Factor Investing and Equity Portfolio Construction[1]

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[1] The materials used in these slides are taken from Cazalet Z. and Roncalli T. (2014), Facts and Fantasies About Factor Investing, Lyxor Research Paper, 116 pages.

Lyxor Research Paper

http://ssrn.com/abstract=2524547

Outline

Risk factors versus factor investing

Risk factor

It is a pattern that explains the cross-section of asset returns and that will explain the cross-section of asset returns in the future.

Risk factors were initially based on systematic and common risks. They embed now other dimensions, such as anomalies or trading strategies.

Risk factors are one of the pillars of performance measurement (Carhart, 1997).

Risk factors versus factor investing

Factor investing (marketing message)

Strategic asset allocation based on asset classes is an issue, because:

- it is difficult to estimate their risk premia;
- correlations between asset classes are time-varying and not stable;
- we don't know if it is the right level of aggregation.

Asset classes are exposed to independent risk factors, which are rewarded on the long-run, meaning that strategic asset allocation based on risk factors is more easy and robust.

Factor investing consists in:

- building factor mimicking portfolios (asset management & index providers);
- allocating between risk factors (investors).

What is the rationale for factor investing?

- At the security level, there is a lot of idiosyncratic risk or alpha:

	Common Risk	Idiosyncratic Risk
GOOGLE	47%	53%
NETFLIX	24%	76%
MASTERCARD	50%	50%
NOKIA	32%	68%
TOTAL	89%	11%
AIRBUS	56%	44%

What is the rationale for factor investing?

- Idiosyncratic risk decreases with the size of the investment portfolios:

Portfolio	Common Risk	Idiosyncratic Risk
Renaissance Europe	69.2%	30.8%
Threadneedle Pan European SC	87.5%	12.5%
Franklin Mutual European	90.2%	9.8%
SG Actions Euro Value	91.7%	8.3%
Metropole Selection	91.8%	8.2%
Allianz Europe Equity Growth	92.0%	8.0%

What is the rationale for factor investing?

- 2009: Professors' Report on the Norwegian GPFG (Ang, Goetzmann and Schaefer) $\Rightarrow$ Risk factors represent 99.1% of the fund return variation

What lessons can we draw from this?

Idiosyncratic risks and specific bets disappear in (large) diversified portfolios. Performance of institutional investors is then exposed to risk factors.

Alpha is not scalable, but risk factors are scalable.

$\Rightarrow$ Risk factors are the only bets that are compatible with diversification.

Is that really true?

The cross-section of expected returns

Cross-section of expected returns

The objective is to explain the dispersion of asset returns at time t (not the time-variation).

- 10 value-weighted (or equally-weighted) sorted portfolios (by deciles)
- 25 VW/EW sorted portfolios
  - e.g. independent sorts into 5 size groups and 5 B/M groups
- Universe of stocks

Two statistical tools are used:

- t-stat (are asset returns sensitive to the factor?)
- R^2 (how many variance is explained?)

If XMY (e.g. HML) is a risk factor, -XMY (e.g. LMH) is a risk factor.

Risk premium versus risk premia

CAPM

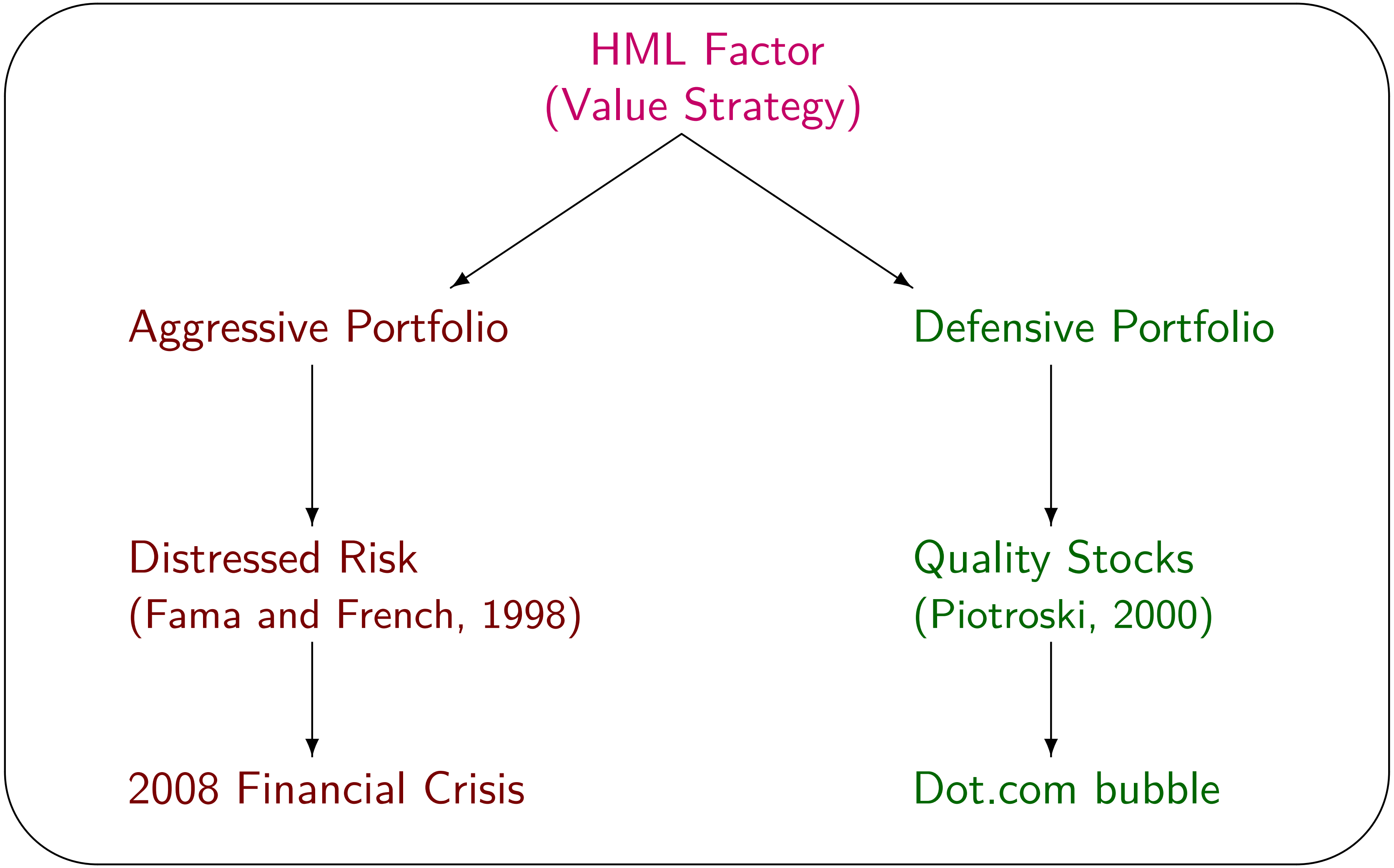
There is one risk premium, which can be captured by the market portfolio.

Factor investing

There are other risk premia than the market risk premium. They correspond to rewarded risk factors.

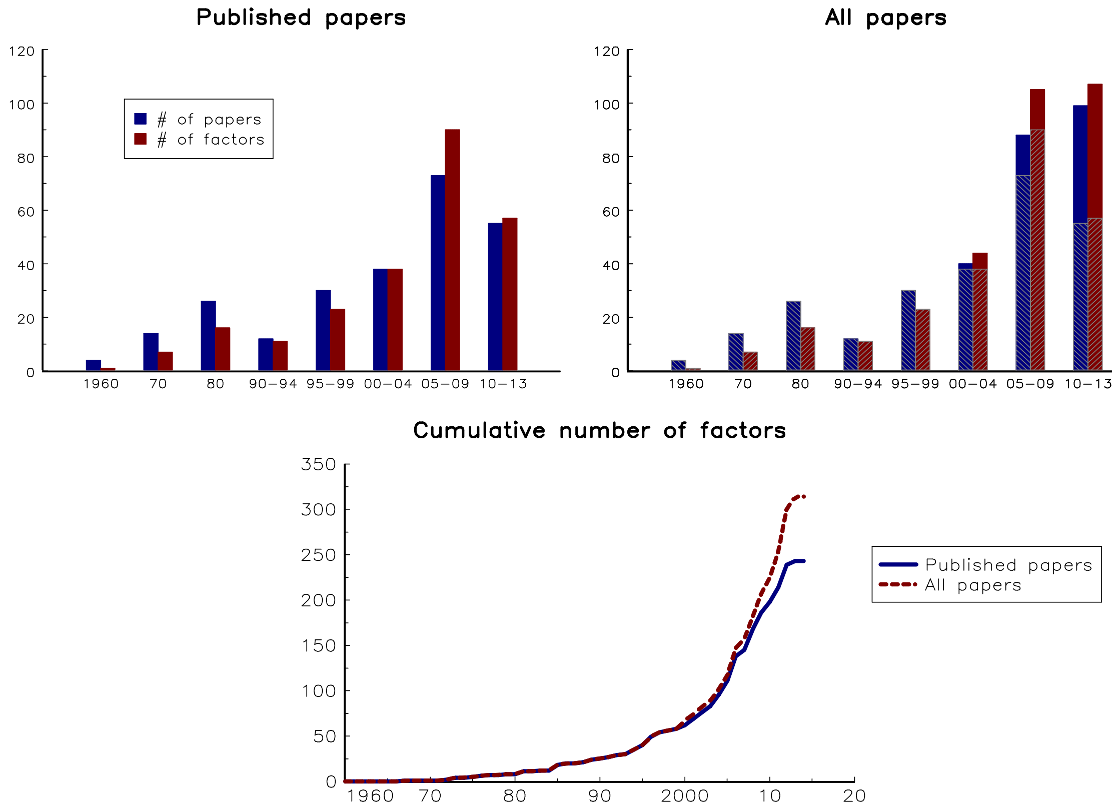
- If XMY (e.g. HML) is a risk premium, -XMY (e.g. LMH) is not a risk premium.
- XMY is a risk premium $\Rightarrow$ XMY is a risk factor.
- XMY is a risk factor $\nRightarrow$ XMY is a risk premium.

Risk factors and anomalies



Factor zoo

Figure: Harvey *et al.* (2014)



"*Now we have a zoo of new factors*" (Cochrane, 2011).

Factors, factors everywhere

"Standard predictive regressions fail to reject the hypothesis that the party of the U.S. President, the weather in Manhattan, global warming, El Niño, sunspots, or the conjunctions of the planets, are significantly related to anomaly performance. These results are striking, and quite surprising. In fact, some readers may be inclined to reject some of this paper's conclusions solely on the grounds of plausibility. I urge readers to consider this option carefully, however, as doing do so entails rejecting the standard methodology on which the return predictability literature is built."(Novy-Marx, 2014).

$\Rightarrow$ MKT, SMB, HML, WML, STR, LTR, VOL, IVOL, BAB, QMJ, LIQ, TERM, CARRY, DIV, JAN, CDS, GDP, INF, etc.

The alpha puzzle (Cochrane, 2011)

- Chaos

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i}$$

- Sharpe (1964)

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f)$$

- Chaos again

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i} + \beta_i^m (\mathbb{E}[R_m] - R_f)$$

- Fama and French (1992)

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \beta_i^{hml} \mathbb{E}[R_{hml}]$$

This is not the end of the story...

The alpha puzzle (Cochrane, 2011)

It's just the beginning!

- Chaos again

$$\mathbb{E}[R_i] - R_f = \boxed{\alpha_i} + \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \beta_i^{hml} \mathbb{E}[R_{hml}]$$

- Carhart (1997)

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \beta_i^{hml} \mathbb{E}[R_{hml}] + \beta_i^{wml} \mathbb{E}[R_{wml}]$$

- Chaos again

$$\begin{aligned} \mathbb{E}[R_i] - R_f &= \boxed{\alpha_i} + \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \\ & \quad \beta_i^{hml} \mathbb{E}[R_{hml}] + \beta_i^{wml} \mathbb{E}[R_{wml}] \end{aligned}$$

- Etc.

How can alpha always come back?

Facts and fantasies

Main fact

Risk factors are a powerful tool to understand the cross-section of (expected) returns.

✔

Facts and fantasies

Fact

- Common risk factors explain more variance than idiosyncratic risks in diversified portfolios.
- Some risk factors are more relevant than others, for instance SMB, HML and WML.
- Risk premia are time-varying and low-frequency mean-reverting. The length of a cycle is between 3 and 10 years.
- The explanatory power of risk factors other than the market risk factor has declined over the last few years, because Beta has been back since 2003.

✔

Facts and fantasies

Fact

- Long-only and long/short risk factors have not the same behavior. This is for example the case of BAB and WML factors.
- Risk factors are local, not global. It means that risk factors are not homogeneous. For instance, the value factors in US and Japan cannot be compared (distressed stocks versus quality stocks).
- Factor investing is not a new investment style. It has been largely used by asset managers and hedge fund managers for a long time.

✔

Facts and fantasies

Main fantasy

- There are many rewarded risk factors.

✘

Facts and fantasies

Fantasy

- Risk factors are not dependent on size. It is a fantasy. Some risk factors present a size bias, like the HML risk factor.
- HML is much more rewarded than WML.
- WML exhibits a CTA option profile. This is wrong. The option profile of a CTA is a long straddle whereas WML presents some similarities to a short call exposure.
- Long-only risk factors are more risky than long/short risk factors. This is not always the case. For instance, the risk of the long/short WML factor is very high.

✗

Facts and fantasies

Fantasy

- HML is riskier than WML. It is generally admitted in finance that contrarian strategies are riskier than trend-following strategies. However, this is not always the case, such as with the WML factor, which is exposed to momentum crashes.

- Strategic asset allocation with risk factors is easier than strategic asset allocation with asset classes. This is not easy, in particular in a long-only framework. Estimating the alpha, beta and idiosyncratic volatility of a long-only risk factor remains an issue, implying that portfolio allocation is not straightforward.

✘

Fama-French risk factors

Fama-French three-factor model

We have:

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \beta_i^{hml} \mathbb{E}[R_{hml}]$$

where R_{smb} is the return of small stocks minus the return of large stocks, and R_{hml} is the return of stocks with high book-to-market values minus the return of stocks with low book-to-market values.

The factors are defined as follows:

$$\mathrm{SMB}_t = \frac{1}{3}(R_t(\mathrm{SV}) + R_t(\mathrm{SN}) + R_t(\mathrm{SG})) - \frac{1}{3}(R_t(\mathrm{BV}) + R_t(\mathrm{BN}) + R_t(\mathrm{BG}))$$

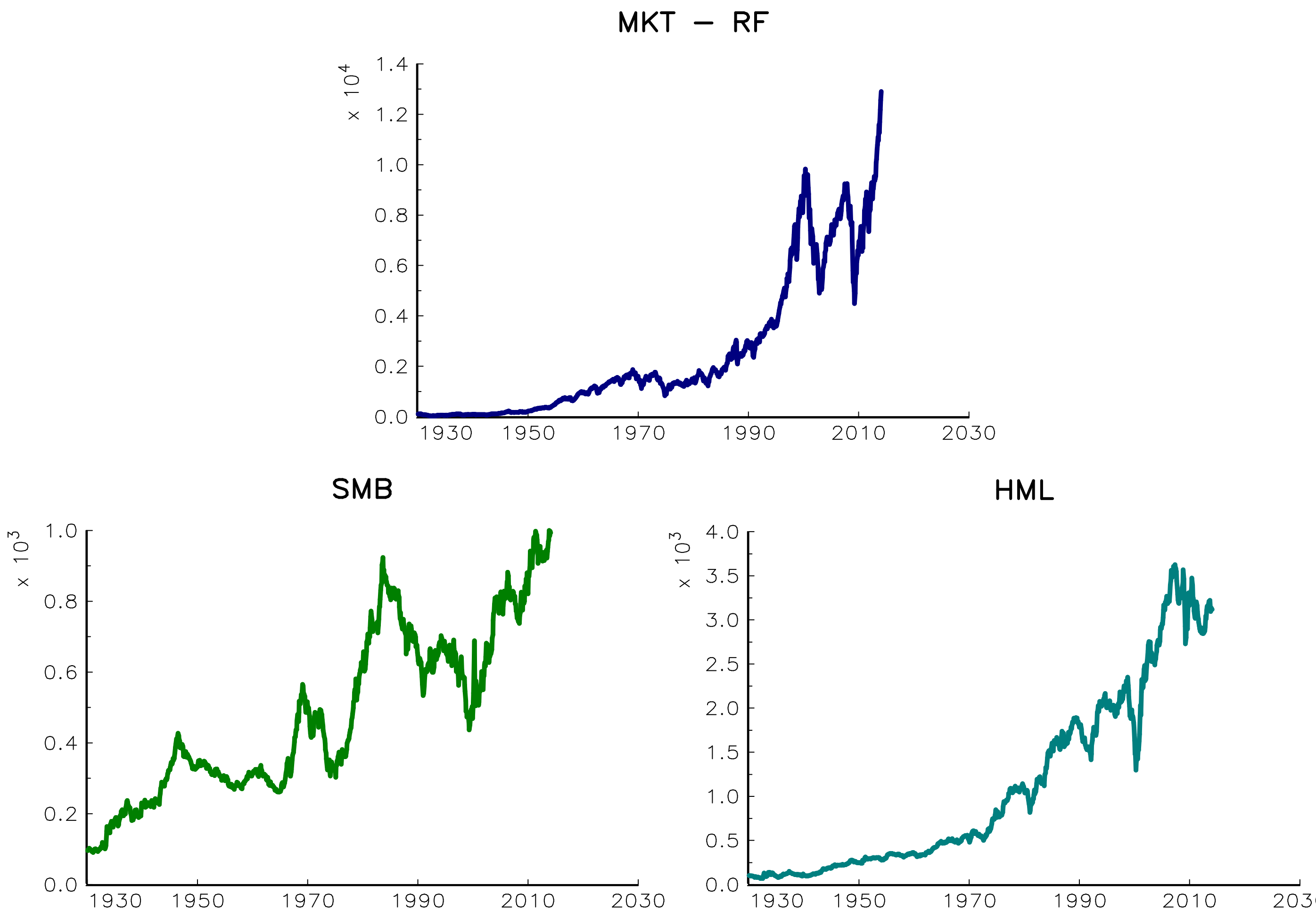
$$\mathrm{HML}_t = \frac{1}{2}(R_t(\mathrm{SV}) + R_t(\mathrm{BV})) - \frac{1}{2}(R_t(\mathrm{SG}) + R_t(\mathrm{BG}))$$

with the following 6 portfolios:

	Value	Neutral	Growth
Small	SV	SN	SG
Big	BV	BN	BG

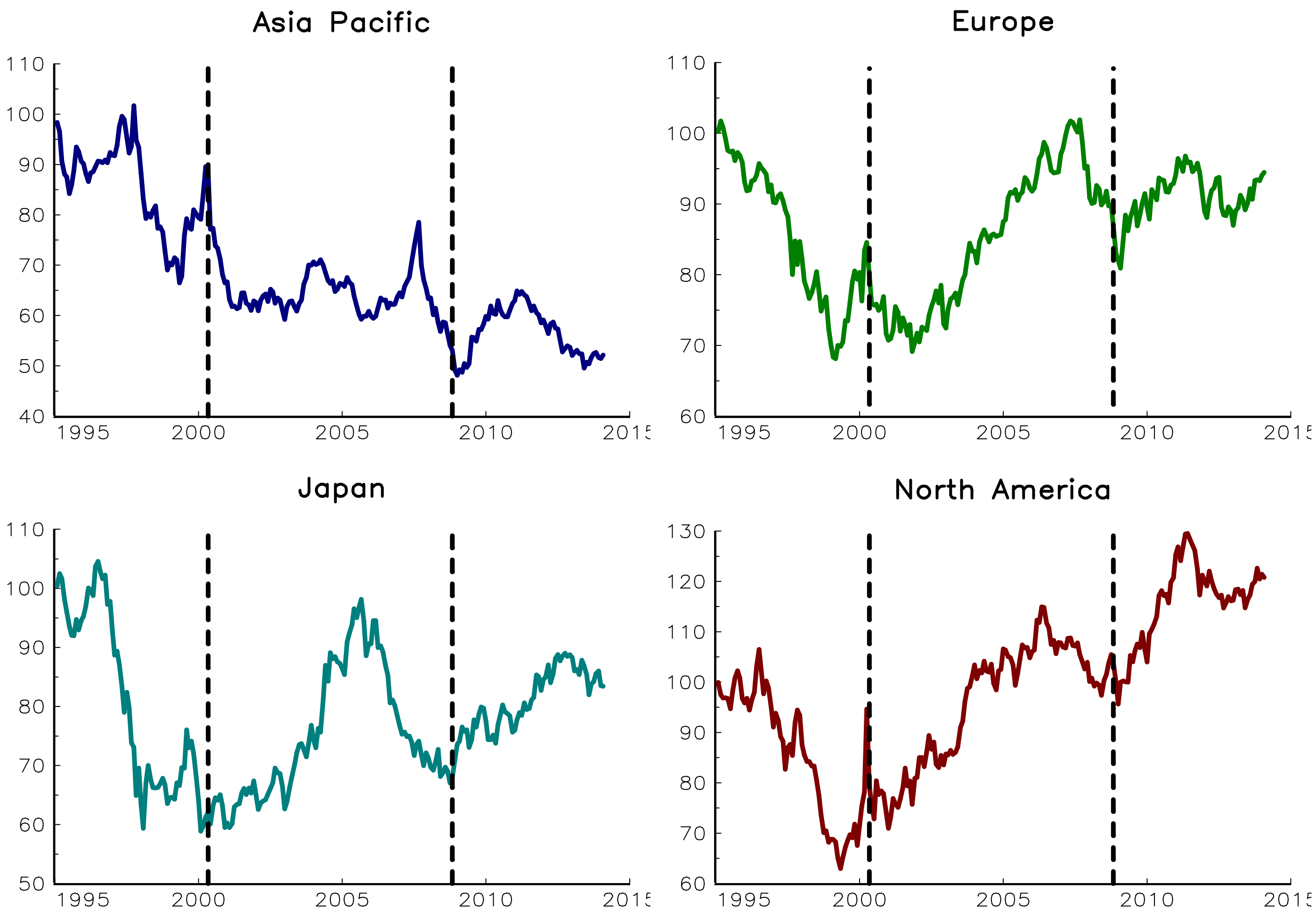
Dynamics of the three risk factors

Figure: Fama-French US risk factors (1930 - 2013)



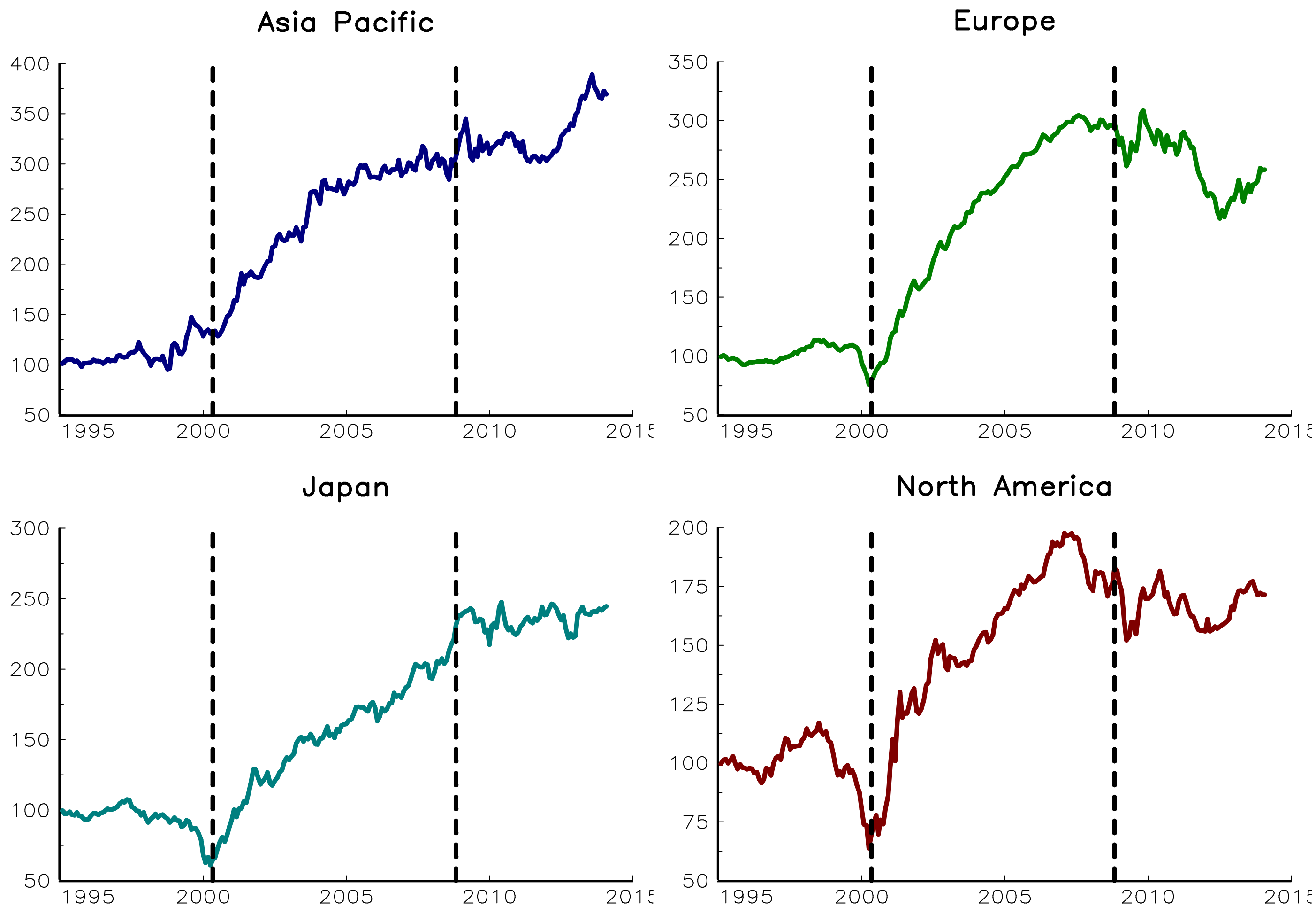
Dynamics of the three risk factors

Figure: Fama-French SMB factor (1995 – 2013)



Dynamics of the three risk factors

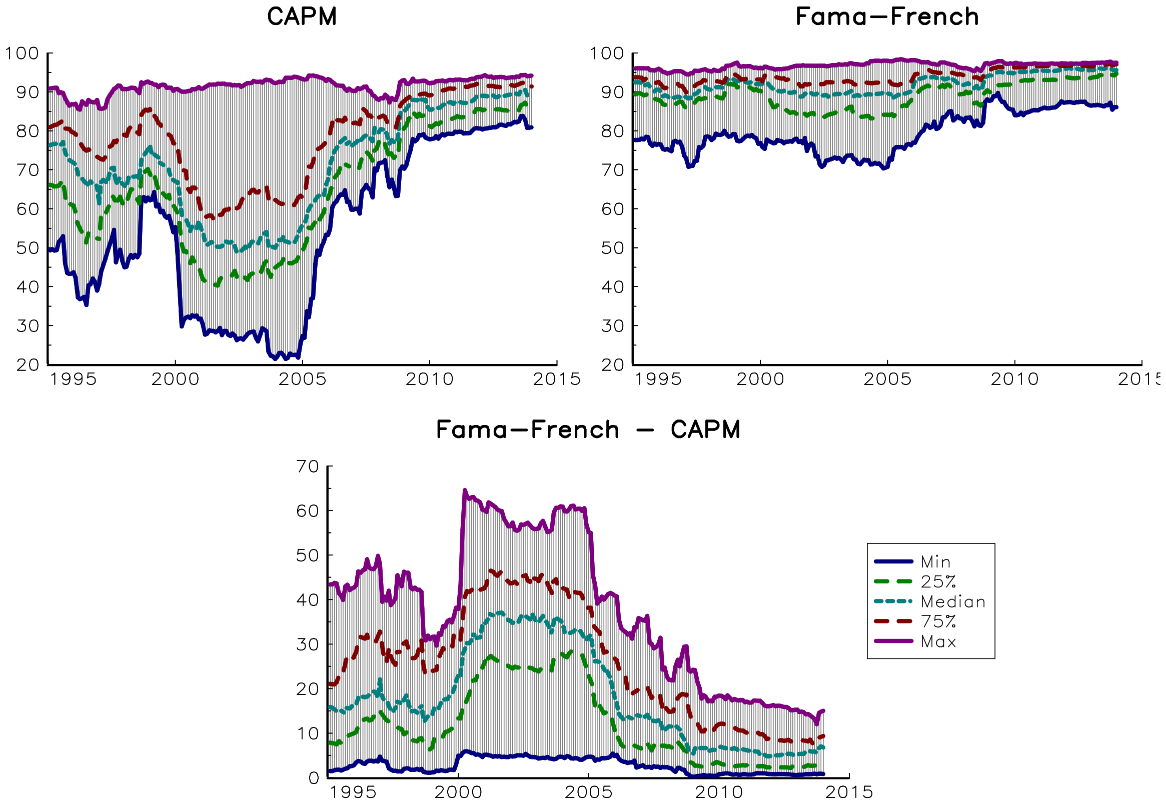
Figure: Fama-French HML factor (1995 – 2013)



The cross-section of asset returns

Cross-section = 100 value-weighted portfolios (independent sorts into 10 size groups and 10 B/M groups

Figure: R^2 coefficient (in %) – US



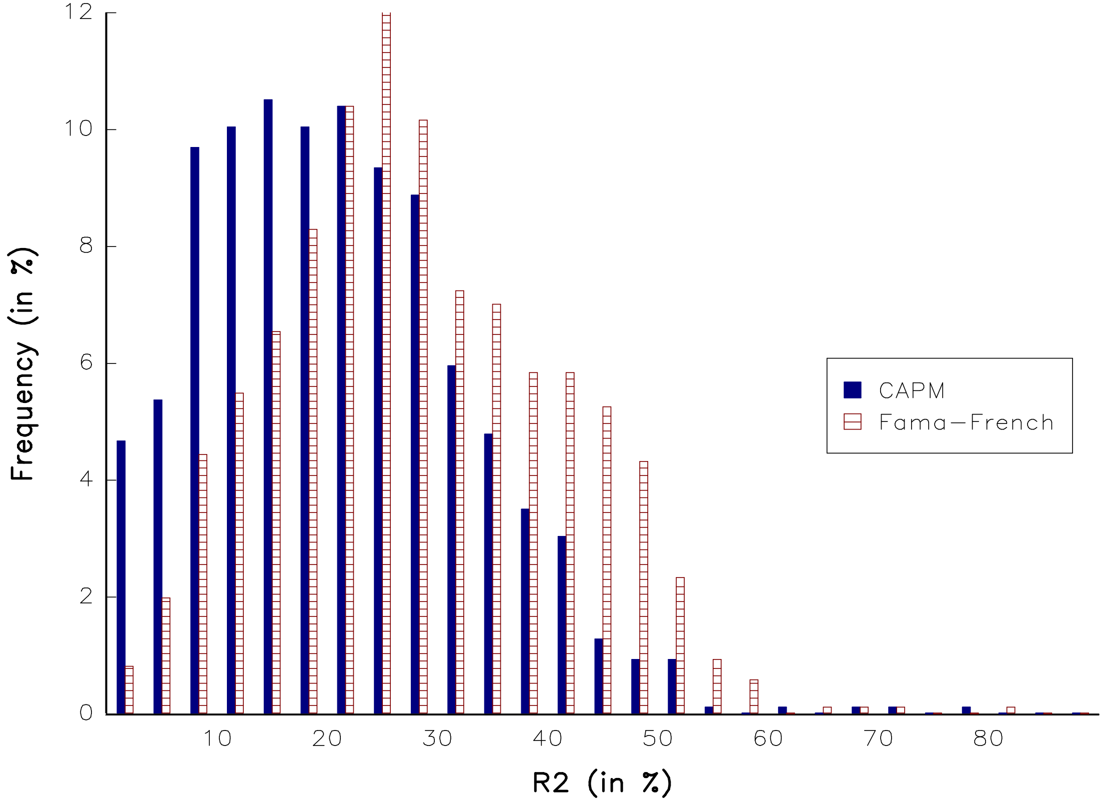
The cross-section of asset returns

Table: Average of $R^2_{\mathrm{FF}} - R^2_{\mathrm{CAPM}}$ (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	12.1	13.7	10.0	17.9	18.0
1996	11.7	14.4	9.8	22.5	23.0
1997	12.7	17.6	11.1	22.4	20.2
1998	13.0	19.1	14.0	21.1	18.4
1999	12.8	19.9	15.2	19.2	19.2
2000	13.1	27.2	20.4	29.5	31.6
2001	13.0	26.4	21.1	30.3	36.1
2002	12.3	23.4	20.9	28.6	35.0
2003	13.3	20.3	19.4	27.3	34.4
2004	13.5	17.5	19.3	27.1	33.2
2005	11.5	11.6	13.9	17.7	23.7
2006	11.3	8.8	14.2	13.0	15.7
2007	12.5	7.5	15.4	11.3	13.6
2008	9.6	6.3	15.8	10.0	11.4
2009	6.1	5.0	15.5	7.1	7.8
2010	5.9	5.7	15.0	6.8	7.9
2011	5.4	5.1	14.1	5.9	6.9
2012	4.8	4.9	13.7	5.3	6.3
2013	5.3	5.1	12.1	5.3	6.3

The cross-section of asset returns

Figure: Frequency of the R^2 coefficient with S&P 500 stocks (1995-2013)



$\Rightarrow$ Alpha (or idiosyncratic risk) exists!

The size effect in the HML risk factor

- SHML is the HML factor for small stocks
- BHML is the HML factor for big stocks

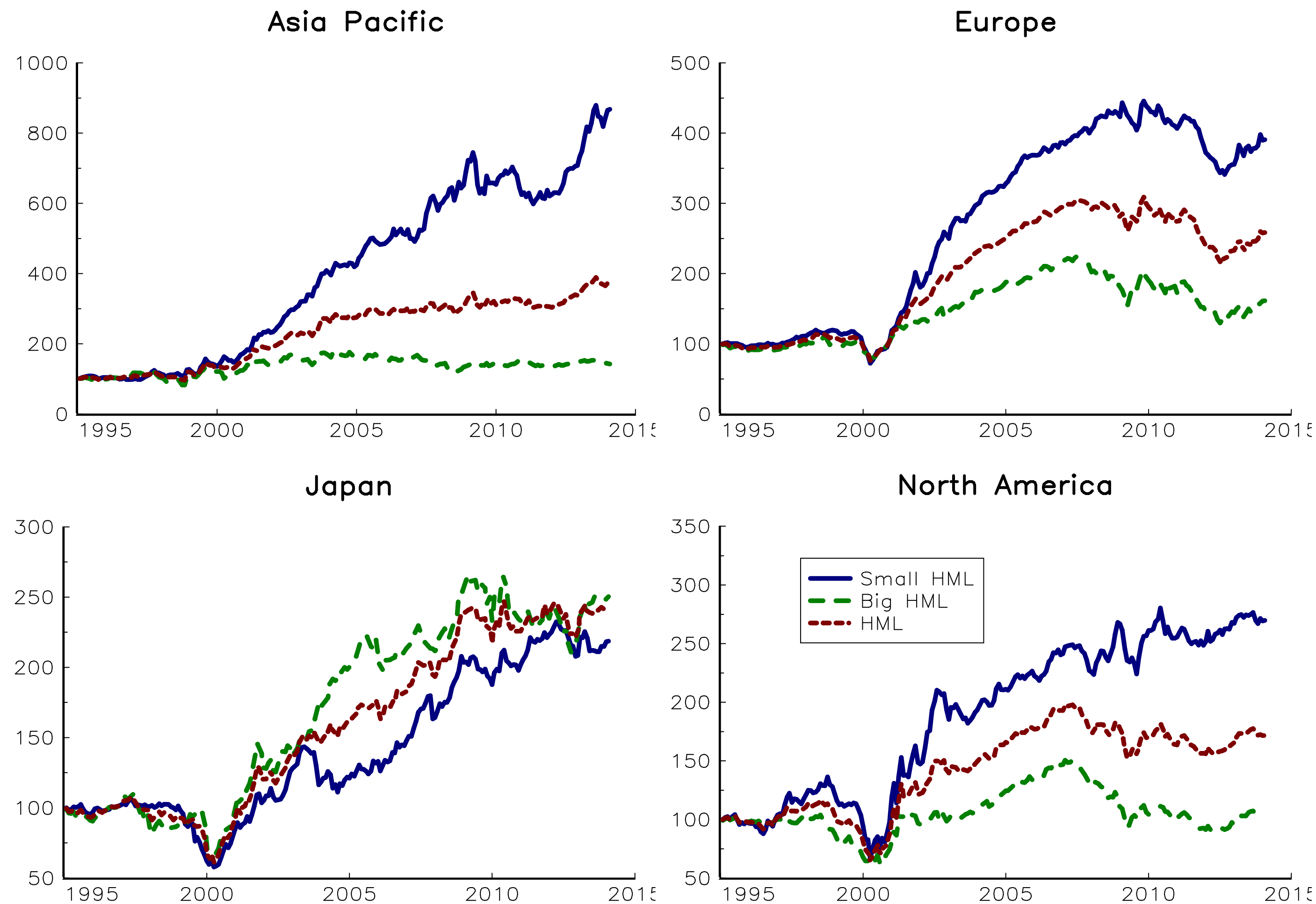
$$\begin{aligned}
\mathrm{HML}_t &= \frac{1}{2}\left(R_t(\mathrm{SV}) + R_t(\mathrm{BV})\right) - \frac{1}{2}\left(R_t(\mathrm{SG}) + R_t(\mathrm{BG})\right) \\
&= \frac{1}{2}\left(R_t(\mathrm{SV}) - R_t(\mathrm{SG})\right) + \frac{1}{2}\left(R_t(\mathrm{BV}) - R_t(\mathrm{BG})\right) \\
&= \frac{1}{2}\mathrm{SHML}_t + \frac{1}{2}\mathrm{BHML}_t
\end{aligned}$$

$\Rightarrow$ The HML factor may be biased toward a size factor because of two effects:

- the SHML factor contributes more than the BHML factor;
- the BHML factor is itself biased by a size effect.

The size effect in the HML risk factor

Figure: Fama-French SHML, BHML and HML factors (1995 – 2013)



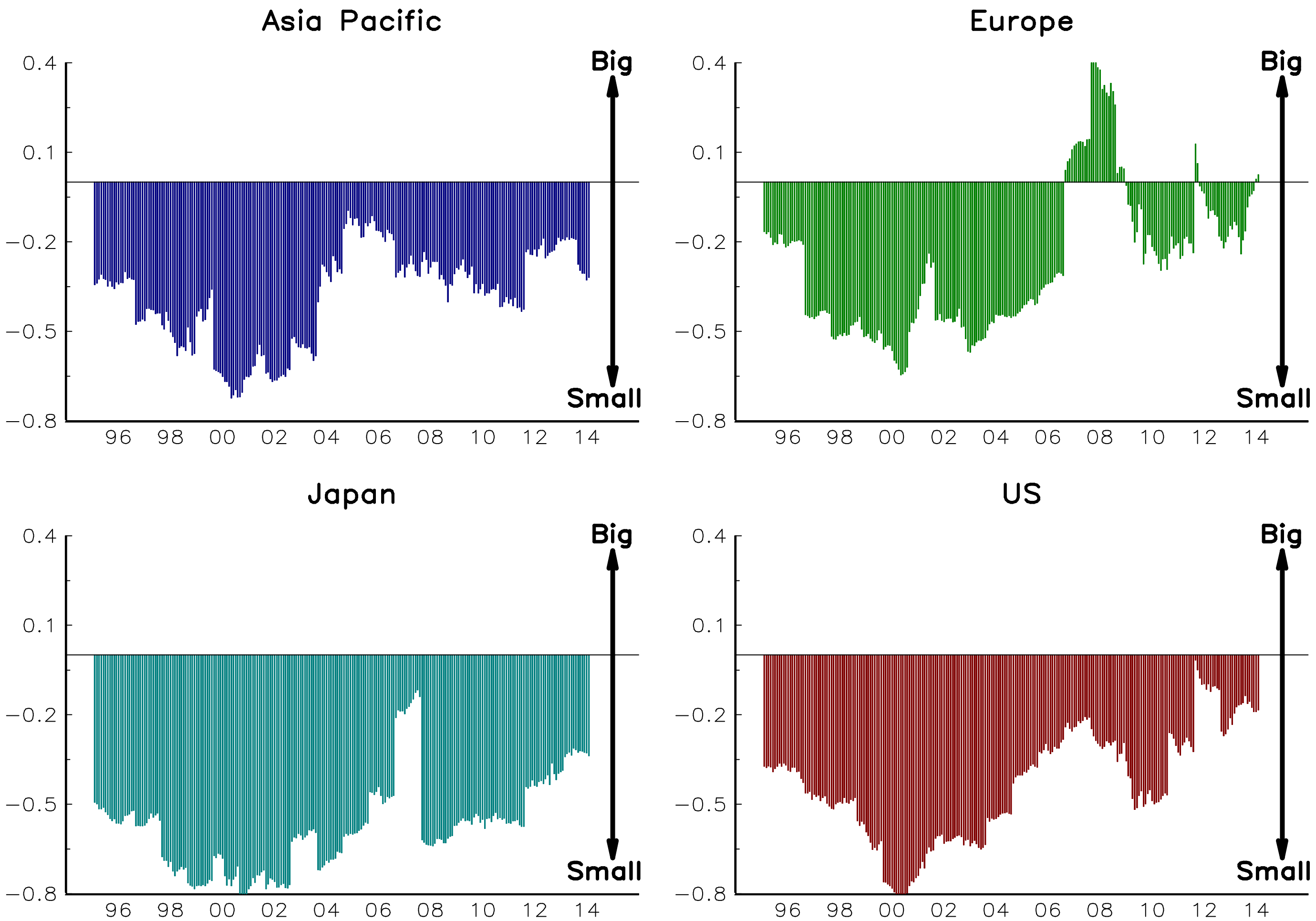
The size effect in the HML risk factor

Table: Performance of the SHML, BHML and HML factors (1995 – 2013)

Statistic	Factor	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	SHML	12.0	7.4	4.2	5.4	5.1
	BHML	1.8	2.6	5.0	0.2	−0.6
	HML	7.1	5.2	4.8	2.9	2.4
$\sigma(x)$	SHML	11.7	10.0	11.0	15.2	13.4
	BHML	15.2	11.0	13.3	11.2	11.9
	HML	11.5	9.0	10.3	12.1	11.5
$\mathrm{SR}(x \mid r)$	SHML	1.03	0.74	0.38	0.35	0.38
	BHML	0.12	0.24	0.38	0.02	−0.05
	HML	0.61	0.57	0.47	0.24	0.20

The size bias of the BHML factor

Figure: Size ratio between the big value and the big growth portfolios



Stock-based versus fund-based risk factors

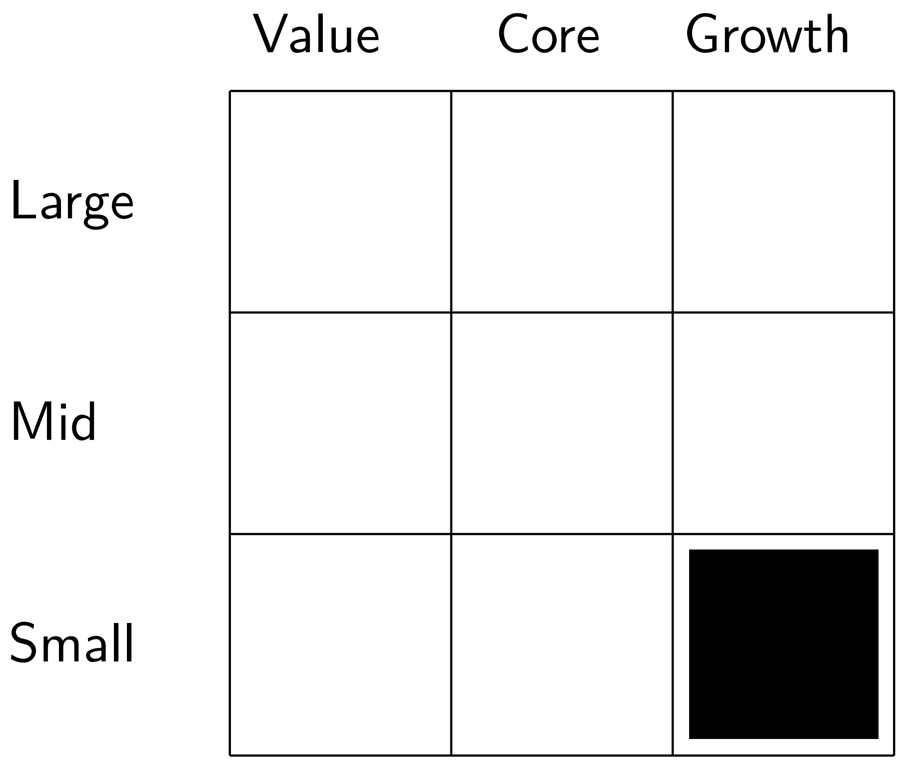
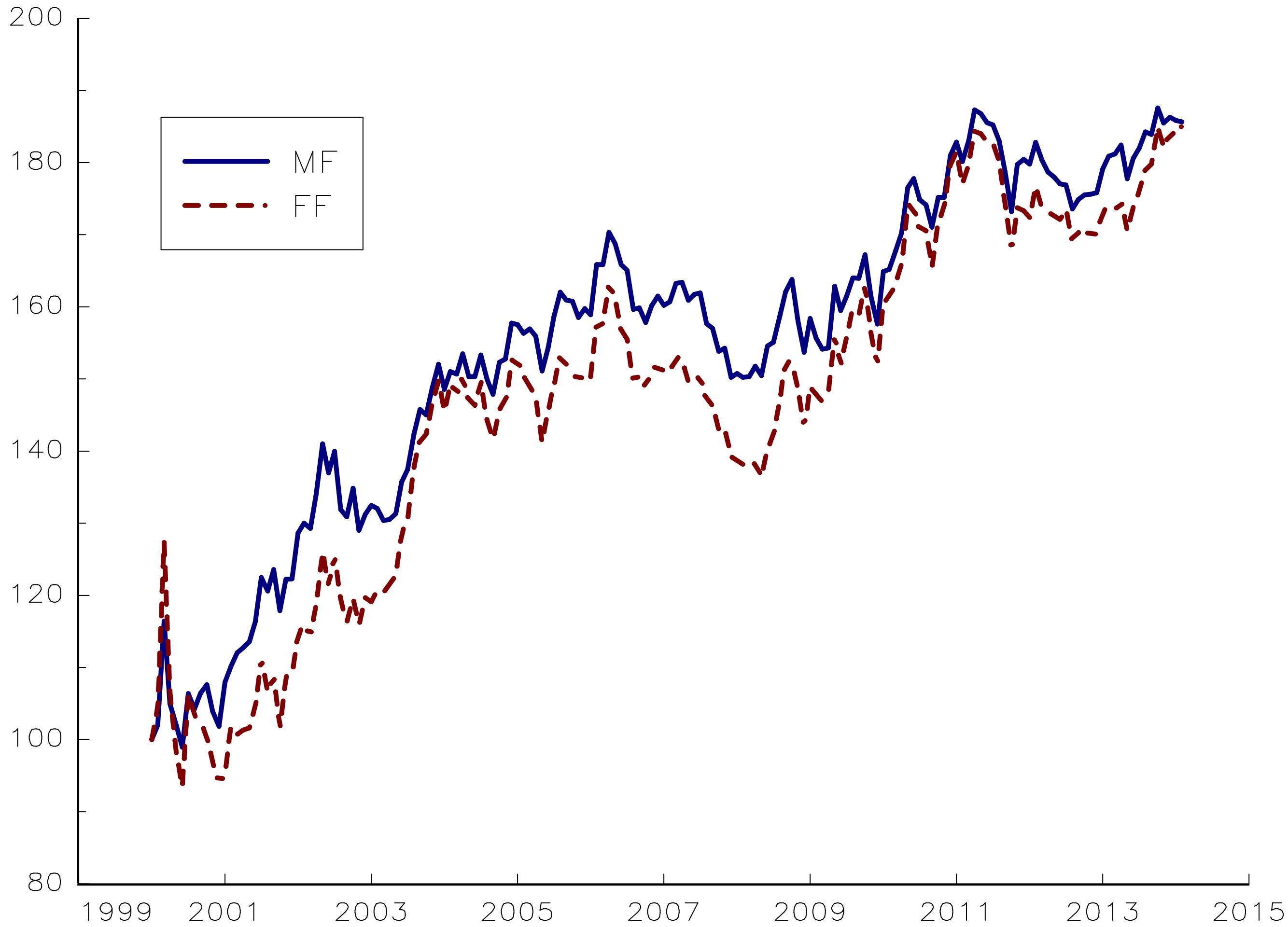


Figure: The Morningstar style box

⇒ We can build SMB and HML risk factors by using the performance of mutual funds.

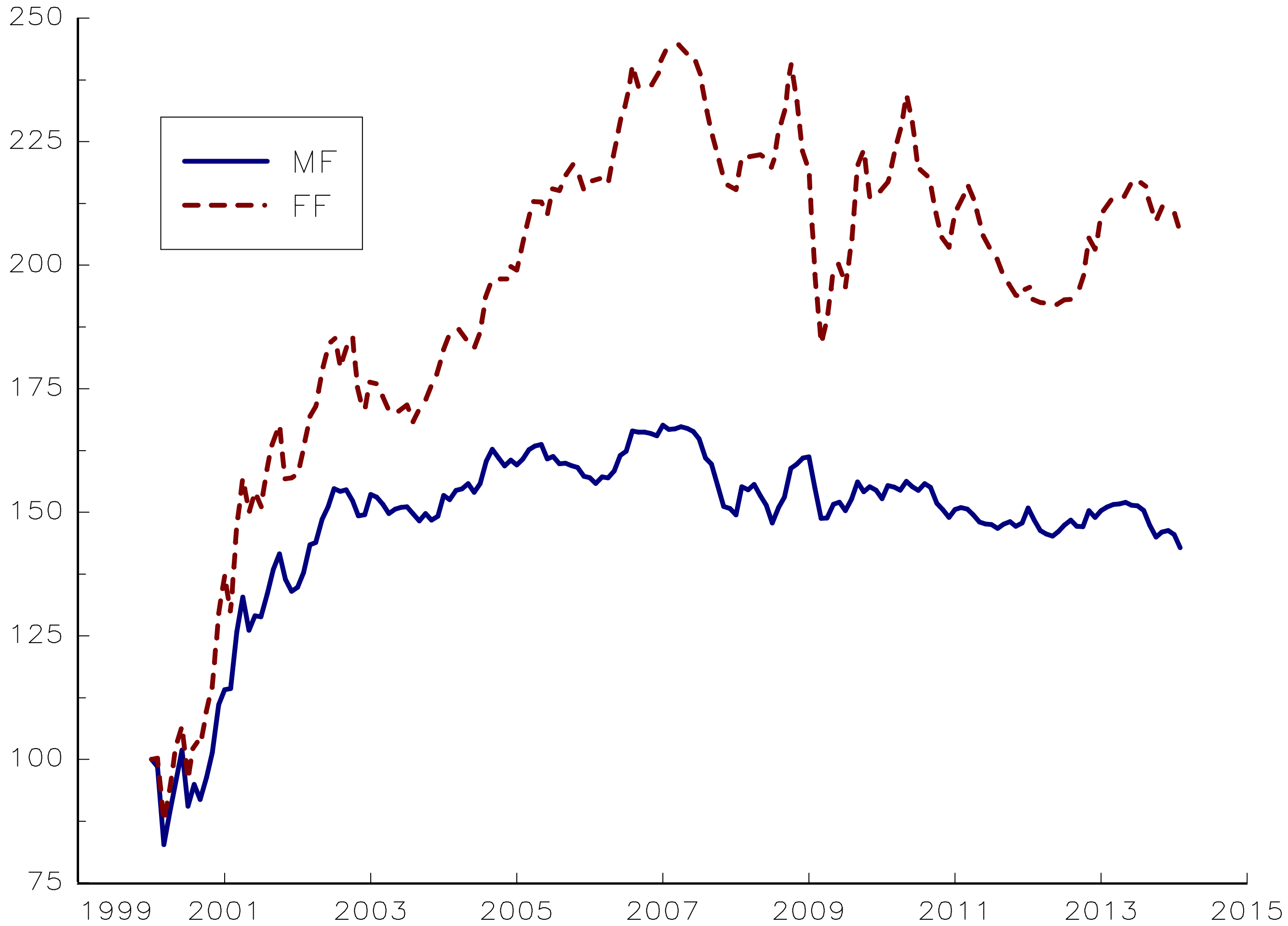
Stock-based versus fund-based risk factors

Figure: Comparison between FF and MF SMB risk factors (US, 1999-2014))



Stock-based versus fund-based risk factors

Figure: Comparison between FF and MF HML risk factors (US, 1999-2014))



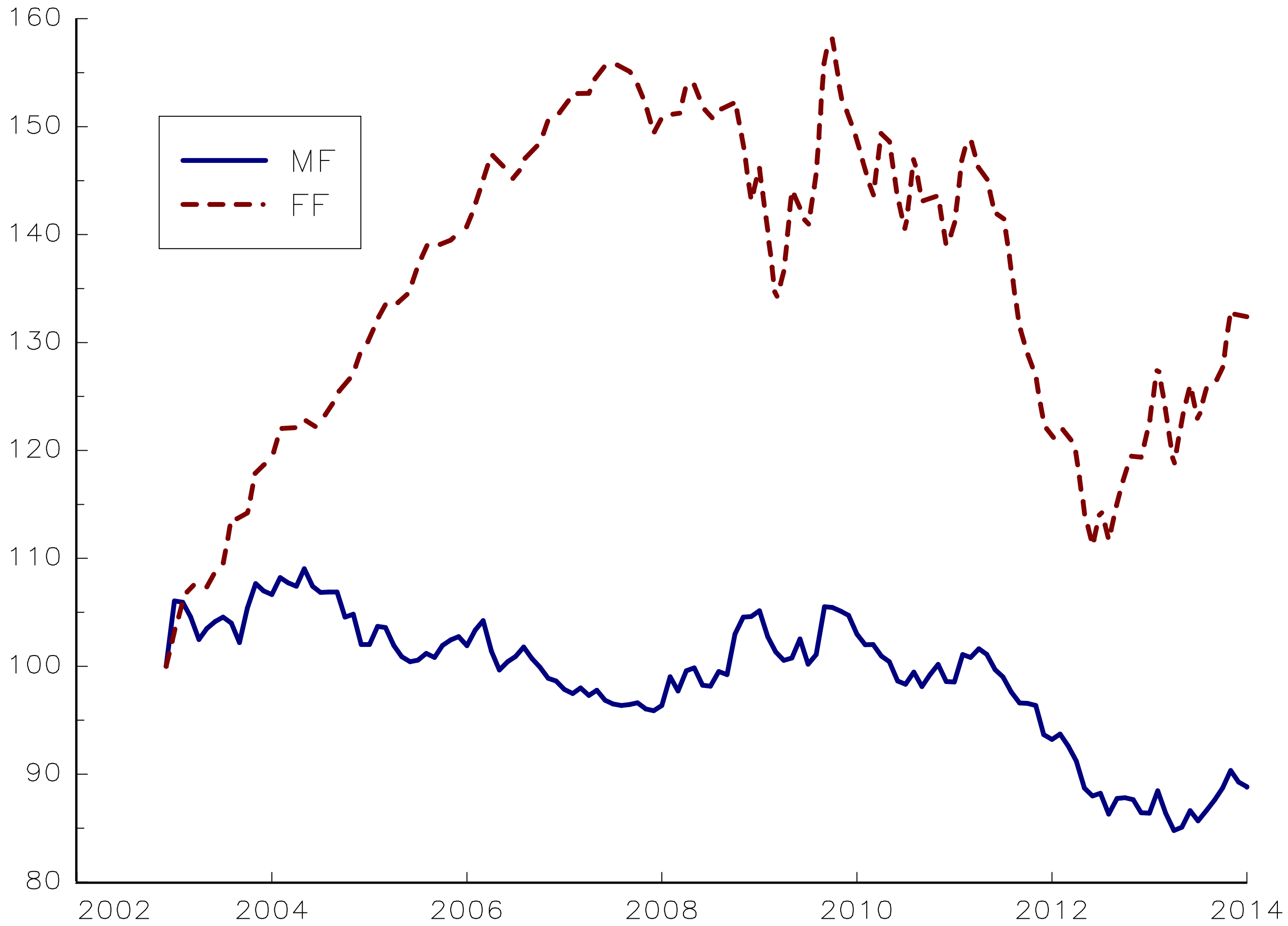
Stock-based versus fund-based risk factors

Table: Correlation between FF and MF risk factors (1999 – 2013)

Factor	Europe	Japan	US
SMB	79.8	86.0	93.9
HML	55.5	54.3	84.8

Stock-based versus fund-based risk factors

Figure: Comparison between FF and MF HML risk factors (Europe, 1999-2014))



Momentums?

Short-term reversal	Trend-following	Long-term reversal
1 Day - 1 Month	2 Months - 2 Years	2 Years - 5 Years

Jegadeesh and Titman (1993)

Lehman (1990)

De Bondt and Thaler (1985)

Carhart four-factor model

Carhart four-factor model

We have:

$$\mathbb{E}[R_i] - R_f = \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb} \mathbb{E}[R_{smb}] + \beta_i^{hml} \mathbb{E}[R_{hml}] + \beta_i^{wml} \mathbb{E}[R_{wml}]$$

where R_{wml} is the return difference of winner and loser stocks of the past twelve months.

Fama and French (2012) considered six portfolios:

	Loser	Average	Winner
Small	SL	SA	SW
Big	BL	BA	BW

They then define the WML factor as follows:

$$\mathrm{WML}_t = \frac{1}{2}(R_t(\mathrm{SW}) + R_t(\mathrm{BW})) - \frac{1}{2}(R_t(\mathrm{SL}) + R_t(\mathrm{BL}))$$

Performance of the WML factor

Table: Performance of the WML factor

Statistic	Period	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	#1	3.8	19.9	8.7	22.6	17.6
	#2	11.9	11.5	−0.3	1.4	3.7
	#3	5.6	2.9	−2.0	−4.5	−9.3
$\sigma(x)$	#1	24.7	12.8	22.1	18.7	14.5
	#2	12.7	15.9	14.1	20.0	20.1
	#3	15.2	17.3	14.0	15.3	19.9
$\mathrm{SR}(x \mid r)$	#1	0.15	1.56	0.40	1.21	1.22
	#2	0.93	0.72	−0.02	0.07	0.19
	#3	0.37	0.17	−0.14	−0.30	−0.47

#1 January 1995 – March 2000

#2 April 2000 – March 2009

#3 April 2009 – December 2013

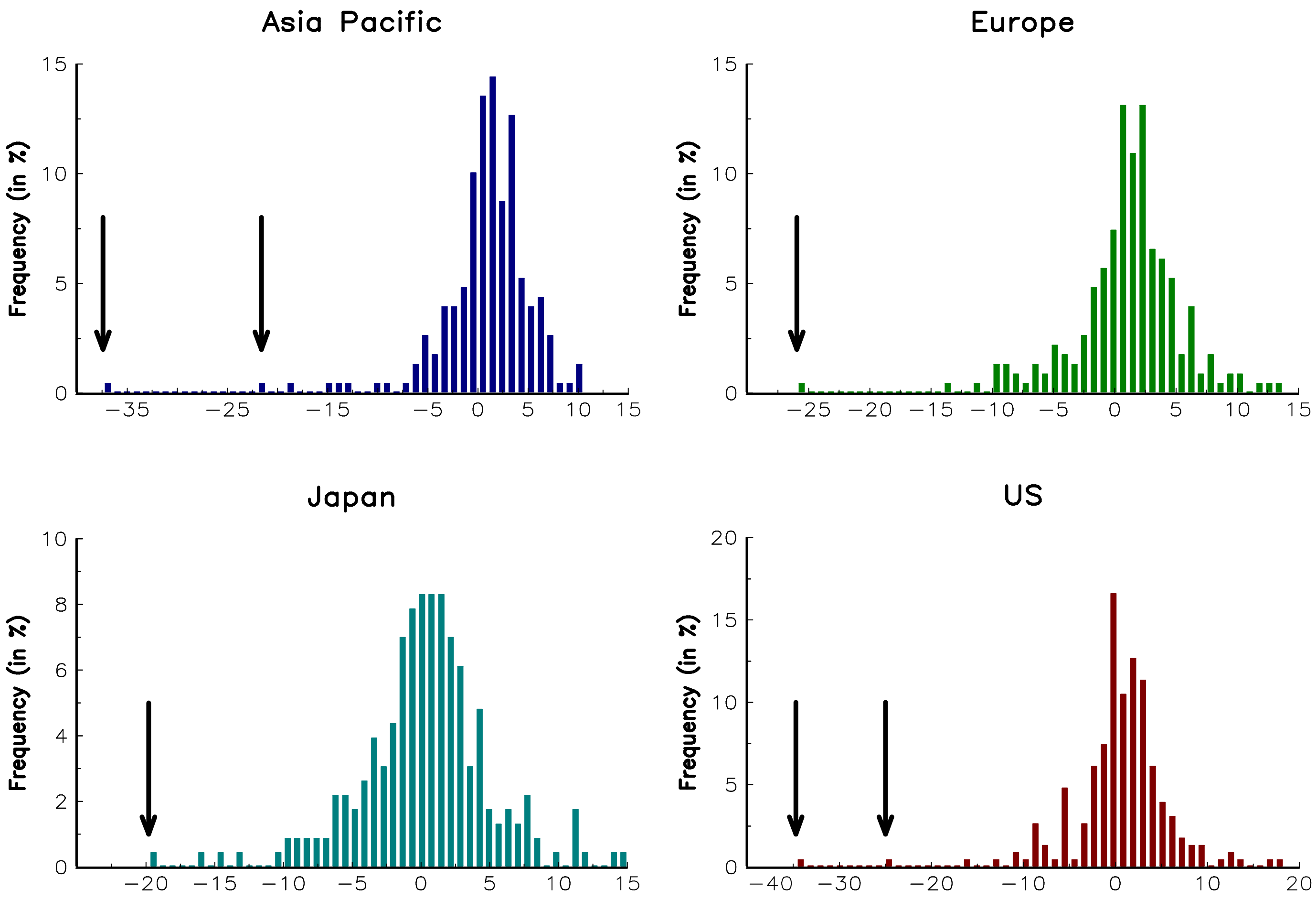
Performance of the WML factor

Table: Yearly return of the WML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	2.3	24.9	−15.4	13.1	14.6
1996	20.2	19.2	−6.5	4.5	5.5
1997	25.9	11.4	53.9	11.6	9.5
1998	−30.2	17.4	−16.6	24.1	22.2
1999	2.6	30.9	66.8	51.3	29.0
2000	−15.9	−23.5	−30.8	−9.7	16.9
2001	27.8	22.2	16.0	−8.3	−10.4
2002	40.7	53.1	−6.2	29.5	28.1
2003	11.8	−11.5	−15.1	−10.7	−17.8
2004	18.1	7.7	7.3	2.2	−0.3
2005	9.7	17.8	21.3	19.7	15.3
2006	26.3	13.1	−3.8	−4.0	−6.5
2007	13.6	20.2	10.0	22.0	22.8
2008	3.4	27.5	15.3	5.9	18.3
2009	−39.5	−37.6	−33.0	−42.0	−52.7
2010	4.8	30.3	−3.3	6.6	5.7
2011	14.3	9.5	3.5	5.1	8.4
2012	19.6	3.6	2.3	0.9	−1.1
2013	38.0	20.7	16.1	12.9	6.2

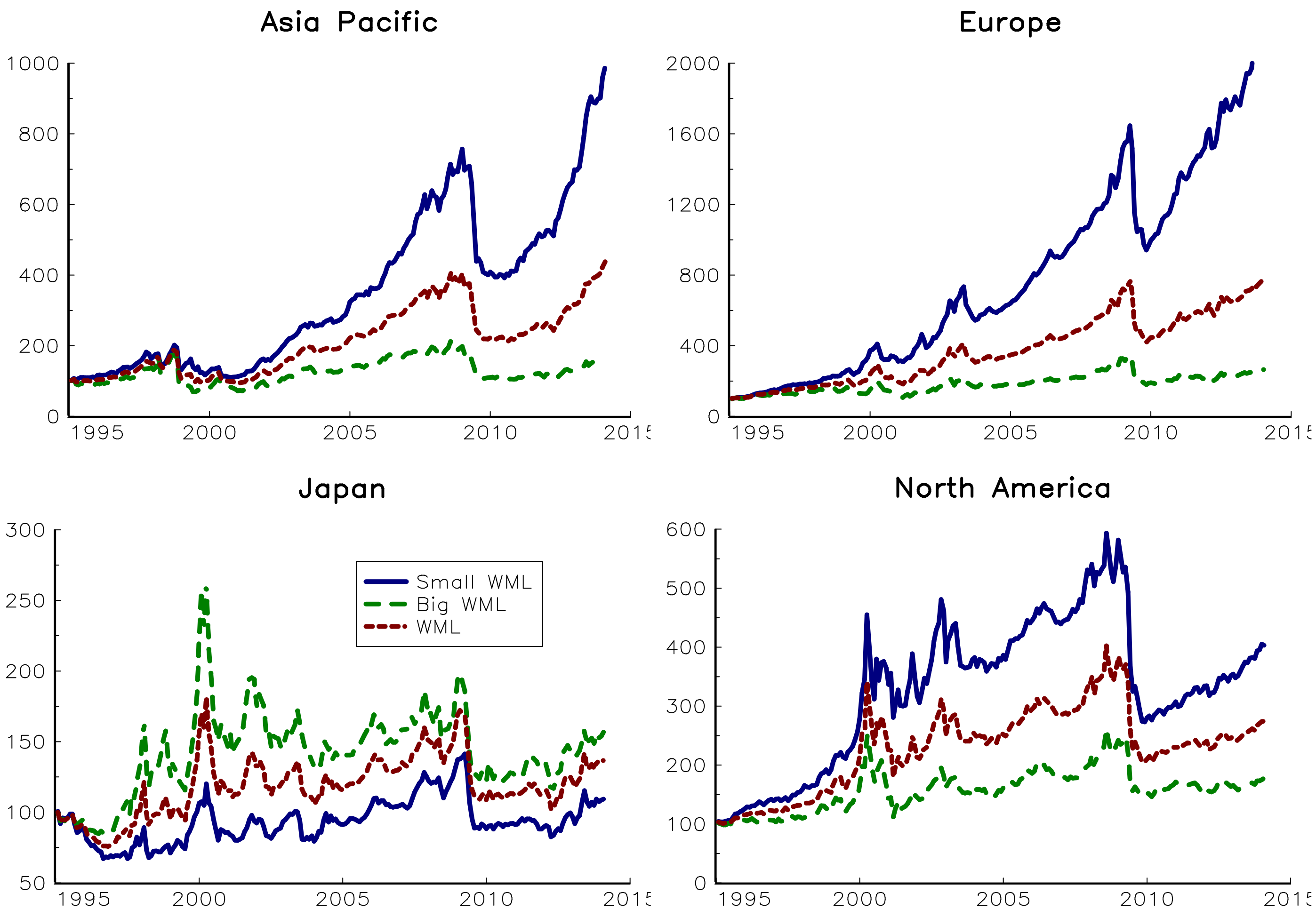
Momentum crashes (Daniel and Moskowitz, 2013)

Figure: Distribution of WML monthly returns



The size effect in the WML risk factor

Figure: The SWML, BWML and WML factors (1995 – 2013)



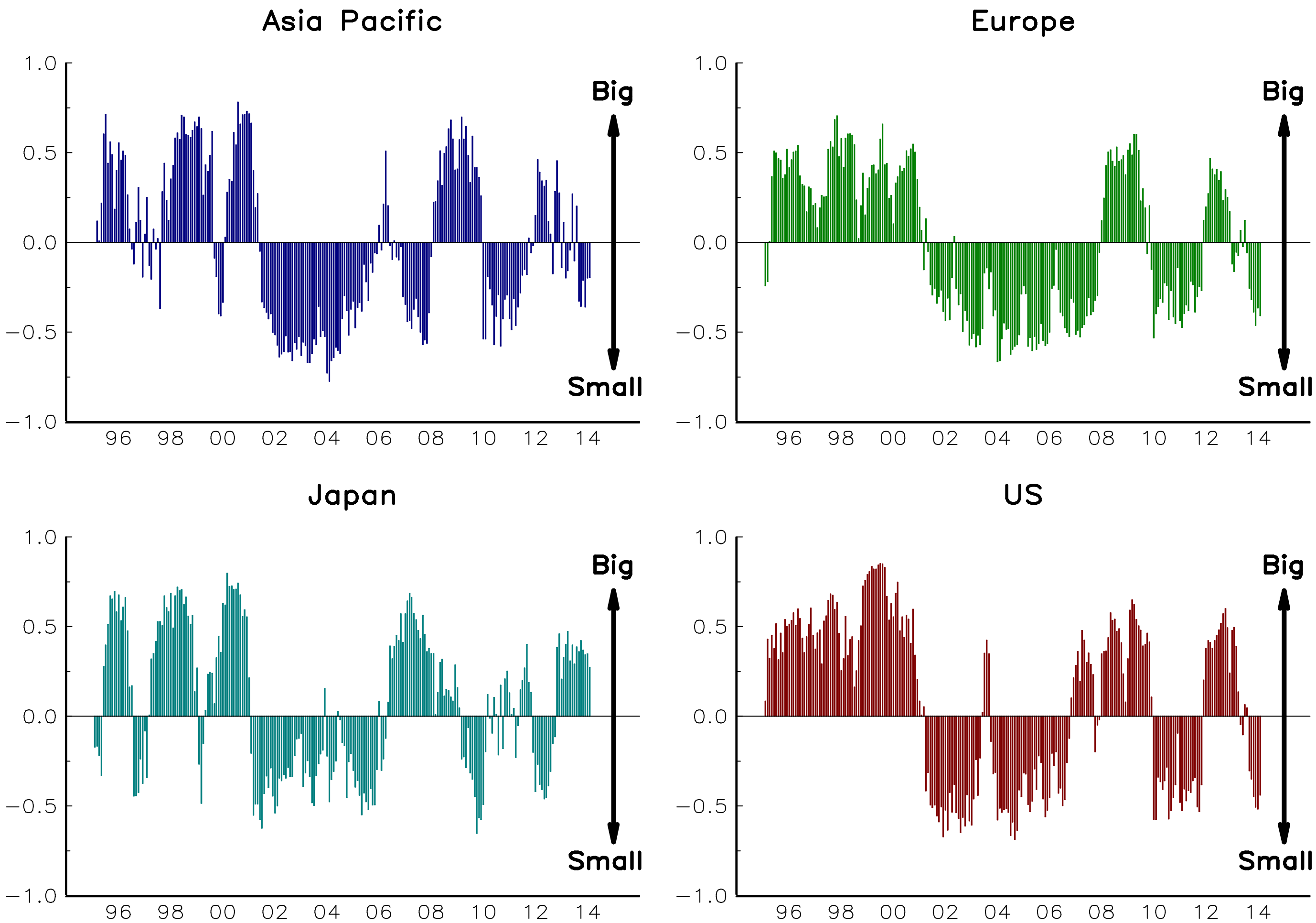
The size effect in the WML risk factor

Table: Performance of the SWML, BWML and WML factors (1995 – 2013)

Statistic	Factor	Asia Pacific	Europe	Japan	North America	US
$\mu(x)$	SWML	12.7	17.7	0.4	7.4	4.9
	BWML	2.9	5.3	2.4	3.0	2.5
	WML	8.0	11.5	1.7	5.3	3.8
$\sigma(x)$	SWML	16.2	14.1	15.3	19.0	19.4
	BWML	20.9	18.3	20.4	19.8	19.7
	WML	17.3	15.5	16.6	18.7	18.8
$\mathrm{SR}(x \mid r)$	SWML	0.78	1.25	0.03	0.39	0.25
	BWML	0.14	0.29	0.12	0.15	0.13
	WML	0.46	0.74	0.10	0.28	0.20

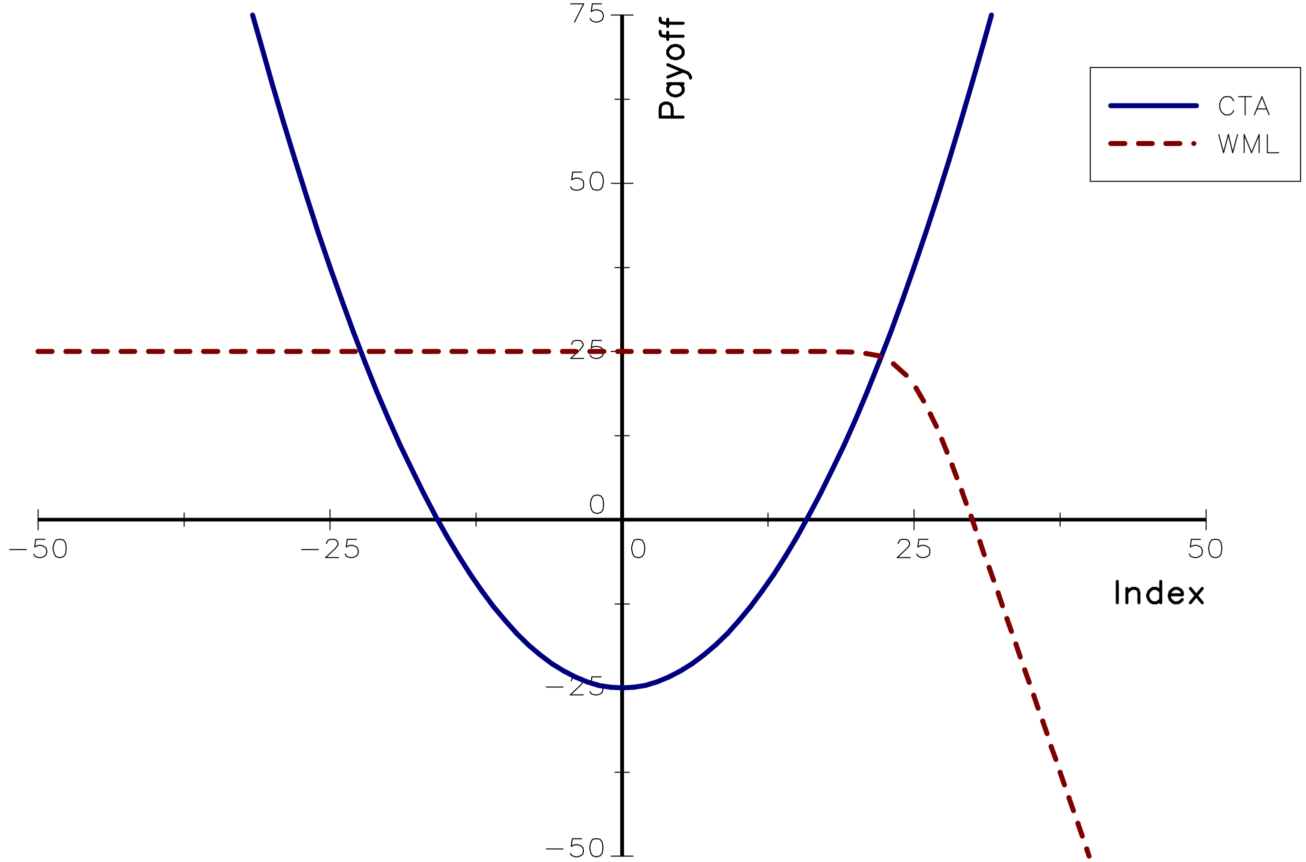
The size neutrality of the BWML factor

Figure: Size ratio between the big value and the big growth portfolios



WML does not exhibit a CTA option profile

Figure: Payoff of CTA and conditional payoff of WML



Volatility

Three anomalies

- Low volatility anomaly
- Idiosyncratic volatility anomaly
- Low beta anomaly

$\Rightarrow$ They are strongly related.

Low volatility anomaly

CAPM

Let x_1 and x_2 be two diversified portfolios. The expected return is an increasing function of the volatility of the portfolio:

$$\sigma(x_2) > \sigma(x_1) \Rightarrow \mu(x_2) > \mu(x_1)$$

$\Rightarrow$ Not always verified (Haugen and Baker, 1991; Clarke *et al.*, 2006; Blitz and van Vliet, 2007).

$\Rightarrow$ Minimum variance portfolio, rank-based portfolios.

Idiosyncratic volatility anomaly

Ang *et al.* (2006) defined IVOL as the volatility of the idiosyncratic risk $\epsilon_i(t)$ corresponding to the residual of the Fama-French regression:

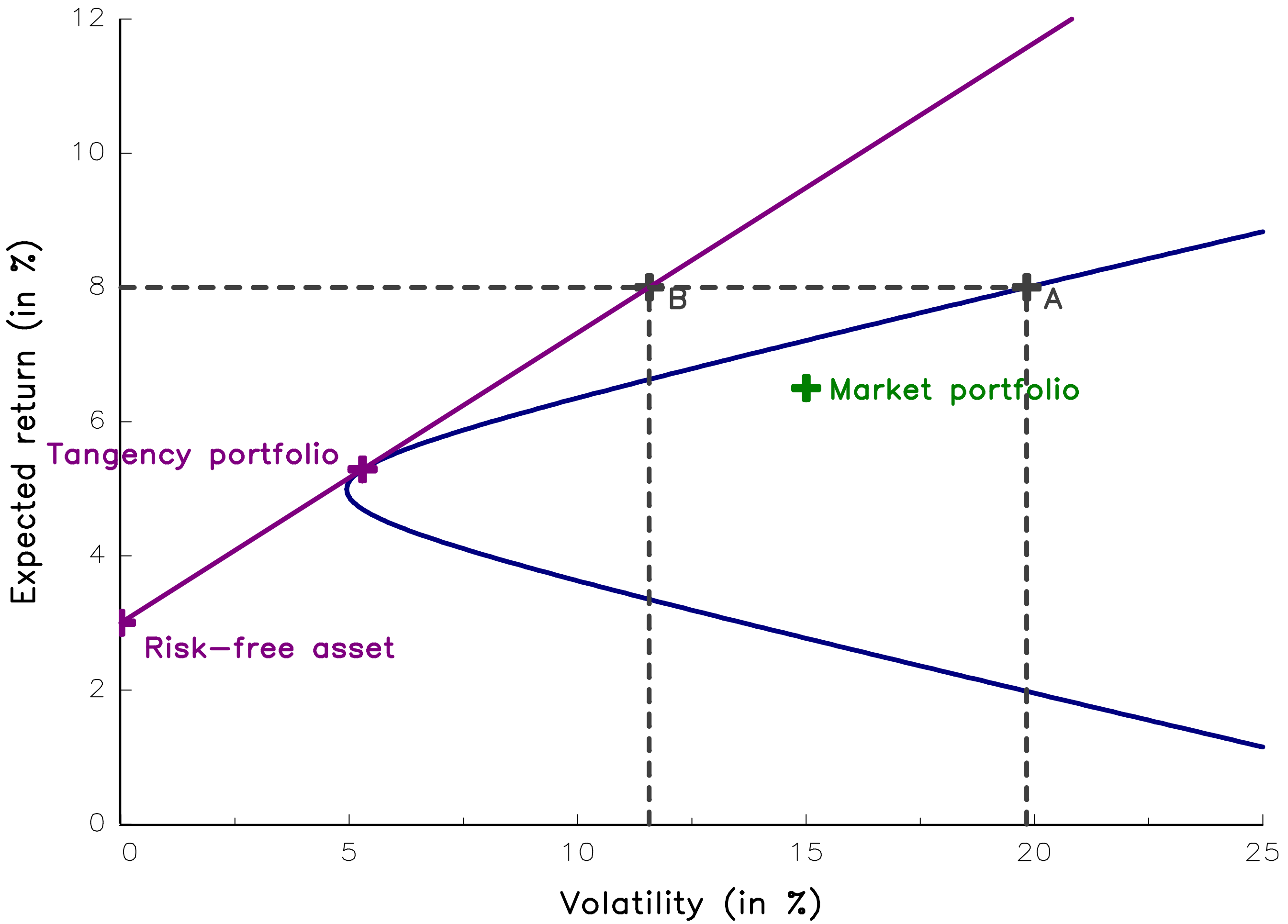
$$R_i(t) = \alpha_i + \beta_i^m R_m(t) + \beta_i^{smb} R_{smb}(t) + \beta_i^{hml} R_{hml}(t) + \epsilon_i(t)$$

By sorting stocks by exposure to IVOL, Ang *et al.* (2006) observed that the return difference between the first quintile portfolio and the last quintile portfolio was 1.06% per month in the United States, and that these results cannot be attributed to size, value, momentum or liquidity factors (Ang *et al.*, 2009).

$\Rightarrow$ Robustness of the results? Bali and Cakini (2008), Fu (2009).

Low beta anomaly

Figure: What is the impact of borrowing constraints on the market portfolio?



Low beta anomaly

Frazzini and Pedersen (2014)

If the investors face some borrowing contraints, the relationship between the risk premium and the beta of asset i becomes:

$$\mathbb{E}\left[R_i\right] - R_f = \alpha_i + \beta_i^m \left(\mathbb{E}\left[R_m\right] - R_f\right)$$

where $\alpha_i = \psi \left(1 - \beta_i^m\right)$ is a decreasing function of β_i.

This can be linked to the empirical evidence of Black *et al.* (1972), which found that the slope of the security market line is lower than the theoretical slope given by the CAPM.

Low beta anomaly

Example

We consider four assets where $\mu_1 = 5\%$, $\mu_2 = 6\%$, $\mu_3 = 8\%$, $\mu_4 = 6\%$, $\sigma_1 = 15\%$, $\sigma_2 = 20\%$, $\sigma_3 = 25\%$ and $\sigma_4 = 20\%$. The correlation matrix C is equal to:

$$C = \begin{pmatrix} 1.00 & & & \\ 0.10 & 1.00 & & \\ 0.20 & 0.60 & 1.00 & \\ 0.40 & 0.50 & 0.50 & 1.00 \end{pmatrix}$$

The risk-free rate is set to 2%.

Table: Tangency portfolio $x^\star$ without any constraints

Asset	$x_i^\star$	$\beta_i(x^\star)$	$\pi_i(x^\star)$
1	47.50%	0.74	3.00%
2	19.83%	0.98	4.00%
3	27.37%	1.47	6.00%
4	5.30%	0.98	4.00%

Low beta anomaly

Let us suppose that the market includes two investors. The first investor cannot leverage his risky portfolio, whereas the second investor must hold 50% of his wealth in cash. We obtain:

Asset	$x_{m,i}$	α_i	$\beta_i(x_m)$	$\pi_i(x_m)$	$\alpha_i + \pi_i(x_m)$
1	42.21%	0.32%	0.62	2.68%	3.00%
2	15.70%	0.07%	0.91	3.93%	4.00%
3	36.31%	−0.41%	1.49	6.41%	6.00%
4	5.78%	0.07%	0.91	3.93%	4.00%

Table: Betting-against-beta (BAB) portfolios

Portfolio	#1	#2	#3	#4
$\tilde{x}_1$	1	0	1	5
$\tilde{x}_2$	0	1	1	0
$\tilde{x}_3$	−1	0	−3	−5
$\tilde{x}_4$	0	−1	1	0
$\mathbb{E}[R(\tilde{x})]$	0.79%	0.00%	1.51%	3.94%
$\sigma(R(\tilde{x}))$	26.45%	21.93%	46.59%	132.24%

Low beta anomaly

Table: Performance of the BAB factor (1995-2013)

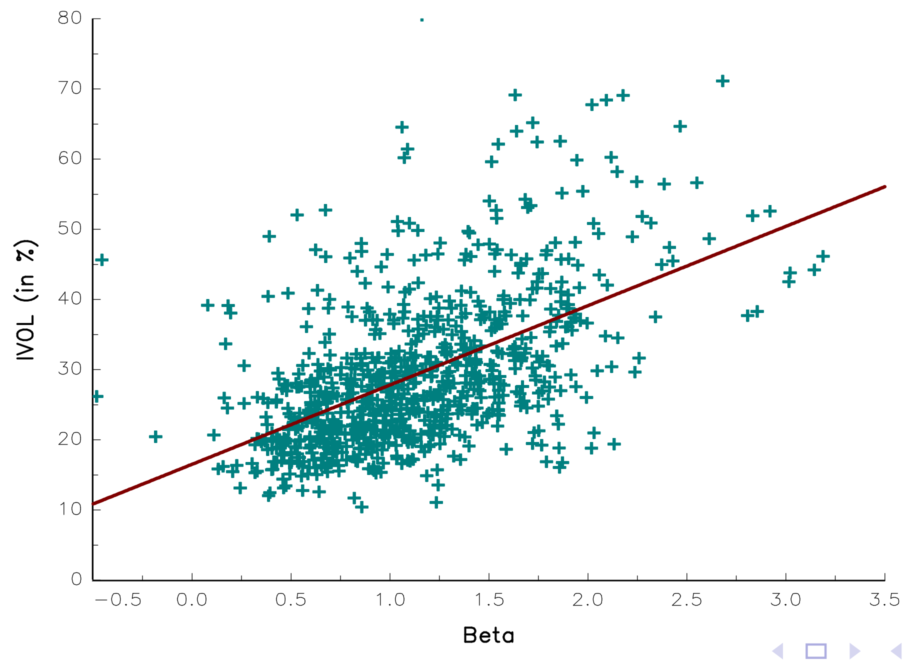
Asset class	$\mu(x)$	$\sigma(x)$	$\mathrm{SR}(x \mid r)$
USD Equities	9.04%	14.96%	0.60
JPY Equities	2.65%	13.12%	0.20
DEM Equities	6.38%	17.98%	0.36
FRF Equities	−3.03%	26.26%	−0.12
GBP Equities	5.31%	14.41%	0.37
International Equities	7.73%	8.20%	0.94
US Treasury Bonds	1.73%	2.95%	0.59
US Corporate Bonds	5.43%	10.81%	0.50
Currencies	1.12%	8.64%	0.13
Commodities	−4.78%	17.76%	−0.27
All assets	5.36%	4.34%	1.24

Links between VOL, IVOL and BAB

CAPM

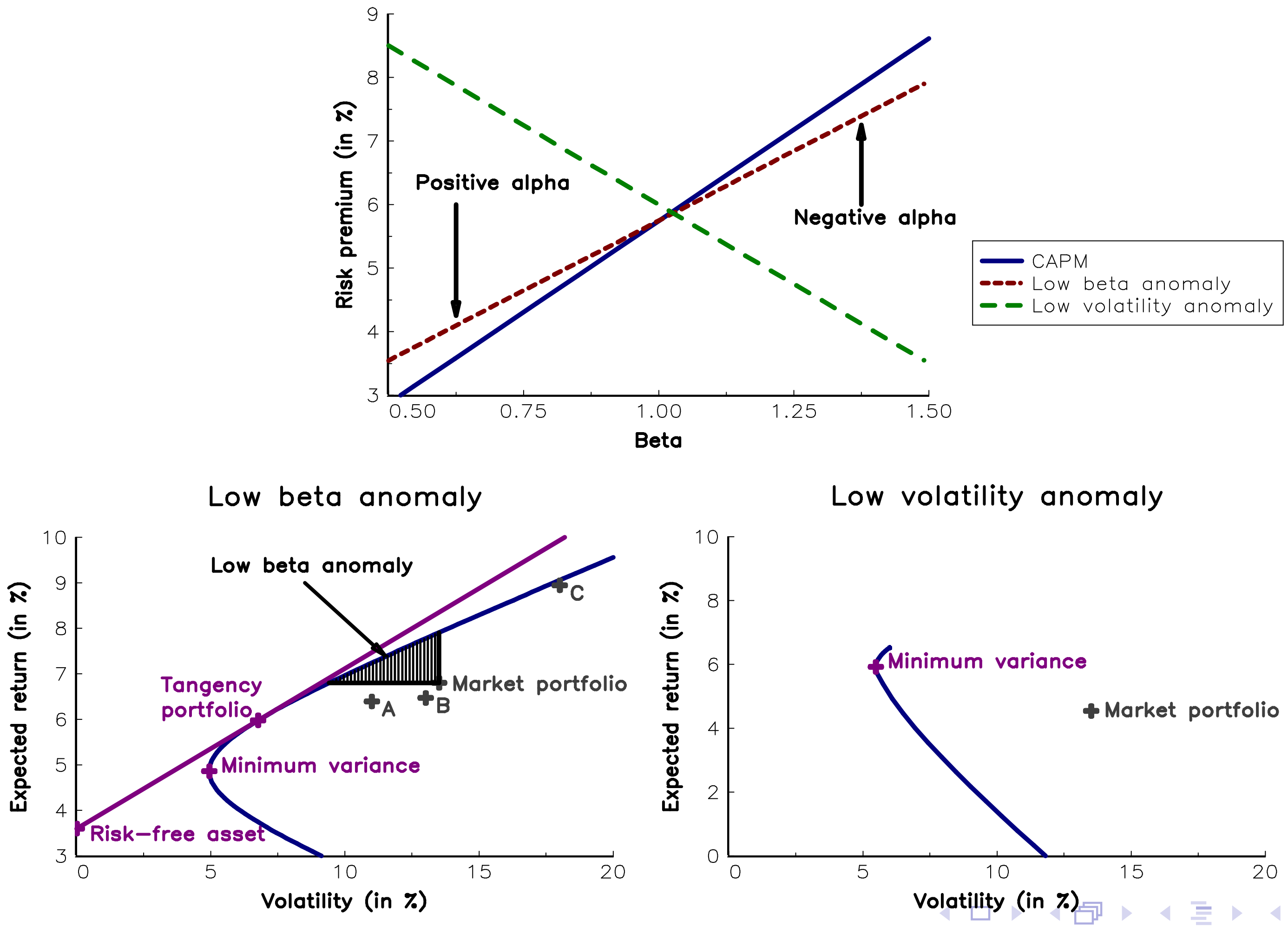
$$\underbrace{\sigma_i^2}_{\text{VOL}} = \underbrace{\left(\beta_i^m\right)^2 \sigma_m^2}_{\text{BETA}} + \underbrace{\tilde{\sigma}_i^2}_{\text{IVOL}}$$

Figure: Relation between β_i^m and IVOL_i (Fama-French)



Links between VOL, IVOL and BAB

Figure: Difference between the low beta and low volatility anomalies

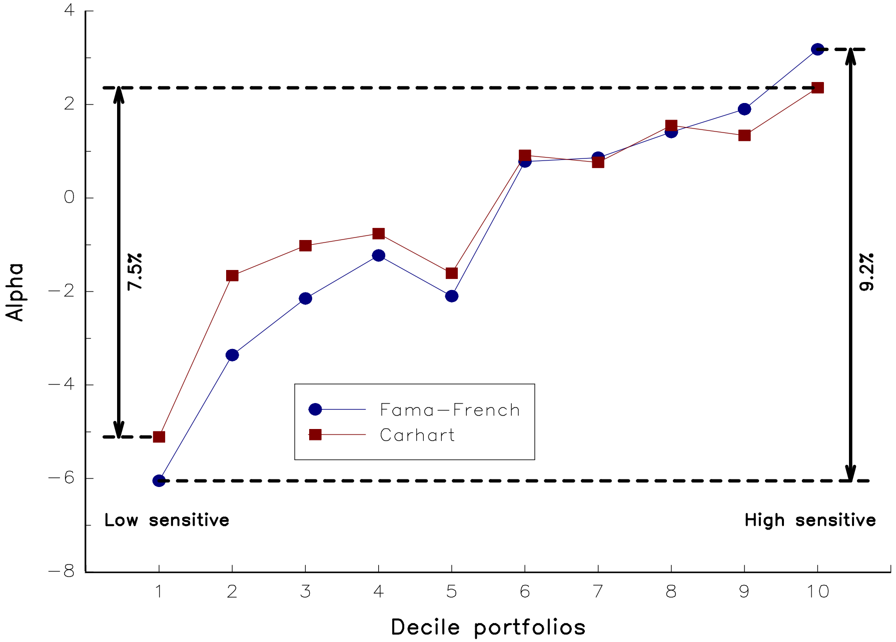


Liquidity

Pàstor and Stambaugh (2003) suggested including a liquidity premium in the Fama-French-Carhart model:

$$\begin{aligned}\mathbb{E}[R_i] - R_f &= \beta_i^m (\mathbb{E}[R_m] - R_f) + \beta_i^{smb}\mathbb{E}[R_{smb}] + \beta_i^{hml}\mathbb{E}[R_{hml}] + \\ & \quad \beta_i^{wml}\mathbb{E}[R_{wml}] + \beta_i^{liq}\mathbb{E}\left[R_{liq}\right]\end{aligned}$$

where LIQ measures the shock or innovation of the aggregate liquidity.



Alphas of decile portfolios sorted on predicted liquidity betas

Long Q10 / Short Q1:

- 9.2% wrt 3F
- 7.5% wrt 4F

Carry

Let X_t be the capital allocated at time t to finance a futures position on asset S_t. Koijen *et al.* (2013) showed that the expected excess return is the sum of the carry and the expected price change:

$$\mathbb{E}_t\left[R_{t+1}(X)\right] - R_f = C_t + \frac{\mathbb{E}_t\left[\Delta S_{t+1}\right]}{X_t}$$

where $C_t = (S_t - F_t)/X_t$ is the carry.

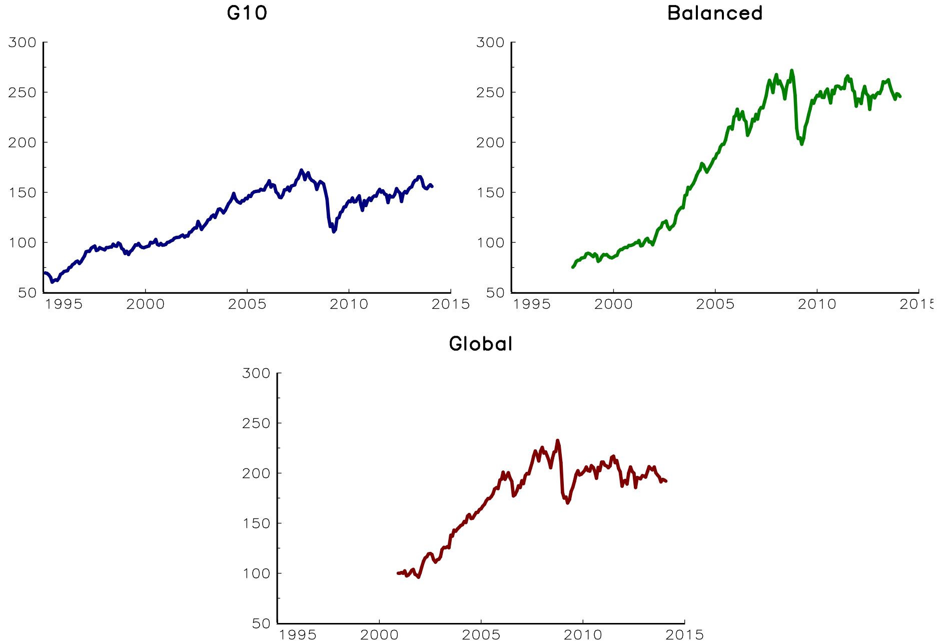
- Currencies:
$$C_t \simeq i_t^* - i$$
- Equities:
$$C_t \simeq DY_t - R_f$$
- Bonds
  - Roll-down strategy
  - Carry of the slope: $C_t \simeq R_t^{10\mathrm{Y}} - R_t^{2\mathrm{Y}}$

Carry

Table: Performance of DB currency carry strategies (1995-2013)

Universe	$\mu(x)$	$\sigma(x)$	$\mathrm{SR}(x \mid r)$
G10	4.31%	10.48%	0.41
Balanced	7.44%	10.87%	0.68
Global	5.02%	11.68%	0.43

Figure: Performance of DB currency carry indices



Quality

Piotroski (2000) argues that the success of the value strategy is explained by the strong performance of quality stocks, and not by the performance of distressed stocks.

Scoring system:

1. Piotroski (2000): profitability, leverage/liquidity, operating efficiency.
2. Novy-Marx (2013): gross profitability.
3. Asness *et al.* (2013): profitability, payout ratio, required return, growth.

Asness *et al.* (2013) defined the QMJ factor as follows:

$$\mathrm{QMJ}_t = \frac{1}{2}\left(R_t\left(\mathrm{SQ}\right) + R_t\left(\mathrm{BQ}\right)\right) - \frac{1}{2}\left(R_t\left(\mathrm{SJ}\right) + R_t\left(\mathrm{BJ}\right)\right)$$

with the following six portfolios:

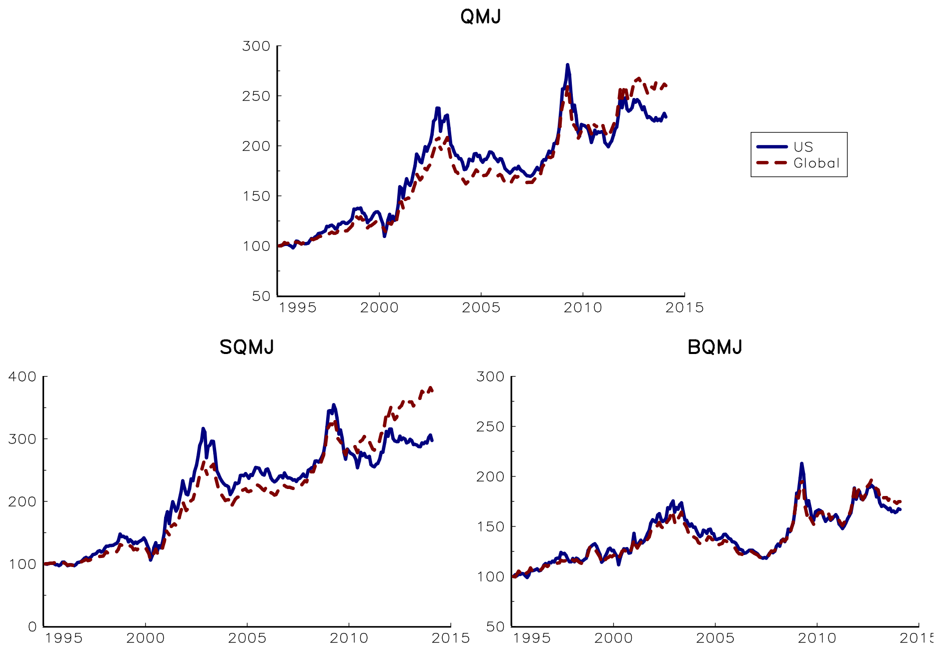
	Junk	Median	Quality
Small	SJ	SM	SQ
Big	BJ	BM	BQ

Quality

Table: Statistics for the SQMJ, BQMJ and QMJ factors (1995 – 2013)

Statistic	US SQMJ	US BQMJ	US QMJ	Global SQMJ	Global BQMJ	Global QMJ
$\mu(x)$	5.9	2.7	4.4	7.2	3.0	5.2
$\sigma(x)$	13.5	10.4	10.8	10.0	8.6	8.5
$\mathrm{SR}(x \mid r)$	0.44	0.26	0.41	0.73	0.35	0.60

Figure: Performance of the QMJ, SQMJ and BQMJ factors



How to define factor indexes?

Asset universe: academics versus investors

Academics generally use a large asset universe provided by the Center for Research in Security Prices (CRSP) or Standard and Poor's (Compustat and Xpressfeed).

Table: Average number of stocks to compute FF HML factor

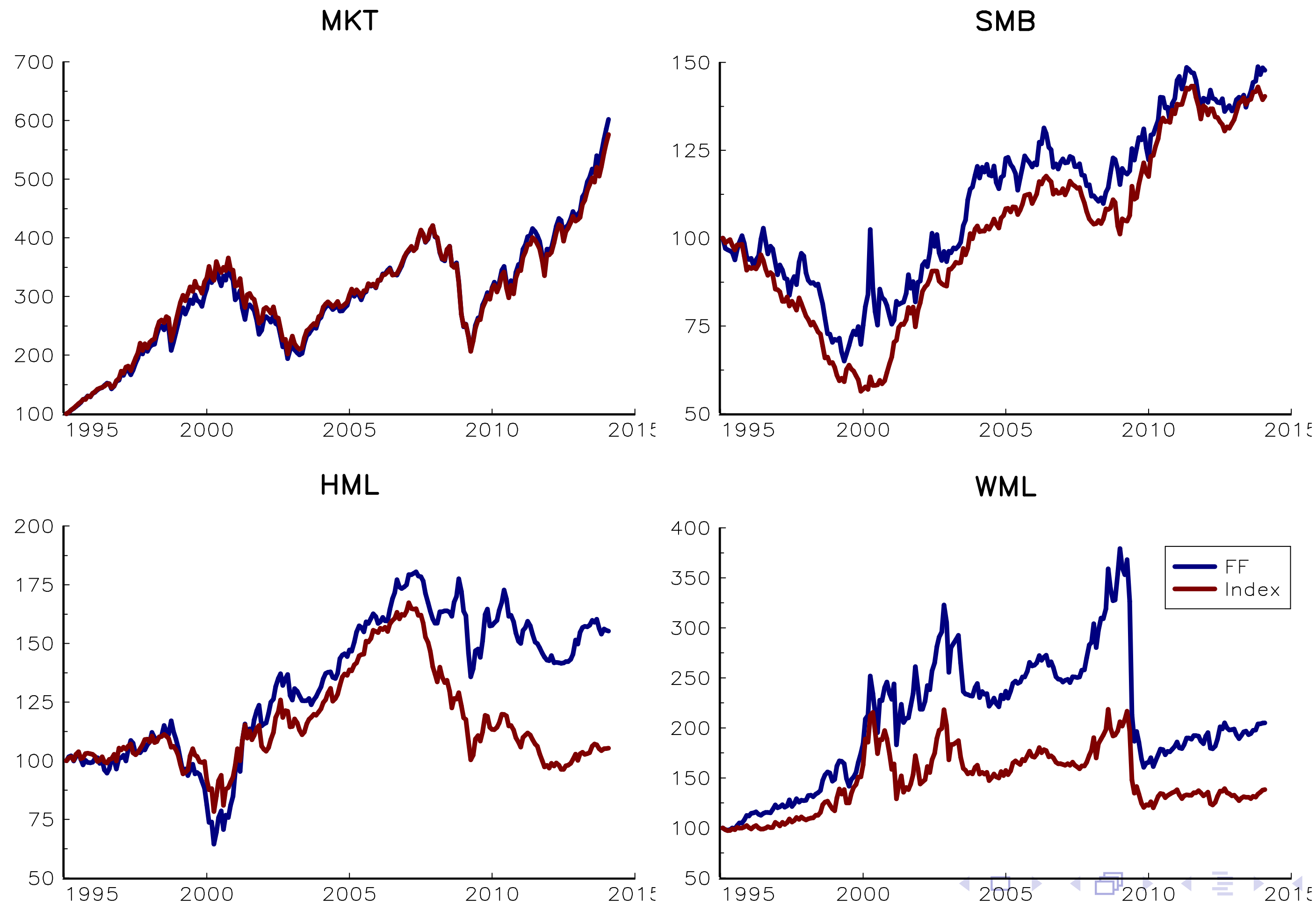
	Asia Pacific	Europe	Japan	North America	US
Big	326	615	591	888	846
Small	2646	4093	1840	2982	2385
Total	2972	4708	2431	3870	3231

Remark

NBIM had about 1900 *and* 1300 *American and Japanese stocks in its portfolio at the end of December 2013.*

How to define factor indexes?
Asset universe

Figure: Performance of risk factors with the S&P 500 index (1995 – 2013)



How to define factor indexes?

Weighting scheme

Three weighting methods:

1. Value-weighted (VW) portfolios:

$$w_i \propto \begin{cases} -ME_i & \text{if } R_i < Q_1 \\ +ME_i & \text{if } R_i > Q_2 \end{cases}$$

   where Q_1 and Q_2 are two numbers such that $Q_1 < \bar{R} < Q_2$.

2. Equally-weighted (EW) portfolio:

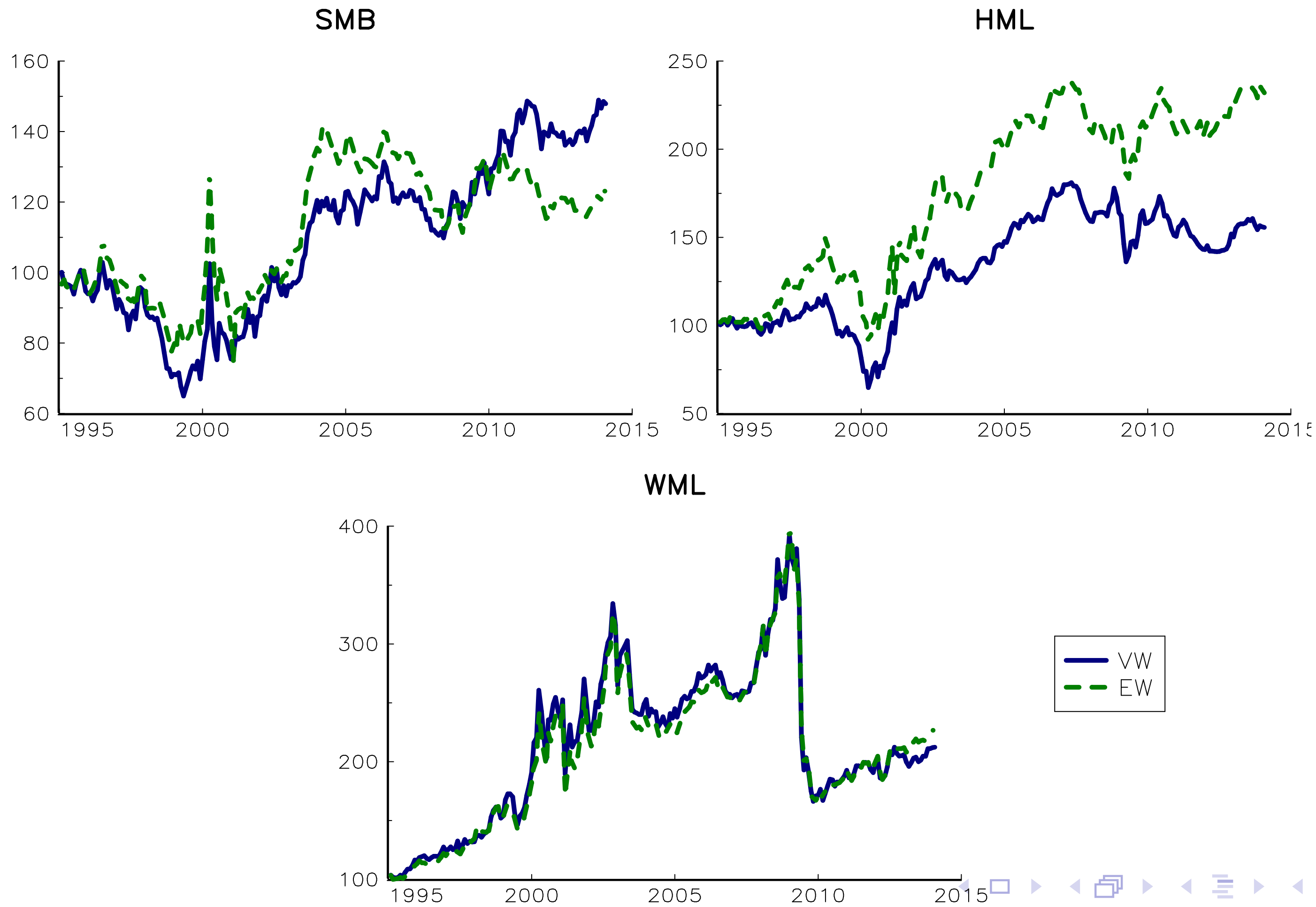
$$w_i \propto \begin{cases} -1 & \text{if } R_i < Q_1 \\ +1 & \text{if } R_i > Q_2 \end{cases}$$

3. Rank-weighted portfolios:

$$w_i \propto \begin{cases} -\left|R_i - \bar{R}\right| & \text{if } R_i < Q_1 \\ +\left|R_i - \bar{R}\right| & \text{if } R_i > Q_2 \end{cases}$$

How to define factor indexes?
Weighting scheme

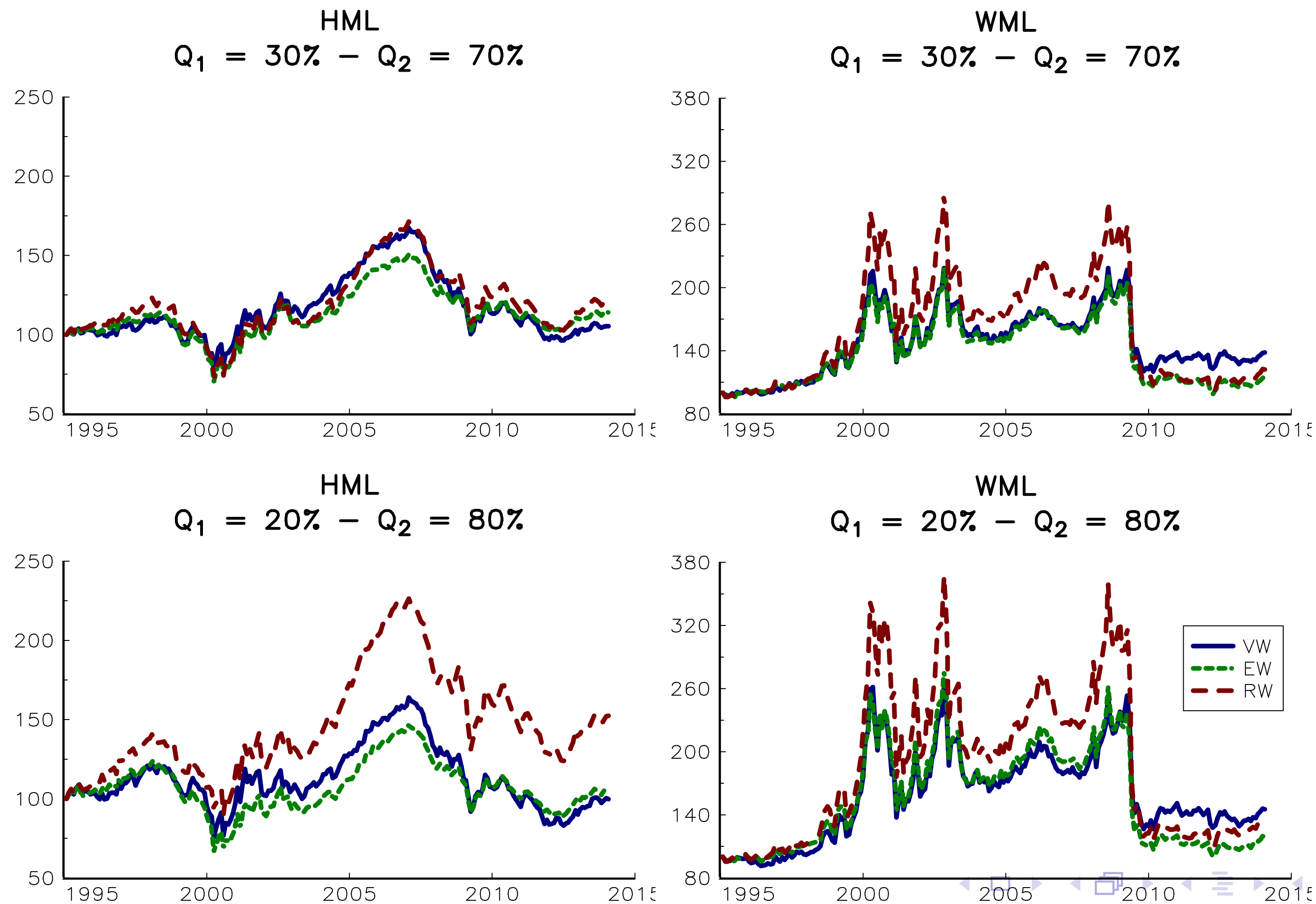
Figure: Comparison of VW and EW risk factors (US, 1995 – 2013)



How to define factor indexes?

Weighting scheme

Figure: Impact of (Q_1, Q_2) on HML and WML factors (S&P 500, 1995 – 2013)



How to define factor indexes?

Factor replication

Factor model

We consider a set of n assets $\{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ and a set of m risk factors $\{\mathcal{F}_1, \ldots, \mathcal{F}_m\}$. We denote by R the $(n \times 1)$ vector of asset returns at time t, while Σ is its associated covariance matrix. We also denote by $\mathcal{F}$ the $(m \times 1)$ vector of factor returns at time t and Ω its associated covariance matrix. We assume the following linear factor model:

$$R = \alpha + B\mathcal{F} + \varepsilon$$

where α is a $(n \times 1)$ vector, B is a $(n \times m)$ matrix and ε is a $(n \times 1)$ centered random vector of covariance D.

$\Rightarrow$ The beta β_i^j of asset i with respect to factor $\mathcal{F}_j$ is $(B)_{i,j}$.

How to define factor indexes?

Factor replication

Example

We consider $n = 6$ assets and $m = 3$ factors. The loadings matrix is:

$$B = \begin{pmatrix} 0.9 & 0.3 & 2.5 \\ 1.1 & 0.5 & -1.5 \\ 1.2 & 0.6 & 3.4 \\ 0.8 & -0.8 & -1.2 \\ 0.8 & -0.2 & 2.1 \\ 0.7 & -0.4 & -5.2 \end{pmatrix}$$

The three factors are uncorrelated and their volatilities are equal to 20%, 15% and 1%. We consider a diagonal matrix D with specific volatilities 10%, 13%, 5%, 8%, 18% and 8%.

$\Rightarrow$ We have to estimate the replication portfolio x in order to define the replicated factor $\mathcal{F}_j^\star = \sum_{i=1}^n x_i R_i$.

How to define factor indexes?

Factor replication

Table: Minimizing the tracking error volatility

Portfolio	#1			#2		
Factor	1	2	3	1	2	3
x_1	8.4	1.2	0.7	8.5	1.3	1.3
x_2	6.3	10.8	−1.5	6.4	12.0	−2.6
x_3	39.9	39.6	2.0	40.5	44.0	3.6
x_4	23.2	−56.3	1.5	23.6	−62.5	2.6
x_5	3.2	−5.7	0.5	3.2	−6.4	0.9
x_6	19.2	−13.3	−4.2	19.5	−14.8	−7.5
β_1	97.0	1.5	0.1	98.5	1.7	0.1
β_2	2.7	81.0	1.1	2.8	90.0	1.9
β_3	26.3	246.1	32.4	26.7	273.4	56.9
$\mathrm{RC}_1^\star$	100.0	0.0	0.0	100.0	0.0	0.0
$\mathrm{RC}_2^\star$	0.0	100.0	0.0	0.0	100.0	0.0
$\mathrm{RC}_3^\star$	0.0	0.0	100.0	0.0	0.0	100.0
$\sigma\left(\mathcal{F}_j^\star \mid \mathcal{F}_j\right)$	3.5	6.5	0.8	3.5	6.7	0.9
$\sigma\left(\mathcal{F}_j^\star\right)$	19.7	13.5	0.6	20.0	15.0	1.0

#1 = without constraints.

#2 = same volatility than the original factor.

How to define factor indexes?

Factor replication

Table: Comparison of the three approaches

Approach	Sensitivity			Beta			Risk contribution		
Factor	1	2	3	1	2	3	1	2	3
x_1	16.7	16.3	2.2	17.4	4.2	2.7	15.2	13.9	2.1
x_2	20.4	27.1	−1.3	17.9	40.4	−3.7	18.6	42.9	−2.9
x_3	22.3	32.6	2.9	17.7	16.0	1.0	19.3	2.3	2.5
x_4	14.9	−43.4	−1.0	17.8	−55.7	0.5	19.6	−68.5	−0.1
x_5	14.9	−10.9	1.8	17.4	−29.9	3.6	16.0	−15.3	2.6
x_6	13.0	−21.7	−4.5	17.7	1.6	−3.9	16.4	5.5	−5.4
β_1	97.1	25.0	1.5	97.1	0.0	0.0	97.3	−0.7	−0.1
β_2	8.5	83.6	4.0	0.0	81.0	0.0	0.0	82.7	2.4
β_3	32.7	252.8	45.6	0.0	0.0	42.7	0.4	0.0	51.7
$\mathrm{RC}_1^\star$	99.6	11.0	9.8	99.8	0.0	0.0	100.0	0.0	0.0
$\mathrm{RC}_2^\star$	0.3	91.2	25.3	0.0	100.5	0.0	0.0	100.0	0.0
$\mathrm{RC}_3^\star$	0.1	−2.2	64.8	0.0	0.0	89.5	0.0	0.0	100.0
$\sigma\left(\mathcal{F}_j^\star \mid \mathcal{F}_j\right)$	4.8	8.6	1.0	4.8	9.2	1.1	4.7	8.8	1.0
$\sigma\left(\mathcal{F}_j^\star\right)$	20.0	15.0	1.0	20.0	15.0	1.0	20.0	15.0	1.0

From long/short to long-only solutions

Factor replication

Table: Impact of the long-only constraint

Approach	Tracking error			Sensitivity			Beta		
Factor	1	2	3	1	2	3	1	2	3
x_1	8.5	0.0	0.8	16.7	13.4	1.4	17.4	0.0	0.0
x_2	6.4	0.0	0.0	20.4	22.4	0.0	17.9	40.3	0.0
x_3	40.5	57.0	2.9	22.3	26.9	2.0	17.7	18.2	1.7
x_4	23.6	0.0	0.0	14.9	0.0	0.0	17.8	0.0	0.0
x_5	3.2	0.0	0.5	14.9	0.0	1.2	17.4	0.0	2.8
x_6	19.5	0.0	0.0	13.0	0.0	0.0	17.7	0.0	0.0
β_1	98.5	68.3	4.7	97.1	68.9	4.6	97.1	66.2	4.2
β_2	2.8	34.2	1.9	8.5	31.3	1.4	0.0	31.1	0.4
β_3	26.7	193.7	13.1	32.7	91.3	12.9	0.0	1.5	11.6
$\mathrm{RC}_1^\star$	100.0	83.9	89.2	99.6	87.7	90.7	99.8	81.9	78.7
$\mathrm{RC}_2^\star$	0.0	12.1	7.0	0.3	12.7	2.4	0.0	14.6	−1.1
$\mathrm{RC}_3^\star$	0.0	2.1	4.1	0.1	−0.5	6.1	0.0	0.0	8.7
$\sigma\left(\mathcal{F}_j^\star \mid \mathcal{F}_j\right)$	3.5	17.2	1.3	4.8	17.6	1.3	4.8	17.6	1.3
$\sigma\left(\mathcal{F}_j^\star\right)$	20.0	15.0	1.0	20.0	15.0	1.0	20.0	15.0	1.0

The correlation matrix between replicated portfolios becomes:

$$C = \begin{pmatrix} 1.00 & & \\ 0.93 & 1.00 & \\ 0.95 & 0.99 & 1.00 \end{pmatrix}$$

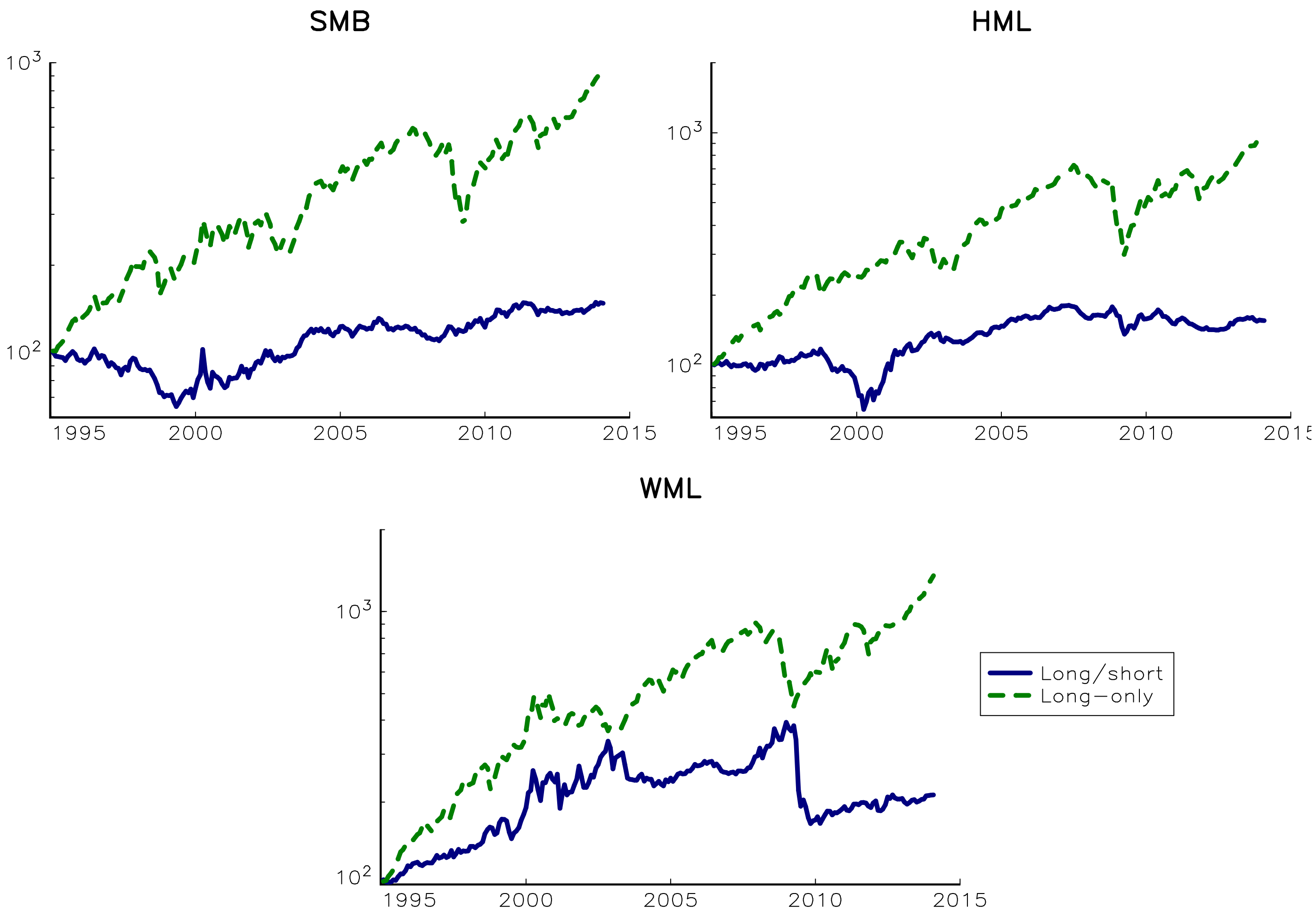
From long/short to long-only solutions

We define the following three risk factors in the case of the Fama-French-Carhart model:

$$\begin{aligned}
\mathrm{SMB}_t^+ &= \frac{1}{3}\left(R_t\left(\mathrm{SV}\right)+R_t\left(\mathrm{SN}\right)+R_t\left(\mathrm{SG}\right)\right) \\
\mathrm{HML}_t^+ &= \frac{1}{2}\left(R_t\left(\mathrm{SV}\right)+R_t\left(\mathrm{BV}\right)\right) \\
\mathrm{WML}_t^+ &= \frac{1}{2}\left(R_t\left(\mathrm{SW}\right)+R_t\left(\mathrm{BW}\right)\right)
\end{aligned}$$

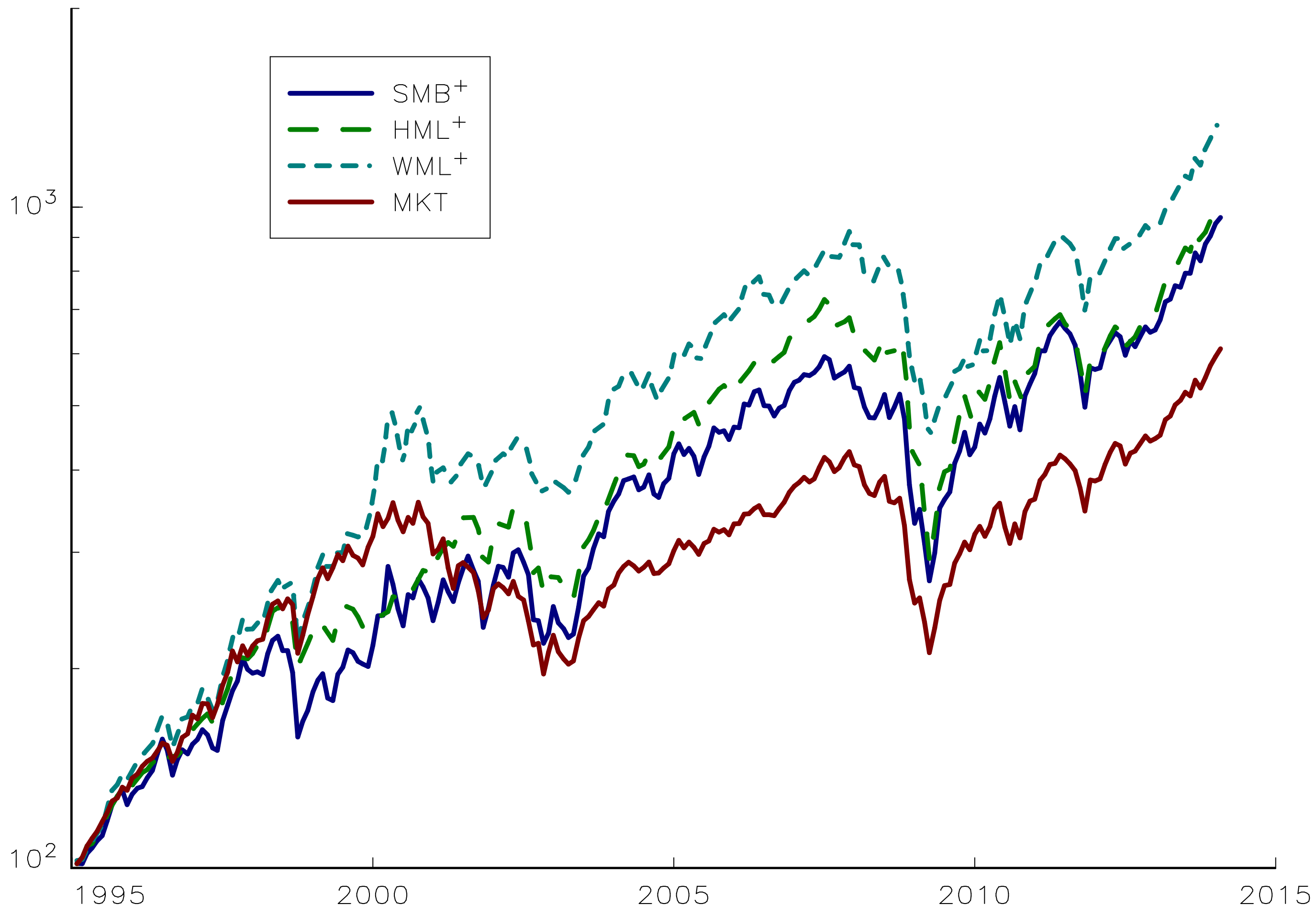
From long/short to long-only solutions

Figure: Performance of long/short and long-only risk factors (US, 1995 – 2013)



From long/short to long-only solutions

Figure: Performance of long-only risk factors (US, 1995 – 2013)



From long/short to long-only solutions

Table: Correlation matrix of risk factors (US, 1995 – 2013)

Factor	MKT	SMB	HML	WML	SMB$^+$	HML$^+$	WML$^+$
Volatility	15.9	12.1	11.5	18.8	20.4	17.7	18.4
MKT	100						
SMB	25	100					
HML	−23	−36	100				
WML	−28	8	−15	100			
SMB$^+$	87	66	−18	−22	100		
HML$^+$	87	33	19	−34	90	100	
WML$^+$	89	53	−29	7	92	81	100

From long/short to long-only solutions

- Long/short portfolio $x^{\pm}$: 100% of market risk and α% of long/short risk factors.
- Long-only portfolio x^{+}: $(100-\alpha)$% of market risk and α% of long-only risk factors.

Table: Statistics (in %) of long/short and long/only portfolios (US, 1995 – 2013)

Portfolio	#0	#1	#2	#3	#4	#5	#6	#7	#8
SMB	0.0	10.0	20.0	0.0	20.0	30.0	0.0	50.0	100.0
HML	0.0	10.0	20.0	20.0	20.0	30.0	0.0	50.0	100.0
WML	0.0	10.0	0.0	20.0	20.0	30.0	60.0	50.0	100.0
$\mu\left(x^{\pm}\right)$	9.9	11.2	11.1	12.0	12.5	13.7	13.5	16.0	21.0
$\mu\left(x^{+}\right)$		11.0	11.2	11.5	12.1	13.2	12.8	13.5	13.5
$\mu\left(x^{+} \mid x^{\pm}\right)$		−0.2	0.0	−0.5	−0.4	−0.5	−0.8	−2.5	−7.5
$\sigma\left(x^{\pm}\right)$	15.9	15.6	16.2	14.8	15.5	15.9	16.7	17.3	24.5
$\sigma\left(x^{+}\right)$		16.2	16.5	16.1	16.9	17.7	17.0	18.1	18.1
$\sigma\left(x^{+} \mid x^{\pm}\right)$		1.7	1.0	3.5	3.5	5.2	8.6	8.0	18.1
$\rho\left(x^{+}, x^{\pm}\right)$		99.5	99.8	97.8	98.0	95.8	86.9	89.8	67.8

Capacity and liquidity

- Lesmond *et al.* (2004): momentum profits are offset by trading costs.
- Korajczyk and Sadka (2004): the break-even fund sizes for long-only momentum strategies are between \$2 and \$5 billion (relative to December 1999 market capitalization).
- Frazzini *et al.* (2012) estimate the following break-even sizes (in \$ billion) for long/short risk factors:

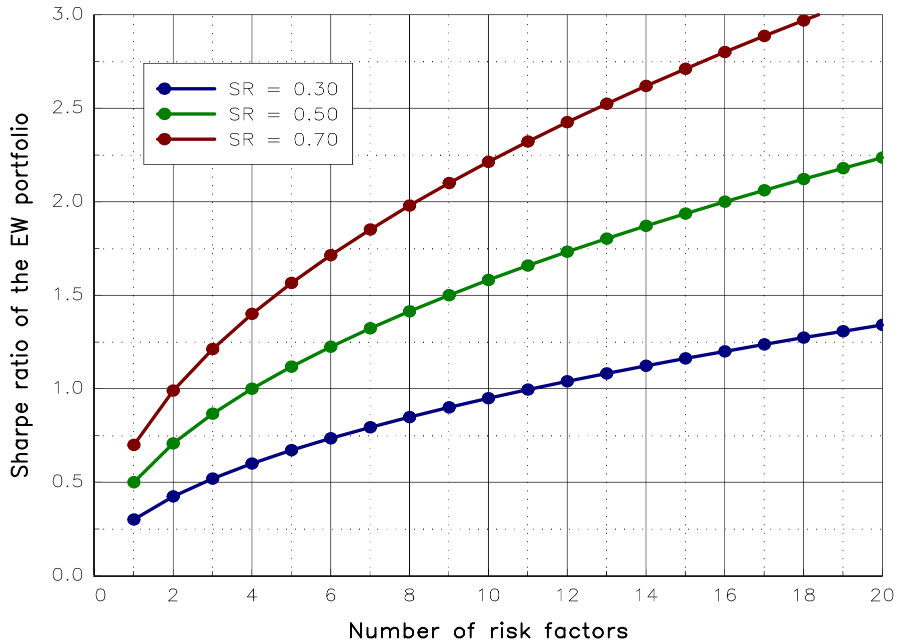
Factor	SMB	HML	WML	STR
US	103	83	52	9
Global	156	190	89	13

$\Rightarrow$ The issue for long-term investors is the **absolute** value of transaction costs, not the relative value.

- ☹ alpha = 5%, TC = 1%
- ☺ alpha = 3%, TC = 1 bp

A magical world

Figure: The arithmetic of Sharpe ratio



In the case of long/short risk factors, we have $\mathrm{SR}(x) \approx \sqrt{m} \cdot \overline{\mathrm{SR}(\mathcal{F})}$ where $\overline{\mathrm{SR}(\mathcal{F})}$ is the average Sharpe ratio.

A magical world

The cash + long/short 5F portfolio

- We consider a 5F long/short portfolio with SMB, HML, WML, BAB and QMJ risk factors.
- The targeted volatility is equal to 10%.

Table: Performance of the 5F and MKT portfolios (1995 – 2013)

Statistic	Asia Pacific		Europe		Japan		North America		US	
	5F	MKT	5F	MKT	5F	MKT	5F	MKT	5F	MKT
$\mu(x)$	13.2	9.2	14.3	9.2	6.8	0.6	11.2	10.2	10.0	9.9
$\sigma(x)$	10.0	21.6	10.0	18.1	10.0	18.6	10.0	15.9	10.0	15.9
$\mathrm{SR}(x \mid r)$	1.04	0.29	1.14	0.35	0.40	−0.12	0.83	0.47	0.71	0.45
$\mathrm{MDD}(x)$	21.6	60.2	19.9	58.9	21.4	58.1	17.7	50.9	21.4	50.4

A magical world

The cash + long/short 5F portfolio

MKT

The correlation matrix between MKT portfolios for the 5 regions is:

$$C = \begin{pmatrix} 1.00 & & & & \\ 0.78 & 1.00 & & & \\ 0.56 & 0.51 & 1.00 & & \\ 0.77 & 0.84 & 0.50 & 1.00 & \\ 0.76 & 0.83 & 0.49 & 1.00 & 1.00 \end{pmatrix}$$

5F

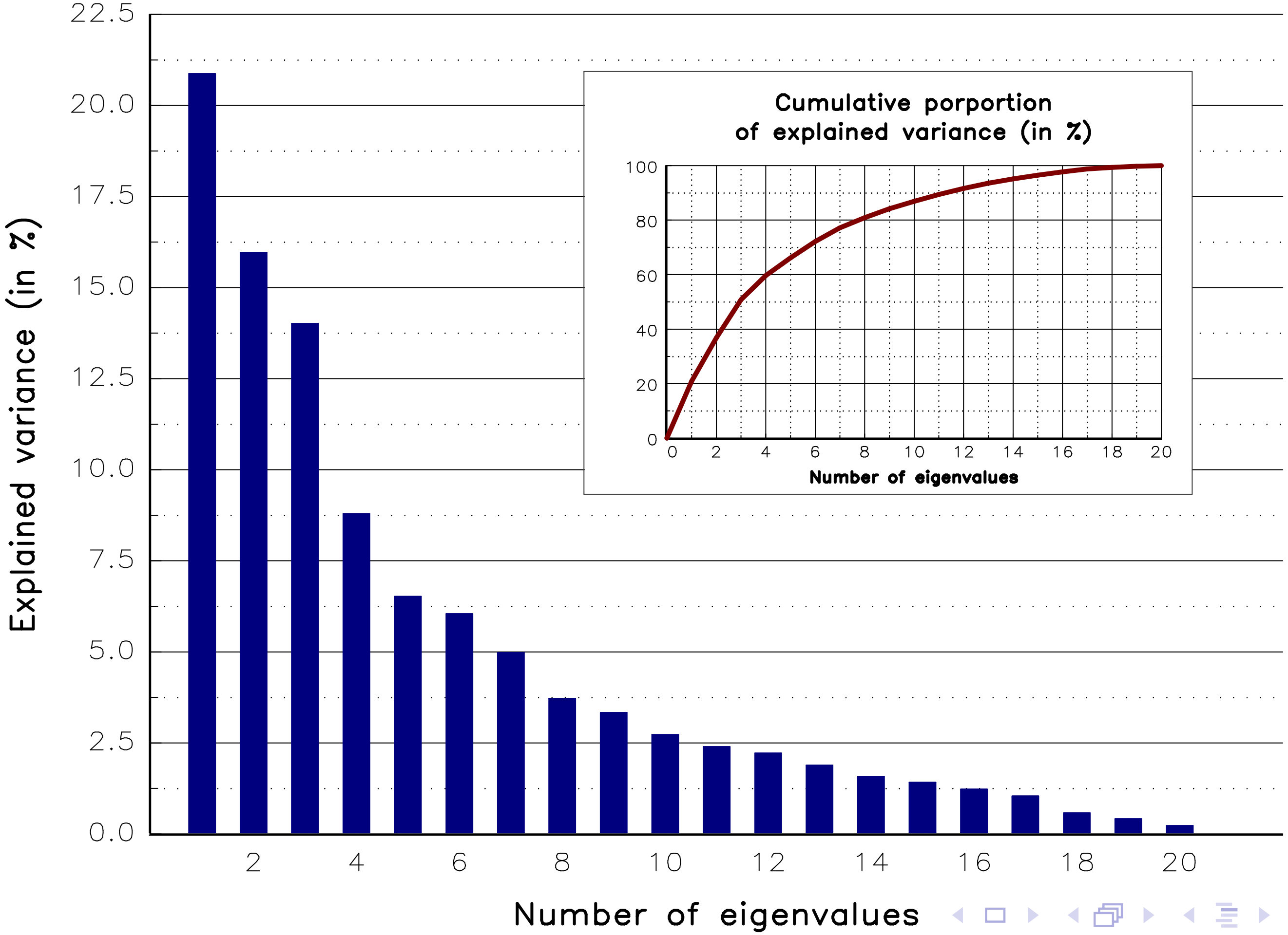
The correlation matrix between 5F portfolios for the 5 regions is:

$$C = \begin{pmatrix} 1.00 & & & & \\ 0.48 & 1.00 & & & \\ 0.56 & 0.38 & 1.00 & & \\ 0.43 & 0.74 & 0.34 & 1.00 & \\ 0.43 & 0.74 & 0.38 & 0.98 & 1.00 \end{pmatrix}$$

A magical world

The cash + long/short 5F portfolio

Figure: Eigenvalues of the risk factors

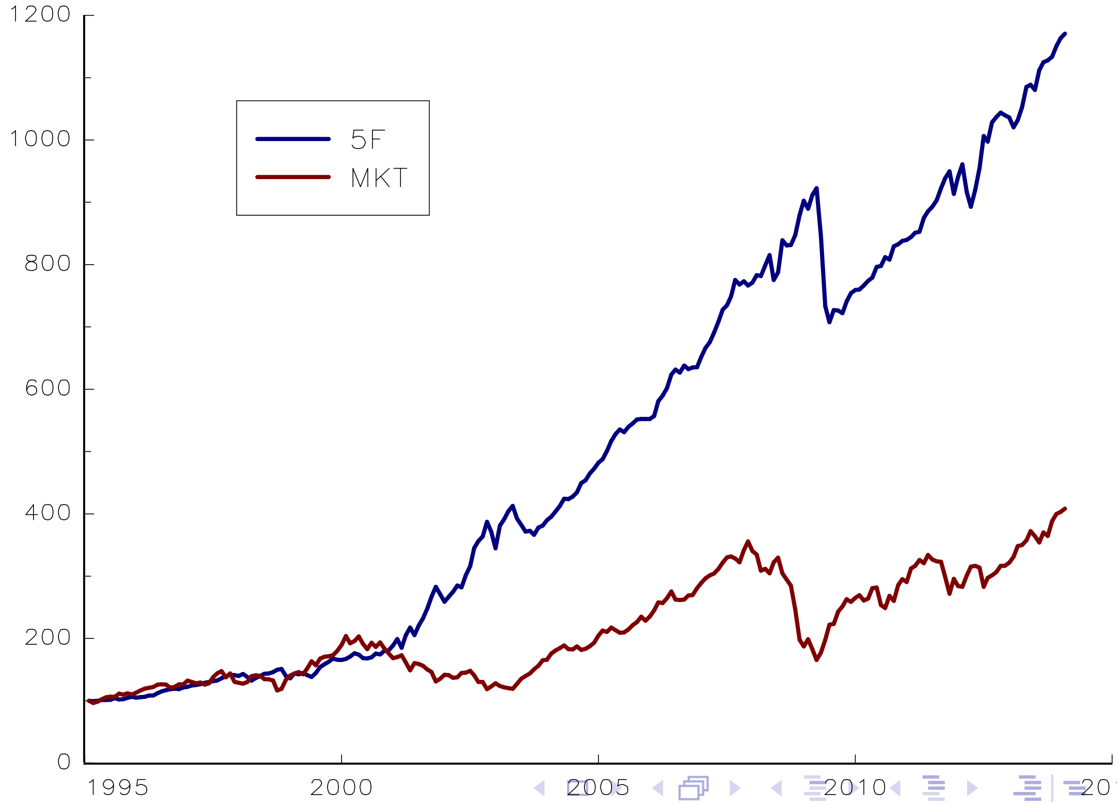


A magical world

The cash + long/short 5F portfolio

Performance of equally-weighted 5F and MKT global portfolios (1995 – 2013)

Statistic	5F	MKT
$\mu(x)$	13.8	7.7
$\sigma(x)$	10.0	16.0
$\mathrm{SR}(x \mid r)$	1.10	0.31
$\mathrm{MDD}(x)$	23.3	53.4



A magical world

The MKT + long/short 5F portfolio

Table: Performance of the MKT + long/short 5F portfolio (1995 – 2013)

Statistic	Asia Pacific	Europe	Japan	North America	US	Global
$\mu(x)$	20.9	22.2	5.2	19.9	18.5	20.1
$\sigma(x)$	21.1	16.8	17.8	14.6	14.0	14.2
$\mathrm{SR}(x \mid r)$	0.85	1.16	0.13	1.18	1.12	1.22
$\mathrm{MDD}(x)$	55.3	53.6	55.9	49.6	46.1	45.5

Table: Performance of the MKT portfolio (1995 – 2013)

Statistic	Asia Pacific	Europe	Japan	North America	US	Global
$\mu(x)$	9.2	9.2	0.6	10.2	9.9	7.7
$\sigma(x)$	21.6	18.1	18.6	15.9	15.9	16.0
$\mathrm{SR}(x \mid r)$	0.29	0.35	−0.12	0.47	0.45	0.31
$\mathrm{MDD}(x)$	60.2	58.9	58.1	50.9	50.4	53.4

A magical world

The long/only 5F portfolio

Table: Performance of the long-only 5F portfolio (1995 – 2013)

Statistic	Asia Pacific	Europe	Japan	North America	US	Global
$\mu(x)$	13.4	15.5	3.1	15.8	14.3	11.3
$\sigma(x)$	21.9	16.9	18.1	15.2	16.3	15.6
$\mathrm{SR}(x \mid r)$	0.48	0.75	0.02	0.86	0.71	0.54
$\mathrm{MDD}(x)$	60.5	58.1	58.1	52.5	55.3	54.5

$\Rightarrow$ **Bad times are not always uncorrelated!**

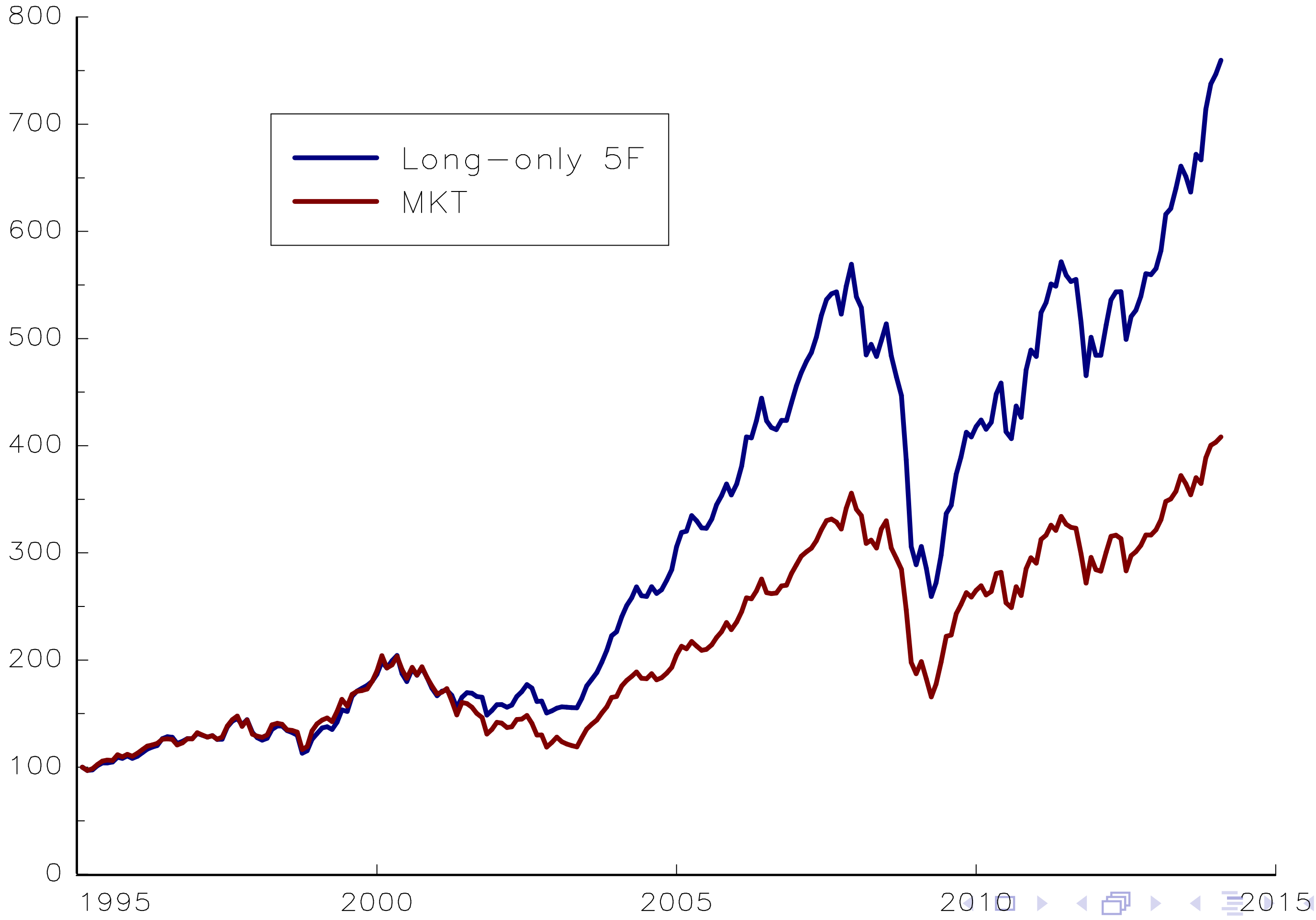
Table: Performance of the MKT portfolio (1995 – 2013)

Statistic	Asia Pacific	Europe	Japan	North America	US	Global
$\mu(x)$	9.2	9.2	0.6	10.2	9.9	7.7
$\sigma(x)$	21.6	18.1	18.6	15.9	15.9	16.0
$\mathrm{SR}(x \mid r)$	0.29	0.35	-0.12	0.47	0.45	0.31
$\mathrm{MDD}(x)$	60.2	58.9	58.1	50.9	50.4	53.4

A magical world

The long/only 5F portfolio

Figure: Performance of long-only 5F and MKT global portfolios



Optimal allocation

Long/short solution

MVO The optimal solution is:

$$x^{\star}(\phi) \propto \Omega^{-1}\mu(\mathcal{F})$$

MVO* The risk factors are independent implying that:

$$x_j^{\star} \propto \frac{\mu(\mathcal{F}_j)}{\sigma^2(\mathcal{F}_j)}$$

ERC If the Sharpe ratio is the same for all risk factors, we obtain the ERC portfolio:

$$x_j^{\star} \propto \frac{1}{\sigma(\mathcal{F}_j)}$$

EW If we assume that expected returns and volatilities are the same for all the factors, the solution is the EW portfolio:

$$x_j^{\star} = \frac{1}{m}$$

Optimal allocation

Long/short solution

Table: Performance and weights of long/short 5F global portfolios (1995 – 2013)

		EW	ERC	MVO*	MVO
Statistic	$\mu(x)$	13.8	14.0	14.7	15.3
	$\sigma(x)$	10.0	10.0	10.0	10.0
	$\mathrm{SR}(x \mid r)$	1.10	1.11	1.19	1.24
	$\mathrm{MDD}(x)$	23.3	19.8	19.7	21.9
Weight	SMB	20.0	25.1	0.0	0.0
	HML	20.0	22.6	31.1	46.3
	WML	20.0	12.8	13.7	20.6
	BAB	20.0	18.2	31.9	26.6
	QMJ	20.0	21.4	23.4	6.5

Optimal allocation

Long-only solution

Optimal portfolio (maximum Sharpe ratio)

$$x_j^\star \propto \frac{\max\left(\mu_j\left(\mathcal{F}_j^+\right) - r - \beta_j \lambda^\star, 0\right)}{\left(\tilde{\sigma}_j^+\right)^2} \quad \text{or} \quad x_j^\star \propto \frac{\max\left(\alpha_j^+ + \beta_j\left(\mu_m - r^\star\right), 0\right)}{\left(\tilde{\sigma}_j^+\right)^2}$$

where $\lambda^\star$ is a weighted average of risk premia and $r^\star = r + \lambda^\star$.

What is an optimal long-only risk factors?

- High alpha;
- Low beta if $\mu_m \simeq 0$ but high beta otherwise;
- Low idiosyncratic volatility.

Optimal allocation

Long-only solution

Optimal portfolio (tracking error)

$$x_j^\star \propto \frac{\left(\mu\left(\mathcal{F}_j^+\right) - \beta_j \mu_m + \tilde{\lambda}_j^\star\right)}{\left(\tilde{\sigma}_j^+\right)^2} \quad \text{or} \quad x_j^\star \propto \frac{\alpha_j^+ + (1-\beta_j)\, r + \tilde{\lambda}_j^\star}{\left(\tilde{\sigma}_j^+\right)^2}$$

where $\tilde{\lambda}_j^\star$ is the gain or cost on the risk factor $\mathcal{F}_k^+$ due to long-only constraints.

The allocation in the market risk factor is the complementary allocation of the other risk factors.

Robustness

Figure: Comparison of Long/short and long-only solutions

Long/short solution

$$x_j^\star \propto \frac{\max\left(\mathrm{RP}_j, 0\right)}{\mathrm{VOL}_j^2}$$

Long-only solution (SR)

$$x_j^\star \propto \frac{\max\left(\mathrm{RP}_j - \beta_j \lambda^\star, 0\right)}{\mathrm{IVOL}_j^2}$$

Long-only solution (TE)

$$x_j^\star \propto \frac{\mathrm{RP}_j - \beta_j \mathrm{RP}_m + \tilde{\lambda}_j^\star}{\mathrm{IVOL}_j^2}$$

Robustness

Stability

Example

We consider a universe of three risk factors:

	α_j^-	α_j^+	$\tilde{\sigma}_j^-$	$\tilde{\sigma}_j^+$	β_j
$\mathcal{F}_1$	2%	2%	7%	7%	1.10
$\mathcal{F}_2$	3%	3%	10%	10%	0.90
$\mathcal{F}_3$	3%	3%	12%	12%	1.00

The other parameters are $\mu_m = 6\%$, $\sigma_m = 20\%$ and $r = 2\%$. This initial parameter set is disturbed as follows:

Set	#0	#1	#2	#3	#4	#5	#6
α_2^- / α_2^+		4%					0%
$\tilde{\sigma}_3^-$ / $\tilde{\sigma}_3^+$			8%				
β_2				0.70			
σ_m					10%		
μ_m						2%	

Robustness

Stability

Table: Long/short solution

Set	#0	#1	#2	#3	#4	#5	#6
$x_1^\star$	44.54	40.15	34.68	44.54	44.54	44.54	66.21
$x_2^\star$	32.73	39.35	25.49	32.73	32.73	32.73	0.00
$x_3^\star$	22.73	20.50	39.83	22.73	22.73	22.73	33.79
$\mathrm{SR}(x^\star \mid r)$	0.68	0.78	0.79	0.68	0.68	0.68	0.54

Robustness

Stability

Table: Long-only solution (SR)

Set	#0	#1	#2	#3	#4	#5	#6
$x_1^\star$	0.00	0.00	0.00	0.00	33.40	0.00	30.50
$x_2^\star$	64.39	87.44	47.81	72.74	40.16	74.19	0.00
$x_3^\star$	35.61	12.56	52.19	27.26	26.44	25.81	69.50
$\mathrm{SR}(x^\star \mid r)$	0.33	0.37	0.34	0.35	0.58	0.15	0.31
$\mu(x^\star \mid b)$	2.74	3.52	2.81	2.13	2.64	3.00	2.82
$\sigma(x^\star \mid b)$	7.83	9.04	6.42	9.09	5.63	8.18	8.63
$\mathrm{IR}(x^\star \mid b)$	0.35	0.39	0.44	0.23	0.47	0.37	0.33

Robustness

Stability

Table: Long-only solution (TE, $\phi = 1$)

Set	#0	#1	#2	#3	#4	#5	#6
$x_1^\star$	21.44	0.00	0.00	42.64	21.08	0.00	42.64
$x_2^\star$	29.83	82.26	14.29	0.00	30.15	58.06	0.00
$x_3^\star$	48.73	17.74	85.71	57.36	48.78	41.94	57.36
$x_b^\star$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mathrm{SR}(x^\star \mid r)$	0.32	0.37	0.33	0.30	0.56	0.15	0.30
$\mu(x^\star \mid b)$	2.75	3.49	2.94	2.74	2.75	3.00	2.74
$\sigma(x^\star \mid b)$	6.74	8.65	7.01	7.55	6.75	7.77	7.55
$\mathrm{IR}(x^\star \mid b)$	0.41	0.40	0.42	0.36	0.41	0.39	0.36

Robustness
Stability

Table: Long-only solution (TE, $\phi = 20$)

Set	#0	#1	#2	#3	#4	#5	#6
$x_1^\star$	23.65	24.02	23.65	24.63	24.26	20.01	22.64
$x_2^\star$	13.41	18.23	13.41	8.79	13.11	15.19	0.00
$x_3^\star$	10.42	10.42	23.44	10.42	10.42	10.42	10.42
$x_b^\star$	52.52	47.33	39.50	56.16	52.21	54.38	66.94
$\mathrm{SR}(x^\star \mid r)$	0.26	0.27	0.28	0.25	0.50	0.06	0.24
$\mu(x^\star \mid b)$	1.23	1.55	1.62	1.06	1.24	1.17	0.86
$\sigma(x^\star \mid b)$	2.48	2.78	2.85	2.30	2.49	2.42	2.07
$\mathrm{IR}(x^\star \mid b)$	0.50	0.56	0.57	0.46	0.50	0.48	0.41

Robustness

SAA versus TAA

Constant mix strategy = right answer?

$\Rightarrow$ Not obvious if risk premia are time-varying and mean-reverting.

BUT

How to diversify bad times (or skewness premia)?

Robustness

Scalability

Scalability of risk factors?

$\Rightarrow$ Index-based or fund-based management (execution)?

Conclusion

Factor investing = a powerful tool, but not so easy to manipulate:

- The zoo of factors (Cochrane, 2011)
- Factor investment products (indexes, strategies & funds) $\neq$ risk factors
- Allocating between risk factors is not straightforward.

Factor investing = a complementary approach and not a substitute to traditional asset allocation

Investment universe for managing large portfolios
=
Beta (or asset classes) + Risk Factors (or new betas)

Conclusion

Table: Definition of Smart Beta

Risk Factors:	Market Risk Factor	Other Risk Factors
Beta:	Traditional Beta (Old Beta)	Alternative Betas (New Betas)
Smart Beta:	CW, EW, MDP, ERC	SMB, HML, WML, BAB, QMJ

MV?

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Tables

For Year 2014, all computations are done on the full year except:

- $^{(\dagger)}$: January-November 2014;
- $^{(\ddagger)}$: January-October 2014.

The source of data are the following:

- Kenneth French library for MKT, SMB, HML and WML;
- AQR library for BAB and QMJ.

Yearly return of the MKT factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	14.9	19.3	−2.5	35.7	36.8
1996	22.9	21.9	−16.1	22.3	21.1
1997	−20.6	19.9	−28.7	30.8	31.2
1998	−6.7	25.5	7.5	22.9	24.3
1999	46.6	19.7	81.7	23.3	25.2
2000	−15.6	−9.9	−32.9	−7.8	−11.6
2001	−8.1	−20.0	−28.9	−10.7	−11.4
2002	−7.0	−14.0	−7.8	−21.3	−21.1
2003	50.5	42.7	41.0	32.5	31.8
2004	28.6	23.6	17.3	13.1	11.9
2005	13.4	11.9	27.2	7.9	6.1
2006	33.8	37.0	1.0	15.6	15.4
2007	36.5	14.1	−4.9	7.8	5.7
2008	−51.1	−45.7	−26.0	−37.9	−36.7
2009	77.4	35.5	5.2	31.9	28.3
2010	22.4	6.1	16.2	18.3	17.5
2011	−15.2	−13.0	−10.3	−0.7	0.5
2012	24.6	21.1	6.5	15.6	16.4
2013	6.2	28.5	26.7	32.3	35.2
2014	−1.6	−6.5	−2.6	10.7	11.7

Tables

Yearly return of the SMB factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	−10.3	−8.3	−3.5	−3.1	−5.7
1996	4.0	−1.9	−8.6	−3.1	−2.0
1997	−9.3	−12.3	−33.1	−8.8	−4.8
1998	−15.8	−13.6	8.9	−19.6	−19.3
1999	12.9	11.7	−8.9	9.8	13.4
2000	−20.1	−7.0	1.1	−3.1	−4.8
2001	−3.6	−0.7	5.1	16.3	20.4
2002	1.4	6.5	2.1	0.6	3.9
2003	13.5	8.8	14.3	17.6	22.2
2004	−6.2	7.5	17.0	5.7	5.0
2005	−9.6	5.0	10.7	0.0	−1.8
2006	7.1	6.1	−20.4	0.6	0.5
2007	−0.3	−8.0	−7.3	−5.8	−7.8
2008	−22.4	−10.2	6.3	−0.7	7.0
2009	20.4	8.6	0.3	9.6	7.9
2010	9.4	9.7	4.5	15.7	12.9
2011	−10.6	−8.7	9.3	−6.1	−5.0
2012	−9.1	1.0	1.3	−0.6	0.5
2013	−1.0	6.2	−3.1	2.0	6.0
2014	−4.1	−2.0	6.8	−7.4	−7.0

Tables

Yearly return of the HML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	1.0	−5.0	−3.5	−2.0	0.8
1996	7.1	1.6	9.0	5.2	1.4
1997	−2.4	12.2	−12.8	11.4	9.6
1998	11.3	0.6	1.7	−13.6	−10.2
1999	11.5	−17.0	−32.5	−25.3	−26.7
2000	23.7	33.0	60.1	48.9	37.3
2001	18.0	33.0	20.2	11.7	14.8
2002	18.3	26.9	14.4	18.2	12.6
2003	13.7	15.6	9.2	4.6	3.2
2004	8.3	9.2	8.6	9.0	8.7
2005	1.5	8.0	−0.3	7.0	8.6
2006	2.1	7.9	14.5	11.6	12.7
2007	4.5	−0.6	5.9	−12.4	−11.6
2008	9.0	−3.1	21.3	0.2	2.0
2009	−5.1	1.7	−3.8	−1.7	−1.8
2010	−1.1	−5.3	−0.3	−1.1	−2.1
2011	−1.9	−14.1	5.5	−4.5	−6.8
2012	14.9	1.4	−1.6	5.6	6.8
2013	5.0	7.9	2.3	0.7	0.4
2014	9.2	−6.1	−0.3	−5.6	−3.5

Tables

Yearly return of the SHML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	2.9	−3.1	−0.7	−3.6	2.4
1996	−3.3	2.4	3.9	8.6	10.6
1997	11.0	14.3	−1.6	19.4	22.7
1998	8.1	4.2	−3.5	−7.2	−3.1
1999	16.5	−22.8	−39.0	−28.6	−29.8
2000	30.4	33.4	49.3	57.5	40.0
2001	29.6	50.1	21.3	14.1	15.6
2002	34.6	45.2	19.4	31.3	29.4
2003	23.2	13.1	−6.5	0.9	6.1
2004	12.1	10.3	2.6	6.5	5.8
2005	10.4	11.0	1.7	5.2	9.7
2006	0.2	4.4	19.5	11.8	12.2
2007	22.7	6.6	11.7	−9.6	−15.5
2008	19.4	8.3	20.9	16.9	11.4
2009	−6.8	−2.9	−3.7	−0.1	1.0
2010	−7.8	−4.4	4.7	−0.7	0.1
2011	1.9	−10.3	9.6	0.5	−3.4
2012	18.8	−1.4	−1.6	4.5	5.5
2013	15.8	7.3	−1.3	−0.9	−2.5
2014	4.2	−1.4	1.9	−5.9	−3.9

Tables

Yearly return of the BHML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	−1.2	−6.9	−6.3	−0.5	−0.8
1996	18.1	0.8	14.2	1.7	−7.2
1997	−14.7	10.1	−23.4	3.6	−2.7
1998	13.5	−3.4	6.8	−19.9	−17.0
1999	6.5	−11.0	−27.0	−22.3	−23.9
2000	14.7	31.7	70.9	38.9	34.0
2001	6.9	16.7	18.5	8.6	13.6
2002	3.6	9.7	9.2	5.5	−3.1
2003	4.6	18.0	26.2	8.3	0.3
2004	4.2	8.0	14.3	11.4	11.8
2005	−7.0	5.0	−2.6	8.6	7.5
2006	3.8	11.4	9.7	11.3	13.0
2007	−11.7	−7.5	0.1	−15.1	−7.5
2008	−1.2	−13.6	21.4	−14.6	−7.0
2009	−3.8	5.8	−4.2	−3.5	−5.0
2010	5.7	−6.5	−5.3	−1.5	−4.4
2011	−5.8	−17.9	1.5	−9.3	−10.2
2012	10.9	4.0	−1.8	6.6	8.0
2013	−5.1	8.4	5.9	2.3	3.2
2014	14.4	−10.6	−2.6	−5.4	−3.2

Tables

Yearly return of the WML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	2.3	24.9	−15.4	13.1	14.6
1996	20.2	19.2	−6.5	4.5	5.5
1997	25.9	11.4	53.9	11.6	9.5
1998	−30.2	17.4	−16.6	24.1	22.2
1999	2.6	30.9	66.8	51.3	29.0
2000	−15.9	−23.5	−30.8	−9.7	16.9
2001	27.8	22.2	16.0	−8.3	−10.4
2002	40.7	53.1	−6.2	29.5	28.1
2003	11.8	−11.5	−15.1	−10.7	−17.8
2004	18.1	7.7	7.3	2.2	−0.3
2005	9.7	17.8	21.3	19.7	15.3
2006	26.3	13.1	−3.8	−4.0	−6.5
2007	13.6	20.2	10.0	22.0	22.8
2008	3.4	27.5	15.3	5.9	18.3
2009	−39.5	−37.6	−33.0	−42.0	−52.7
2010	4.8	30.3	−3.3	6.6	5.7
2011	14.3	9.5	3.5	5.1	8.4
2012	19.6	3.6	2.3	0.9	−1.1
2013	38.0	20.7	16.1	12.9	6.2
2014	11.2	4.9	−3.6	−1.2	1.4

Tables

Yearly return of the SWML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	14.7	30.0	−19.5	23.8	25.8
1996	19.1	31.2	−14.6	10.3	6.3
1997	25.7	12.2	29.0	16.5	15.1
1998	−19.7	28.2	−17.1	24.6	25.0
1999	−4.6	52.5	45.2	56.2	32.0
2000	−15.2	−16.6	−21.8	10.3	43.6
2001	41.1	26.7	13.3	−14.1	−17.2
2002	43.8	64.6	1.5	34.8	36.9
2003	14.7	−11.3	−15.3	−10.8	−16.7
2004	24.3	12.1	12.0	3.3	−0.3
2005	11.6	27.2	19.8	19.0	15.6
2006	35.6	15.9	−3.8	−1.8	−3.9
2007	24.1	21.8	14.1	21.9	18.0
2008	12.4	32.0	14.3	2.9	7.9
2009	−42.2	−34.7	−34.5	−49.1	−60.8
2010	11.0	36.4	1.5	5.5	1.9
2011	17.8	17.8	−0.9	11.9	16.5
2012	32.3	9.8	1.4	4.0	3.4
2013	41.3	25.6	18.9	15.9	5.8
2014	25.0	15.1	−2.5	−3.3	1.2

Tables

Yearly return of the BWML factor (in %)

Year	Asia Pacific	Europe	Japan	North America	US
1995	−9.1	19.7	−11.3	3.2	4.2
1996	20.9	8.2	2.2	−1.2	4.5
1997	25.4	10.3	81.7	6.6	3.9
1998	−41.2	6.8	−16.8	23.4	18.9
1999	9.3	11.0	88.5	45.7	25.3
2000	−17.3	−30.3	−39.4	−27.2	−5.8
2001	15.1	17.0	17.8	−2.5	−3.4
2002	37.0	42.0	−13.6	23.9	19.2
2003	8.8	−11.9	−15.3	−10.7	−18.9
2004	12.0	3.5	2.7	1.0	−0.3
2005	7.7	9.0	22.5	20.1	14.8
2006	17.4	10.3	−4.1	−6.3	−9.1
2007	3.5	18.6	5.7	21.9	27.7
2008	−5.3	22.8	15.7	8.6	29.3
2009	−36.8	−40.5	−31.7	−34.1	−43.2
2010	−1.4	24.3	−7.9	7.6	9.5
2011	10.6	1.5	8.0	−1.3	0.7
2012	7.8	−2.5	2.9	−2.1	−5.6
2013	34.0	15.9	13.3	9.8	6.4
2014	−1.5	−4.5	−4.8	0.7	1.6

Yearly return of the BAB factor (in %)

Year	Asia Pacific$^{(\ddagger)}$	Europe$^{(\ddagger)}$	Japan$^{(\ddagger)}$	North America$^{(\ddagger)}$	US$^{(\ddagger)}$
1995	−10.6	9.6	−10.8	22.6	23.7
1996	0.1	13.4	−4.8	31.1	31.8
1997	−9.8	−5.1	−15.6	45.8	47.1
1998	−11.0	9.9	−8.5	−13.0	−13.2
1999	29.9	9.8	30.7	−34.2	−35.3
2000	−5.4	23.4	−13.4	14.0	14.3
2001	6.8	17.9	4.5	12.6	12.3
2002	12.7	52.6	7.4	37.0	35.4
2003	6.7	21.4	−1.6	16.2	10.4
2004	22.7	46.3	21.6	31.8	30.3
2005	14.6	4.1	13.5	14.8	12.9
2006	7.4	28.2	−3.9	12.6	11.0
2007	30.9	14.8	−2.9	−4.2	−6.8
2008	0.1	−20.7	23.2	−33.9	−35.0
2009	21.9	−6.3	−10.6	16.2	9.9
2010	22.1	4.4	3.1	8.4	6.4
2011	11.1	17.8	22.9	5.6	4.1
2012	7.8	3.6	0.6	16.7	16.0
2013	12.0	16.8	−7.5	20.6	20.3
2014	19.8	8.3	15.3	13.0	12.2

Tables

Yearly return of the QMJ factor (in %)

Year	Asia Pacific(‡)	Europe(‡)	Japan(‡)	North America(‡)	US(‡)
1995	−2.5	7.6	−2.9	3.6	3.3
1996	6.0	3.4	8.4	8.7	9.0
1997	22.7	−5.3	22.3	8.5	8.0
1998	1.1	6.6	−1.1	13.4	13.5
1999	2.4	−8.5	4.5	−7.1	−7.8
2000	13.1	6.7	6.9	24.4	24.0
2001	15.7	14.9	16.9	17.0	16.1
2002	11.3	17.0	7.4	23.4	22.6
2003	−19.8	−12.5	−26.2	−17.9	−18.2
2004	−5.3	4.3	−8.1	0.7	0.0
2005	−9.5	−3.0	−16.9	1.1	−0.4
2006	17.1	−2.4	28.0	−5.1	−4.8
2007	8.0	11.6	13.5	8.0	6.9
2008	25.8	31.4	15.6	38.7	38.1
2009	−5.9	−5.7	2.7	−14.7	−14.4
2010	2.5	2.6	−1.2	−7.7	−7.6
2011	16.0	24.7	14.6	22.4	21.8
2012	3.2	0.5	−9.0	−3.7	−5.8
2013	2.6	2.7	−10.4	0.1	−2.0
2014	7.4	9.2	11.2	4.5	3.1

Tables

Yearly return of the SQMJ and BQMJ factors (in %) – US

Year	SQMJ$^{(\ddagger)}$	BQMJ$^{(\ddagger)}$	QMJ$^{(\ddagger)}$
1995	0.5	6.0	3.3
1996	10.0	7.9	9.0
1997	16.0	0.3	8.0
1998	11.2	15.6	13.5
1999	−8.6	−7.4	−7.8
2000	40.4	8.0	24.0
2001	14.9	16.3	16.1
2002	36.6	9.5	22.7
2003	−22.4	−13.9	−18.2
2004	6.1	−5.9	0.0
2005	3.7	−4.4	−0.4
2006	−3.3	−6.4	−4.8
2007	5.7	8.1	6.9
2008	37.4	38.5	38.1
2009	−19.7	−9.1	−14.4
2010	−6.6	−8.6	−7.6
2011	22.0	21.4	21.8
2012	−5.7	−6.1	−5.8
2013	0.0	−3.9	−1.9
2014	2.0	4.1	3.1

Tables

Yearly return of the SQMJ and BQMJ factors (in %) – Global

Year	SQMJ(‡)	BQMJ(‡)	QMJ(‡)
1995	0.4	5.5	2.9
1996	7.1	5.9	6.6
1997	11.0	3.3	7.1
1998	9.1	10.4	9.8
1999	−6.7	−5.1	−5.8
2000	28.7	8.3	18.5
2001	20.0	12.5	16.4
2002	33.5	8.4	20.4
2003	−19.1	−13.9	−16.5
2004	6.3	−4.3	0.9
2005	1.4	−5.4	−2.0
2006	1.8	−2.4	−0.3
2007	9.4	8.9	9.2
2008	33.0	34.8	34.0
2009	−11.5	−8.3	−9.8
2010	−0.2	−4.8	−2.5
2011	22.8	20.7	21.8
2012	2.1	−4.5	−1.2
2013	5.0	−2.5	1.2
2014	7.1	6.0	6.5