Measuring Efficiency of Exchange Traded Funds[1]

An Issue of Performance, Tracking Error and Liquidity

Thierry Roncalli[†]

[†]Evry University & Lyxor Asset Management, France

Joint work with Marlène Hassine

The full paper can be downloaded from SSRN:

http://ssrn.com/abstract=2212596

February 5, 2013

[1]The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.

Main results

1. Current rating systems are not adapted to index funds.
2. The information ratio could not be used to measure the performance of trackers.
3. The efficiency measure of an exchange traded fund is a function of three main parameters: excess return, tracking error volatility and liquidity spread:

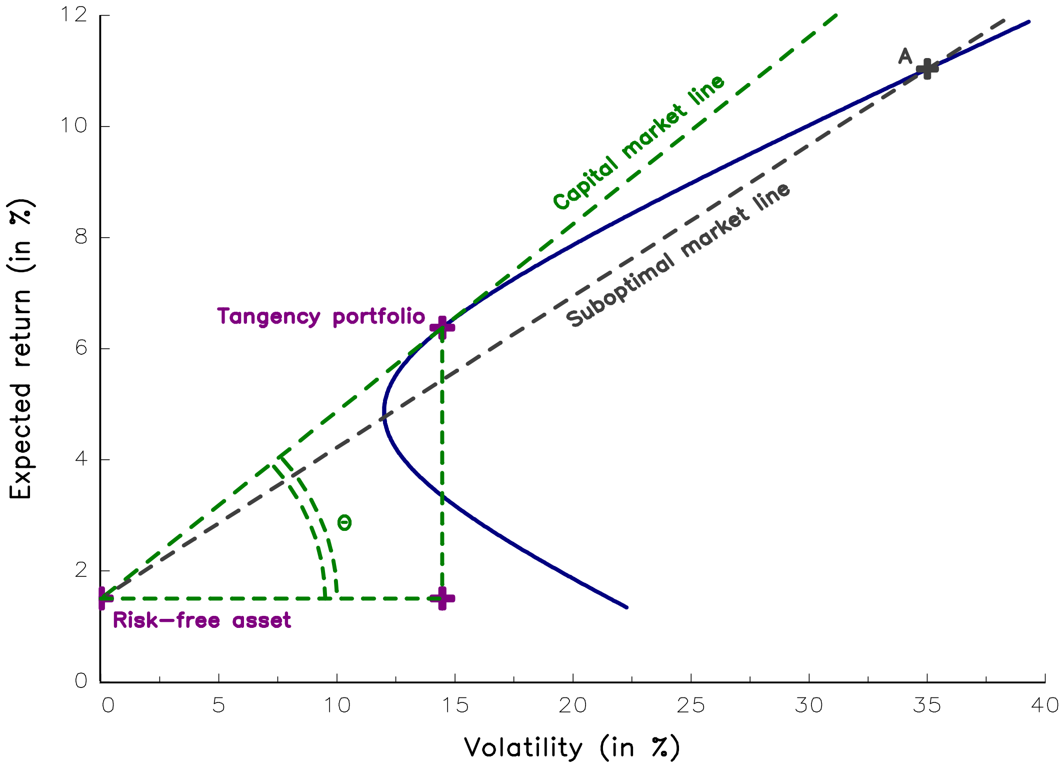
$$\zeta_{\alpha}(x \mid b) = \mu(x \mid b) - s(x \mid b) - \Phi^{-1}(\alpha)\,\sigma(x \mid b)$$

4. The efficiency measure is the right statistic to measure the performance of trackers.
5. For institutional investors and active managers, the efficiency measure is principally driven by the liquidity:

$$\lim_{m \to \infty} \zeta_{\alpha}(x \mid b) = -m \cdot s_N(x \mid b)$$

Outline

Why index funds?



- Main result of Sharpe (1964): Tangency Portfolio = Market (Capitalization) Portfolio.
- Jensen (1968): No alpha in mutual funds.
- Wells Fargo Bank (1971): First (private) index fund.
- Wells Fargo/American National Bank in Chicago (1973): First S&P 500 index fund.
- Carhart (1997): No persistence in mutual fund performance.

Performance or efficiency?

Fund picking process

- Current rating systems = measure the alpha and its persistence with respect to the right risk factors
  1. How to define the universe of funds?
  2. How to measure the alpha?

- Fund picking is different with passive management.
  1. The categorization of funds is not an issue.
  2. α is not the relevant measure to assess the performance of index funds.

What is a good tracker?

A fund that presents no risk wrt. to the index.

Portfolio optimization with a benchmark

We consider a universe of n assets. μ and Σ are the vector of expected returns and the covariance matrix of asset returns. We note b the benchmark (or the index) and x the portfolio. The tracking error is:

$$e = R(x) - R(b) = (x-b)^\top R$$

The expected tracking error is then:

$$\mu(x \mid b) = (x-b)^\top \mu$$

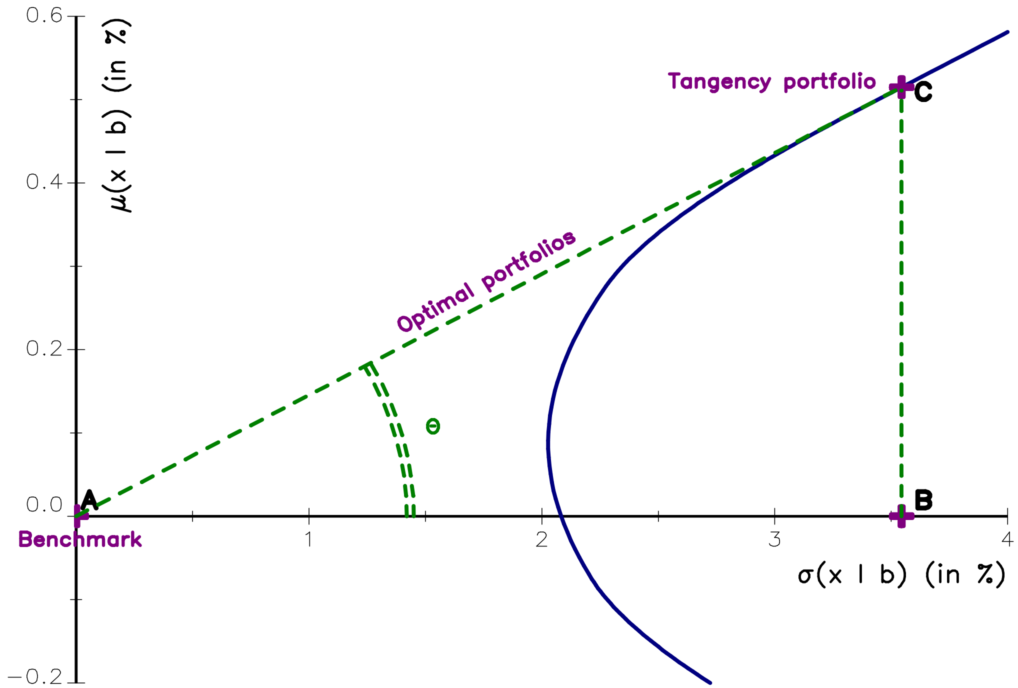
whereas tracking error volatility is equal to:

$$\sigma(x \mid b) = \sqrt{(x-b)^\top \Sigma (x-b)}$$

The objective of the investor is to maximize the expected tracking error with a constraint on the tracking error volatility:

$$\begin{aligned} x^\star &= \arg\max (x-b)^\top \mu \\ \text{u.c.} &\quad \mathbf{1}^\top x = 1 \text{ and } \sigma(e) \leq \sigma^\star \end{aligned}$$

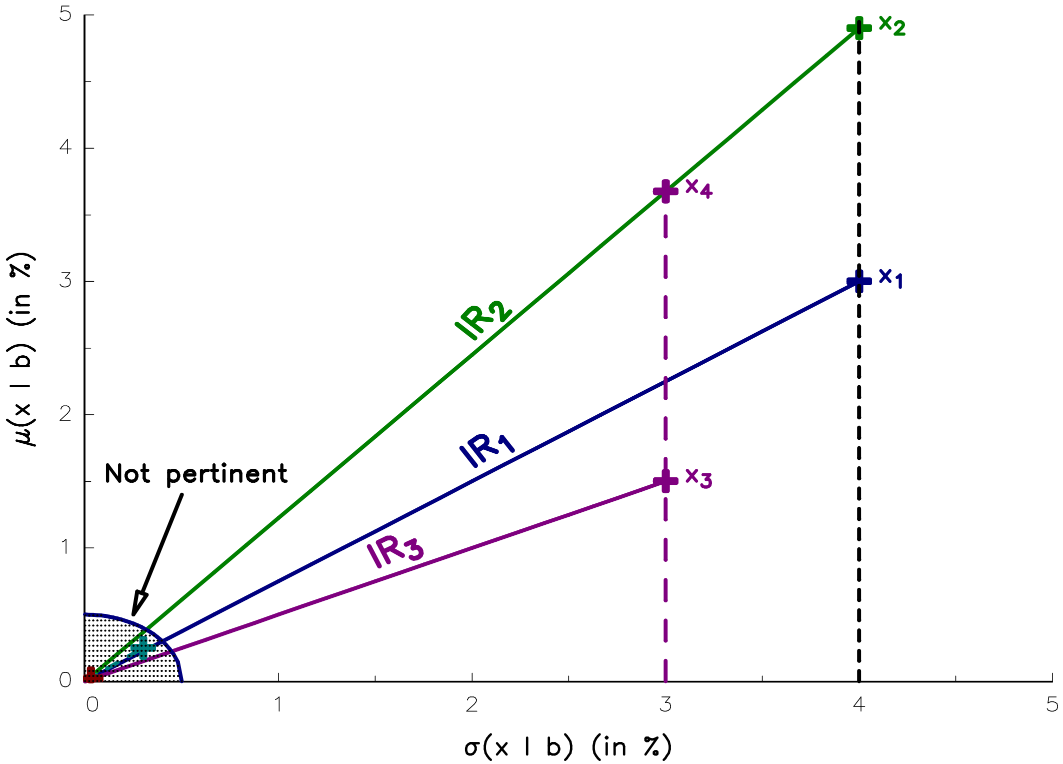
The geometry of the information ratio



The tangency portfolio is the portfolio that maximizes the information ratio:

$$\tan\theta = \frac{BC}{AB} = \frac{\mu(x \mid b)}{\sigma(x \mid b)} = \mathrm{IR}(x \mid b)$$

Comparing benchmarked portfolios



- $x_2 \succ x_1$ because it has a better excess-return performance
- $x_2 \succ x_3$ because $x_4 \succ x_3$ with:

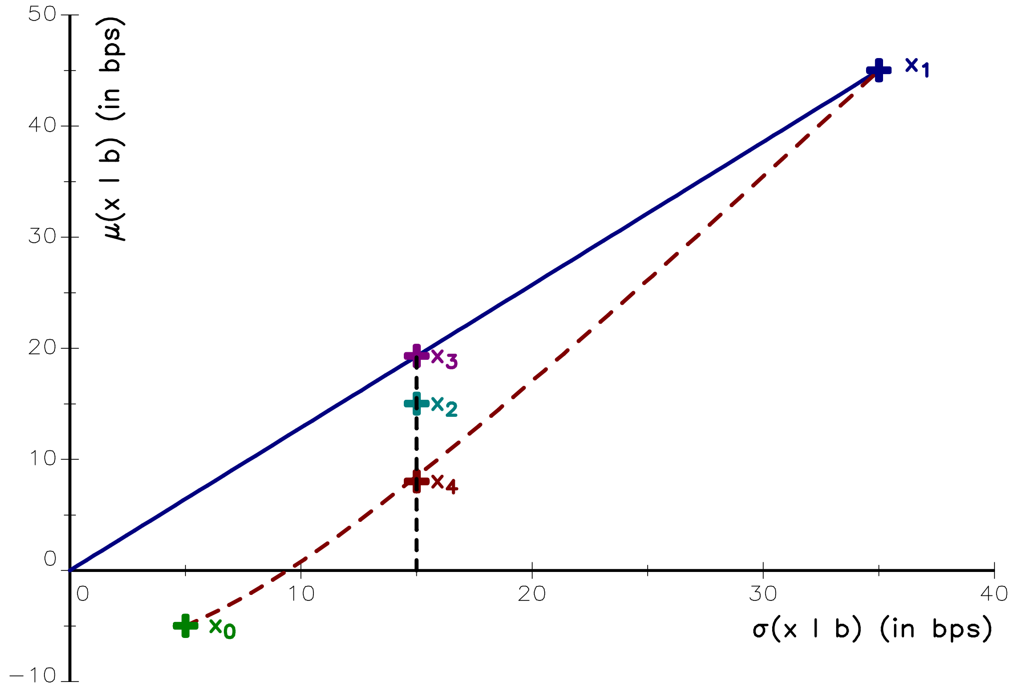
$$\begin{cases} x_4 = (1-\alpha)\, b + \alpha x_2 \\ \alpha = \sigma(x_3 \mid b) / \sigma(x_2 \mid b) \end{cases}$$

Fundamental rule of benchmarked portfolios

$$x \succ y \Leftrightarrow \mathrm{IR}(x \mid b) \geq \mathrm{IR}(y \mid b)$$

The irrelevance of the information ratio for trackers

- Using the previous rule, we have $x_3 \succ x_2 \Rightarrow x_1 \succ x_2$.
- The problem is that we cannot replicate the benchmark exactly. In real life, we need to use a tracker x_0 to proxy the benchmark.
- In the real life, $x_3 \equiv x_4$ and $x_2 \succ x_1$.



- For benchmarked funds with low tracking-error volatility:

$$\mathrm{IR}(x \mid b) > \mathrm{IR}(y \mid b) \nRightarrow x \succ y$$

- If we consider the information ratio, investors will never chose the tracker x_0!

The framework

The two-period trading model

- The investor buy the tracker x at time $t=0$ and sells it at time $t=1$. Note the corresponding tracking error e. The relative PnL of the investor with respect to the benchmark b is:

$$\Pi(x \mid b) = e - s(x \mid b)$$

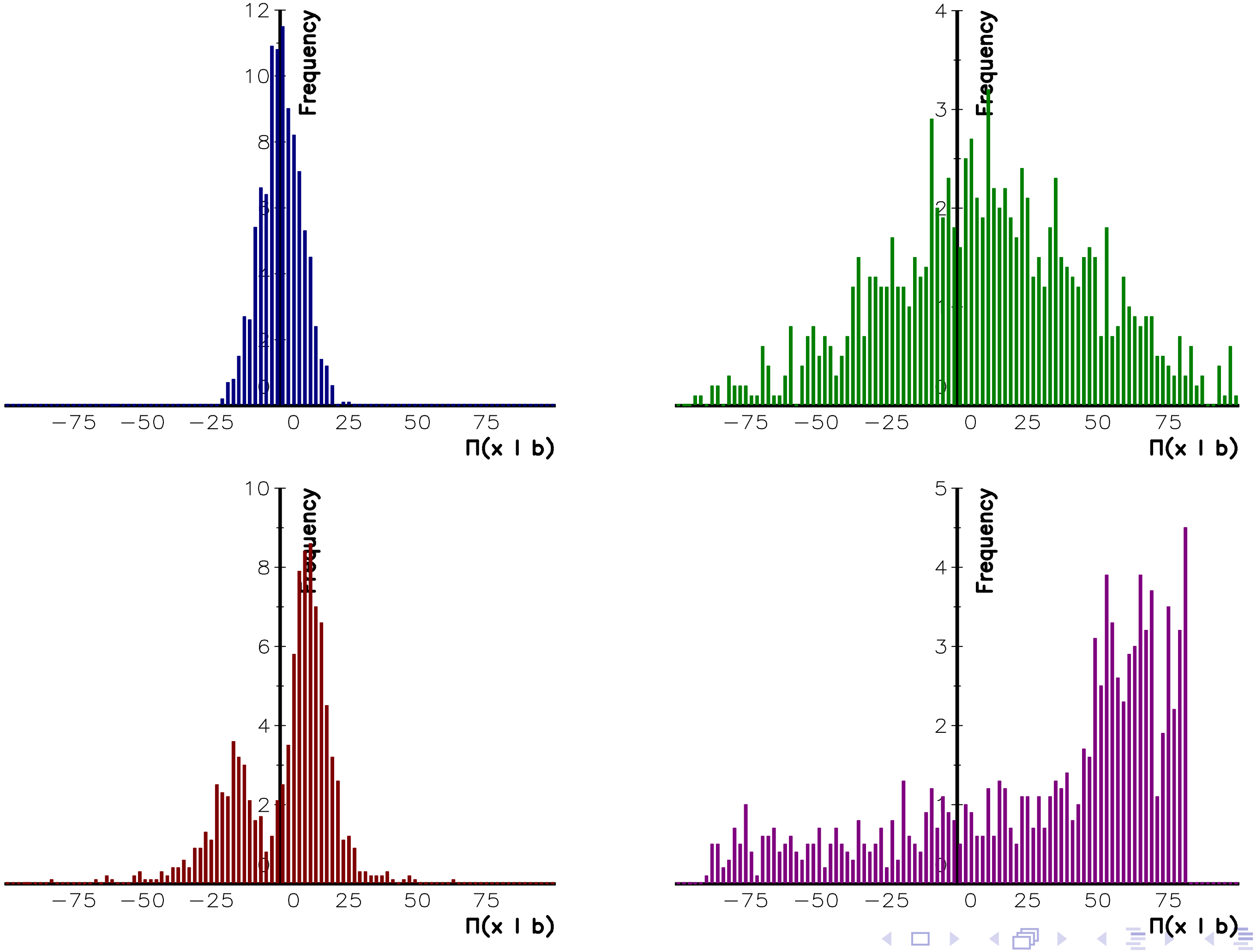
  where $s(x \mid b)$ is the bid-ask spread of the tracker.
- The loss $\mathscr{L}(x \mid b)$ of the investor is defined as follows:

$$\mathscr{L}(x \mid b) = -\Pi(x \mid b)$$

- The tracker efficiency measure is a risk measure applied to the loss function $\mathscr{L}(x \mid b)$ of the investor.

Illustration

What is your best tracker?



Definition of the efficiency measure

We propose to use value-at-risk, which is today commonly accepted as a standard risk measure. In this case, the efficiency measure $\zeta_\alpha(x \mid b)$ is defined as follows:

$$\zeta_\alpha(x \mid b) = -\{\inf \zeta : \Pr\{\mathscr{L}(x \mid b) \leq \zeta\} \geq \alpha\}$$

Definition

The efficiency measure $\zeta_\alpha(x \mid b)$ of the tracker x with respect to the benchmark b corresponds to:

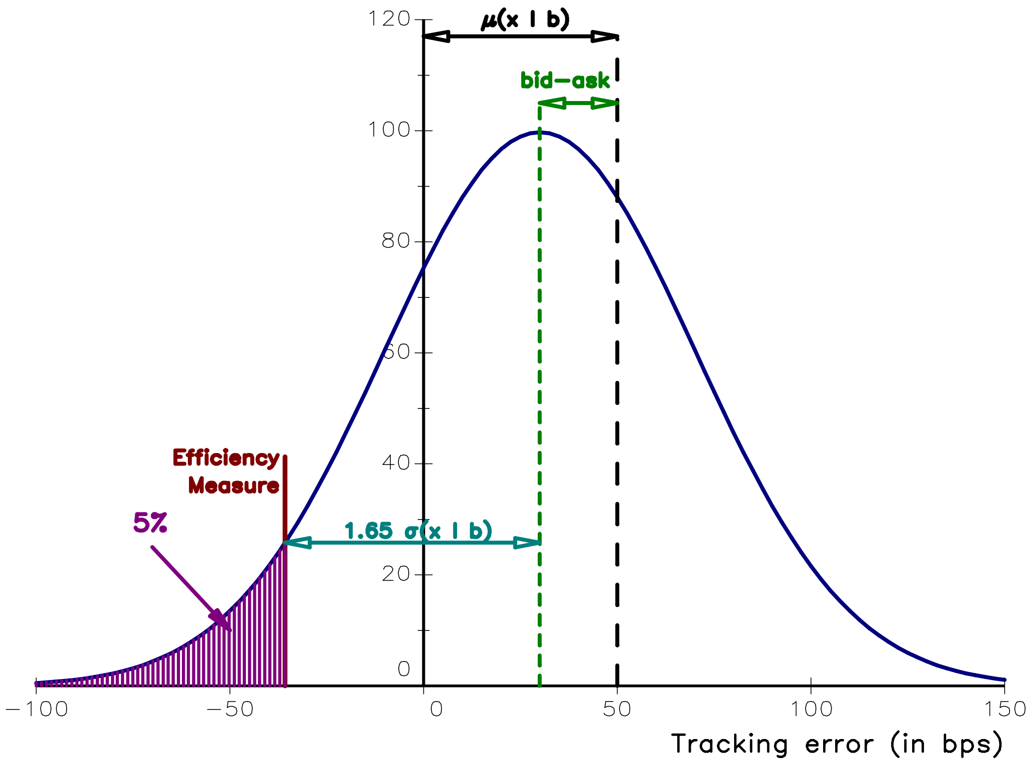
$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - \Phi^{-1}(\alpha)\,\sigma(x \mid b)$$

where $\mu(x \mid b)$ is the expected value of the tracking error, $s(x \mid b)$ is the bid-ask spread and $\sigma(x \mid b)$ is the volatility of the tracking error$^{(*)}$.

(*) IOSCO terminology: $\mu(x \mid b)$ = Tracking Difference (TD) & $\sigma(x \mid b)$ = Tracking Error (TE).

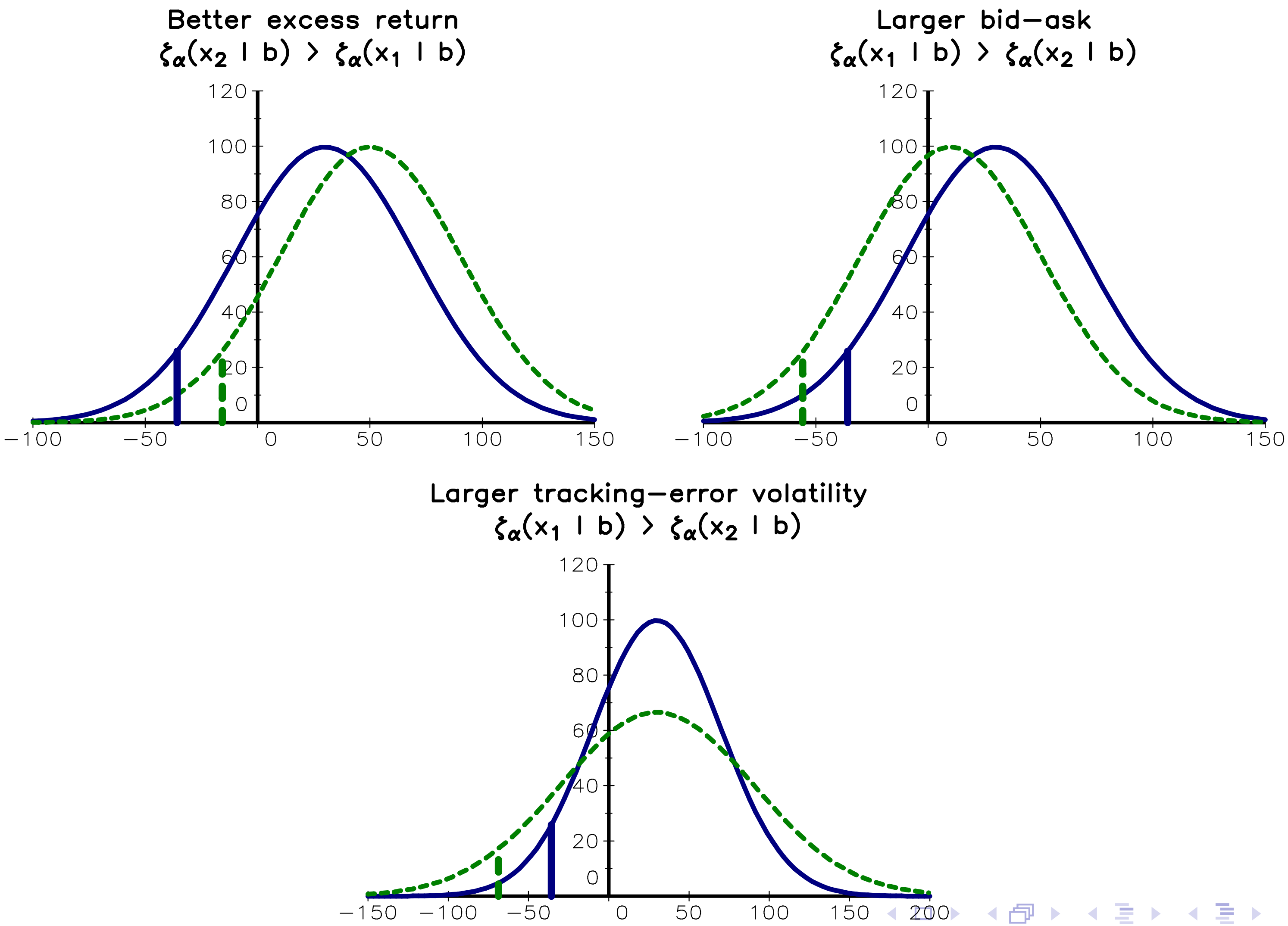
Computing the efficiency measure

We assume that $\mu(x \mid b) = 50$ bps, $\sigma(x \mid b) = 40$ bps and $s(x \mid b) = 20$ bps. The confidence level α is set to 95%.



$\Rightarrow$ The efficiency measure of the tracker $\zeta_\alpha(x \mid b)$ is -35.79 bps.

Impact of parameters on the efficiency measure



Impact of parameters on the efficiency measure

- $\alpha = 95\%$.
- Study period: Nov., 30$^{\text{th}}$ 2011 – No., 30$^{\text{th}}$ 2012.
- We compute the best spread of the first limit order for each listing place and each trading day t. The daily spread is then the weighted average by considering the daily volume of the different listing places. The spread $s(x \mid b)$ is therefore the average of daily spreads.
- We rebuild the net asset value of the ETF by incorporating dividends in order to compute the excess return $\mu(x \mid b)$ and the tracking error volatility $\sigma(x \mid b)$.
- We consider € as the default currency.

Results

Tracker	$\hat{\mu}(x)$	$\hat{\mu}(x \mid b)$	$\hat{s}(x \mid b)$	$\hat{\sigma}(x)$	$\hat{\sigma}(x \mid b)$	$\zeta_\alpha(x \mid b)$
Amundi	15.29	62.56	9.84	22.33	11.97	32.98
db X-trackers	15.33	65.97	12.27	22.86	7.31	41.64
iShares (DE)	15.05	37.88	7.96	21.62	56.54	−63.38
iShares	15.25	58.46	10.39	21.92	19.62	15.70
Lyxor	15.30	63.51	8.48	22.01	14.89	30.47
Source	14.90	23.51	15.38	22.23	7.25	−3.83
Eurostoxx 50	14.67			22.09		

Tracker	$\hat{\mu}(x)$	$\hat{\mu}(x \mid b)$	$\hat{s}(x \mid b)$	$\hat{\sigma}(x)$	$\hat{\sigma}(x \mid b)$	$\zeta_\alpha(x \mid b)$
Amundi	19.49	9.19	16.97	12.59	3.14	−12.97
Credit Suisse	19.57	16.99	18.23	12.50	4.63	−8.88
db X-trackers	19.56	16.04	18.26	12.79	4.65	−9.90
HSBC	19.68	28.20	20.58	12.68	3.45	1.92
iShares	19.34	−6.10	7.45	12.56	4.90	−21.63
Lyxor	19.60	19.87	13.56	12.59	0.98	4.69
Source	19.34	−5.30	17.81	12.87	1.78	−26.04
UBS	19.40	0.02	41.13	12.85	0.59	−42.09
S&P 500	19.40			12.76		

Results

Tracker	$\hat{\mu}(x)$	$\hat{\mu}(x \mid b)$	$\hat{s}(x \mid b)$	$\hat{\sigma}(x)$	$\hat{\sigma}(x \mid b)$	$\zeta_\alpha(x \mid b)$
Amundi	17.40	−20.75	23.91	10.32	3.67	−50.72
Commerzbank	17.42	−18.25	18.72	10.83	3.56	−42.85
db X-trackers	17.36	−24.46	13.20	10.30	25.18	−79.20
iShares	17.18	−42.08	13.75	10.14	50.80	−139.65
Lyxor	17.37	−23.09	11.93	10.09	1.55	−37.58
Source	17.08	−52.64	24.83	10.31	1.69	−80.25
UBS	17.36	−23.98	31.25	10.39	14.51	−79.16
MSCI World	17.60			10.18		

Tracker	$\hat{\mu}(x)$	$\hat{\mu}(x \mid b)$	$\hat{s}(x \mid b)$	$\hat{\sigma}(x)$	$\hat{\sigma}(x \mid b)$	$\zeta_\alpha(x \mid b)$
Credit Suisse	13.20	−205.79	30.05	13.18	150.68	−484.46
db X-trackers	14.13	−112.34	15.85	13.07	12.83	−149.35
iShares	14.18	−107.56	17.90	13.07	160.21	−389.80
Lyxor	14.45	−80.01	20.72	13.07	14.87	−125.28
Source	14.16	−109.20	50.30	13.22	3.96	−166.02
MSCI EM	15.25			13.12		

The case of different benchmarks

- Problem
  ETF providers do not choose the same benchmark to give access to an asset class (e.g. Japanese equities with Topix and MSCI Japan).
- Answer
  Use the Amenc and Martellini (2002) PCA method to build a reference index.

Tracker	$\hat{\mu}(x)$	$\hat{\mu}(x \mid b)$	$\hat{s}(x \mid b)$	$\hat{\sigma}(x)$	$\hat{\sigma}(x \mid b)$	$\zeta_\alpha(x \mid b)$
db X-trackers	−0.72	−35.87	15.82	16.58	256.73	−475.29
iShares	−0.76	−40.64	19.98	15.97	66.84	−170.90
Lyxor	−1.29	−92.96	14.98	16.10	62.81	−211.58
Source	−0.67	−31.40	35.11	16.58	65.30	−174.25
Japanese equities	−0.36			15.84		

Choosing another risk measure

- Semi-variance:

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - 1.65 \cdot \sqrt{2} \cdot \sigma_-(x \mid b)$$

- Historical value-at-risk:

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - \mathbf{F}_0^{-1}(\alpha)$$

where $\mathbf{F}_0$ is the probability distribution of centered tracking errors.

- Cornish-Fisher value-at-risk

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - z_\alpha \cdot \sigma(x \mid b)$$

where z_α depends on the skewness and kurtosis of centered tracking errors.

- Expected shortfall

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - s(x \mid b) - \mathbb{E}\left[L_0 \mid L_0 \geq \mathbf{F}_0^{-1}(\alpha)\right]$$

where L_0 is the (random) centered tracking error.

Choosing another risk measure

Results

Eurostoxx 50

Tracker	$\sigma(x \mid b)$	$\sigma_-(x \mid b)$	$\mathrm{VaR}_\alpha(x \mid b)$	$\mathrm{ES}_\alpha(x \mid b)$	$\mathrm{CF}_\alpha(x \mid b)$
Amundi	32.98	44.72	47.75	47.25	62.54
db X-trackers	41.64	44.12	45.90	41.02	47.15
iShares (DE)	−63.38	−64.32	−59.40	−102.51	−62.72
iShares	15.70	20.38	34.77	12.21	27.93
Lyxor	30.47	41.63	47.97	38.26	61.48
Source	−3.83	3.74	5.00	4.63	19.17

S&P 500

Tracker	$\sigma(x \mid b)$	$\sigma_-(x \mid b)$	$\mathrm{VaR}_\alpha(x \mid b)$	$\mathrm{ES}_\alpha(x \mid b)$	$\mathrm{CF}_\alpha(x \mid b)$
Amundi	−12.97	−10.64	−10.24	−10.55	−8.04
Credit Suisse	−8.88	−8.26	−6.60	−10.37	−7.35
db X-trackers	−9.90	−10.13	−8.88	−11.77	−10.02
HSBC	1.92	2.92	4.24	2.23	3.73
iShares	−21.63	−21.15	−19.10	−23.59	−20.23
Lyxor	4.69	4.73	4.88	4.25	4.71
Source	−26.04	−25.67	−25.50	−26.00	−25.37
UBS	−42.09	−41.98	−41.93	−42.09	−41.95

The liquidity issue

Issue with the previous spread definition

Institutional investors buy or sell a notional N, that can not generally be executed via the best first limit orders.

Definition of the liquidity spread

We then consider another spread measure $s_N(x \mid b)$ corresponding to intraday spreads weighted by the duration between two ticks for a given notional.

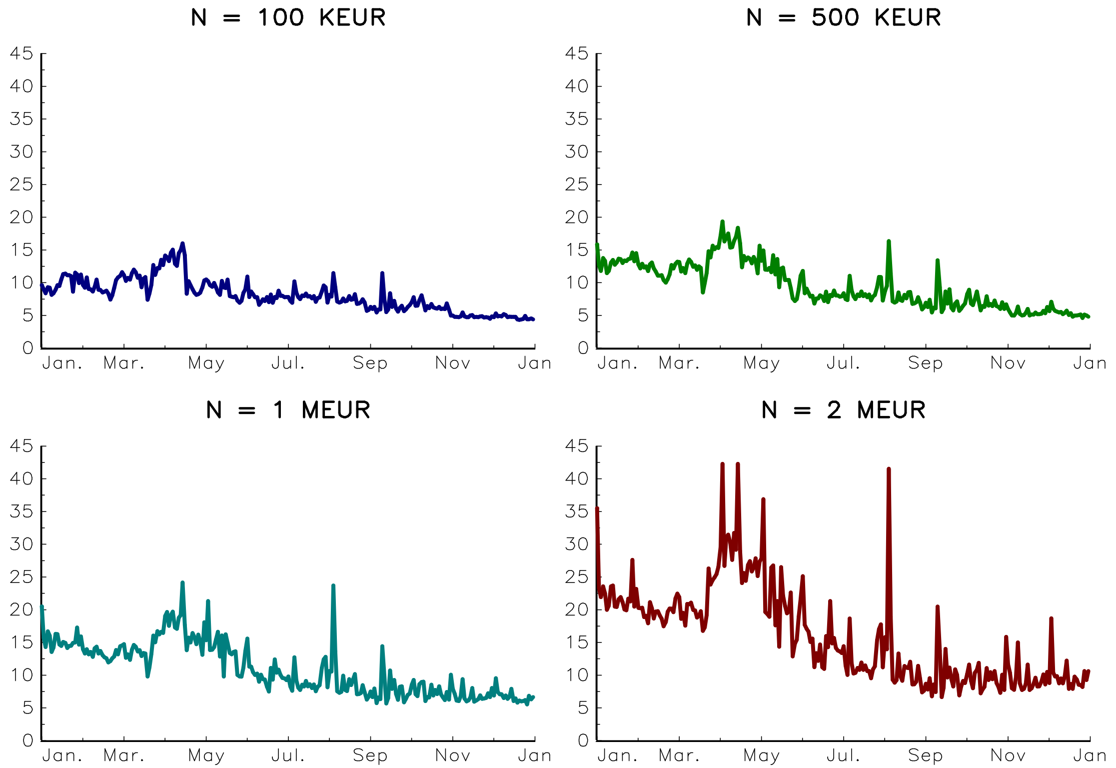
We have:

$$s_N(x \mid b) = \frac{\sum_{j=\text{open}}^{\text{close}} s_j \left(t_{j+1} - t_j\right)}{\sum_{j=\text{open}}^{\text{close}} \left(t_{j+1} - t_j\right)}$$

where s_j is the spread of the j^{th} tick in order to trade a notional N and $t_{j+1} - t_j$ the elapsed time between two consecutive ticks.

The liquidity issue

Evolution of the spread of the Amundi Eurostoxx 50 tracker

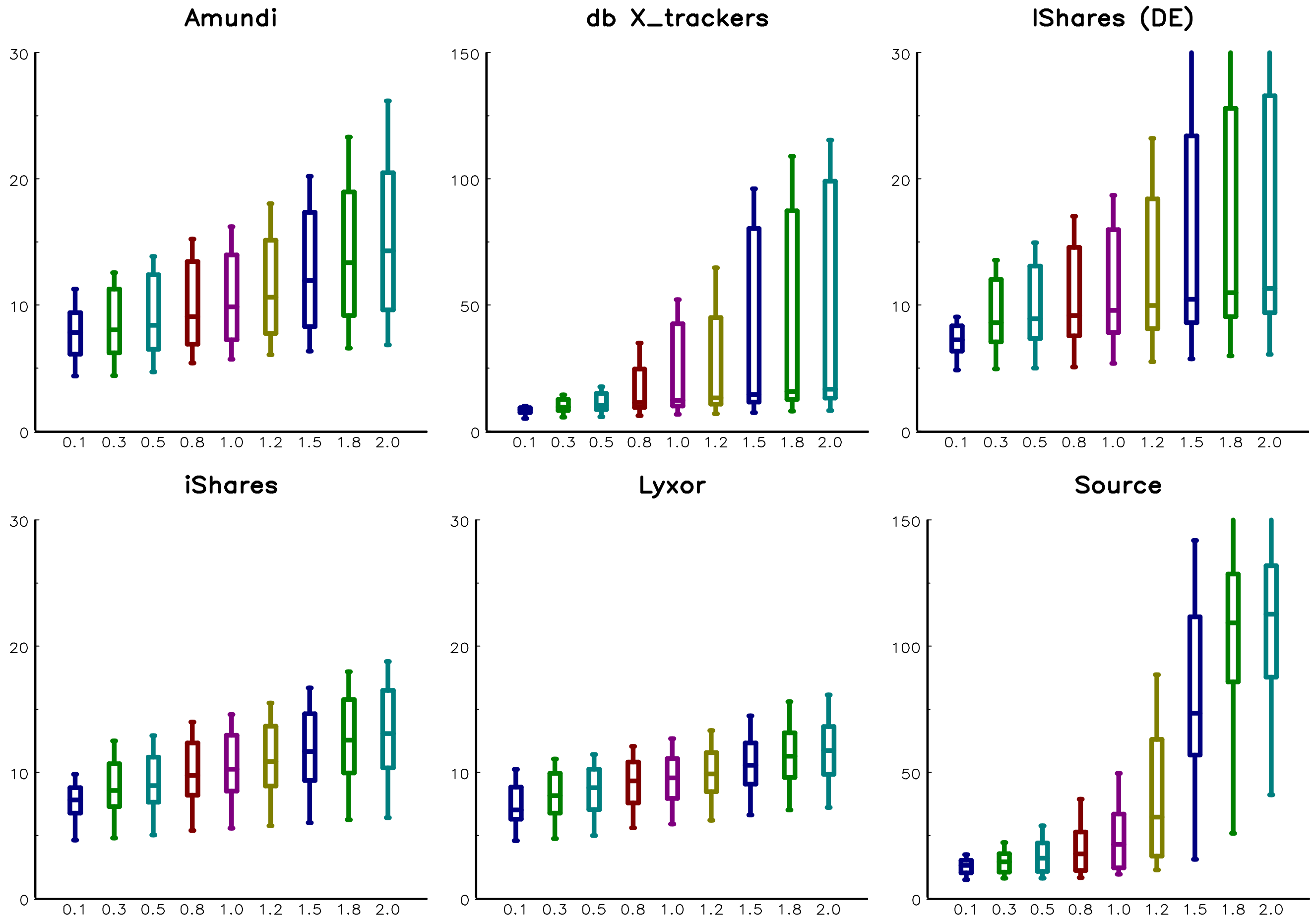


The liquidity spread increases with the notional:

$$N_1 \geq N_2 \Rightarrow s_{N_1}(x \mid b) \geq s_{N_2}(x \mid b)$$

The liquidity issue

Boxplot(*) of Eurostoxx 50 ETF spreads



(*) The boxplot indicates the minimum value, the quartile range, the median and the last decile.

The liquidity issue

Impact of the liquidity on the efficiency measure (Eurostoxx 50)

The efficiency measure becomes:

$$\zeta_{\alpha}(x \mid b) = \mu(x \mid b) - \mathbf{F}_{s_N}^{-1}(\alpha) - \Phi^{-1}(\alpha)\,\sigma(x \mid b)$$

where $\mathbf{F}_{s_N}$ is the distribution of the liquidity spread s_N.

Tracker	100 KEUR 95%	1 MEUR 95%	2 MEUR 95%
Amundi	30.19	25.45	15.00
db X-trackers	43.28	−2.39	−66.68
iShares (DE)	−65.05	−77.04	−98.73
iShares	15.72	10.33	5.31
Lyxor	27.89	24.98	19.97
Source	−8.80	−64.29	−193.50

$\Rightarrow$ The efficiency measure is not the same for retail investors and institutional investors!

The liquidity issue

What about active managers?

Generalization to the multi-period model

If we consider a multi-period model with m trades, the performance measure becomes:

$$\zeta_\alpha(x \mid b) = \mu(x \mid b) - m \cdot s_N(x \mid b) - \Phi^{-1}(\alpha)\,\sigma(x \mid b)$$

This formula highlights the importance of liquidity for active managers.

Remark

A highly active manager will only be interested in the spread measure because:

$$\lim_{m \to \infty} \zeta_\alpha(x \mid b) = -m \cdot s_N(x \mid b)$$

Conclusion

1. Current rating systems are not adapted to index funds.
2. The information ratio could not be used to measure the performance of trackers.
3. The efficiency measure of an exchange traded fund is a function of three main parameters: excess return, tracking error volatility and liquidity spread:

$$\zeta_{\alpha}(x \mid b) = \mu(x \mid b) - s(x \mid b) - \Phi^{-1}(\alpha)\,\sigma(x \mid b)$$

4. The efficiency measure is the right statistic to measure the performance of trackers.
5. For institutional investors and active managers, the efficiency measure is principally driven by the liquidity:

$$\lim_{m \to \infty} \zeta_{\alpha}(x \mid b) = -m \cdot s_N(x \mid b)$$

Analytical expression of the spread $s_N(x \mid b)$

We define the daily spread $s_N(x \mid b)$ as a weighted average of intraday spreads:

$$s_N(x \mid b) = \frac{\sum_{j=\text{open}}^{\text{close}} s_j \left(t_{j+1} - t_j\right)}{\sum_{j=\text{open}}^{\text{close}} \left(t_{j+1} - t_j\right)}$$

where s_j is the spread of the j^{th} tick and $t_{j+1} - t_j$ the elapsed time between two consecutive ticks:

$$s_j = c_j \frac{\left(\bar{P}_j^+ - \bar{P}_j^-\right)}{\bar{P}_j^0}$$

We have also:

$$\bar{P}_j^\bullet = \frac{\sum_{k=1}^K \bar{Q}_{j,k}^\bullet P_{j,k}^\bullet}{\sum_{k=1}^K \bar{Q}_{j,k}^\bullet}$$

where $P_{j,k}^+$ (resp. $P_{j,k}^-$) is the ask (or bid) price at t_j for the k^{th} limit order. The average mid price $\bar{P}_j^0$ corresponds to:

$$\bar{P}_j^0 = \frac{\bar{P}_j^+ + \bar{P}_j^-}{2}$$

Analytical expression of the spread $s_N(x \mid b)$

The quantity $\bar{Q}^{+}_{j,k}$ and $\bar{Q}^{-}_{j,k}$ are defined as follows:

$$\bar{Q}^{\bullet}_{j,k} = \max\left(0, \min\left(Q^{\bullet}_{j,k}, Q^{\star}_{j} - \sum_{l=1}^{k-1} Q^{\bullet}_{j,l}\right)\right)$$

Here, $Q^{+}_{j,k}$ and $Q^{-}_{j,k}$ are the ask and bid volumes of the k^{th} limit order. The reference quantity $Q^{\star}_{j}$ is the ratio between the trading notional N and the mid price:

$$Q^{\star}_{j} = \frac{N}{\bar{P}^{0}_{j}}$$

Sometimes it may appear that the trading volume on the order book is lower than the notional N. That is why the factor c_j may be greater than one:

$$c_j = \max\left(1, \frac{Q^{\star}_{j}}{\min\left(\sum_{k=1}^{K} Q^{+}_{j,k}, \sum_{k=1}^{K} Q^{-}_{j,k}\right)}\right)$$

For instance, if we wish to execute an order of 2 MEUR and there is only a trading volume of 1 MEUR, we multiply the spread by two.

For each trading day, we compute the daily spread for the different listing places using the previous formulas and we take the best spread.

Example

The limit order book

k	Buy orders $Q_{j,k}^{-}$	Buy orders $P_{j,k}^{-}$	Sell orders $Q_{j,k}^{+}$	Sell orders $P_{j,k}^{+}$
1	900	85.90	600	86.05
2	200	85.85	300	86.06
3	57	85.82	400	86.20
4	18	85.75	213	86.21
5	117	85.74	73	86.22
6	1000	85.73	200	86.23
7	3000	85.72	1500	86.25

It corresponds to a notional $N = Q^{\star} \times \bar{P}_j^0$ of 85 981 €.

Computing the spread for $Q^{\star} = 1000$

k	Buy orders $\bar{Q}_{j,k}^{-}$	Buy orders $P_{j,k}^{-}$	Sell orders $\bar{Q}_{j,k}^{+}$	Sell orders $P_{j,k}^{+}$
1	900	85.90	600	86.05
2	100	85.85	300	86.06
3	0	85.82	100	86.20
$\sum_{k=1}^{K} \bar{Q}_{j,k}^{\bullet}$	1000		1000	
$\bar{P}_j^{\bullet}$		85.89		86.07

We deduce that:

$$\bar{P}_j^0 = \frac{85.89 + 86.07}{2} = 85.98$$

and:

$$s_j = \frac{86.07 - 85.89}{85.98} = 20.12 \text{ bps}$$

Example

Given a notional N, we find the optimal value of $Q^\star$ by solving the nonlinear inequality:

$$Q^\star = \inf \left\{ Q \in \mathbb{N} : Q \bar{P}_j^0 \geq N \right\}$$

k	$N = 100$ KEUR				$N = 500$ KEUR			
	Buy orders		Sell orders		Buy orders		Sell orders	
	$\bar{Q}_{j,k}^-$	$P_{j,k}^-$	$\bar{Q}_{j,k}^+$	$P_{j,k}^+$	$\bar{Q}_{j,k}^-$	$P_{j,k}^-$	$\bar{Q}_{j,k}^+$	$P_{j,k}^+$
1	900	85.90	600	86.05	900	85.90	600	86.05
2	100	85.85	300	86.06	200	85.85	300	86.06
3	57	85.82	263	86.20	57	85.82	400	86.20
4	6	85.75	0	86.21	18	85.75	213	86.21
5	0	85.74	0	86.22	117	85.74	73	86.21
6	0	85.73	0	86.23	1000	85.73	200	86.23
7	0	85.72	0	86.25	3000	85.72	1500	86.25
$\sum_{k=1}^K \bar{Q}_{j,k}^\bullet$	1163		1163		5292		3286	
$\bar{P}_j^\bullet$		85.89		86.09		85.76		86.19
s_j	23.24 bps				87.81 bps			