INTERPRETATION AND ESTIMATION OF DEFAULT CORRELATIONS

Petit Déjeuner de la Finance

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Agenda

- Motivations
- 1st Case: Default Correlations and Loss distribution of a Credit Book
  1. Raroc and credit pricing
  2. Credit portfolio management
  3. Definition of default correlations
  4. MLE of default correlations
- 2nd Case: Default Correlations and Credit Basket Pricing/Hedging
  1. Duality between factor models and copula models
  2. Default correlations and spread jumps
  3. Trac-X implied correlation
  4. Implications for CDO pricing

Interpretation and Estimation of Default Correlations
1 Motivations

Correlations = Parameter of the multivariate Normal distribution / Linear dependence between gaussian random variables

⇒ The second point of view is the reference in finance (regression, factor analysis, etc.)

In asset management, correlations are used to represent the dependence between returns. 
Objective: computing risk/return of portfolios.

⇒ Correlation = a good tool for credit risk modelling?
Our point of view = Correlation is a mathematical tool to define the loss of credit portfolios (CreditRisk+).

Default correlation ≠ dependence between default times (KMV, CreditMetrics).

⇒ Credit Portfolio Management = Loss distribution of a portfolio (Credit VaR, Risk Contribution, Stress Testing, etc.)

⇒ Credit Derivatives Pricing/Hedging = Loss distribution of a tranche & Spread dynamics
2 Default Correlations and Loss distribution of a Credit Book
2.1 Raroc and credit pricing

Some notations:

\[
\begin{align*}
L & \quad \text{Loss of a loan or a portfolio} \\
EL &= \mathbb{E}[L] & \quad \text{Expected Loss or risk cost} \\
UL & \quad \text{Unexpected Loss} \\
UL &= \text{VaR}[L; \alpha] - EL
\end{align*}
\]

Definition of Raroc:

\[
\text{Raroc} = \frac{\text{Expected Return}}{\text{Economic Capital}} = \frac{\text{PNB} - \text{Cost} - \text{EL}}{\text{UL}}
\]

Objective:

Target on Return on Equity $\iff$ Target on Raroc
2.1.1 Ex-Ante Raroc of a loan

Economic Capital = Risk contribution of the loan to the total risk of the portfolio

\[
\text{Raroc} = \frac{\text{Expected Return}}{\text{Risk Contribution of the loan}}
\]

Problems: What is the target portfolio of the bank? Given this portfolio, how to calibrate the parameters of the Raroc model? How to approximate the risk contribution when the credit is well modelled (ex-ante raroc)?
2.1.2 An example with an infinitely fine-grained portfolio and a one factor model

Let $UL = \text{VaR} [L; \alpha] - EL$. If the portfolio is infinitely fine-grained, we have $RC_i = \mathbb{E} [L_i | L = EL + UL] - \mathbb{E} [L_i]$. We consider the following proxy $UL^* = k \times \sigma (L)$. Because we have:

$$\sigma (L) = \sum_i \sigma (L_i) \frac{\text{cov} (L, L_i)}{\sigma (L) \sigma (L_i)} = \sum_i f_i \times \sigma (L_i)$$

we deduce that a proxy of the risk contribution is $RC_i^* = k \times f_i \times \sigma (L_i)$. $f_i$ is called the diversification factor, because it depends on the dependence structure of the portfolio. In the case of one-factor model and an homogeneous portfolio, we obtain:

$$f = \sqrt{\frac{C (PD, PD; \rho) - PD^2}{\frac{\sigma^2 [\text{LGD}]}{\mathbb{E}^2 [\text{LGD}]} PD + (PD - PD^2)}}$$

$\Rightarrow f$ depends on the default correlation $\rho$. 

Interpretation and Estimation of Default Correlations
Default Correlations and Loss distribution of a Credit Book 2-4
2.2 Credit Portfolio Management

⇒ Moving the original portfolio to obtain the target portfolio

• the original portfolio without management is generally concentrated (either at a name, industry or geography level, etc.)
• the target portfolio is generally an infinitely fine-grained portfolio which has some other good properties (= optimise the capital)

Dis-investment / Re-investment

• Single-name hedges (CDS) / Multi-name hedges (F2D, CDO)
• Securitisations (CBO)
• Investment opportunities (CDS / CDO)

⇒ CPM needs default correlations.
2.3 Definition of default correlations

- Default time correlation $\rho(\tau_1, \tau_2)$
- Default event correlations $\rho(1\{\tau_1 \leq t_1\}, 1\{\tau_2 \leq t_2\})$
- Spread jumps $s_1(t_1 \mid \tau_2 = t_2, \tau_1 \geq t_2)$
- Asset / Equity correlations

⇒ How to calibrate correlations needed by CPM?

In the target portfolio, the credit risk is principally a risk on default rates.

- probability of default ⇔ mean of default rates
- default correlations ⇔ volatility of default rates
2.4 Data

- History of annual default rates by risk class
- Risk classes are typically industrial sectors, rating grades, geographical zones, ...

For example: S&P provides this data, between 1980 and 2002 by industrial sector and by rating.
2.5 The model

- Merton model: obligor $n$ defaults if and only if $Z_n \leq B_n$.
- The latent variable $Z_n$ is gaussian.
- Homogeneity of risk classes: $B^n = B^c$.
- Within a given class of risk the correlation between two firms is constant, that is:
  \[
  \rho_{m,n} = \rho_c, \quad \forall m, n \in c
  \]
- Given any pair of risk classes $(c, d)$ there is a unique correlation between any couple of firms $(m, n)$ belonging to each class, that is:
  \[
  \rho_{m,n} = \rho_{c,d}, \quad \forall m \in c, n \in d
  \]
Let’s define

$$
\Sigma = \begin{pmatrix}
\rho_1 & \rho_{1,2} & \cdots & \rho_{1,C} \\
\rho_{2,1} & \rho_2 & \ddots & \vdots \\
\vdots & \ddots & \ddots & \rho_{C-1,C} \\
\rho_{C,1} & \cdots & \rho_{C,C-1} & \rho_C
\end{pmatrix}
$$

then we can rewrite $Z_n$ as a linear function of $F$ factors $X_f$ (with $A^\top A = \Sigma$)

$$
Z_n = \sum_{f=1}^{F} A_{f,c} X_f + \sqrt{1 - \rho_c \varepsilon_n}, \quad n \in c
$$
2.6 MLE of default correlations

The number of default in risk class $D_c \mid X = x \sim \mathcal{B}(n_t^c; P_c(x))$. The default probability conditionally to the factors $X$ is:

$$P_c(x) = \Phi \left( \frac{B_c - \sum_{f=1}^{F} A_{f,c} x_f}{\sqrt{1 - \rho_c}} \right)$$

The unconditional log-likelihood is then:

$$\ell_t(\theta) = \ln \int \cdots \int_{\mathbb{R}^F} \prod_{c=1}^{C} \text{Bin}_{c,t}(x) \ d\Phi(x)$$

with:

$$\text{Bin}_{c,t}(x) = \binom{n_t^c}{d_t^c} P_c(x)^{d_t^c} (1 - P_c(x))^{n_t^c - d_t^c}$$

$\Rightarrow$ The loglikelihood is not tractable (in particular when $C$ increases), due to the multi-dimensional integration.
2.7 Constrained Model

\[
\Sigma = \begin{pmatrix}
\rho_1 & \rho & \cdots & \rho \\
\rho & \rho_2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \rho \\
\rho & \cdots & \rho & \rho_C
\end{pmatrix}
\]

\[
Z_n = \sqrt{\rho}X + \sqrt{\rho_c - \rho}X_c + \sqrt{1 - \rho_c} \varepsilon_n
\]

Interpretation: \(Z_n\) is explained by a common factor \(X\) and by a specific factor \(X_c\) depending on the risk class.

Why: robustness of estimation; this assumption seems intuitive

\[
P_c(x, x_c) = \Phi \left( \frac{B_C - \sqrt{\rho}x - \sqrt{\rho_c - \rho}x_c}{\sqrt{1 - \rho_c}} \right)
\]
2.7.1 Binomial MLE

The conditional likelihood is first computed and then integrated successively on the distribution of each sectorial factor and on the distribution of the common factor:

\[
\ell_t(\theta) = \ln \int_{\mathbb{R}} \left( \prod_{c=1}^{C} \int_{\mathbb{R}} \text{Bin}_{c,t}(x, x_c) \, d\Phi(x_c) \right) \, d\Phi(x)
\]

This is the 'binomial' MLE.
2.7.2 Asymptotic MLE

Let \( \mu_t^c = \frac{d_t^c}{n_t^c} \) be the default rate at time \( t \) in class \( c \).

\[
\mu_t^c \mid X = x, X_c = x_c \rightarrow P(x, x_c)
\]

The loglikelihood function is then:

\[
\ell_t(\theta) = \ln \int_0^1 \prod_{c=1}^C \phi(f(y)) \frac{\sqrt{1 - \rho_c}}{\sqrt{\rho_c - \rho}} \frac{1}{\Phi^{-1}(\mu_t^c)} \, dy
\]

with:

\[
f(y) = \frac{B^c - \sqrt{1 - \rho_c} \Phi^{-1}(\mu_t^c) - \sqrt{\rho} \Phi^{-1}(y)}{\sqrt{\rho_c - \rho}}
\]
## 2.8 Monte Carlo simulations

### Single-factor

- $T = 20$ years, number of firms $n_t = N = 200$, homogeneous class (PD = 200 bp), $\rho = 25\%$

- MLE1: full information estimator ($B = \Phi^{-1}(PD)$ is known)
- MLE2: limited information estimator ($B$ is estimated)

<table>
<thead>
<tr>
<th>Statistics (in %)</th>
<th>Asymptotic</th>
<th>Binomial</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MLE1</td>
<td>MLE2</td>
</tr>
<tr>
<td>mean</td>
<td>23.7</td>
<td>22.5</td>
</tr>
<tr>
<td>std error</td>
<td>5.8</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Statistics of the estimates (PD = 200 bp)

⇒ Bias for Asymptotic estimators

⇒ Downward bias for MLE2

⇒ Standard error is important
Impact of $N$ on binomial MLE

$p$ (in %)

$N = 50$
$N = 200$
$N = 500$
$N = 20000$
Impact of $N$ on asymptotic MLE
## Two risk classes

\[
\Sigma = \begin{pmatrix} \rho_1 & \rho \\ \rho & \rho_2 \end{pmatrix} = \begin{pmatrix} 20\% & 7\% \\ 7\% & 10\% \end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Statistics (in %)</th>
<th>Asymptotic</th>
<th>Binomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>$\rho_1$</td>
<td>$\rho_2$</td>
</tr>
<tr>
<td></td>
<td>19.9</td>
<td>12.9</td>
</tr>
<tr>
<td>std error</td>
<td>4.8</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>4.3</td>
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</table>

### Remark 1

*The bias seems lower than in the one risk class experiment.*
### 2.9 Estimation using S&P data

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\bar{N}_c$</th>
<th>$\bar{\mu}_c$</th>
<th>two-factor</th>
<th></th>
<th></th>
<th>Single-factor</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Aerospace / Automobile</td>
<td>301</td>
<td>2.08%</td>
<td>13.3%</td>
<td>13.9%</td>
<td></td>
<td>13.7%</td>
<td>11.6%</td>
<td></td>
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<tr>
<td>Consumer / Service sector</td>
<td>355</td>
<td>2.37%</td>
<td>12.2%</td>
<td>10.6%</td>
<td></td>
<td>12.2%</td>
<td>8.9%</td>
<td></td>
</tr>
<tr>
<td>Energy / Natural resources</td>
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<td>2.10%</td>
<td>23.2%</td>
<td>25.5%</td>
<td></td>
<td>16.2%</td>
<td>14.5%</td>
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<tr>
<td>Financial institutions</td>
<td>424</td>
<td>0.57%</td>
<td>17.0%</td>
<td>16.4%</td>
<td></td>
<td>12.0%</td>
<td>9.5%</td>
<td></td>
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<tr>
<td>Forest / Building products</td>
<td>282</td>
<td>1.90%</td>
<td>18.1%</td>
<td>18.8%</td>
<td></td>
<td>28.6%</td>
<td>31.5%</td>
<td></td>
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<td>Health</td>
<td>135</td>
<td>1.27%</td>
<td>12.9%</td>
<td>10.6%</td>
<td></td>
<td>13.1%</td>
<td>13.2%</td>
<td></td>
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<tr>
<td>High technology</td>
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<td>1.66%</td>
<td>15.0%</td>
<td>16.4%</td>
<td></td>
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<td>10.6%</td>
<td></td>
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<td>Insurance</td>
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<td>26.3%</td>
<td>34.3%</td>
<td></td>
<td>13.6%</td>
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<td></td>
<td>17.2%</td>
<td>12.0%</td>
<td></td>
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<td>Real estate</td>
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<td>1.01%</td>
<td>43.2%</td>
<td>52.4%</td>
<td></td>
<td>48.7%</td>
<td>53.0%</td>
<td></td>
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<td>22.9%</td>
<td>29.1%</td>
<td></td>
<td>27.0%</td>
<td>34.0%</td>
<td></td>
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<td>17.7%</td>
<td>11.1%</td>
<td></td>
<td>12.8%</td>
<td>10.4%</td>
<td></td>
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<tr>
<td>Utilities</td>
<td>206</td>
<td>0.43%</td>
<td>14.4%</td>
<td>18.7%</td>
<td></td>
<td>10.4%</td>
<td>17.5%</td>
<td></td>
</tr>
<tr>
<td>Inter-sector</td>
<td></td>
<td></td>
<td>7.2%</td>
<td>9.4%</td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

- we extend the study of Gordy and Heitfield (2002)
- we apply our methodology to S&P data
- there is a downward bias that one could try to correct

Application to Stress-Testing

⇒ Pillar II.
3 Default Correlations and Credit Basket Pricing/Hedging
3.1 Duality between factor models and copula models

Let \( Z_i = \sqrt{\rho}X + \sqrt{1 - \rho} \varepsilon_i \) be a latent variable with \( X \) the common factor and \( \varepsilon_i \) the specific factor. We have

\[
D_i(t) = 1 \iff Z_i < B_i = \Phi^{-1}(PD_i(t))
\]

Let \( \Sigma = C(\rho) \) be the constant correlation matrix. We have

\[
S(t_1, \ldots, t_I) = \Pr \{ \tau_1 > t_1, \ldots, \tau_I > t_I \}
\]
\[
= \Pr \left\{ Z_1 > \Phi^{-1}(PD_1(t_1)), \ldots, Z_I > \Phi^{-1}(PD_I(t_I)) \right\}
\]
\[
= C(1 - PD_1(t_1), \ldots, 1 - PD_I(t_I); \Sigma)
\]
\[
= C(S_1(t_1), \ldots, S_I(t_I); \Sigma)
\]

where \( C \) is the Normal copula.

**Remark 2** Let \( \tau_1 \) et \( \tau_2 \) be two default times with the joint survival function \( S(t_1, t_2) = \bar{C}(S_1(t_1), S_2(t_2)) \). We have

\[
S_1(t | \tau_2 = t^*) = \partial_2 \bar{C}(S_1(t), S_2(t^*)).
\]

If \( C \neq C_{\perp} \), default probability of one firm changes when the other has defaulted.
Example 1  The next figures show jumps of the hazard function
\( \lambda(t) = f(t) / S(t) \) of the annual S&P transition matrix. With a Normal copula and \( \Sigma = C_I(\rho) \), we have

\[
S_1(t \mid \tau_2 = t^*) = \Phi \left( \frac{\Phi^{-1}(S_1(t)) - \rho \Phi^{-1}(S_2(t^*))}{\sqrt{1 - \rho^2}} \right)
\]
Hazard rate of the ratings
A firm rated AAA defaults - \( \rho = 5\% \)
A firm rated AAA defaults $- \rho = 50\%$
A firm rated BB defaults \(- \rho = 50\%\)
A firm rated CCC defaults $- \rho = 50\%$
3.2 Default correlations and spread jumps

We assume an exponential default model with intensity $\lambda$. Let $s$ and $R$ be the spread of the CDS and the recovery rate. We have

$$s = \lambda (1 - R)$$

It comes that the default probability is

$$PD(t) = 1 - \exp \left( -\frac{s}{1 - R} t \right)$$

The conditional probability of the first name given that the second name has defaulted at time $t^*$ is then

$$PD_1(t \mid \tau_2 = t^*) = \partial_2 C(PD_1(t), PD_2(t^*)) \quad (t \geq t^*)$$

we deduce that the spread of the first name after the default of the second name becomes

$$s_1(t \mid \tau_2 = t^*, \tau_1 \geq t^*) = -\frac{(1 - R_1)}{(t - t^*)} \ln \left( 1 - PD_1(t \mid \tau_2 = t^*, \tau_1 \geq t^*) \right)$$
Correlation implied to Ahold default

<table>
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<th>Start</th>
<th>Wide</th>
<th>Jump</th>
<th>Correlation implied</th>
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<tbody>
<tr>
<td></td>
<td>Recovery</td>
<td>28/10/2002</td>
<td>28/02/2003</td>
<td>Normal</td>
</tr>
<tr>
<td><strong>AHO</strong>LD</td>
<td>40%</td>
<td>235</td>
<td>1205</td>
<td>970</td>
</tr>
<tr>
<td>CASINO</td>
<td>40%</td>
<td>235</td>
<td>152</td>
<td>-83</td>
</tr>
<tr>
<td>SAINSBURY</td>
<td>40%</td>
<td>48</td>
<td>95</td>
<td>47</td>
</tr>
<tr>
<td>CARREFOUR</td>
<td>40%</td>
<td>60</td>
<td>47</td>
<td>-13</td>
</tr>
<tr>
<td>KROGER</td>
<td>40%</td>
<td>127,5</td>
<td>108</td>
<td>-19,5</td>
</tr>
<tr>
<td>SAFEWAY</td>
<td>40%</td>
<td>66,5</td>
<td>145</td>
<td>78,5</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Start</th>
<th>Wide</th>
<th>Jump</th>
<th>Correlation implied</th>
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<tbody>
<tr>
<td></td>
<td>Recovery</td>
<td>20/02/2003</td>
<td>28/02/2003</td>
<td>Normal</td>
</tr>
<tr>
<td><strong>AHO</strong>LD</td>
<td>40%</td>
<td>195</td>
<td>1205</td>
<td>1010</td>
</tr>
<tr>
<td>CASINO</td>
<td>40%</td>
<td>135</td>
<td>160</td>
<td>25</td>
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<td>SAINSBURY</td>
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<td>95</td>
<td>27</td>
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<td>CARREFOUR</td>
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<td>47</td>
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<td>KROGER</td>
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<td>90</td>
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<td>SAFEWAY</td>
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<td>195</td>
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## Correlation implied to Worldcom default

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<tr>
<th></th>
<th>Start Recovery</th>
<th>Wide 05/07/2001</th>
<th>Jump 01/05/2002</th>
<th>Correlation implied Normal</th>
<th>Correlation implied T4</th>
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<tr>
<td>WORLDCOM</td>
<td>15%</td>
<td>165</td>
<td>1700</td>
<td>1535</td>
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<tr>
<td>TELECOM</td>
<td>15%</td>
<td>165</td>
<td>130</td>
<td>-35</td>
<td>-5</td>
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<tr>
<td>TELEFONI</td>
<td>15%</td>
<td>95</td>
<td>80</td>
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<tr>
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<td>490</td>
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<td>TELECOM</td>
<td>15%</td>
<td>185</td>
<td>345</td>
<td>160</td>
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## Correlation implied to TXU Corp. default

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<th>Start Recovery</th>
<th>Wide 13/08/2002</th>
<th>Jump 10/10/2002</th>
<th>Correlation implied Normal</th>
<th>Correlation implied T4</th>
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<td>TXU Corp.</td>
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<td>1250</td>
<td>800</td>
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<td>400</td>
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<td>7</td>
</tr>
<tr>
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<td>225</td>
<td>55</td>
<td>5</td>
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<tr>
<td>SUEZ</td>
<td>40%</td>
<td>105</td>
<td>130</td>
<td>25</td>
<td>4</td>
</tr>
<tr>
<td>AMELECOPO</td>
<td>40%</td>
<td>380</td>
<td>925</td>
<td>545</td>
<td>20</td>
</tr>
<tr>
<td>RWEAG</td>
<td>40%</td>
<td>67</td>
<td>98</td>
<td>31</td>
<td>6</td>
</tr>
<tr>
<td>ENEL</td>
<td>40%</td>
<td>68</td>
<td>87</td>
<td>19</td>
<td>4</td>
</tr>
</tbody>
</table>

---

Interpretation and Estimation of Default Correlations
Default Correlations and Credit Basket Pricing/Hedging

3-5
3.3 Trac-X implied correlation

Model: \( Z_i = \beta X + \sqrt{1 - \beta^2} \varepsilon_i. \Rightarrow \beta = \sqrt{\rho}. \)

Expectation of losses (5Y maturity)

Value of the floating leg (5Y maturity)

Implied correlation

Attachment points: \( 0 = A_0 < A_1 < \ldots < A_M \leq 1 \)

Marked spread: \( s(A_{i-1}, A_i)^{obs} \) for the tranche \([A_{i-1}, A_i]\)

The implied correlation for the tranche \([A_{i-1}, A_i]\) verify:

\[
\forall i \neq 1, \quad s(A_{i-1}, A_i, \rho(A_{i-1}, A_i)) = s(A_{i-1}, A_i)^{obs},
\]

(correction for the equity tranche because of upfront payment)
Pourcentage de pertes

- Tranche equity: 0% -> 3%
- Tranche mezzanine: 3% -> 12%
- Tranche senior: 12% -> 100%

Correlation
Jambe Variable

correlation (%)

- tranche equity: 0%->3%
- tranche mezzanine: 3%->12%
- tranche senior: 12%->100%
Example: Trac-X Euro 02/06/2004.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Upfront payment</th>
<th>Running spread (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>3%</td>
<td>34%</td>
<td>500</td>
</tr>
<tr>
<td>3%</td>
<td>6%</td>
<td>279</td>
<td></td>
</tr>
<tr>
<td>6%</td>
<td>9%</td>
<td>114</td>
<td></td>
</tr>
<tr>
<td>9%</td>
<td>12%</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>12%</td>
<td>22%</td>
<td>23</td>
<td></td>
</tr>
</tbody>
</table>

**Implied correlation of Trac-X**

**Loss distribution for the second tranche**

**Base correlation of Trac-X**

**Implied correlation of Trac-X (T9 copula)**
correlation implicite

tranches
% pertes sur la tranche

probabilité

corrélation implicite=0,04

corrélation implicite=0,75
point d'attachement haut (%)

base correlation

0 3 6 9 12 15 18 21 24
La corrélation implicite pour les tranche de taux d'intérêt suivants :
- 0->3% : 0,1
- 3->6% : 0,8
- 6->9% : 0,1
- 9->12% : 0,1
- 12->22% : 0,1
Gaussian factors with three types of names (spread = 50 bp, 150 bp and 250 bp).

Three structures of correlation:

\[ \Sigma_1 = \begin{pmatrix} 1 & 0.3^2 & 0.3^2 \\ 0.3^2 & 1 & 0.3^2 \\ 0.3^2 & 0.3^2 & 1 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 1 & 0.1 \times 0.3 & 0.1 \times 0.5 \\ 0.1 \times 0.3 & 1 & 0.3 \times 0.5 \\ 0.1 \times 0.5 & 0.3 \times 0.5 & 1 \end{pmatrix} \]

\[ \Sigma_3 = \begin{pmatrix} 1 & 0.5 \times 0.3 & 0.1 \times 0.5 \\ 0.5 \times 0.3 & 1 & 0.3 \times 0.1 \\ 0.1 \times 0.5 & 0.3 \times 0.1 & 1 \end{pmatrix} \]
Corrélation implicite

*Betans corrélés*

*Betans anticorrélés*

*Betas=0,3*

Tranches

0->3% 3->6% 6->9% 9->12% 12->22% 22->100%
3.4 Implications for CDO pricing

Implied correlation = not useful for CDO pricing.

Implied correlation of CDO ≠ Implied correlation of spread of two equity indices

A new dimension = TRAC-X PORTFOLIO.

What is the meaning of implied correlation?
⇒ the mathematical root of an equation