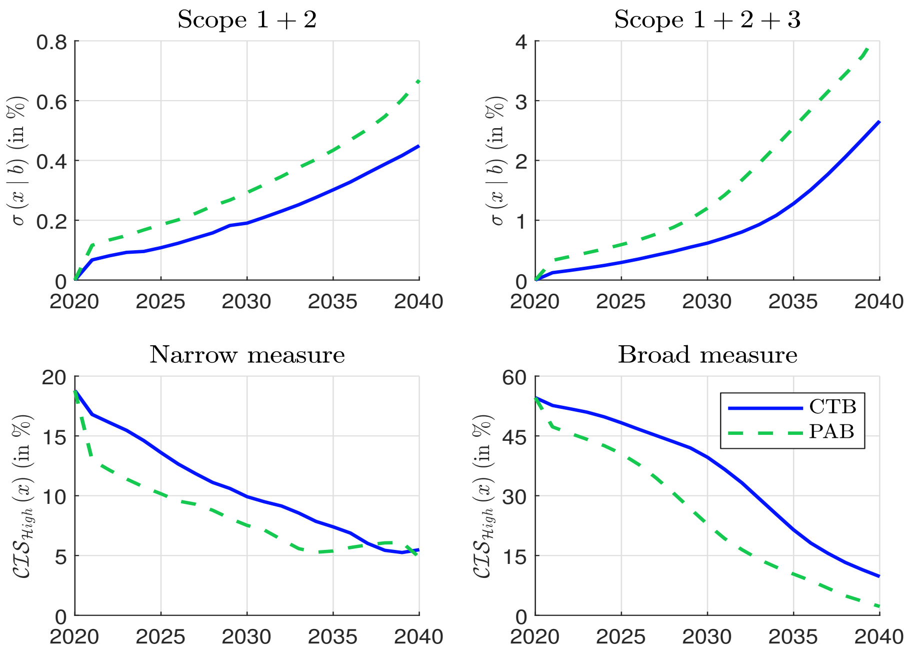
We notice that the computation of $x^\star(t)$ at time t requires us to know the covariance matrix $\Sigma(t)$, the carbon intensities $\boldsymbol{\mathcal{CI}}_i(t)$, the investable universe $b(t)$ and the previous optimized portfolio $x^\star(t-1)$. At the current year t_1, these observations are only available for the dates between t_0 and t_1. Therefore, it is impossible to compute $x^\star(t_1+1)$, $x^\star(t_1+2)$, etc. Nevertheless, we can do an exercise by assuming that the world does not change. In this case, the covariance matrix of asset returns, the carbon intensities of issuers and the investable universe remain constant, implying that $\Sigma(t) = \Sigma(t_0)$, $\boldsymbol{\mathcal{CI}}_i(t) = \boldsymbol{\mathcal{CI}}_i(t_0)$ and $b(t) = b(t_0)$. The results of the sequential optimization process starting from the base year $t_0 = 2021$ and ending at the target date $T = 2040$ are given in Figures 18 and 19, and Table 9 when the turnover aversion coefficient λ is set to zero. We can make a number of observations. First, we have optimized the portfolios by only considering the decarbonization pathway of CTB and PAB labels in Figure 18. The difference between the top-left and top-right panels is the choice of the scope. The top-left panel presents the tracking error risk $\sigma\left(x(t) \mid b(t)\right)$ of optimized portfolios when we consider scope $1+2$. When we include scope 3, we obtain the top-right panel. We observe that the magnitude is very different. Indeed, the tracking error risk is less than 80 bps for $t \leq 2040$ when we consider scope $1+2$, whereas it can reach 4% for the PAB label when we include scope 3. In the sequel, we only consider this last case since CTB and PAB labels require all CO_2 emissions to be covered, and in particular upstream and downstream emissions. In this context, we have computed the HCIS weight $\mathcal{CIS}_{\mathcal{H}igh}\left(x^\star(t)\right)$ of scope $1+2+3$ optimized portfolios. We verify that the exposure to high climate impact sectors decreases over time when we impose a higher carbon emission reduction. For example, $\mathcal{CIS}_{\mathcal{H}igh}\left(x^\star(2040)\right)$ is equal to 5.49% (narrow) and 9.73% (broad) for the CTB label[32], whereas it is equal to 18.79% (narrow) and 54.59% (broad) for the current investable universe (see Table 8 on page 31). Therefore, we introduce the HCIS constraint and obtain the results given in Figure 19. We observe that the constraint using the broad HCIS universe is tighter than the constraint using the narrow HCIS universe. Indeed, the tracking error risks are similar with or without the narrow HCIS universe[33]. This is not the case when we compare the tracking error risks obtained with or without the broad HCIS universe[34]. For the CTB label, $\sigma\left(x^\star(2040) \mid b(2040)\right)$ is respectively equal to 2.66% when we only consider the carbon emissions reduction, 2.83% when we add the narrow HCIS constraint and 4.43% when we consider the broad HCIS constraint (see Table 9). The

[32] For the PAB label, $\mathcal{CIS}_{\mathcal{H}igh}\left(x^\star(2040)\right)$ is equal to 4.86% (narrow) and 2.23% (broad).

[33] We have to compare the top-right panel in Figure 18 and the top-left panel in Figure 19.

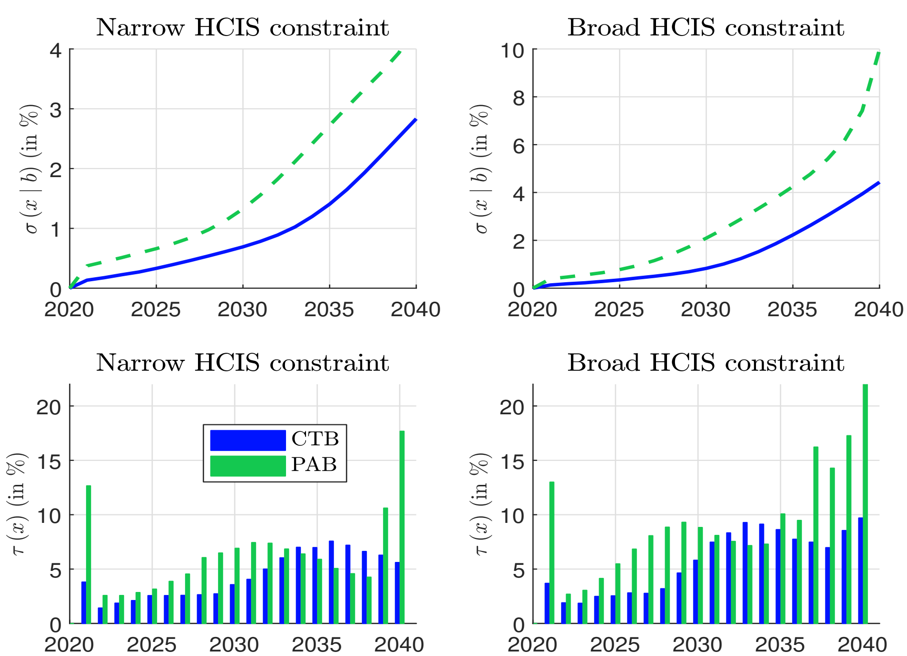
[34] We have to compare the top-right panel in Figure 18 and the top-right panel in Figure 19.

Figure 18: The impact of scope 3 on CTB and PAB labels



Source: Trucost reporting year 2020 & Authors' calculations.

Figure 19: The impact of the HCIS constraint on CTB and PAB labels



Source: Trucost reporting year 2020 & Authors' calculations.

figures become 4.19%, 4.43% and 9.97% for the PAB label. Therefore, the introduction of the HCIS constraint is not neutral. This is particularly true for the broad universe in the medium term ($t > 2025$) and the narrow measure in the long term ($t > 2040$). In Figure 19, we have also reported the one-way turnover $\tau\left(x^{\star}(t) \mid x^{\star}(t-1)\right)$. We observe a high turnover at the beginning of the period[35] and at the end of the period. In the last case, the carbon reductions are very high, and we obtain concentrated portfolios that are far to be diversified. In order to illustrate this lack of diversification, we compute the effective number of bets which is the inverse of the Herfindahl index $\mathcal{H}(x) = \sum_{i=1}^{n} x_i^2$. It is currently equal to 70.56 for the S&P 500 index, meaning the S&P 500 index portfolio is equivalent to an equally-weighted portfolio of 70.56 stocks (see Table 10). Until 2030, the effective number of bets of optimized portfolios is relatively close to this of the S&P 500 index and greater than 65. Nevertheless, we observe a drop in terms of diversification in 2040. For instance, it is equal to 5.48 for the PAB label with the broad HCIS constraint.

Table 9: Tracking error risk of CTB and PAB labels

Year	CTB			PAB		
	Scope 3	Narrow	Broad	Scope 3	Narrow	Broad
2021	0.12%	0.13%	0.13%	0.33%	0.37%	0.39%
2022	0.16%	0.18%	0.19%	0.39%	0.44%	0.47%
2023	0.20%	0.23%	0.23%	0.46%	0.51%	0.56%
2024	0.24%	0.27%	0.28%	0.52%	0.59%	0.65%
2025	0.29%	0.33%	0.35%	0.59%	0.66%	0.78%
2030	0.62%	0.69%	0.83%	1.20%	1.33%	2.09%
2035	1.28%	1.40%	2.23%	2.55%	2.72%	4.25%
2040	2.66%	2.83%	4.43%	4.19%	4.43%	9.97%

Table 10: Effective number of bets

Year	CTB			PAB		
	Scope 3	Narrow	Broad	Scope 3	Narrow	Broad
2020	70.56	70.56	70.56	70.56	70.56	70.56
2021	69.95	70.29	70.15	68.37	69.30	68.78
2022	69.77	70.24	70.53	68.06	68.93	68.30
2023	69.48	70.22	69.88	67.59	68.43	67.95
2024	69.07	69.73	69.68	67.02	68.12	67.11
2025	68.66	69.52	69.08	66.58	67.46	66.75
2030	66.26	67.24	66.42	66.64	68.82	66.85
2035	67.36	69.61	66.15	76.35	76.42	44.72
2040	76.26	75.67	41.45	49.97	42.61	5.48

Remark 6. *In our implementation, the CTB and PAB labels only differ because of the minimum reduction $\boldsymbol{\mathcal{R}}^{-}$. In this case, we can show that the CTB approach is lagging behind the PAB approach by 4.64 years*[36]*. This explains the difference between the two labels.*

Remark 7. *In practice, the ex-post tracking error risk will be lower than those computed in this section, in particular when t is greater than 2025. The reason is that we assume that*

[35] Because of the implementation of the minimum reduction $\boldsymbol{\mathcal{R}}^{-}$.
[36] See Appendix A.4.2 on page 46.

the future investable universe $b(t)$ is equal to the current investable universe $b(t_0)$. In fact, we expect that the market-cap benchmark will follow a decarbonization pathway at a level, which will be certainly lower than the CTB and PAB labels (e.g. $2-4\%$ instead of the 7% objective).

Remark 8. *Our empirical results are very different from those obtained by Bolton et al. (2021). Indeed, these authors found that the tracking error of optimized portfolios is less than 2% by 2050. In fact, their calculations use "scope $1+2$, and only scope 3 upstream first-tier data, which only covers the direct supply chain", whereas our computation is based on the total sum of carbon scopes and includes full indirect upstream emissions. On the other hand, "first-tier indirect emissions are defined as GHG Protocol scope 2 emissions, plus the company's first-tier upstream supply chain — their direct suppliers". Therefore, they omitted a significant part of the upstream emissions, in particular as we get close to the final product. This makes a big difference. On pages 50 and 51, we report the tracking error of CTB and PAB labels when we consider scope $1+2$ and then add scope 3 direct upstream[37] or the full scope 3 (see Figures 26, 27 and 28). By only considering scope 3 direct upstream emissions, the tracking error risk is approximately equal to the figure obtained with scope $1+2$. In this case, we confirm that the tracking error is less than 2% by 2050 for the CTB label. Nevertheless, the EU regulation considers the entire scope 3, and not only the direct upstream. Moreover, we also observe that adding the broad HCIS constraint is a supplementary source of tracking error risk. These results show that the cost for investors may be higher when we consider both scope 3 and the HCIS constraint. Nevertheless, as argued before, we can assume that the investable universe in ten, twenty or thirty years' time will be very different and more aligned than the current one.*

So far, we have illustrated the sequential optimization process when the turnover aversion risk λ is set to 0. In fact, the optimization process is static since we don't need to know the optimized portfolio $x^\star(t-1)$ to compute $x^\star(t)$. If $\lambda > 0$, the computation of $x^\star(t)$ depends on the past values $x^\star(t-1), x^\star(t-2), \ldots, x^\star(t_0)$ and the optimization process becomes sequential. In this case, it is a special case of the multi-period optimization problem.

Multi-period optimization problem In this framework, the optimization problem becomes:

$$
\begin{aligned}
x^\star(t) &= \underset{x(t),x(t+1),\ldots}{\arg\min} \sum_{s=t}^{T} e^{-\rho(s-t)} \left(\frac{1}{2}\sigma^2\left(x(s)\mid b(s)\right) + \lambda\tau\left(x(s)\mid x(s-1)\right)\right) \qquad (53) \\
&\text{s.t.} \left\{
\begin{array}{l}
\mathbf{1}_n^\top x(s) = 1 \\
x(s) \geq \mathbf{0}_n \\
\boldsymbol{\mathcal{CI}}\left(x(s)\right) \leq \left(1-\boldsymbol{\mathcal{R}}(t_0,s)\right)\cdot \boldsymbol{\mathcal{CI}}\left(b(t_0)\right) \\
\mathcal{CIS}_{\mathcal{H}igh}\left(x(s)\right) \geq \varphi_{\mathcal{CIS}} \cdot \mathcal{CIS}_{\mathcal{H}igh}\left(\tilde{b}(s)\right) \\
\left|\sum_{i\in\mathcal{S}ector_j} x_i(s) - \sum_{i\in\mathcal{S}ector_j} \tilde{b}_i(s)\right| \leq \delta_j \\
\sigma\left(x(s)\mid b(s)\right) = \sqrt{\left(x(s)-\tilde{b}(s)\right)^\top \tilde{\Sigma}(s)\left(x(s)-\tilde{b}(s)\right)} \\
\tau\left(x(s)\mid x(s-1)\right) = \dfrac{1}{2}\left\|x(s)-x(s-1)\right\|_1 \\
\boldsymbol{\mathcal{CI}}\left(x(s)\right) = \sum_{i=1}^n x_i(s)\,\widetilde{\boldsymbol{\mathcal{CI}}}_i(s)
\end{array}
\right.
\end{aligned}
$$

[37]To be more precise, we use scope 1 and first-tier indirect emissions. Trucost defines these latter as "*GHG Protocol scope 2 emissions, plus the company's first-tier upstream supply chain — their direct suppliers. The goal of this enhancement is to include some of the company's most relevant upstream scope 3 emissions, while limiting the extent of the double counting of emissions. The upstream supply chain of companies in Trucost's database is calculated using its EEIO model*".

where ρ is the discount rate. This problem is complex because three variables are stochastic: the benchmark $\tilde{b}(s)$, the covariance matrix $\tilde{\Sigma}(s)$ and the carbon intensities $\widetilde{\mathcal{CI}}_i(s)$. Therefore, we face an optimization problem with a stochastic objective function and some stochastic constraints. In order to solve this problem, the underlying idea is to integrate all the stochastic constraints into the objective function and minimize the mathematical expectation. Another approach consists in replacing the stochastic variables by their mathematical expectations: $b(s) = \mathbb{E}\left[\tilde{b}(s) \middle| \mathcal{F}_t\right]$, $\Sigma(s) = \mathbb{E}\left[\tilde{\Sigma}(s) \middle| \mathcal{F}_t\right]$ and $\mathcal{CI}_i(s) = \mathbb{E}\left[\widetilde{\mathcal{CI}}_i(s) \middle| \mathcal{F}_t\right]$ where $\mathcal{F}_t$ is the filtration.

Solving Problem (53) requires choosing the time horizon T. We recall that the objective is to smooth the allocation in order to reduce the turnover between two rebalancing dates. Therefore, it is not optimal to choose T far away from the current date t. For instance, it is better to set $T-t$ equal to two, three or five years. The underlying idea is to avoid some rebalancing decisions at time t that will not be consistent with the future rebalancing decisions at time $s \in [t+1,\ldots,T]$. This approach is appealing since we can also impose that[38] $b(s) \approx b(t)$, $\Sigma(s) \approx \Sigma(t)$ and $\mathcal{CI}_i(s) \approx \mathcal{CI}_i(t)$ for $s \in [t+1,\ldots,T]$. In this case, at the current date t, we have to estimate $n \times (T-t+1)$ variables — $x(t), x(t+1), \ldots, x(T)$ — where n is the number of assets in the investable universe. Therefore, the dimension of the optimization problem is relatively high. For instance, in the case of the S&P 500 index, we have to estimate $500 \times 5 = 2\,500$ variables if the current date t is 2021 and the time horizon T is 2025. Another difficulty lies in the treatment of the ℓ_1-penalty function, which corresponds to the turnover measure. A first method to numerically solve Problem (53) is to use the augmented variables technique (Bourgeron *et al.*, 2018; Perrin and Roncalli, 2020). In this case, we write $x(s+1) = x(s) + \Delta x^+(s) - \Delta x^-(s)$ where $\Delta x^+(s) \geq \mathbf{0}_n$ and $\Delta x^-(s) \geq \mathbf{0}_n$. The ℓ_1-penalty function becomes then:

$$\lambda \sum_{s=t}^{T} e^{-\rho(s-t)} \tau\left(x(s) \mid x(s-1)\right) = \frac{\lambda}{2} \sum_{s=t}^{T} e^{-\rho(s-t)} \left(\sum_{i=1}^{n} \Delta x_i^+(s) + \sum_{i=1}^{n} \Delta x_i^-(s)\right) \tag{54}$$

With this technique, Problem (53) remains a QP problem but with $3n \times (T-t+1)$ variables: $x(t), \Delta x^+(t), \Delta x^-(t), x(t+1), \Delta x^+(t+1), \Delta x^-(t+1) \ldots, x(T), \Delta x^+(T), \Delta x^-(T)$. If we consider the previous example, we have to estimate $3 \times 500 \times 5 = 7\,500$ variables if the current date t is 2021 and the time horizon T is 2025. Here, we face a dimensionality curse. For instance, this approach is not adapted for the MSCI World index. The second method is to solve Problem (53) by using the ADMM algorithm and the proximal operator of the ℓ_1-penalty function (Perrin and Roncalli, 2020). This method is very efficient since the x-step consists[39] in solving $(T-t+1)$ QP problems[40] of dimension n and the y-step is reduced to apply the soft-thresholding operator. In the case of the S&P 500 index, we need less than one second to obtain the solution when $T-t$ is set to five years. On average, the multi-period optimization problem helps to reduce the turnover of the single-period optimization problem by a factor of $15-20\%$.

Remark 9. *When $\lambda > 0$, Problem (50) is equivalent to Problem (53) by imposing that the time horizon T is equal to the current date t.*

So far, we have assumed that the carbon intensity is constant. We now consider an approach where we introduce the dynamics of carbon intensity for each issuer. In particular,

[38] We can also forecast the trend of the carbon intensity in order to estimate $\mathcal{CI}_i(s)$. We apply this approach in the next section.

[39] We notice that the function $f_x(x)$ is fully separable since there is no interaction between $x(s)$ and $x(s+1)$.

[40] These QP problems have the same form as Problem (50) where λ is equal to zero.

the underlying idea is to penalize issuers who do not make any effort to reduce their carbon footprint.

3.3.2 Net zero objective

In Section 3.3.1 on page 28, we have performed a portfolio alignment by considering a global decarbonization path for the portfolio but without taking into account the decarbonization path of the issuers. In this section, we go further by using net zero metrics. The simplest solution is to consider the carbon intensity trend for each issuer. Nevertheless, we can use other more sophisticated measures (Lombard *et al.*, 2022).

A simple net zero metric Using the historical Trucost data between 2013 and 2019, we project the trend of total absolute emissions (scope $1+2+3$) between 2020 and 2050 — for some assets, we have the historical data for the year 2020 and the trend is then projected between 2021 and 2050. Among the 500 constituents of the S&P500 index in October 2021, 450 issuers have full reporting allowing us to build a consistent trajectory. Filling in upstream/downstream values around reported values allows us to improve the coverage up to 493 issuers which corresponds to 98.6% of the index with a synthetic trend trajectory[41]. The trend is normalized at the initial year t_0 such that we have:

$$\widehat{\boldsymbol{\mathcal{CI}}}_i^{\text{trend}}(t) = m_i^{\text{trend}}(t) \cdot \boldsymbol{\mathcal{CI}}_i(t_0) \tag{55}$$

where the multiplicative factor $m_i^{\text{trend}}(t)$ is linear:

$$m_i^{\text{trend}}(t) = 1 + \beta_i^{\text{trend}} \cdot (t - t_0) \tag{56}$$

Therefore, we have $m_i^{\text{trend}}(t_0) = 1$ for all the assets. Since we have the following relationship between the multiplicative factor and the reduction:

$$m_i^{\text{trend}}(t) = 1 - \boldsymbol{\mathcal{R}}_i(t_0, t) \tag{57}$$

the reduction $\boldsymbol{\mathcal{R}}_i(t_0, t)$ between t_0 and t of the asset i is then the opposite of the projected trend:

$$\boldsymbol{\mathcal{R}}_i(t_0, t) = -\beta_i^{\text{trend}} \cdot (t - t_0) \tag{58}$$

In Figure 20, we have reported the multiplicative factor $m_i^{\text{trend}}(t)$ for the 500 constituents of the S&P 500 index. 70% of trends are upward, meaning that only 30% of the issuers have begun to substantially reduce their carbon emissions.

Portfolio metrics of carbon reduction Before considering the net zero optimization problem, we define the following reference reduction measures:

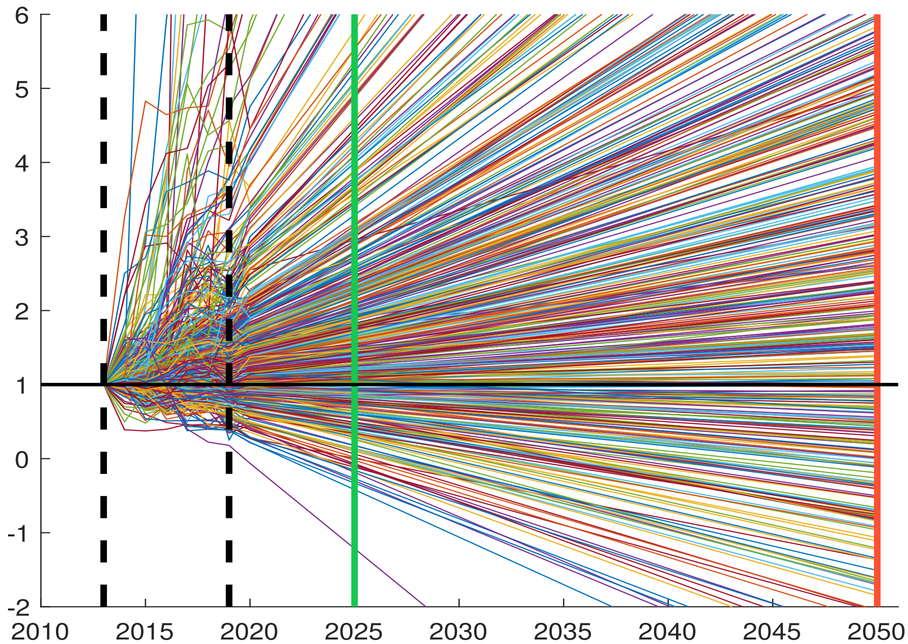
- The capitalization-weighted (or portfolio) reduction is given by:

$$\boldsymbol{\mathcal{R}}_x^{\text{CW}}(t_0, t) = \sum_{i=1}^{n} x_i(t_0) \cdot \boldsymbol{\mathcal{R}}_i(t_0, t) \tag{59}$$

  This measure controls the portfolio weighted average reduction.

[41] It is important to note that the pivot year, from which we shift from historical emissions to projected emissions, is an important parameter that plays a key role, in particular in the context of a dynamic use. This problem is illustrated in Lombard *et al.* (2022). Nevertheless, we consider the 2019 pivot year because it corresponds to the last reporting year with a high coverage.

Figure 20: Carbon emission trends $m_i^{\text{trend}}(t)$ of the S&P index constituents



Source: Trucost reporting year 2019/2020 & Authors' calculations.

- The equally-weighted reduction is equal to:

$$\mathcal{R}_x^{\text{EW}}(t_0, t) = \frac{1}{n} \sum_{i=1}^{n} \mathcal{R}_i(t_0, t) \tag{60}$$

This measure does not depend on the portfolio weights. It restricts the universe to issuers whose equally-weighted aggregation of the trajectories meets a given requirement. For instance, the concept of *net zero alignment* has *a priori* no reason to depend on the size of a company or its weight in the portfolio.

- The intensity-weighted reduction measure is defined as:

$$\mathcal{R}_x^{\text{IW}}(t_0, t) = \frac{\sum_{i=1}^{n} \mu(\mathcal{CI}_i) \cdot \mathcal{R}_i(t_0, t)}{\sum_{i=1}^{n} \mu(\mathcal{CI}_i)} \tag{61}$$

where $\mu(\mathcal{CI})$ is an increasing function of the carbon intensity. Similarly, as the equally-weighted reduction, this measure restricts the investment universe to issuers whose intensity-weighted aggregation of the trajectories meets a given requirement. The rationale is that a net zero trajectory can only be met if high-intensive issuers, which in terms of weight are relatively underrepresented in investment indices, have reduction pathways in line with the requirement. This measure may be used to build a quantitative engagement strategy.

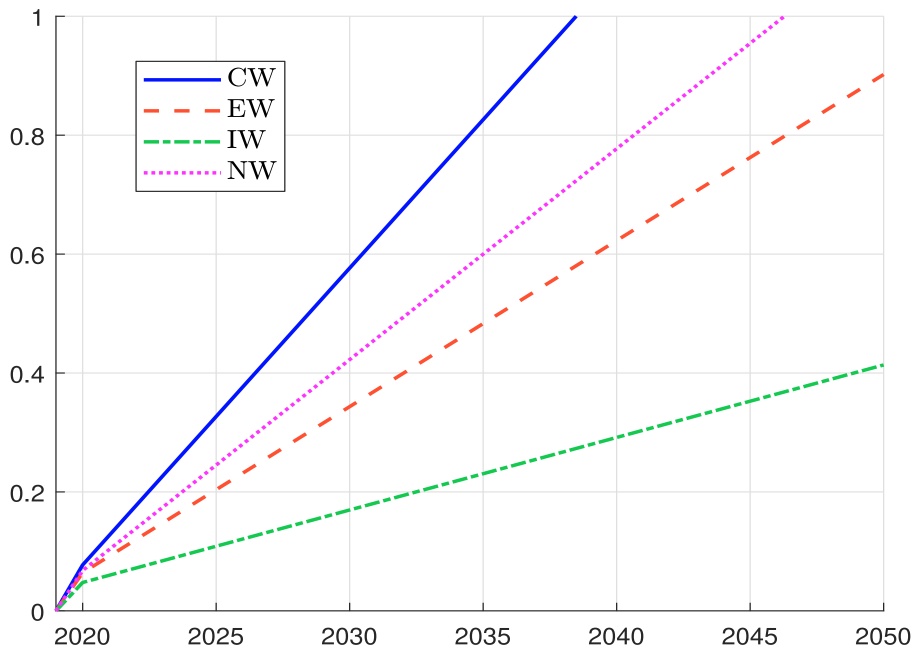
- Finally, the inverse intensity-weighted reduction is the mirror of the previous one:

$$\mathcal{R}_x^{\text{NW}}(t_0, t) = \frac{\sum_{i=1}^{n} \mu(\mathcal{CI}_i)^{-1} \cdot \mathcal{R}_i(t_0, t)}{\sum_{i=1}^{n} \mu(\mathcal{CI}_i)^{-1}} \tag{62}$$

It ensures that low-intensive issuers remain in line and do not increase their emissions. This corresponds to a pure decarbonization strategy.

In Figure 21, we have reported the (opposite of) NZE reduction metrics of the S&P 500 index. Since the values have a positive sign, this indicates that we observe a positive carbon emissions trend at the portfolio level whatever the climate risk measure[42] (capitalization-, equally-, intensity- or inverse intensity-weighted). Based on the trends between 2013 and 2019, the S&P 500 index is clearly not on track to achieve carbon neutrality by 2050.

Figure 21: NZE reduction metrics of the S&P 500 index (carbon intensity, scope $1+2+3$)



Source: Trucost reporting year 2019 & Authors' calculations.

NZE optimization problem The optimization problem is the same as previously except that we explicitly introduce the NZE trajectories for the individual carbon intensity trajectories. For instance, we can formulate the NZE problem as follows:

$$
\begin{aligned}
x^{\mathrm{NZE}}(t) &= \arg\min_x \frac{1}{2}(x-b)^\top \Sigma (x-b) \\
&\text{s.t.} \quad \left\{ \begin{array}{l} \mathbf{1}_n^\top x = 1 \\ x \geq \mathbf{0}_n \\ \boldsymbol{\mathcal{CI}}^{\mathrm{NZE}}(x) \leq \left(1 - \boldsymbol{\mathcal{R}}(t_0, t)\right) \cdot \boldsymbol{\mathcal{CI}}^{\mathrm{NZE}}\left(b(t_0)\right) \end{array} \right.
\end{aligned} \tag{63}
$$

where $\boldsymbol{\mathcal{CI}}^{\mathrm{NZE}}(x)$ uses the projected trends:

$$
\boldsymbol{\mathcal{CI}}^{\mathrm{NZE}}(x) = \sum_{i=1}^n x_i \cdot \widehat{\boldsymbol{\mathcal{CI}}}_i^{\mathrm{trend}}(t) \tag{64}
$$

[42]We consider the square root function $\mu(\boldsymbol{\mathcal{CI}}_i) = \sqrt{\boldsymbol{\mathcal{CI}}_i}$ to compute $\boldsymbol{\mathcal{R}}_x^{\mathrm{IW}}(t_0, t)$ and $\boldsymbol{\mathcal{R}}_x^{\mathrm{NW}}(t_0, t)$.

We can compare the solution $x^{\text{NZE}}(t)$ with the one $x^{\text{DCN}}(t)$ obtained by considering the current carbon intensities $\mathcal{CI}_i(t_0)$ instead of the estimated values $\widehat{\mathcal{CI}}_i^{\text{trend}}(t)$. This latter can be viewed as a pure decarbonization portfolio. In order to measure the discrepancy between the NZE portfolio $x^{\text{NZE}}(t)$ and the decarbonization portfolio $x^{\text{DCN}}(t)$, we compute the active share between the weights of the two portfolios:

$$\mathcal{AS}\left(x^{\text{NZE}}(t), x^{\text{DCN}}(t)\right) = \frac{1}{2}\left\| x^{\text{NZE}}(t) - x^{\text{DCN}}(t)\right\|_1 \tag{65}$$

The results are given in Table 11. We observe that the divergence between the NZE portfolio and the decarbonization portfolio increases with the target date and the reduction level $\mathcal{R}(t_0, t)$.

Table 11: Active share between the NZE portfolio and the decarbonization portfolio

Year	$\mathcal{R}(t_0,t)$							
	10%	20%	30%	40%	50%	60%	70%	80%
2025	1.0%	1.3%	2.0%	3.2%	4.4%	7.8%	18.4%	48.9%
2030	1.1%	1.4%	2.7%	4.8%	9.2%	15.3%	28.5%	58.3%
2035	1.1%	1.8%	3.3%	5.9%	11.0%	17.3%	30.6%	60.0%
2040	1.1%	2.0%	3.6%	6.3%	11.5%	18.0%	31.3%	60.6%
2045	1.2%	2.1%	3.8%	6.5%	11.8%	18.3%	31.6%	60.8%
2050	1.2%	2.1%	3.8%	6.7%	11.9%	18.5%	31.7%	60.9%

Remark 10. *This divergence will increase in the future when we have more exhaustive data and robust NZE measures. For instance, Lombard et al. (2022) use issuer targets and trend slopes to perform NZE portfolios and show that they are highly sensitive to the ambition of carbon issuers.*

3.4 The case of bonds

In the case of an equity portfolio, we define the tracking risk as the tracking error variance of Portfolio x with respect to the Benchmark b:

$$\mathcal{TR}(x \mid b) = \sigma^2(x \mid b) = (x-b)^\top \Sigma (x-b) \tag{66}$$

For a bond portfolio, it can be replaced by the active risk, which can be measured with respect to the weights, the modified duration and the duration-time-spread:

$$\mathcal{TR}(x \mid b) = \sum_{s=1}^{n_{\mathcal{S}ector}} \left(\left| \sum_{i \in s} (x_i - b_i) \right| + \left| \sum_{i \in s} (x_i - b_i) \operatorname{MD}_i \right| + \left| \sum_{i \in s} (x_i - b_i) \operatorname{DTS}_i \right| \right) \tag{67}$$

where $n_{\mathcal{S}ector}$ is the number of sectors, s is the sector index, MD_i is the modified duration of Bond i and DTS_i is the duration-times-spread of Bond i. As argued by Ben Slimane (2021), the active risk is not sufficient to define the objective function. Indeed, the portfolio solution must also take into account the liquidity cost, otherwise we can obtain theoretical bond portfolios that cannot be implemented in practice. Therefore, the optimization procedure for bonds is less straightforward than for stocks. For instance, it is not realistic to rebalance the portfolio at a given frequency, e.g. every six months or every year. It may be more relevant to define a long-term decarbonization or NZE portfolio and implement a management transition from the current portfolio to the target portfolio. This issue is left for future research.

4 Conclusion

In this survey dedicated to portfolio construction with climate risk measures, we first present several metrics and then show how climate risk measures can be integrated into traditional portfolio allocation problems. Some years ago, carbon emissions and intensity were the two main climate risk measures. In this case, portfolio optimization reduced to portfolio decarbonization. Since 2018, we have observed major development of new climate risk metrics. For example, one of the main challenges is to define carbon transition pathways. This means defining a carbon reduction scenario at the global level and the carbon trajectory of corporate issuers. Even if this research axis is not mature, it completely changes the practice of portfolio alignment, whose earlier approaches can be viewed as an enhanced method of portfolio decarbonization. Alongside the usefulness of this survey for both academics and professionals, this research paper also shows two important results. First, decarbonization is more difficult when we consider scope 3 carbon emissions. This is a big issue because asset owners and managers are encouraged by regulators and the society to go beyond scope $1 + 2$. Second, portfolio alignment requires new metrics that are more difficult to estimate, implying more uncertainties about the portfolio solution. As a result, these two combined factors (scope 3 and portfolio alignment) will have significant impact on investors in the coming years. Implementing a net zero carbon portfolio alignment with scope 3 emissions and carbon trajectories is therefore a big challenge for investors. In particular, we can predict that carbon investing will have a major effect on asset allocation with respect to traditional investing.

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A Appendix

A.1 Notations

Table 12: Climate risk measures

Symbol	Description
$\mathcal{C}_i$	Generic climate risk measure
$\mathcal{CE}_i$	Carbon emission
$\mathcal{CI}_i$	Carbon intensity
$\mathcal{CE}_{i,j}$, $\mathcal{CI}_{i,j}$	Carbon scope j
$\mathcal{R}_i$	Carbon reduction
$\mathcal{T}_i$	Climate temperature objective
$\mathcal{CPM}_i$	Carbon price margin
$\mathcal{CB}_i$	Carbon beta of the stock i
$\mathcal{CS}_i$	Climate score
$\mathcal{CM}_i$	Climate metric or KPI

A.2 Data

Table 13 gives the correspondence between the NACE code the GICS code (Sector/Industry Group/Industry) when we consider the narrow measure of high climate impact sectors.

Table 13: The narrow measure of high climate impact sectors

NACE		GICS	
Code	Sector	Code	Sector
A	Agriculture, Forestry & Fishing	302020	Food Products
B	Mining & Quarrying	10 151040	Energy Metals & Mining
C	Manufacturing	20	Industrials
D	Electricity, Gas, Steam & Air Conditioning Supply	55	Utilities
E	Water Supply Sewerage, Waste Management & Remediation Activities	55	Utilities
F	Construction	151020	Construction Materials
G	Wholesale & retail trade Repair of Motor Vehicles & Motorcycles	301010	Food & Staples Retailing
H	Transportation & Storage	2030	Transportation
L	Real Estate Activities	60	Real Estate

Source: Authors' calculations.

A.3 Scope 3 emissions

Scope 3 emissions of Trucost are defined as follows: other indirect greenhouse gas emissions, such as from the extraction and production of purchased materials and fuels, transport-related activities in vehicles not owned or controlled by the reporting entity, electricity-related activities (e.g. T&D losses) not covered in scope 2, outsourced activities, waste disposal, etc (in line with GHG protocol standards). It seems that this definition does not contain use of sold product, end of life treatment of sold product, downstream transportation and distribution, etc.

A.4 Mathematical results

A.4.1 Relationship between x_i, ω_i and Y_i

In the case $n = 2$, the direct measure of the carbon intensity (WACI) is equal to:

$$\boldsymbol{\mathcal{CI}}_j(x) = x_1 \boldsymbol{\mathcal{CI}}_{1,j} + x_2 \boldsymbol{\mathcal{CI}}_{2,j} \tag{68}$$

whereas its exact measure is defined as:

$$\boldsymbol{\mathcal{CI}}_j(x) = \frac{\omega_1 \dfrac{\boldsymbol{\mathcal{CE}}_{1,j}}{\mathcal{MV}_1} + \omega_2 \dfrac{\boldsymbol{\mathcal{CE}}_{2,j}}{\mathcal{MV}_2}}{\omega_1 \dfrac{Y_1}{\mathcal{MV}_1} + \omega_2 \dfrac{Y_2}{\mathcal{MV}_2}} \tag{69}$$

Since we have $x_1 + x_2 = 1$ and $\omega_1 + \omega_2 = 1$, we obtain:

$$\frac{\omega_1 \dfrac{\boldsymbol{\mathcal{CE}}_{1,j}}{\mathcal{MV}_1} + (1-\omega_1) \dfrac{\boldsymbol{\mathcal{CE}}_{2,j}}{\mathcal{MV}_2}}{\omega_1 \dfrac{Y_1}{\mathcal{MV}_1} + (1-\omega_1) \dfrac{Y_2}{\mathcal{MV}_2}} = x_1 \frac{\boldsymbol{\mathcal{CE}}_{1,j}}{Y_1} + (1-x_1) \frac{\boldsymbol{\mathcal{CE}}_{2,j}}{Y_2} \tag{70}$$

We deduce that:

$$x_1 = \frac{\omega_1 \dfrac{Y_1}{\mathcal{MV}_1}}{\omega_1 \dfrac{Y_1}{\mathcal{MV}_1} + (1-\omega_1) \dfrac{Y_2}{\mathcal{MV}_2}} \tag{71}$$

and:

$$\omega_1 = \frac{x_1 \left(\dfrac{Y_1}{\mathcal{MV}_1}\right)^{-1}}{x_1 \left(\dfrac{Y_1}{\mathcal{MV}_1}\right)^{-1} + (1-x_1) \left(\dfrac{Y_2}{\mathcal{MV}_2}\right)^{-1}} \tag{72}$$

More generally, we have:

$$x_i = \frac{\omega_i \dfrac{Y_i}{\mathcal{MV}_i}}{\sum_{k=1}^{n} \omega_k \dfrac{Y_k}{\mathcal{MV}_k}} \tag{73}$$

and:

$$\omega_i = \frac{x_i \left(\dfrac{Y_i}{\mathcal{MV}_i}\right)^{-1}}{\sum_{k=1}^{n} x_k \left(\dfrac{Y_k}{\mathcal{MV}_k}\right)^{-1}} \tag{74}$$

A.4.2 Equivalence between CTB and PAB pathways

Since we have:

$$\boldsymbol{\mathcal{R}}_{\mathrm{CTB}}\left(t_0,t\right)=1-\left(1-7\%\right)^{t-t_0}\left(1-\boldsymbol{\mathcal{R}}_{\mathrm{CTB}}^{-}\right) \tag{75}$$

and:

$$\boldsymbol{\mathcal{R}}_{\mathrm{PAB}}\left(t_0,t\right)=1-\left(1-7\%\right)^{t-t_0}\left(1-\boldsymbol{\mathcal{R}}_{\mathrm{PAB}}^{-}\right) \tag{76}$$

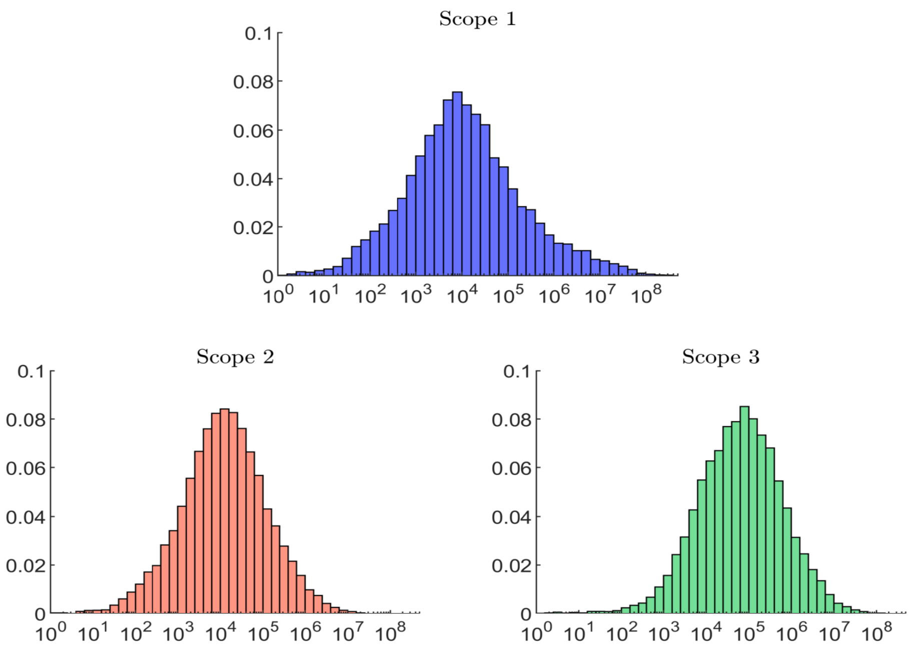
we deduce that there is a date t_{CTB} such that $\boldsymbol{\mathcal{R}}_{\mathrm{CTB}}\left(t_0,t_{\mathrm{CTB}}\right)=\boldsymbol{\mathcal{R}}_{\mathrm{PAB}}\left(t_0,t_{\mathrm{PAB}}\right)$. It follows that:

$$\begin{aligned} t_{\mathrm{CTB}} &= t_{\mathrm{PAB}}+\frac{\ln\left(\dfrac{1-\boldsymbol{\mathcal{R}}_{\mathrm{PAB}}^{-}}{1-\boldsymbol{\mathcal{R}}_{\mathrm{CTB}}^{-}}\right)}{\ln\left(1-7\%\right)} \\ &= t_{\mathrm{PAB}}+4.6365\text{ years} \end{aligned} \tag{77}$$

A.5 Complementary results

A.5.1 Additional figures and tables

Figure 22: Histogram of carbon emission (log scale, tCO_2e)



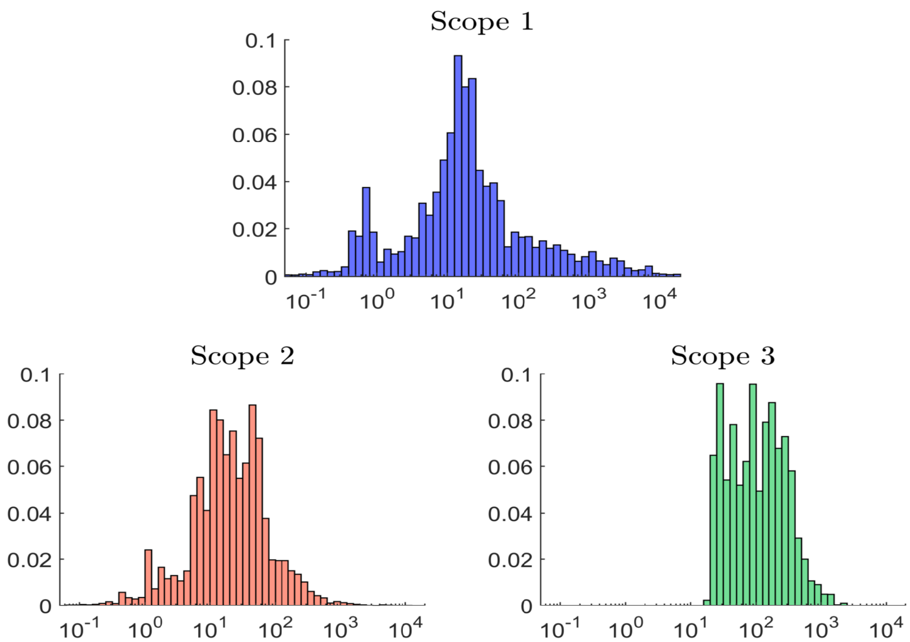
Source: Trucost reporting year 2019 & Authors' calculations.

Table 14: Rank correlation matrix (in %) of carbon metrics

	$\mathcal{CE}_1$	$\mathcal{CE}_2$	$\mathcal{CE}_3$	$\mathcal{CI}_1$	$\mathcal{CI}_2$	$\mathcal{CI}_3$
$\mathcal{CE}_1$	100.0					
$\mathcal{CE}_2$	78.1	100.0				
$\mathcal{CE}_3$	81.9	81.9	100.0			
$\mathcal{CI}_1$	70.3	32.8	32.0	100.0		
$\mathcal{CI}_2$	38.0	55.3	18.1	54.4	100.0	
$\mathcal{CI}_3$	55.5	36.6	55.6	66.6	44.7	100.0

Source: Trucost reporting year 2019 & Authors' calculations.

Figure 23: Histogram of carbon intensity (log scale, $tCO_2e/\$$ mn)



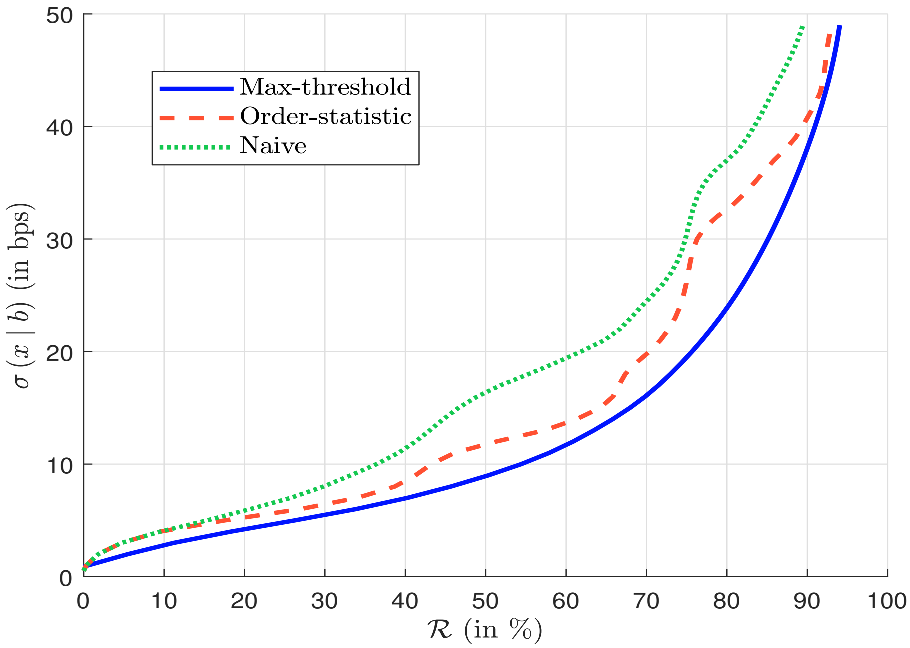
Source: Trucost reporting year 2019 & Authors' calculations.

Table 15: Rank correlations per sector

Sector	$\mathcal{CE}_1$ $\mathcal{CE}_2$	$\mathcal{CE}_2$ $\mathcal{CE}_3$	$\mathcal{CE}_1$ $\mathcal{CE}_3$	$\mathcal{CI}_1$ $\mathcal{CI}_2$	$\mathcal{CI}_2$ $\mathcal{CI}_3$	$\mathcal{CI}_1$ $\mathcal{CI}_3$
Communication Services	89.2	91.8	89.2	36.3	27.1	11.6
Consumer Discretionary	80.4	76.2	80.3	34.8	7.7	29.1
Consumer Staples	74.1	81.3	76.0	39.4	30.4	56.6
Energy	69.6	75.8	72.9	35.6	0.7	9.6
Financials	83.7	87.5	87.1	26.4	58.3	−4.3
Health Care	93.9	94.7	94.3	26.4	−12.4	25.7
Industrials	69.4	82.2	69.5	27.9	42.3	12.7
Information Technology	83.1	92.2	81.7	66.2	72.9	55.4
Materials	79.3	79.7	73.4	50.1	4.5	28.9
Real Estate	76.5	69.2	79.9	9.5	−27.3	29.9
Utilities	34.7	55.9	83.1	−14.0	−4.8	68.5

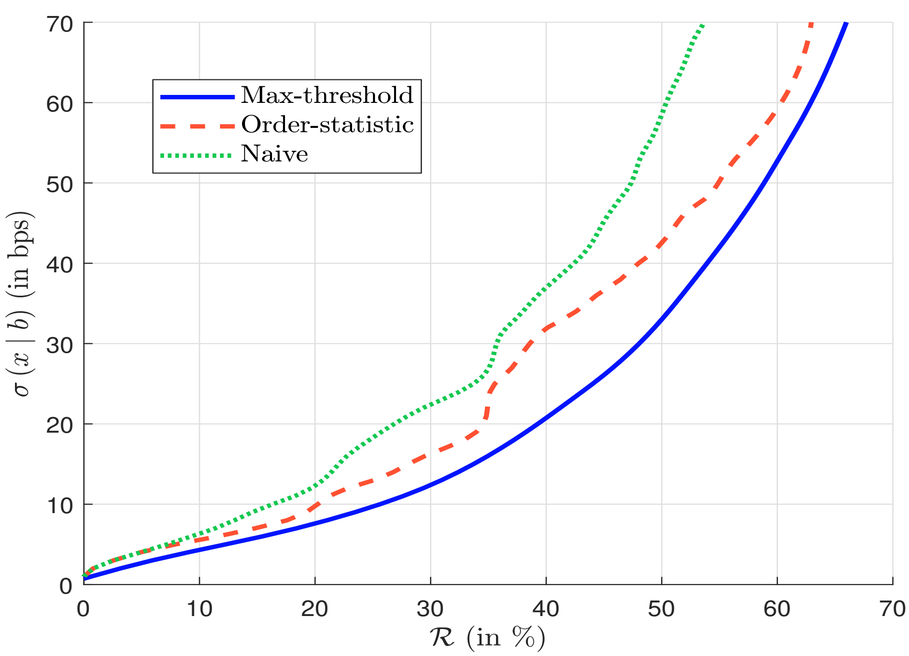
Source: Trucost reporting year 2019 & Authors' calculations.

Figure 24: Efficient frontier of optimal decarbonization portfolios (S&P 500 index, October 2021, scope 1)



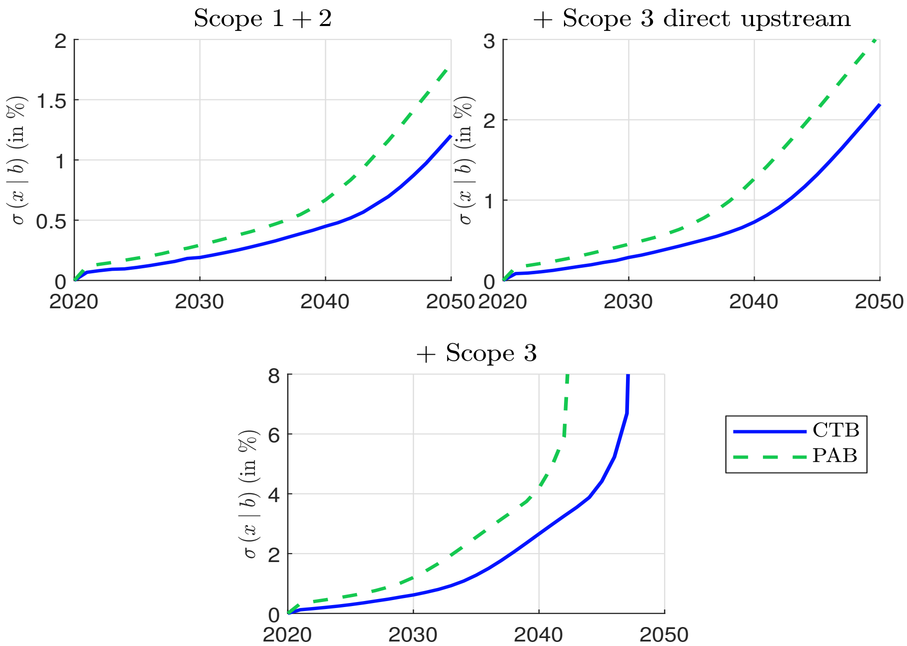
Source: Trucost reporting year 2020 & Authors' calculations.

Figure 25: Efficient frontier of optimal decarbonization portfolios (S&P 500 index, October 2021, scope $1+2+3$)



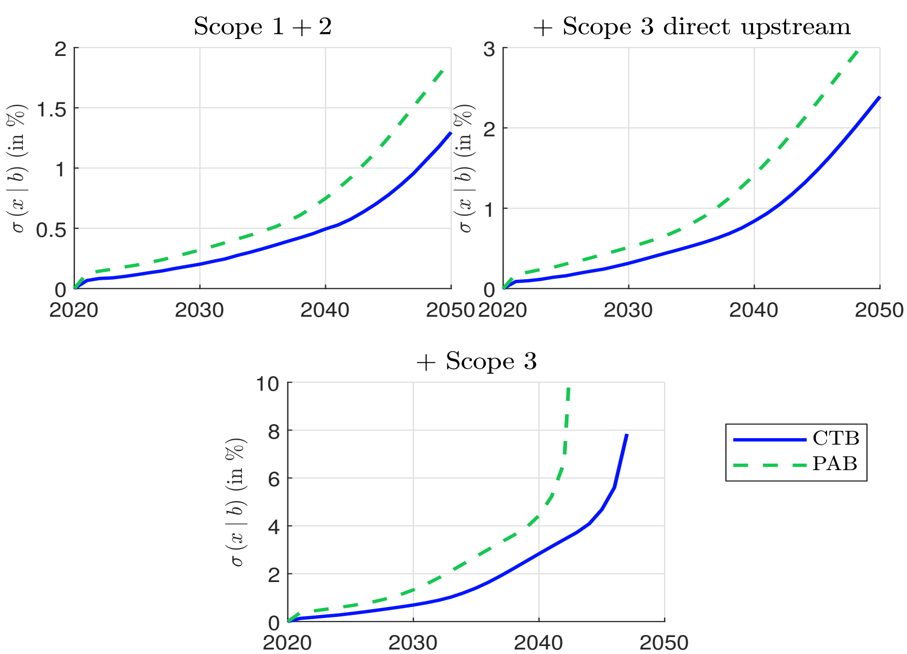
Source: Trucost reporting year 2020 & Authors' calculations.

Figure 26: Tracking error of CTB and PAB labels when implementing the decarbonization pathway



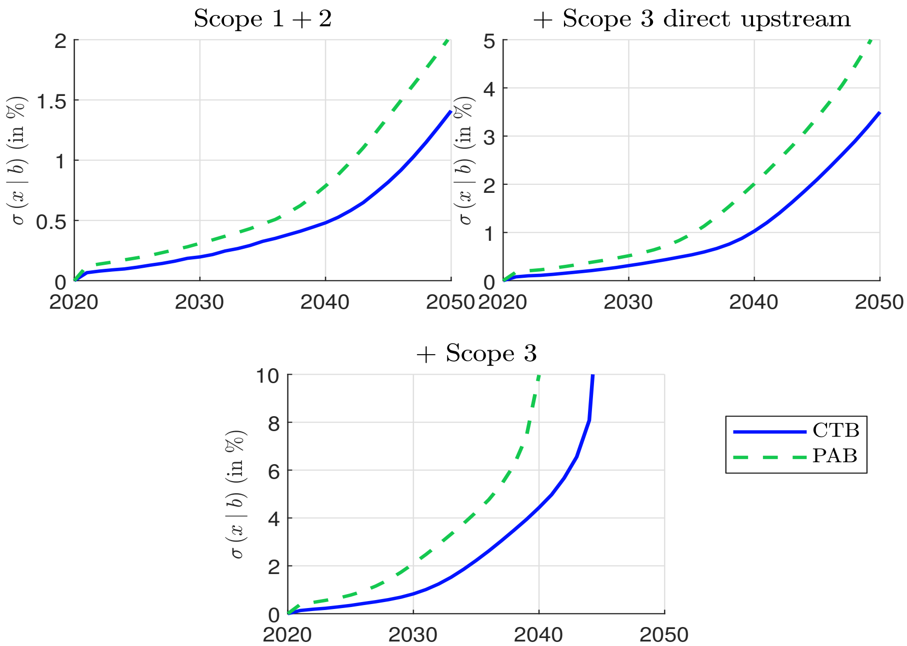
Source: Trucost reporting year 2020 & Authors' calculations.

Figure 27: Tracking error of CTB and PAB labels when implementing the decarbonization pathway and the narrow HCIS constraint



Source: Trucost reporting year 2020 & Authors' calculations.

Figure 28: Tracking error of CTB and PAB labels when implementing the decarbonization pathway and the broad HCIS constraint



Source: Trucost reporting year 2020 & Authors' calculations.

A.5.2 Computation of the carbon trajectory

Using the observations in Table 16, we obtain the following estimates: $\hat{\beta}_0 = 3479.77$ and $\hat{\beta}_1 = -1.7055$. We deduce that:

$$\begin{aligned} \mathcal{CE}_{i,1}^{\mathrm{trend}}(2019+h) &= \hat{\beta}_0 + \hat{\beta}_1 (2019+h) \\ &= 36.33 - 1.7055h \end{aligned}$$

We have: $\mathcal{CE}_{i,1}^{\mathrm{trend}}(2020) = 34.62$, $\mathcal{CE}_{i,1}^{\mathrm{trend}}(2021) = 32.92$, $\mathcal{CE}_{i,1}^{\mathrm{trend}}(2030) = 17.57$, $\mathcal{CE}_{i,1}^{\mathrm{trend}}(2040) = 0.51$, etc.

Table 16: Carbon emissions (scope 1) of the company $\mathbf{\Lambda}$ in $MtCO_2e$

Year	2006	2007	2008	2009	2010	2011	2012
$\mathcal{CE}_{i,1}(t)$	57.80	58.46	57.90	55.13	51.63	46.34	47.09
Year	2013	2014	2015	2016	2017	2018	2019
$\mathcal{CE}_{i,1}(t)$	46.08	44.37	41.75	39.40	36.26	40.71	40.91

Source: Trucost reporting year 2019.