Financial Risk Management

Tutorial Class — Session 3

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3 Counterparty Credit Risk and Collateral Risk

3.1 Impact of netting agreements in counterparty credit risk

The table below gives the current mark-to-market of 7 OTC contracts between Bank A and Bank B:

	Equity			Fixed income		FX	
	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$
A	$+10$	-5	$+6$	$+17$	-5	-5	$+1$
B	-11	$+6$	-3	-12	$+9$	$+5$	$+1$

The table should be read as follows: Bank A has a mark-to-market equal to 10 for the contract $\mathcal{C}_1$ whereas Bank B has a mark-to-market equal to -11 for the same contract, Bank A has a mark-to-market equal to -5 for the contract $\mathcal{C}_2$ whereas Bank B has a mark-to-market equal to $+6$ for the same contract, etc.

1. (a) Explain why there are differences between the MtM values of a same OTC contract.
   (b) Calculate the exposure at default of Bank A.
   (c) Same question if there is a global netting agreement.
   (d) Same question if the netting agreement only concerns equity products.

2. In the following, we measure the impact of netting agreements on the exposure at default.

   (a) We consider a first OTC contract $\mathcal{C}_1$ between Bank A and Bank B. The mark-to-market $\mathrm{MtM}_1(t)$ of Bank A for the contract $\mathcal{C}_1$ is defined as follows:

   $$\mathrm{MtM}_1(t) = x_1 + \sigma_1 W_1(t)$$

   where $W_1(t)$ is a Brownian motion. Calculate the potential future exposure of Bank A.

   (b) We consider a second OTC contract between Bank A and Bank B. The mark-to-market is also given by the following expression:

   $$\mathrm{MtM}_2(t) = x_2 + \sigma_2 W_2(t)$$

   where $W_2(t)$ is a second Brownian motion that is correlated with $W_1(t)$. Let ρ be this correlation such that $\mathbb{E}[W_1(t) W_2(t)] = \rho t$. Calculate the expected exposure of bank A if there is no netting agreement.

   (c) Same question when there is a global netting agreement between Bank A and Bank B.

   (d) Comment on these results.

3.2 Calculation of the capital charge for counterparty credit risk

We denote by $e(t)$ the potential future exposure of an OTC contract with maturity T. The current date is set to $t = 0$. Let N and σ be the notional and the volatility of the underlying contract. We assume that $e(t) = N\sigma\sqrt{t}X$ with $0 \leq X \leq 1$, $\Pr\{X \leq x\} = x^{\gamma}$ and $\gamma > 0$.

1. Calculate the peak exposure $\mathrm{PE}_{\alpha}(t)$, the expected exposure $\mathrm{EE}(t)$ and the effective expected positive exposure $\mathrm{EEPE}(0;t)$.
2. The bank manages the credit risk with the foundation IRB approach and the counterparty credit risk with an internal model. We consider an OTC contract with the following parameters: N is equal to \$3 mn, the maturity T is one year, the volatility σ is set to 20% and γ is estimated at 2.
   (a) Calculate the exposure at default EAD knowing that the bank uses the regulatory value for the parameter α.
   (b) The default probability of the counterparty is estimated at 1%. Calculate then the capital charge for counterparty credit risk of this OTC contract[1].

4 Operational Risk

4.1 Estimation of the loss severity distribution

We consider a sample of n individual losses $\{x_1, \ldots, x_n\}$. We assume that they can be described by different probability distributions:

(i) X follows a log-normal distribution $\mathcal{LN}(\mu, \sigma^2)$.

(ii) X follows a Pareto distribution $\mathcal{P}(\alpha, x^{-})$ defined by:

$$\Pr\{X \leq x\} = 1 - \left(\frac{x}{x_{-}}\right)^{-\alpha}$$

with $x \geq x_{-}$ and $\alpha > 0$.

(iii) X follows a gamma distribution $\Gamma(\alpha, \beta)$ defined by:

$$\Pr\{X \leq x\} = \int_0^x \frac{\beta^{\alpha} t^{\alpha-1} e^{-\beta t}}{\Gamma(\alpha)} \, \mathrm{d}t$$

with $x \geq 0$, $\alpha > 0$ and $\beta > 0$.

1. We consider the case *(i)*.
   (a) Show that the probability density function is:
   $$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)$$
   (b) Calculate the two first moments of X. Deduce the orthogonal conditions of the generalized method of moments.
   (c) Find the maximum likelihood estimators $\hat{\mu}$ and $\hat{\sigma}$.
2. We consider the case *(ii)*.

[1] We will take a value of 70% for the LGD parameter and a value of 20% for the default correlation. We can also use the approximations $-1.06 \approx -1$ and $\Phi(-1) \approx 16\%$.

(a) Calculate the two first moments of X. Deduce the GMM conditions for estimating the parameter α.

(b) Find the maximum likelihood estimator $\hat{\alpha}$.

3. We consider the case *(iii)*. Write the log-likelihood function associated to the sample of individual losses $\{x_1, \ldots, x_n\}$. Deduce the first-order conditions of the maximum likelihood estimators $\hat{\alpha}$ and $\hat{\beta}$.

4. We now assume that the losses $\{x_1, \ldots, x_n\}$ have been collected beyond a threshold H meaning that $X \geq H$.

   (a) What becomes the generalized method of moments in the case *(i)*.

   (b) Calculate the maximum likelihood estimator $\hat{\alpha}$ in the case *(ii)*.

   (c) Write the log-likelihood function in the case *(iii)*.