Climate Finance

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Deadline: ??

We consider an investment universe of 8 issuers. In the table below, we report the carbon emissions $\mathcal{CE}_{i,j}$ of these companies and their revenues Y_i, and we indicate in the last row whether the company belongs to sector $\mathcal{S}_1$ or $\mathcal{S}_2$:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
$\mathcal{CE}_{i,1}$ (in $ktCO_2e$)	75	5 000	720	50	2 500	25	30 000	5
$\mathcal{CE}_{i,2}$ (in $ktCO_2e$)	75	5 000	1 030	350	4 500	5	2000	64
Y_i (in \$ bn)	300	328	125	100	200	102	107	25
Sector	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_2$

The benchmark b of this investment universe is defined as:

$$b = (22\%, 19\%, 17\%, 13\%, 11\%, 8\%, 6\%, 4\%)$$

In what follows, we consider **long-only portfolios**.

1. We want to compute the carbon intensity of the benchmark.
   (a) Deduce the carbon intensities $\mathcal{CI}_{i,1+2}$ of each company i for scope $1+2$ emissions.
   (b) Deduce the weighted average carbone intensity (WACI) of the benchmark.

2. We would like to manage **an equity portfolio** with respect to the previous investment universe and reduce the **weighted average carbon intensity** of the benchmark by the rate $\mathcal{R}$. We assume that the volatility of the stocks is respectively equal to 22%, 20%, 25%, 18%, 40%, 23%, 13% and 29%. The correlation matrix between these stocks is given by:

$$\rho = \begin{pmatrix} 100\% & & & & & & & \\ 80\% & 100\% & & & & & & \\ 70\% & 75\% & 100\% & & & & & \\ 60\% & 65\% & 80\% & 100\% & & & & \\ 70\% & 50\% & 70\% & 85\% & 100\% & & & \\ 50\% & 60\% & 70\% & 80\% & 60\% & 100\% & & \\ 70\% & 50\% & 70\% & 75\% & 80\% & 50\% & 100\% & \\ 60\% & 65\% & 70\% & 75\% & 65\% & 70\% & 60\% & 100\% \end{pmatrix}$$

   (a) Compute the covariance matrix Σ.
   (b) Write the optimization problem if the objective function is to minimize the tracking error risk under the constraint of carbon intensity.
   (c) Give the QP formulation of the optimization problem.

(d) $\boldsymbol{\mathcal{R}}$ is equal to 20%. Find the optimal portfolio if we target scopes $1+2$. What is the value of the tracking error volatility?

(e) Same question if $\boldsymbol{\mathcal{R}}$ is equal to 30%, 50%, and 70%.

3. We would like to manage **a bond portfolio** with respect to the previous investment universe and reduce the **weighted average carbon intensity** of the benchmark by the rate $\boldsymbol{\mathcal{R}}$ (scope $1+2$ intensity). In the table below, we report the modified duration MD_i and the duration times spread DTS_i of each corporate bond i:

Issuer	#1	#2	#3	#4	#5	#6	#7	#8
MD_i (in year)	3.56	7.48	6.54	10.23	2.40	2.30	9.12	7.96
DTS_i (in bps)	103	155	75	796	89	45	320	245
Sector	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_2$

In what follows, we use the following numerical values: $\varphi_{\text{AS}} = 100$, $\varphi_{\text{MD}} = 25$ and $\varphi_{\text{DTS}} = 1$. The reduction rate $\boldsymbol{\mathcal{R}}$ of the weighted average carbon intensity is set to 50% for scopes 1, 2 and 3.

(a) Compute the modified duration $\text{MD}(b)$ and the duration times spread $\text{DTS}(b)$ of the benchmark.

(b) Let x be the equally-weighted portfolio. Compute[1] $\text{MD}(x)$, $\text{DTS}(x)$, $\sigma_{\text{AS}}(x \mid b)$, $\sigma_{\text{MD}}(x \mid b)$ and $\sigma_{\text{DTS}}(x \mid b)$.

(c) Solve the following optimization problem[2]:

$$x^\star = \arg\min \frac{1}{2}\mathcal{R}_{\text{AS}}(x \mid b) \quad \text{s.t.} \quad \left\{\begin{array}{l} \sum_{i=1}^n x_i = 1 \\ \text{MD}(x) = \text{MD}(b) \\ \text{DTS}(x) = \text{DTS}(b) \\ \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CI}}(b) \\ 0 \leq x_i \leq 1 \end{array}\right.$$

Compute $\text{MD}(x^\star)$, $\text{DTS}(x^\star)$, $\sigma_{\text{AS}}(x^\star \mid b)$, $\sigma_{\text{MD}}(x^\star \mid b)$ and $\sigma_{\text{DTS}}(x^\star \mid b)$.

(d) Solve the following optimization problem:

$$x^\star = \arg\min \frac{1}{2}\varphi_{\text{AS}}\mathcal{R}_{\text{AS}}(x \mid b) + \frac{1}{2}\varphi_{\text{MD}}\mathcal{R}_{\text{MD}}(x \mid b) \quad \text{s.t.} \quad \left\{\begin{array}{l} \sum_{i=1}^n x_i = 1 \\ \text{DTS}(x) = \text{DTS}(b) \\ \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CI}}(b) \\ 0 \leq x_i \leq 1 \end{array}\right.$$

Compute $\text{MD}(x^\star)$, $\text{DTS}(x^\star)$, $\sigma_{\text{AS}}(x^\star \mid b)$, $\sigma_{\text{MD}}(x^\star \mid b)$ and $\sigma_{\text{DTS}}(x^\star \mid b)$.

(e) Solve the following optimization problem:

$$x^\star = \arg\min \frac{1}{2}\mathcal{R}(x \mid b) \quad \text{s.t.} \quad \left\{\begin{array}{l} \sum_{i=1}^n x_i = 1 \\ \boldsymbol{\mathcal{CI}}(x) \leq (1-\boldsymbol{\mathcal{R}})\,\boldsymbol{\mathcal{CI}}(b) \\ 0 \leq x_i \leq 1 \end{array}\right.$$

Compute $\text{MD}(x^\star)$, $\text{DTS}(x^\star)$, $\sigma_{\text{AS}}(x^\star \mid b)$, $\sigma_{\text{MD}}(x^\star \mid b)$ and $\sigma_{\text{DTS}}(x^\star \mid b)$.

(f) Comment on the results obtained in Question 3.(c), 3.(d) and 3.(e).

[1] Precise the corresponding unit (years, bps or %) for each metric.

[2] You can use any numerical nonlinear solvers in Questions 3.(c), 3.(d) and 3.(e), not necessarily a QP solver.

Reminders

- The MD of portfolio x is equal to:

$$\mathrm{MD}(x) = \sum_{i=1}^{n} x_i \cdot \mathrm{MD}_i$$

- The DTS of portfolio x is equal to:

$$\mathrm{DTS}(x) = \sum_{i=1}^{n} x_i \cdot \mathrm{DTS}_i$$

- The active share risk is equal to:

$$\sigma_{\mathrm{AS}}(x \mid b) = \sqrt{\sum_{i=1}^{n} (x_i - b_i)^2}$$

- The MD-based tracking error risk is equal to:

$$\sigma_{\mathrm{MD}}(x \mid b) = \sqrt{\left(\sum_{i \in \mathcal{S}_1} (x_i - b_i)\,\mathrm{MD}_i\right)^2 + \left(\sum_{i \in \mathcal{S}_2} (x_i - b_i)\,\mathrm{MD}_i\right)^2}$$

- The DTS-based tracking error risk is equal to:

$$\sigma_{\mathrm{DTS}}(x \mid b) = \sqrt{\left(\sum_{i \in \mathcal{S}_1} (x_i - b_i)\,\mathrm{DTS}_i\right)^2 + \left(\sum_{i \in \mathcal{S}_2} (x_i - b_i)\,\mathrm{DTS}_i\right)^2}$$

- We note $\mathcal{R}_{\mathrm{AS}}(x \mid b) = \sigma^2_{\mathrm{AS}}(x \mid b)$, $\mathcal{R}_{\mathrm{MD}}(x \mid b) = \sigma^2_{\mathrm{MD}}(x \mid b)$ and $\mathcal{R}_{\mathrm{DTS}}(x \mid b) = \sigma^2_{\mathrm{MD}}(x \mid b)$
- The synthetic risk measure is the combination of AS, MD and DTS active risks:

$$\mathcal{R}(x \mid b) = \varphi_{\mathrm{AS}}\mathcal{R}_{\mathrm{AS}}(x \mid b) + \varphi_{\mathrm{MD}}\mathcal{R}_{\mathrm{MD}}(x \mid b) + \varphi_{\mathrm{DTS}}\mathcal{R}_{\mathrm{DTS}}(x \mid b)$$

  where $\varphi_{\mathrm{AS}} \geq 0$, $\varphi_{\mathrm{MD}} \geq 0$ and $\varphi_{\mathrm{DTS}} \geq 0$ indicate the weight of each risk