Portfolio Diversification & Asset Allocation
What Does It Mean?

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¹The opinions expressed in this presentation are those of the authors and are not meant to represent the opinions or official positions of Amundi Asset Management.
Which method for diversifying?

- Portfolio optimization (Markowitz)
- Risk budgeting
Mean-variance optimized portfolios

Let $\mu$ and $\Sigma$ be the vector of expected returns and the covariance matrix of asset returns. The optimization problem is:

$$x^* = \arg\max_x x^\top \mu$$

u.c. $\sqrt{x^\top \Sigma x} \leq \sigma^*$

This problem is equivalent to the QP problem:

$$x^* = \arg\min_x \frac{1}{2} x^\top \Sigma x - \gamma x^\top \mu$$

$$= \gamma \Sigma^{-1} \mu$$
MVO portfolios are sensitive to arbitrage factors

MVO portfolios are of the following form: \( x^* \propto f(\Sigma^{-1}) \).

The important quantity is then the information matrix \( J = \Sigma^{-1} \).

We have \( \Sigma = \Lambda \Lambda^\top \) and \( \Sigma^{-1} = (\Lambda \Lambda^\top)^{-1} = \Lambda^{-1} \Lambda^{-1} \Lambda^{-1} = \Lambda^{-1} \Lambda^{-1} \Lambda^{-1} \).

If we consider the following example: \( \sigma_1 = 20\% \), \( \sigma_2 = 21\% \), \( \sigma_3 = 10\% \) and \( \rho_{i,j} = 80\% \), we obtain the following eigendecomposition:

<table>
<thead>
<tr>
<th>Asset / Factor</th>
<th>Covariance matrix ( \Sigma )</th>
<th>Information matrix ( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>65.35%</td>
<td>-72.29%</td>
</tr>
<tr>
<td>2</td>
<td>69.38%</td>
<td>69.06%</td>
</tr>
<tr>
<td>3</td>
<td>30.26%</td>
<td>-2.21%</td>
</tr>
<tr>
<td>Eigenvalue</td>
<td>8.31%</td>
<td>0.84%</td>
</tr>
<tr>
<td>% cumulated</td>
<td>88.29%</td>
<td>97.20%</td>
</tr>
</tbody>
</table>

\[ 12.04 \equiv \frac{1}{8.31\%} \]

Reverse order of eigenvectors
**Common factors versus idiosyncratic factors**

**Figure:** PCA applied to the stocks of the FTSE index (June 2012)

Why traditional shrinkage methods do not work?
Arbitrage factors and hedging portfolios

We consider the following regression model:

\[ R_{i,t} = \beta_0 + \beta_i^\top R_{t}^{(-i)} + \epsilon_{i,t} \]

- \( R_{t}^{(-i)} \) denotes the vector of asset returns \( R_t \) excluding the \( i^{th} \) asset
- \( \epsilon_{i,t} \sim \mathcal{N}(0, s_i^2) \)
- \( R_i^2 \) is the \( R \)-squared of the linear regression

Information matrix

Stevens (1998) shows that the information matrix is given by:

\[
\mathcal{I}_{i,i} = - \frac{1}{\hat{\sigma}_i^2 (1 - R_i^2)}
\]

\[
\mathcal{I}_{i,j} = - \frac{\hat{\beta}_{i,j}}{\hat{\sigma}_i^2 (1 - R_i^2)} = - \frac{\hat{\beta}_{j,i}}{\hat{\sigma}_j^2 (1 - R_j^2)}
\]
Arbitrage factors and hedging portfolios

Table: Hedging portfolios (in %) at the end of 2006

<table>
<thead>
<tr>
<th></th>
<th>SPX</th>
<th>SX5E</th>
<th>TPX</th>
<th>RTY</th>
<th>EM</th>
<th>US HY</th>
<th>EMBI</th>
<th>EUR</th>
<th>JPY</th>
<th>GSCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPX</td>
<td>58.6</td>
<td>6.0</td>
<td>150.3</td>
<td>-30.8</td>
<td>-0.5</td>
<td>5.0</td>
<td>-7.3</td>
<td>15.3</td>
<td>-25.5</td>
<td></td>
</tr>
<tr>
<td>SX5E</td>
<td>9.0</td>
<td>-1.2</td>
<td>-1.3</td>
<td>35.2</td>
<td>0.8</td>
<td>3.2</td>
<td>-4.5</td>
<td>-5.0</td>
<td>-1.5</td>
<td></td>
</tr>
<tr>
<td>TPX</td>
<td>0.4</td>
<td>-0.6</td>
<td>-2.4</td>
<td>38.1</td>
<td>1.1</td>
<td>-3.5</td>
<td>-4.9</td>
<td>-0.8</td>
<td>-0.3</td>
<td></td>
</tr>
<tr>
<td>RTY</td>
<td>48.6</td>
<td>-2.7</td>
<td>-10.4</td>
<td>26.2</td>
<td>-0.6</td>
<td>1.9</td>
<td>0.2</td>
<td>-6.4</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>EM</td>
<td>-4.1</td>
<td>30.9</td>
<td>69.2</td>
<td>10.9</td>
<td>0.9</td>
<td>4.6</td>
<td>9.1</td>
<td>3.9</td>
<td>33.1</td>
<td></td>
</tr>
<tr>
<td>ÜS HY</td>
<td>-5.0</td>
<td>53.5</td>
<td>160.0</td>
<td>-18.8</td>
<td>69.5</td>
<td></td>
<td>95.6</td>
<td>48.4</td>
<td>31.4</td>
<td>-211.7</td>
</tr>
<tr>
<td>EMBI</td>
<td>10.8</td>
<td>44.2</td>
<td>-102.1</td>
<td>12.3</td>
<td>73.4</td>
<td>19.4</td>
<td></td>
<td>-5.8</td>
<td>40.5</td>
<td>86.2</td>
</tr>
<tr>
<td>EUR</td>
<td>-3.6</td>
<td>-14.7</td>
<td>-33.4</td>
<td>0.3</td>
<td>33.8</td>
<td>2.3</td>
<td>-1.4</td>
<td>56.7</td>
<td>48.2</td>
<td></td>
</tr>
<tr>
<td>JPY</td>
<td>6.8</td>
<td>-14.5</td>
<td>-4.8</td>
<td>-8.8</td>
<td>12.7</td>
<td>1.3</td>
<td>8.4</td>
<td>50.4</td>
<td>-33.2</td>
<td></td>
</tr>
<tr>
<td>GSCI</td>
<td>-1.1</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0.8</td>
<td>10.7</td>
<td>-0.9</td>
<td>1.8</td>
<td>4.2</td>
<td>-3.3</td>
<td></td>
</tr>
<tr>
<td>s_i</td>
<td>0.3</td>
<td>0.7</td>
<td>0.9</td>
<td>0.5</td>
<td>0.7</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
<td>1.2</td>
</tr>
<tr>
<td>R^2</td>
<td>83.0</td>
<td>47.7</td>
<td>34.9</td>
<td>82.4</td>
<td>60.9</td>
<td>39.8</td>
<td>51.6</td>
<td>42.3</td>
<td>43.7</td>
<td>12.1</td>
</tr>
</tbody>
</table>
We finally obtain:

$$x_i^*(\gamma) = \gamma \frac{\mu_i - \hat{\beta}_i^\top \mu(-i)}{\hat{s}_i^2}$$

From this equation, we deduce the following conclusions:

1. The better the hedge, the higher the exposure. This is why highly correlated assets produces unstable MVO portfolios.
2. The long-short position is defined by the sign of $\mu_i - \hat{\beta}_i^\top \mu(-i)$. If the expected return of the asset is lower than the conditional expected return of the hedging portfolio, the weight is negative.

Markowitz diversification $\neq$ Diversification of risk factors $\neq$ Concentration on arbitrage factors
The rules of the game

The mean-variance approach is one of the most aggressive active management models: it concentrates the portfolio on a small number of bets (idiosyncratic factors and arbitrage factors).

Traditional shrinkage approaches (RMT, Ledoit-Wolf, etc.) are not sufficient. This is why portfolio managers use discretionary constraints: $Cx \geq D$. Jagannathan and Ma (2003) showed that:

$$\tilde{\Sigma} = \Sigma - \left( C^T \lambda 1^T + 1^T \lambda C^T \right)$$

where $\lambda$ is the vector of Lagrange coefficients associated to $Cx \geq D$.

$\Rightarrow$ Using constraints is equivalent to shrink the covariance matrix (Ledoit-Wolf) or to introduce relative views (Black-Litterman)
Weight budgeting versus risk budgeting

Let $x = (x_1, \ldots, x_n)$ be the weights of $n$ assets in the portfolio. Let $\mathcal{R}(x_1, \ldots, x_n)$ be a coherent and convex risk measure. We have:

\[
\mathcal{R}(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i \cdot \frac{\partial \mathcal{R}(x_1, \ldots, x_n)}{\partial x_i}
\]

\[
= \sum_{i=1}^{n} \text{RC}_i(x_1, \ldots, x_n)
\]

Let $b = (b_1, \ldots, b_n)$ be a vector of budgets such that $b_i \geq 0$ and $\sum_{i=1}^{n} b_i = 1$. We consider two allocation schemes:

1. Weight budgeting (WB)
   \[x_i = b_i\]

2. Risk budgeting (RB)
   \[\text{RC}_i = b_i \cdot \mathcal{R}(x_1, \ldots, x_n)\]
Let $\Sigma$ be the covariance matrix of the assets returns. We assume that the risk measure $R(x)$ is the volatility of the portfolio $\sigma(x) = \sqrt{x^\top \Sigma x}$. We have:

\[
\frac{\partial R(x)}{\partial x} = \frac{\Sigma x}{\sqrt{x^\top \Sigma x}}
\]

\[
RC_i(x_1, \ldots, x_n) = x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}}
\]

\[
\sum_{i=1}^{n} RC_i(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i \cdot \frac{(\Sigma x)_i}{\sqrt{x^\top \Sigma x}} = x^\top \frac{\Sigma x}{\sqrt{x^\top \Sigma x}} = \sigma(x)
\]

The risk budgeting portfolio is defined by this system of equations:

\[
\begin{aligned}
x_i \cdot (\Sigma x)_i &= b_i \cdot (x^\top \Sigma x) \\
x_i &\geq 0 \\
\sum_{i=1}^{n} x_i &= 1
\end{aligned}
\]
**Illustration**

- **3 assets**
- Volatilities are equal to 30%, 20% and 15%
- Correlations are set to 80% between the 1\textsuperscript{st} asset and the 2\textsuperscript{nd} asset, 50% between the 1\textsuperscript{st} asset and the 3\textsuperscript{rd} asset and 30% between the 2\textsuperscript{nd} asset and the 3\textsuperscript{rd} asset
- Budgets are set to 50%, 20% and 30%
- For the ERC (Equal Risk Contribution) portfolio, all the assets have the same risk budget

<table>
<thead>
<tr>
<th>Asset</th>
<th>Weight</th>
<th>Marginal Risk</th>
<th>Risk Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Absolute</td>
</tr>
<tr>
<td>1</td>
<td>50.00%</td>
<td>29.40%</td>
<td>14.70%</td>
</tr>
<tr>
<td>2</td>
<td>20.00%</td>
<td>16.63%</td>
<td>3.33%</td>
</tr>
<tr>
<td>3</td>
<td>30.00%</td>
<td>9.49%</td>
<td>2.85%</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td>20.87%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Asset</th>
<th>Weight</th>
<th>Marginal Risk</th>
<th>Risk Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Absolute</td>
</tr>
<tr>
<td>1</td>
<td>31.15%</td>
<td>28.08%</td>
<td>8.74%</td>
</tr>
<tr>
<td>2</td>
<td>21.90%</td>
<td>15.97%</td>
<td>3.50%</td>
</tr>
<tr>
<td>3</td>
<td>46.96%</td>
<td>11.17%</td>
<td>5.25%</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td>17.49%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Asset</th>
<th>Weight</th>
<th>Marginal Risk</th>
<th>Risk Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Absolute</td>
</tr>
<tr>
<td>1</td>
<td>19.69%</td>
<td>27.31%</td>
<td>5.38%</td>
</tr>
<tr>
<td>2</td>
<td>32.44%</td>
<td>16.57%</td>
<td>5.38%</td>
</tr>
<tr>
<td>3</td>
<td>47.87%</td>
<td>11.23%</td>
<td>5.38%</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td></td>
<td>16.13%</td>
</tr>
</tbody>
</table>
The logarithmic barrier problem

Roncalli (2013) shows that:

$$x^* = \arg \min R(x) - \lambda \sum_{i=1}^{n} b_i \ln x_i$$

⇒ CCD algorithm (Griveau-Billion et al., 2013).

- The RB portfolio is a combination of the MR and WB portfolios:
  $$x_i / b_i = x_j / b_j \quad \text{(wb)}$$
  $$\partial_{x_i} R(x) = \partial_{x_j} R(x) \quad \text{(mr)}$$
  $$RC_i / b_i = RC_j / b_j \quad \text{(rb)}$$

- The risk of the RB portfolio is between the risk of the MR portfolio and the risk of the WB portfolio:
  $$R(x_{mr}) \leq R(x_{rb}) \leq R(x_{wb})$$

With risk budgeting, we always diminish the risk compared to the weight budgeting.
<table>
<thead>
<tr>
<th>MVO portfolios</th>
<th>RB portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility optimization</td>
<td>Volatility diversification</td>
</tr>
<tr>
<td>Marginal risk</td>
<td>Risk contribution</td>
</tr>
<tr>
<td>Sensitive to $\Sigma^{-1}$</td>
<td>Sensitive to $\Sigma$</td>
</tr>
<tr>
<td>Arbitrage factors</td>
<td>Common risk factors</td>
</tr>
</tbody>
</table>

⇒ Risk parity is the right approach for managing the diversification of **long-only** diversified portfolios.

And in the long/short case?
Which assets (common risk factors) to diversify?

- Traditional assets or risk premia
  - Stocks
  - Bonds
- Equity risk factors
- Alternative risk premia
- Illiquid assets
  - Private equity
  - Private debt
  - Real estate
  - Infrastructure
What is the rationale for factor investing?

How to define risk factors?
Risk factors are common factors that explain the cross-section variance of expected returns:

- 1964: Market or MKT (or BETA) factor
- 1972: Low beta or BAB factor
- 1981: Size or SMB factor
- 1985: Value or HML factor
- 1991: Low volatility or VOL factor
- 1993: Momentum or WML factor
- 2000: Quality or QMJ factor

Factor investing is a subset of smart (new) beta
What is the rationale for factor investing?

At the security level, there is a lot of idiosyncratic risk or alpha:

<table>
<thead>
<tr>
<th></th>
<th>Common Risk</th>
<th>Idiosyncratic Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>GOOGLE</td>
<td>47%</td>
<td>53%</td>
</tr>
<tr>
<td>NETFLIX</td>
<td>24%</td>
<td>76%</td>
</tr>
<tr>
<td>MASTERCARD</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>NOKIA</td>
<td>32%</td>
<td>68%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>89%</td>
<td>11%</td>
</tr>
<tr>
<td>AIRBUS</td>
<td>56%</td>
<td>44%</td>
</tr>
</tbody>
</table>

Carhart’s model with 4 factors, 2010-2014
Source: Author’s research
What is the rationale for factor investing?

- Jensen (1968) – **How to measure the performance of active management?**
  \[ R_t^F = \alpha + \beta R_t^{MKT} + \varepsilon_t \]
  \[ \Rightarrow \bar{\alpha} = -\text{fees} \]

- Hendricks et al. (1993) – **Hot Hands in Mutual Funds**
  \[ \text{cov}(\alpha_t^{\text{Jensen}}, \alpha_{t-1}^{\text{Jensen}}) > 0 \]
  where:
  \[ \alpha_t^{\text{Jensen}} = R_t^F - \beta^{\text{MKT}} R_t^{MKT} \]
  \[ \Rightarrow \text{The persistence of the performance of active management is due to the persistence of the alpha} \]
What is the rationale for factor investing?

- Grinblatt et al. (1995) – **Momentum investors versus Value investors**
  
  “77% of mutual funds are momentum investors”

- Carhart (1997):

\[
\begin{align*}
\text{cov} \left( \alpha_t^{\text{Jensen}}, \alpha_{t-1}^{\text{Jensen}} \right) &> 0 \\
\text{cov} \left( \alpha_t^{\text{Carhart}}, \alpha_{t-1}^{\text{Carhart}} \right) &= 0
\end{align*}
\]

where:

\[
\alpha_t^{\text{Carhart}} = R_t^F - \beta^{\text{MKT}} R_t^{\text{MKT}} - \beta^{\text{SMB}} R_t^{\text{SMB}} - \beta^{\text{HML}} R_t^{\text{HML}} - \beta^{\text{WML}} R_t^{\text{WML}}
\]

⇒ The (short-term) persistence of the performance of active management is due to the (short-term) **persistence of the performance of risk factors**
David Swensen’s rule for effective stock picking

Concentrated portfolio ⇒ No more than 20 bets?

**Figure**: Carhart’s alpha decreases with the number of holding assets

[Graph showing Carhart’s alpha decreasing with the number of holding assets for US equity markets, 2000-2014. Source: Author’s research]
What is the rationale for factor investing?

Figure: What proportion of return variance is explained?

How many bets are there in large portfolios of institutional investors?

1986 Less than 10% of institutional portfolio return is explained by security picking and market timing (Brinson et al., 1986)

2009 Professors’ Report on the Norwegian GPFG: Risk factors represent 99.1% of the fund return variation (Ang et al., 2009)
What is the rationale for factor investing?

What lessons can we draw from this?

Idiosyncratic risks and specific bets disappear in (large) diversified portfolios. Performance of institutional investors is then exposed to risk factors.

**Alpha is not scalable, but risk factors are scalable.**

⇒ Risk factors are the only bets that are compatible with diversification.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Beta(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Concentration</td>
<td>• Diversification</td>
</tr>
<tr>
<td>• Portfolio optimization</td>
<td>• Risk-based allocation</td>
</tr>
<tr>
<td>(e.g. MVO)</td>
<td>(e.g. RB)</td>
</tr>
</tbody>
</table>
A new opportunity for active managers

- Active management does not reduce to stock picking
- Understanding the diversification of equity portfolios
- New tactical products

**Approaches of equity investing**

1. Pure stock picking process (with a limited number of bets)
2. Factor-based stock picking process
3. Allocation between factor-based portfolios
### Figure: Heatmap of risk factors (before 2008) – MSCI Europe

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
<th>Momentum</th>
<th>Low Beta</th>
<th>Size</th>
<th>Value</th>
<th>Low Beta</th>
<th>Size</th>
<th>Value</th>
<th>Low Beta</th>
<th>Market</th>
<th>Quality</th>
<th>Low Beta</th>
<th>Size</th>
<th>Value</th>
<th>Low Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>25.5%</td>
<td>-3.3%</td>
<td>31.1%</td>
<td>66.9%</td>
<td>32.1%</td>
<td>10.1%</td>
<td>39.1%</td>
<td>-40.9%</td>
<td>Low Beta</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>6.2%</td>
<td>-1.7%</td>
<td>40.6%</td>
<td>30.4%</td>
<td>31.5%</td>
<td>2.7%</td>
<td>34.3%</td>
<td>31.5%</td>
<td>30.1%</td>
<td>Market</td>
<td>1.8%</td>
<td>-1.0%</td>
<td>25.5%</td>
<td>26.1%</td>
<td>-9.0%</td>
</tr>
<tr>
<td>2002</td>
<td>Value</td>
<td>-6.8%</td>
<td>Size</td>
<td>Momentum</td>
<td>27.5%</td>
<td>Quality</td>
<td>Value</td>
<td>Low Beta</td>
<td>Size</td>
<td>Market</td>
<td>Quality</td>
<td>Low Beta</td>
<td>Low Beta</td>
<td>Size</td>
<td>Value</td>
</tr>
<tr>
<td>2003</td>
<td>Value</td>
<td>66.9%</td>
<td>Size</td>
<td>Value</td>
<td>30.4%</td>
<td>Value</td>
<td>25.5%</td>
<td>26.5%</td>
<td>Value</td>
<td>Low Beta</td>
<td>26.1%</td>
<td>Quality</td>
<td>24.1%</td>
<td>26.1%</td>
<td>Market</td>
</tr>
<tr>
<td>2004</td>
<td>Low Beta</td>
<td>31.1%</td>
<td>Low Beta</td>
<td>Quality</td>
<td>29.5%</td>
<td>Low Beta</td>
<td>26.1%</td>
<td>25.5%</td>
<td>Low Beta</td>
<td>Value</td>
<td>19.6%</td>
<td>Value</td>
<td>-9.0%</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>Size</td>
<td>32.1%</td>
<td>Size</td>
<td>Low Beta</td>
<td>23.9%</td>
<td>Quality</td>
<td>Size</td>
<td>28.7%</td>
<td>Size</td>
<td>Quality</td>
<td>24.1%</td>
<td>Value</td>
<td>-4.4%</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>Value</td>
<td>31.5%</td>
<td>Value</td>
<td>Value</td>
<td>29.5%</td>
<td>Value</td>
<td>25.5%</td>
<td>26.5%</td>
<td>Value</td>
<td>Low Beta</td>
<td>26.1%</td>
<td>Quality</td>
<td>24.1%</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>Low Beta</td>
<td>10.1%</td>
<td>Value</td>
<td>Low Beta</td>
<td>30.1%</td>
<td>Quality</td>
<td>Low Beta</td>
<td>26.1%</td>
<td>Quality</td>
<td>Low Beta</td>
<td>Value</td>
<td>Value</td>
<td>-53.9%</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>-40.9%</td>
<td>-1.0%</td>
<td>Size</td>
<td>Market</td>
<td>34.3%</td>
<td>Market</td>
<td>-9.0%</td>
<td>-63.6%</td>
<td>Value</td>
<td>Value</td>
<td>-53.9%</td>
<td>Value</td>
<td>Value</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Richard and Roncalli (2015)
**Which method for diversifying?**

**Which assets to diversify?**

**How to diversify?**

**Conclusion**

**Factor investing**

**Alternative risk premia**

---

**A new opportunity for active managers**

---

**Figure:** Heatmap of risk factors (after 2008) – MSCI Europe

![Heatmap of risk factors](image)

Source: Richard and Roncalli (2015)
Risk premia & non-diversifiable risk

Consumption-based model (Lucas, 1978; Cochrane, 2001)

A risk premium is a compensation for accepting (systematic) risk in bad times.

![Graph showing return distribution for trend-following and reversal strategies.]
Characterization of alternative risk premia

- An alternative risk premium (ARP) is a risk premium, which is not traditional
  - Traditional risk premia (TRP): equities, sovereign/corporate bonds
  - Currencies and commodities are not TRP
- The drawdown of an ARP must be positively correlated to bad times
  - Risk premia ≠ insurance against bad times
  - (SMB, HML) ≠ WML
- Risk premia are an increasing function of the volatility and a decreasing function of the skewness

In the market practice, alternative risk premia recovers:

1. **Skewness risk premia** (or pure risk premia), which present high negative skewness and potential large drawdown
2. **Markets anomalies**
Payoff function of alternative risk premia

Figure: Which option profile may be considered as a skewness risk premium?

- Long call (risk adverse)
- Short call (market anomaly)
- Long put (insurance)
- Short put

⇒ SMB, HML, WML, BAB, QMJ
A myriad of alternative risk premia?

**Figure:** Mapping of ARP candidates

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Equities</th>
<th>Rates</th>
<th>Credit</th>
<th>Currencies</th>
<th>Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Carry</strong></td>
<td>Dividend Futures</td>
<td>FRB</td>
<td></td>
<td>FRB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High Dividend Yield</td>
<td>TSS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CTS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td>Amihud liquidity</td>
<td>Turn-of-the-month</td>
<td></td>
<td>Turn-of-the-month</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cross-section</td>
<td>Turn-of-the-month</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td>Cross-section</td>
<td>Time-Series</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time-series</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Reversal</strong></td>
<td>Time-series</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>Value</strong></td>
<td>Value</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>Carry</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Term structure</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Event</strong></td>
<td>Buyback</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Growth</strong></td>
<td>Merger arbitrage</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Low volatility</strong></td>
<td>Low volatility</td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Quality</strong></td>
<td></td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td><strong>Size</strong></td>
<td></td>
<td></td>
<td></td>
<td>Value</td>
<td></td>
</tr>
</tbody>
</table>
The example of bank’s proprietary indices

**Figure:** Graph database of bank’s proprietary indices
The identification problem

What is the problem?

- For traditional risk premia, the cross-correlation between several indices replicating the TRP is higher than 90%
- For alternative risk premia, the cross-correlation between several indices replicating the ARP is between −80% and 100%

Examples (2000-2015)

- In the case of the equities/US traditional risk premium, the cross-correlation between S&P 500, FTSE USA, MSCI USA, Russell 1000 and Russell 3000 indices is between 99.65% and 99.92%
- In the case of the equities/volatility/carry/US risk premium, the cross-correlation between the 14 short volatility indices is between −34.9% and 98.6% (mean = 43.0%, Q₃ − Q₁ > 35%)
Step 1 Define the set of relevant indices (qualitative due diligence).

Step 2 Given an initial set of indices, the underlying idea is to find the subset, whose elements present very similar patterns. For that, we use the deletion algorithm using the $R^2$ statistic:

$$R_{k,t} = \alpha_k + \beta_k R_t^{(-k)} + \varepsilon_{k,t} \Rightarrow R^2_k$$

Step 3 The algorithm stops when the similarity is larger than a given threshold for all the elements of the subset (e.g. $R^2_k > R^2_{\text{min}} = 70\%$).

Step 4 The generic backtest of the ARP is the weighted average of the performance of the subset elements.
Illustration with the volatility carry risk premium

Indices after the 1st step

Selected indices after the 3rd step

Generic cumulative return

- Barclays (BXIISVUE) 90.2%
- Citi (CIISEVCU) 92.4%
- Citi (CIISEVWU) 97.0%
- JP Morgan (AIJPSV1U) 93.4%
- SG (SGIXVPUX) 94.9%
**Figure:** Mapping of **relevant** ARP²

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Equities</th>
<th>Rates</th>
<th>Credit</th>
<th>Currencies</th>
<th>Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carry</td>
<td>Dividend Futures&lt;br&gt;High Dividend Yield</td>
<td>FRB&lt;br&gt;TSS&lt;br&gt;CTS</td>
<td>FRB</td>
<td>FRB</td>
<td>FRB&lt;br&gt;TSS&lt;br&gt;CTS</td>
</tr>
<tr>
<td>Liquidity</td>
<td>Amihud liquidity</td>
<td>Turn-of-the-month&lt;br&gt;Cross-section&lt;br&gt;Term structure</td>
<td>Turn-of-the-month&lt;br&gt;Cross-section&lt;br&gt;Term structure</td>
<td>Turn-of-the-month&lt;br&gt;Cross-section&lt;br&gt;Term structure</td>
<td>Turn-of-the-month&lt;br&gt;Cross-section&lt;br&gt;Term structure</td>
</tr>
<tr>
<td>Momentum</td>
<td>Cross-section&lt;br&gt;Time-series</td>
<td>Cross-section&lt;br&gt;Time-series</td>
<td>Time-Series&lt;br&gt;Time-series</td>
<td>Cross-section&lt;br&gt;Time-series&lt;br&gt;Time-series</td>
<td>Cross-section&lt;br&gt;Time-series&lt;br&gt;Time-series</td>
</tr>
<tr>
<td>Reversal</td>
<td>Time-series&lt;br&gt;Variance</td>
<td>Time-series</td>
<td>Time-series</td>
<td>Time-series</td>
<td>Time-series</td>
</tr>
<tr>
<td>Value</td>
<td>Value</td>
<td>Value</td>
<td>Value</td>
<td>Value</td>
<td>Value&lt;br&gt;PPP&lt;br&gt;Economic model&lt;br&gt;Value</td>
</tr>
<tr>
<td>Volatility</td>
<td>Carry&lt;br&gt;Term structure</td>
<td>Carry&lt;br&gt;Term structure</td>
<td>Carry</td>
<td>Carry</td>
<td>Carry</td>
</tr>
<tr>
<td>Event</td>
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<td>Low volatility</td>
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<td>Quality</td>
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<td>Size</td>
<td>Size</td>
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<td>Size</td>
<td>Size</td>
</tr>
</tbody>
</table>

²Based on bank’s proprietary indices.
Value Carry and momentum everywhere

- Value Carry and momentum everywhere
- Some ARP candidates are not relevant (e.g. liquidity premium in equities, rates and currencies; reversal premium using variance swaps; value premium in rates and commodities; dividend premium; volatility premium in currencies and commodities; correlation premium; seasonality premium.)
- Hierarchy of ARP
  - Equities  value, carry, low volatility, volatility/carry, momentum, quality, growth, size, event, reversal
  - Rates  volatility/carry, momentum, carry
  - Currencies  carry, momentum, value
  - Commodities  carry, momentum, liquidity
- Carry recovers different notions: FRB (Forward Rate Bias), TSS (Term Structure Slope) and CTS (Cross Term Structure).
Volatility diversification

Cumulative inertia (in %)

Factor

TRP
ARP
How to diversify (common risk factors)?

- Volatility diversification
- Skewness diversification
- Liquidity diversification
Consider a portfolio with 2 assets: \( R(x) = x_1 R_1 + x_2 R_2 \). We have:

\[
\text{var}(R(x)) = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho \sigma_1 \sigma_2
\]

**Best solution in terms of volatility diversification**

- Long-only portfolios: \( \rho = -1 \)
- Long/short portfolios: \( \rho = 0 \)

In long-only portfolios, volatility diversification consists in finding assets with negative correlations. In long/short portfolios, volatility diversification consists in finding assets with zero correlations.

**Remark**

In long/short portfolios, a correlation of \(-\rho\) is equivalent to a correlation of \(+\rho\).
Dependence between risk factors

Figure: Value, low beta and carry are not orthogonal risk factors
TRP and non-linear payoff functions

Figure: WML does not exhibit a CTA option profile

- Cross-section momentum ≠ Time-series momentum
- Long-only momentum ≠ Long/short momentum

Source: Cazalet and Roncalli (2014)
Figure: Payoff function of the US short volatility strategy
The skewness puzzle

- ARP are not all-weather strategies:
  - Extreme risks of ARP are high and may be correlated
  - Aggregation of skewness is not straightforward

Skewness aggregation $\neq$ volatility aggregation

When we accumulate long/short skewness risk premia in a portfolio, the volatility of this portfolio decreases dramatically, but its skewness risk generally increases!

- Skewness diversification $\neq$ volatility diversification

\[
\sigma(X + Y) \leq \sigma(X) + \sigma(X) \\
g_1(X + Y) \not\leq g_1(X) + g_1(Y)
\]

$\Rightarrow$ Skewness is not a convex risk measure
Figure: Skewness aggregation of L/S alternative risk premia

Worst skewness coefficients $\gamma_1$

Largest drawdown ratios $DD^*$

MSCI World index

Source: HPRZ (2016)
The skewness puzzle

Figure: Skewness aggregation in the case of the bivariate log-normal distribution

\[
\begin{align*}
\gamma_1(X) &= -1.8, \quad \gamma_1(Y) = -1.8 \\
\gamma_1(X) &= -0.8, \quad \gamma_1(Y) = -0.3 \\
\gamma_1(X) &= -1.8, \quad \gamma_1(Y) = -6.2 \\
\gamma_1(X) &= -0.6, \quad \gamma_1(Y) = -0.6
\end{align*}
\]

Source: HPRZ (2016)
The skewness puzzle

Why?

- Volatility diversification works very well with L/S risk premia:

\[ \sigma(R(x)) \approx \frac{\bar{\sigma}}{\sqrt{n}} \]

- Drawdown diversification don’t work very well because bad times are correlated and are difficult to hedge:

\[ DD(x) \approx \overline{DD} \]
The jump-diffusion representation

- \( n \) risky assets represented by the vector of prices \( S_t = (S_{1,t}, \ldots, S_{n,t}) \) with:
  \[
  \begin{align*}
  dS_t &= \text{diag}(S_t) \, dL_t \\
  dL_t &= \mu \, dt + \Sigma^{1/2} \, dW_t + dZ_t
  \end{align*}
  \]
  where \( Z_t \) is a pure \( n \)-dimensional jump process.
- We assume that the jump process \( Z_t \) is a compound Poisson process:
  \[
  Z_t = \sum_{i=1}^{N_t} Z_i
  \]
  where \( N_t \sim \mathcal{P}(\lambda) \) and \( Z_i \sim \mathcal{N}(\tilde{\mu}, \tilde{\Sigma}) \).

The characteristic function of asset returns \( R_t = (R_{1,t}, \ldots, R_{n,t}) \) for the holding period \( dt \) may be approximated by:
\[
\mathbb{E} \left[ e^{-iu.R_t} \right] \approx (1 - \lambda \, dt) \cdot e^{(iu^T \mu - \frac{1}{2} u^T \Sigma u) \, dt} + (\lambda \, dt) \cdot e^{iu^T (\mu \, dt + \tilde{\mu}) - \frac{1}{2} u^T (\Sigma \, dt + \tilde{\Sigma}) u}
\]
The Gaussian mixture representation

We consider a Gaussian mixture model with two regimes to define $R_t$:

1. The continuous component, which has the probability $(1 - \lambda \ dt)$ to occur, is driven by the Gaussian distribution $\mathcal{N} (\mu \ dt, \Sigma \ dt);$

2. The jump component, which has the probability $\lambda \ dt$ to occur, is driven by the Gaussian distribution $\mathcal{N} \left( \tilde{\mu}, \tilde{\Sigma} \right)$.

The multivariate density function of $R_t$ is:

$$f(y) = \frac{1 - \lambda \ dt}{(2\pi)^{n/2}|\Sigma \ dt|^{1/2}} e^{-\frac{1}{2} (y - \mu \ dt)^\top (\Sigma \ dt)^{-1} (y - \mu \ dt)} +$$

$$\frac{\lambda \ dt}{(2\pi)^{n/2}|\Sigma \ dt + \tilde{\Sigma}|^{1/2}} e^{-\frac{1}{2} (y - (\mu \ dt + \tilde{\mu}))^\top (\Sigma \ dt + \tilde{\Sigma})^{-1} (y - (\mu \ dt + \tilde{\mu}))}$$

The characteristic function of $R_t$ is equal to:

$$\mathbb{E} \left[ e^{-iu \cdot R_t} \right] = (1 - \lambda \ dt) \cdot e^{(iu^\top \mu - \frac{1}{2} u^\top \Sigma u) dt} + (\lambda \ dt) \cdot e^{iu^\top (\mu \ dt + \tilde{\mu}) - \frac{1}{2} u^\top (\Sigma \ dt + \tilde{\Sigma}) u}$$
Let \( x = (x_1, \ldots, x_n) \) be the vector of weights in the portfolio. We have:

\[
R(x) = Y = B_1 \cdot Y_1 + B_2 \cdot Y_2
\]

where:

- \( B_1 \sim B(\pi_1), \ B_2 = 1 - B_1 \sim B(\pi_2), \ \pi_1 = 1 - \lambda \) and \( \pi_2 = \lambda \)
  \( (\mathcal{H} : dt = 1); \)
- \( Y_1 \sim \mathcal{N}(\mu_1(x), \sigma_1^2(x)), \ \mu_1(x) = x^\top \mu \) and \( \sigma_1^2(x) = x^\top \Sigma x; \)
- \( Y_2 \sim \mathcal{N}(\mu_2(x), \sigma_2^2(x)), \ \mu_2(x) = x^\top (\mu + \tilde{\mu}) \) and
  \( \sigma_2^2(x) = x^\top (\Sigma + \tilde{\Sigma}) x. \)

\[ \Rightarrow \] The portfolio’s return \( R(x) \) has the following density function:

\[
f(y) = \pi_1 f_1(y) + \pi_2 f_2(y)
\]

\[
= (1 - \lambda) \frac{1}{\sigma_1(x)} \phi \left( \frac{y - \mu_1(x)}{\sigma_1(x)} \right) + \lambda \frac{1}{\sigma_2(x)} \phi \left( \frac{y - \mu_2(x)}{\sigma_2(x)} \right)
\]
Which method for diversifying?
Which assets to diversify?
How to diversify?
Conclusion

Some issues
The skewness puzzle
What does skewness mean?
Portfolio allocation with skewness risk

Distribution function of the portfolio’s return

![Distribution function graphs](image)

Parameters: $\mu_1 = 5\%$, $\sigma_1 = 10\%$, $\tilde{\mu}_1 = -20\%$, $\tilde{\sigma}_1 = 5\%$, $\mu_2 = 10\%$, $\sigma_2 = 20\%$, $\tilde{\mu}_2 = -40\%$, $\tilde{\sigma}_2 = 5\%$, $\rho = 50\%$, $\tilde{\rho} = 60\%$ and $\lambda = 0.20$. 

Thierry Roncalli
Relationship between jump risk and skewness risk

The skewness of \( R(x) \) is equal to:

\[
\gamma_1 = \frac{(\lambda - \lambda^2) \left( (1 - 2\lambda)(x^\top \tilde{\mu})^3 + 3(x^\top \tilde{\mu})(x^\top \tilde{\Sigma}x) \right)}{\left( x^\top \Sigma x + \lambda x^\top \tilde{\Sigma} x + (\lambda - \lambda^2)(x^\top \tilde{\mu})^2 \right)^{3/2}}
\]

The portfolio exhibits skewness, except for some limit cases:

\[
\gamma_1 = 0 \iff x^\top \tilde{\mu} = 0 \text{ or } \lambda = 0 \text{ or } \lambda = 1
\]

We have:

- If \( x^\top \tilde{\mu} > 0 \), then \( \gamma_1 > 0 \);
- If \( x^\top \tilde{\mu} < 0 \), then \( \gamma_1 < 0 \) in most cases.

\( \Rightarrow \) We retrieve the result of Hamdan et al. (2016):

Skewness risk is maximum when volatility risk is minimum
Relationship between jump risk and skewness risk

Parameters: $\sigma = 20\%$, $\tilde{\mu} = -40\%$, $\tilde{\sigma} = 20\%$ and $\lambda = 25\%$. 
Figure: Cumulative performance of US bonds, US equities and US short volatility
Which method for diversifying?  
Which assets to diversify?  
How to diversify?  
Conclusion

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What does skewness mean?
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Statistics

**Table: Worst returns (in %)**

<table>
<thead>
<tr>
<th>Asset</th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
<th>Annually</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>−1.67</td>
<td>−2.81</td>
<td>−4.40</td>
<td>−3.41</td>
<td>−6.03</td>
</tr>
<tr>
<td>Equities</td>
<td>−9.03</td>
<td>−18.29</td>
<td>−29.67</td>
<td>−49.69</td>
<td>−55.25</td>
</tr>
<tr>
<td>Carry</td>
<td>−6.82</td>
<td>−11.04</td>
<td>−23.43</td>
<td>−23.37</td>
<td>−27.30</td>
</tr>
</tbody>
</table>

**Table: Skewness coefficients**

<table>
<thead>
<tr>
<th>Asset</th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
<th>Annually</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>−0.12</td>
<td>−0.17</td>
<td>0.07</td>
<td>0.22</td>
<td>4.17</td>
</tr>
<tr>
<td>Equities</td>
<td>0.01</td>
<td>−0.44</td>
<td>−0.81</td>
<td>−0.57</td>
<td>18.38</td>
</tr>
<tr>
<td>Carry</td>
<td>−7.24</td>
<td>−5.77</td>
<td>−6.32</td>
<td>−2.23</td>
<td>5.50</td>
</tr>
</tbody>
</table>
### Table: Estimation of the mixture model when $\lambda dt = 0.5\%$ (weekly model)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Asset</th>
<th>$\mu_i$</th>
<th>$\sigma_i$</th>
<th>$\rho_{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>Bonds</td>
<td>5.38</td>
<td>4.17</td>
<td>100.00</td>
</tr>
<tr>
<td></td>
<td>Equities</td>
<td>7.89</td>
<td>15.64</td>
<td>-36.80</td>
</tr>
<tr>
<td></td>
<td>Carry</td>
<td>10.10</td>
<td>2.91</td>
<td>-25.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Jump</td>
<td>Bonds</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
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<td>-1.20</td>
<td>6.76</td>
<td>0.00</td>
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<tr>
<td></td>
<td>Carry</td>
<td>-2.23</td>
<td>2.57</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Impact of the jump component

Figure: PDF of asset returns (weekly model)
The expected shortfall risk measure

Definition of the expected shortfall

\[ ES_\alpha(x) = \mathbb{E}[L(x) \mid L(x) \geq \text{VaR}_\alpha(x)] \]

where \( L(x) = -R(x) \) is the portfolio’s loss.

We obtain:

\[ ES_\alpha(x) = (1 - \lambda) \cdot \phi(\text{VaR}_\alpha(x), \mu_1(x), \sigma_1(x)) + \lambda \cdot \phi(\text{VaR}_\alpha(x), \mu_2(x), \sigma_2(x)) \]

where the function \( \phi(a, b, c) \) is defined by:

\[ \phi(a, b, c) = \frac{c}{1 - \alpha} \Phi \left( \frac{a + b}{c} \right) - \frac{b}{1 - \alpha} \Phi \left( -\frac{a + b}{c} \right) \]

Here, the value-at-risk \( \text{VaR}_\alpha(x) \) is the root of the following equation:

\[ (1 - \lambda) \cdot \Phi \left( \frac{\text{VaR}_\alpha(x) + \mu_1(x)}{\sigma_1(x)} \right) + \lambda \cdot \Phi \left( \frac{\text{VaR}_\alpha(x) + \mu_2(x)}{\sigma_2(x)} \right) = \alpha \]
Analytical expression of risk contributions

We obtain a complicated expression of the risk contribution:

$$ RC_i(x) = x_i \frac{\partial \text{ES}_\alpha(x)}{\partial x_i} = \ldots $$

But it is an analytical formula!

⇒ **No numerical issues for implementing the model**

**Euler decomposition**

We have:

$$ \sum_{i=1}^{n} RC_i(x) = \text{ES}_\alpha(x) $$

⇒ Comparison with the value-at-risk based on the Cornish-Fisher expansion
Risk budgeting portfolios

The RB portfolio is defined by the following non-linear system:

\[
\begin{align*}
RC_i(x) &= b_i R(x) \\
b_i &> 0 \\
x_i &\geq 0 \\
\sum_{i=1}^{n} b_i &= 1 \\
\sum_{i=1}^{n} x_i &= 1
\end{align*}
\]

where \( b_i \) is the ex-ante risk budget of asset \( i \) expressed in relative terms.

Numerical solution of the RB portfolio

\[
y^* = \arg \min_{y} ES_{\alpha}(y) - \sum_{i=1}^{n} b_i \ln y_i \quad \text{u.c.} \quad y \geq 0
\]

The RB portfolio corresponds to the normalized portfolio:

\[
x_i^* = \frac{y_i^*}{\sum_{j=1}^{n} y_j^*}
\]
Figure: Volatility-based ERC portfolio
Risk allocation

Figure: Skewness-based ERC portfolio

Equities

Carry

Bonds

Risk allocation

Figure: Skewness-based ERC portfolio

Equities

Carry

Bonds


0 10 20 30 40 50 60 70 80 90 100
Figure: Comparison of the carry allocation
Volatility hedging versus skewness hedging

Table: Volatility and skewness risks of risk-based portfolios (weekly model)

<table>
<thead>
<tr>
<th>Portfolio Model</th>
<th>MV Gaussian (full sample)</th>
<th>MV Jump model</th>
<th>ERC</th>
<th>MES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>63.26%</td>
<td>36.05%</td>
<td>52.71%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Equities</td>
<td>2.23%</td>
<td>0.00%</td>
<td>10.36%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Carry</td>
<td>34.51%</td>
<td>63.95%</td>
<td>36.93%</td>
<td>0.00%</td>
</tr>
<tr>
<td>(\sigma(x))</td>
<td>2.62%</td>
<td>2.33%</td>
<td>2.75%</td>
<td>4.17%</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>-2.75</td>
<td>-19.81</td>
<td>-6.17</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: BKR (2016)

The arithmetics of skewness

\[- (36.05\% \times 0.17 + 0\% \times 0.44 + 63.95\% \times 5.77) = -19.81\]
The portfolio management of alpha and beta must be different

- Portfolio optimization (MVO) is suitable for managing the concentration of active bets
- Risk-based allocation (RB) is suitable for managing the diversification of risk premia or risk factors

Volatility diversification ≠ skewness diversification

- Volatility hedging ≠ skewness hedging
- Skewness risk = main driver of strategic asset allocation (SAA)
- Volatility risk = main driver of tactical asset allocation (TAA)

Long-only diversification vs long/short diversification

Liquidity issue?

Skewness risk = a **strategic** allocation decision

Volatility risk = a **tactical** allocation decision
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