Financial Risk Management

Tutorial Class — Session 1

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1 Market Risk

1.1 Covariance matrix

We consider a universe of there stocks A, B and C.

1. The covariance matrix of stock returns is:

$$\Sigma = \begin{pmatrix} 4\% & & \\ 3\% & 5\% & \\ 2\% & -1\% & 6\% \end{pmatrix}$$

   (a) Calculate the volatility of stock returns.

   (b) Deduce the correlation matrix.

2. We assume that the volatilities are 10%, 20% and 30%. whereas the correlation matrix is equal to:

$$\rho = \begin{pmatrix} 100\% & & \\ 50\% & 100\% & \\ 25\% & 0\% & 100\% \end{pmatrix}$$

   (a) Write the covariance matrix.

   (b) Calculate the volatility of the portfolio $(50\%, 50\%, 0)$.

   (c) Calculate the volatility of the portfolio $(60\%, -40\%, 0)$. Comment on this result.

   (d) We assume that the portfolio is long \$150 in stock A, long \$500 in stock B and short \$200 in stock C. Find the volatility of this long/short portfolio.

3. We consider that the vector of stock returns follows a one-factor model:

$$R = \beta \mathcal{F} + \varepsilon$$

   We assume that $\mathcal{F}$ and ε are independent. We note $\sigma^2_{\mathcal{F}}$ the variance of $\mathcal{F}$ and $D = \operatorname{diag}\left(\tilde{\sigma}^2_1, \tilde{\sigma}^2_2, \tilde{\sigma}^2_3\right)$ the covariance matrix of idiosyncratic risks ε_t. We use the following numerical values: $\sigma_{\mathcal{F}} = 50\%$, $\beta_1 = 0.9$, $\beta_2 = 1.3$, $\beta_3 = 0.1$, $\tilde{\sigma}_1 = 5\%$, $\tilde{\sigma}_2 = 5\%$ and $\tilde{\sigma}_3 = 15\%$.

   (a) Calculate the volatility of stock returns.

   (b) Calculate the correlation between stock returns.

1.2 Expected shortfall of an equity portfolio

We consider an investment universe, which is composed of two stocks A and B. The current prices of the two stocks are respectively equal to \$100 and \$200. Their volatilities are equal to 25% and 20% whereas the cross-correlation is equal to -20%. The portfolio is long of 4 stocks A and 3 stocks B.

1. Calculate the Gaussian expected shortfall at the 97.5% confidence level for a ten-day time horizon.
2. The eight worst scenarios of daily stock returns among the last 250 historical scenarios are the following:

s	1	2	3	4	5	6	7	8
R_A	-3%	-4%	-3%	-5%	-6%	$+3\%$	$+1\%$	-1%
R_B	-4%	$+1\%$	-2%	-1%	$+2\%$	-7%	-3%	-2%

   Calculate then the historical expected shortfall at the 97.5% confidence level for a ten-day time horizon.

1.3 Value-at-risk of a long/short portfolio

We consider a long/short portfolio composed of a long (buying) position in asset A and a short (selling) position in asset B. The long exposure is \$2 mn whereas the short exposure is \$1 mn. Using the historical prices of the last 250 trading days of assets A and B, we estimate that the asset volatilities σ_A and σ_B are both equal to 20% per year and that the correlation $\rho_{A,B}$ between asset returns is equal to 50%. In what follows, we ignore the mean effect.

1. Calculate the Gaussian VaR of the long/short portfolio for a one-day holding period and a 99% confidence level.
2. How do you calculate the historical VaR? Using the historical returns of the last 250 trading days, the five worst scenarios of the 250 simulated daily P&L of the portfolio are $-58\,700$, $-56\,850$, $-54\,270$, $-52\,170$ and $-49\,231$. Calculate the historical VaR for a one-day holding period and a 99% confidence level.
3. We assume that the multiplication factor m_c is 3. Deduce the required capital if the bank uses an internal model based on the Gaussian value-at-risk. Same question when the bank uses the historical VaR. Compare these figures with those calculated with the standardized measurement method.
4. Show that the Gaussian VaR is multiplied by a factor equal to $\sqrt{7/3}$ if the correlation $\rho_{A,B}$ is equal to -50%. How do you explain this result?
5. The portfolio manager sells a call option on the stock A. The delta of the option is equal to 50%. What does the Gaussian value-at-risk of the long/short portfolio become if the nominal of the option is equal to \$2 mn? Same question when the nominal of the option is equal to \$4 mn. How do you explain this result?
6. The portfolio manager replaces the short position on the stock B by selling a call option on the stock B. The delta of the option is equal to 50%. Show that the Gaussian value-at-risk is minimum when the nominal is equal to four times the correlation $\rho_{A,B}$. Deduce then an expression of the lowest Gaussian VaR. Comment on these results.

1.4 Risk management of exotic options

Let us consider a short position on an exotic option, whose its current value $\mathcal{C}_t$ is equal to \$6.78. We assume that the price S_t of the underlying asset is \$100 and the implied volatility Σ_t is equal to 20%.

1. At time $t+1$, the value of the underlying asset is \$97 and the implied volatility remains constant. We find that the P&L of the trader between t and $t+1$ is equal to \$1.37. Can we explain the P&L by the sensitivities knowing that the estimates of delta $\boldsymbol{\Delta}_t$, gamma $\boldsymbol{\Gamma}_t$ and vega[1] $\boldsymbol{\upsilon}_t$ are respectively equal to 49%, 2% and 40%?

2. At time $t+2$, the price of the underlying asset is \$97 while the implied volatility increases from 20% to 22%. The value of the option $\boldsymbol{\mathcal{C}}_{t+2}$ is now equal to \$6.17. Can we explain the P&L by the sensitivities knowing that the estimates of delta $\boldsymbol{\Delta}_{t+1}$, gamma $\boldsymbol{\Gamma}_{t+1}$ and vega $\boldsymbol{\upsilon}_{t+1}$ are respectively equal to 43%, 2% and 38%?

3. At time $t+3$, the price of the underlying asset is \$95 and the value of the implied volatility is 19%. We find that the P&L of the trader between $t+2$ and $t+3$ is equal to \$0.58. Can we explain the P&L by the sensitivities knowing that the estimates of delta $\boldsymbol{\Delta}_{t+2}$, gamma $\boldsymbol{\Gamma}_{t+2}$ and vega $\boldsymbol{\upsilon}_{t+2}$ are respectively equal to 44%, 1.8% and 38%.

4. What can we conclude in terms of model risk?

[1] measured in volatility points.