# An alternative approach to alternative beta[1]

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## Abstract

Hedge fund replication based on factor models is encountering growing interest. In this paper, we investigate the implications of substituting standard rolling windows regressions, which appear ad-hoc, with more efficient methodologies like the Kalman filter. We show that the copycats constructed this way offer risk-return profiles which share several characteristics with the ones posted by hedge funds indices: Sharpe ratios above buy-and-hold strategies on standard assets, moderate correlation with standard assets, and limited drawdowns during equity downward trends. An interesting result is that the shortfall risk seems less important than with hedge fund indices and regressions-based trackers. We finally propose new breakdowns of hedge fund performance into alpha, traditional beta, and alternative beta.

1 We would like to thank Anne-Sophie Duret for research assistance.

Hedge fund replication has become highly fashionable in recent years. Under this generic term, three main approaches can be identified:

- **Mechanical duplication of strategies —** a growing number of investment banks are proposing products which aim to reproduce in a systematic and quantitative manner a strategy followed by hedge funds. For instance, several investable indices have been proposed which mark-to-market systematic short of put options or variance swaps. Thanks to the positive premia between implied and realized volatilities, this kind of strategy seems able to generate superior Sharpe ratios while exposed to the risk of huge losses when the underlying asset plunges [Lo (2005), Goeztmann et al. (2007)]. Other candidates include trend-following strategies, such as CTA/managed futures, which can be mimicked through lookback straddles [Fung and Hsieh (1997)]; merger arbitrage strategies, which can be replicated by passive investments in all announced mergers [Mitchell and Pulvino (2001)]; fixed income funds that Fung and Hsieh (2002) have shown to be reproducible through credit exposure and constant-maturity products; long-short equity which can be mimicked through beta-one plus small versus large caps exposures; convertible arbitrage [Agarwal et al. (2006)]; or, for Global Macro, systematic FX carry strategies. All in all, Fung and Hsieh (2007) estimate that 75% of net assets of the hedge fund universe are covered by these types of systematic strategies.
- **Replication of distribution —** a very different approach has been advocated by Kat and Palaro (2007). They postulate that investors invest in hedge funds because of their expected returns, volatility, and correlation, not for their month-by-month returns. While this argument might be generalized to other assets, it is probably all the more meaningful as diversification is a major motivation for investing in hedge funds. In practice, the copycat is based on passive futures trading strategies[2]. The investor first determines the existing portfolio (for instance, an equally-weighted mix of U.S. Treasuries and S&P 500), the futures he/she wants to trade, and then the statistical properties in terms of correlation with the existing portfolio, shortfall probability, skewness sign, etc. The investor then receives the line-up of the strategy, that is the daily trading volume of each futures contract he/she has to trade to replicate the fictive option so that in the end the fund has the desired properties. However, there is no indication on how long it will take for this result to materialize and, above all, this is based on the strong implicit assumptions that correlation among underlying futures is constant through time.
- **Factor-based model —** This approach relies on linear regressions of hedge fund returns on a list of market factors, being representative of long-only or spread exposures of hedge funds (equity, credit, and bond indices; value/growth spread, small/big caps spread, etc.). A large empirical literature has studied this type of models [Fung and Hsieh (1997, 2004), Hasanhodzic and Lo (2007), Jaeger and Wagner (2005), Agarwal and Naik (2004)]. Results appear to be mixed and quite different depending on strategies. While directional strategies (equity hedge, emerging markets, global macro) present strong exposures, pure market-neutral/arbitrage ones naturally experience limited exposures. Despite these mixed results, the hedge fund industry seems to be largely exposed as a whole, mainly because of the dominant market share of long/short strategies. For example, based on monthly returns, the correlation of the HFR Index with the S&P 500 index is more than 70% over the period 1990-2006. This evidence has led to the expression 'alternative beta,' which denotes the part of the hedge fund returns that can be attributed to directional investments in standard assets. One huge difference between alternative beta and traditional beta is that for hedge funds exposure is not passive but should integrate non-linear effects. For instance, as they follow dynamic strategies, hedge funds regularly change their exposure, which gives rise to option-like payoffs[3]. To deal with these features, measures of option payoffs, such as long systematic position in straddles, have been incorporated.

These approaches differ in various dimensions. The first two seem more ambitious in that they hope to generate synthetic clones whose performance will be similar to hedge funds (gross of fees). The last approach simply promises to get that fraction of hedge fund returns that can be linked to the alternative beta, which in this paper denotes their time-varying exposures to the standard assets (exposures being either long or short). From this perspective, part of the hedge fund alpha is attributed to this dynamic switching between assets. But all the alpha cannot be reproduced as alpha can stem from investing in illiquid securities or ultra-high frequency trading and thus not reproducible through low frequency trading in liquid futures. Later in this article, we try to disentangle hedge fund returns into alpha, alternative beta, and traditional beta in a more precise way. While this approach appears less ambitious, it has clear advantages, which includes simplicity, transparency, and cost-effectiveness. On the contrary, as they aim to replicate complicated hedge fund strategies, the first two approaches are exposed to operational risks, black-box criticisms, or (possibly) high trading costs[4]. Thus, if the main objective is to get higher transparency, lower regulatory constraints, or liquidity, one should concentrate on the factor-model approach. Consequently, it is not surprising that, to date, this methodology has been the most widespread. Still, most existing approaches have a deficiency in common: they are based on OLS regressions over rolling windows whose time-lag is determined in an ad-hoc fashion. More efficient econometric techniques are available, such as the Kalman filter. This paper investigates whether more robust alternative betas processes can be built using such techniques[5].

2 See the website www.fundcreator.com for more details.
3 In an early analysis, Merton (1981) showed that a manager pursuing a strategy of market timing between several assets is likely to generate similar returns to those of an optional position on one of the assets without nonetheless taking up a position on options.
4 Even in the case of the approach of Kat and Palaro (2007), which is based on liquid futures, the tailor-made specification for each investor might lead the minimum investment size to be very high.
5 Another promising approach has been advocated by Darolles and Mero (2007). Building on the methodology developed by Bai and Ng (2002, 2003), the authors estimate latent factors models from a large sample of individual funds. After extracting the number of appropriate factors, they identify them with standard market factors.

## Kalman filter and dynamic allocation

**Recovering dynamic allocation**

Let $R_t$ and $F_t^{(i)}$ be respectively the return of the hedge fund strategy and the returns of the $i^{th}$ individual factors (i = 1,...m) at time t. We have: $R_t = \Sigma_{i=1 \to m}\, \omega_t^{(i)} F_t^{(i)}$ t = 1,...,T, with $\omega_t^{(i)}$ being the weight of the ith individual strategy. The issue is to recover the time-varying allocation $\omega_t^{(i)}$. We may solve this problem by estimating the local beta of the portfolio with respect to the individual strategies. The regression might or not include an intercept, an aspect we discuss below.

**With rolling OLS**

One of the most used techniques to recover dynamic allocation is the rolling OLS method. In this case, we assume a linear model: $R_t = \Sigma_{i=1 \to m}\, \beta_t^{(i)} F_t^{(i)} + u_t$ and the estimates $\acute{\omega}_t^{(i)}$ correspond to the OLS estimation of $\beta^{(i)}$ on the period [t – h,t], where h is the rolling window. For instance, Hasanhodzic and Lo (2007) use a 24-months lag window. Note that we may use the QP method described in Appendix A to impose some constraints (i.e., $|\beta^{(i)}| \leq 1$) or to center the residuals in the case no intercept is included into the regression[6].

**With Kalman filter**

An alternative approach is to use the Kalman filter described in Appendix B with the model: $R_t = \Sigma_{i=1 \to m}\, \beta_t^{(i)} F_t^{(i)} + \varepsilon_t$; $\beta_t^{(1)} = \beta_{t-1}^{(1)} + \eta_t^{(1)} \cdots \beta_t^{(m)} = \beta_{t-1}^{(m)} + \eta_t^{(m)}$ (1), where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ and $\eta_t^{(i)} \sim N(0, \sigma_i^2)$ are uncorrelated processes. Using the notations introduced in the appendix, the state-space form is $Z_t = (R_t^{(1)} \cdots R_t^{(m)})$, $c_t = 0$, $H_t = \sigma_\varepsilon^2$, $M_t = I_m$, $d_t = 0$, $S_t = I_m$ and $Q_t$ a diagonal matrix such that:

$$Q_t = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_m^2 \end{pmatrix}$$

We assume moreover that $P_0 = 0_m$. It means that we know exactly the initial allocation. We denote by $\theta$ the vector of parameters to be estimated by maximum likelihood with $\theta = (\sigma_\varepsilon\ \sigma_1 \cdots \sigma_m)$. More details concerning estimation and the Kalman filter methodology are given in Appendix B.

**Some examples**

Let us consider an example to illustrate the difference between both methods to evaluate the exposures[7].

We suppose a strategy corresponding to a monthly dynamic allocation between MSCI USA and MSCI EMU since 1990. We assume that the weights are oscillating and are given by $\omega_t^{USA} = {}^1\!/_2[1+\sin(2\pi t/\varpi)]$ and $\omega_t^{EMU} = 1 - \omega_t^{USA}$. We have estimated the model (1) with the Kalman filter and we compare the estimates with those obtained with 24-months rolling OLS. When the frequency $\varpi$ is low, the rolling OLS technique is able to reproduce the dynamic allocation, but with delay. If the allocation frequency is higher, the rolling OLS may estimate weights that are very different from the true weights of the dynamic allocation (see Figure 1 with $\varpi$ = 2Y).

Let us now consider a more realistic example. We assume that weights change by step of 25%, with $\omega_t^{USA} = (\omega \otimes 1)_t$ and $\omega_t^{EMU} = 1 - \omega_t^{USA}$ for $\omega$ = (-1, 0.5, -0.25, 1, 0.25, 0, -0.5, 1,···). The portfolio is 100% long with the possibility of being short in one asset. Results are reported in Figure 2 and confirm the adaptability of the Kalman filter. One criticism is that state vector estimates might be too dependant on initial values. But in practice, this is not truly the case as shown in Figure 3 with the same application. Indeed, one of the properties of the Kalman filter is its ability to quickly adapt to changing conditions.

It is interesting to remark that because $P_0 = 0_m$, the estimates $b_t$ and $b_{t|t-1}$ are homogeneous with respect to the vector of parameters $\theta$; that is they do not change if $\theta$ is scaled by a factor $\kappa$. To show this property, we remark that if $Q^* = \kappa^2 Q$ and $H^* = \kappa^2 H$, it comes that $P^*_{t|t-1} = \kappa^2 P_{t|t-1}$, $F_t^* = \kappa^2 F_t$, and $P^*_t = \kappa^2 P_t$. We also have $P^*_{t|t-1} Z_t^T F_t^{*-1} = P_{t|t-1} Z_t^T F_t^{-1}$, so we prove that $b^*_{t|t-1} = b_{t|t-1}$ and $b^*_t = b_t$. Moreover, we may show that the state vector is invariant if we both scale $R_t$, $F_t^{(i)}$, and $\varepsilon_t$. It means that the parameters of interest are the ratios $\sigma_1/\sigma_\varepsilon, \cdots, \sigma_m/\sigma_\varepsilon$. These ratios give us a direct measure of the dynamic property of the allocation of the individual strategies. The bigger these ratios, the more dynamic is the allocation.

## A comparison of OLS and Kalman filter when applied to hedge fund replication

We try to estimate the beta using the following six underlying exposures: an equity position in the S&P 500 Index, a long/short position between Russell 2000 and S&P 500 indices, a long/short position between Eurostoxx 50 (hedged in USD) and S&P 500 indices, a long/short position between Topix (hedged in USD) and S&P 500 indices, a bond position in the 10-year U.S. Treasury, and an FX position in the EUR/USD. These exposures have been chosen because they correspond to standard exposures identified in the literature [Agarwal and Naik (2004), Amenc et al. (2003), Géhin and Vaissie (2006), Hasanhodzic and Lo (2007), Fung and Hsieh (2007), Jaeger and Wagner (2005)]. However, we limit ourselves to liquid futures markets. For example, we do not include any options or OTC swaps and exclude credit indices for which futures have been recently launched but are not mature enough to be included in this context. The models are estimated over the period 1994-2007 and are done on three well-known indices for matter of comparisons: the Composite HFR Index, the HFR Funds-of-Funds index, and the CSFB-Tremont total Index. All of these indices are non-investable, which increase the interest for the exercise of replication here established. To make the backtest more reliable, we incorporate a lag of one month for incorporating the delays in the release of indices.

We have estimated the dynamic beta using the rolling OLS technique for various frequencies (12 months, 24 months, and 36

6 The case with no intercept makes sense in the replication exercise below, as the alpha can not be replicated through investment in the factors.

7 See Swinkels and Van der Sluis (2005) for a more in-depth analysis in the context of style-based analysis.

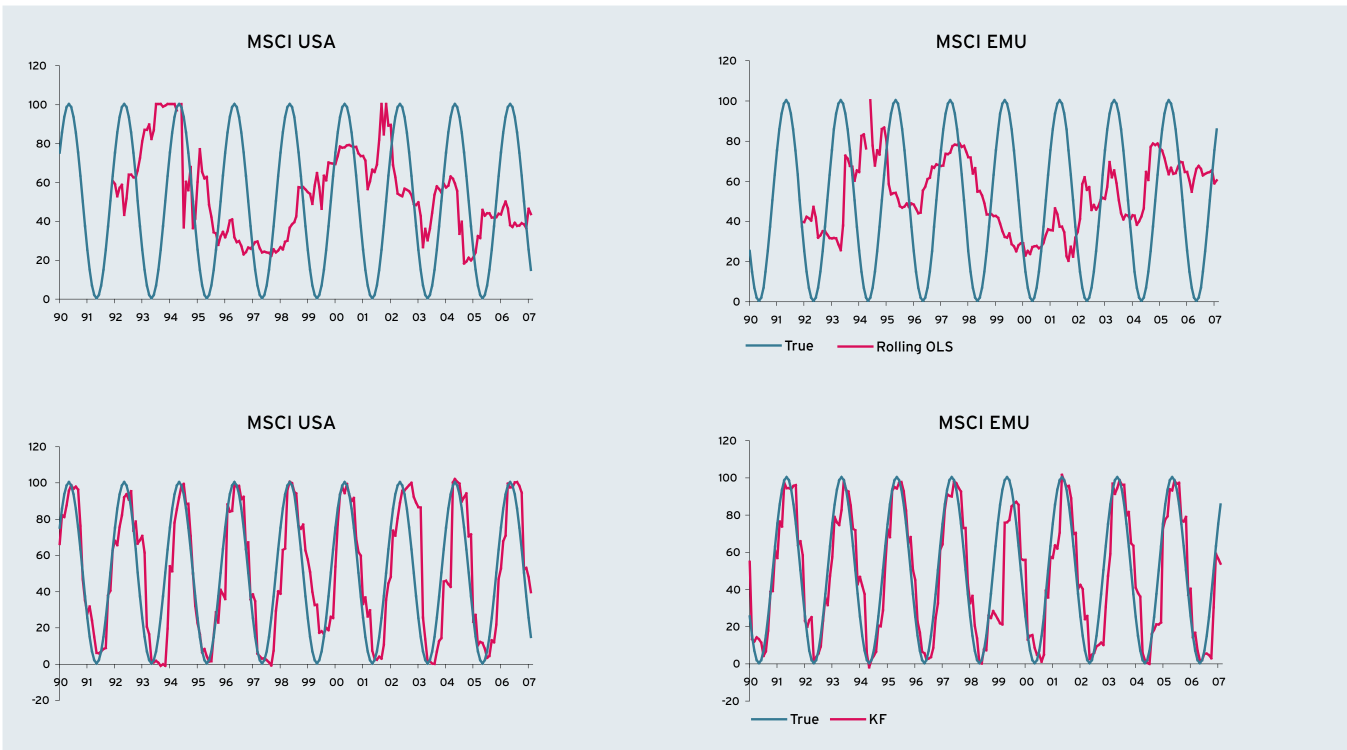


Figure 1 – KF estimates versus rolling OLS estimates – $\varpi$ = 2Y

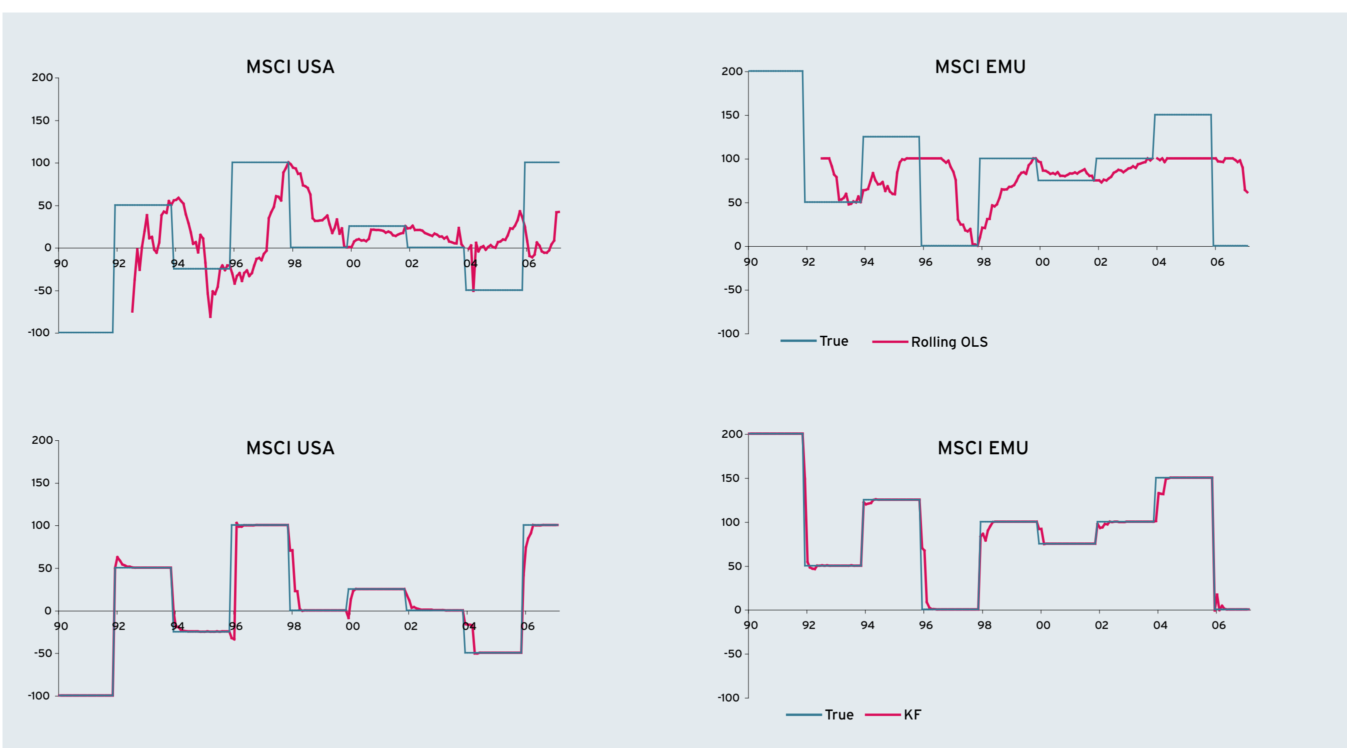


Figure 2 – KF estimates versus rolling OLS estimates – $\varpi$ = 2Y

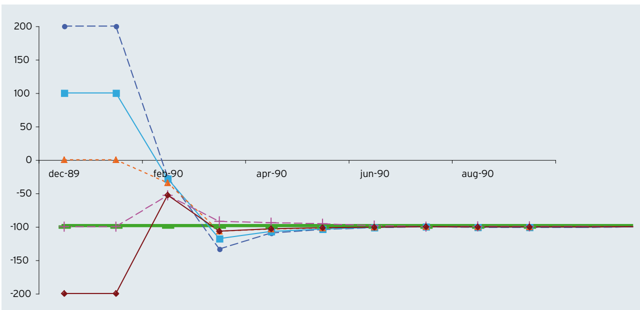


Figure 3 – Adaptative learning of the Kalman filter – $\varpi$ = 2Y

months) and the Kalman filter[8]. Results of the estimations are reported in Figures 4, 5, and 6. We observe that whatever the methodology used, hedge funds considered as a whole have appeared as long equities — as captured by the S&P[9] — and long Russell against S&P. For other factors, exposures vary to a large extent between methods. Nevertheless, whichever factor and index publisher is considered, the variability of exposures is far lower for the Kalman filter. Obviously, as one is attempting to capture the dynamic beta of hedge funds, stability is not an objective as such. But it is not clear whether it is efficient to have quickly changing exposures if true underlying exposures are themselves highly volatile. Indeed, in this case, the danger is to be always 'behind the curve.' For example, numerous inopportune changes in exposures are observed for the 12-month window, such as the large decrease in exposure to S&P at the end of 1997 or the sharp changes in exposures in 2004/2005. At the other end of the spectrum, a very long window could lead to an unnecessarily slow reaction to changing conditions, as is demonstrated here by the S&P exposure during the collapse of the stock market bubble and the recovery period which has followed. Without question, the Kalman filter appears to be a good compromise: it posts smooth changes in exposures but has been able to identify the reduction in exposure to equities markets, which seems to have characterized hedge funds in late 2000-early2001, quite early (even earlier than the 12-month rolling window).

More formally, we report in Figure 7 some statistics on the simulated returns of the various methodologies. It appears that while some window lags lead to higher average excess returns than the Kalman filter method, none globally dominates the latter in terms of risk-adjusted returns. As far as capital preservation is concerned, the Kalman filter also seems to be preferred as it often leads to a higher proportion of positive months and systematically to lower drawdowns. Finally, it offers a higher correlation with the underlying hedge fund index.


Figure 4 – Exposures for the HFR index

8 One should notice that the parameters of the Kalman filter are estimated over the whole sample. However, here, it is not the coefficients but only their variance which are estimated. It is not clear whether this implies look-ahead bias and in practice, we have found that progressive estimations of the Kalman filter are leading to very similar results. Readers familiar with the Kalman filter should not be surprised with this result.

9 The aggregated exposure to the S&P index should be read as the sum of the 'S&P' exposure (as mentioned in the Figure) minus all the exposures of spreads involving the S&P. In fact, the exposure mentioned as 'S&P' recovers the total equity exposure.


Figure 5 – Exposures for the HFR FoF index


Figure 6 – Exposures for the CSFB-Tremont index

| | Rolling OLS 12 M | Rolling OLS 24 M | Rolling OLS 36 M | Kalman filter |
|---|---|---|---|---|
| **HFR Composite Index** | | | | |
| Average excess return (% per year) | 5.30% | 4.98% | 4.09% | 5.28% |
| Standard deviation (% per year) | 7.66% | 8.07% | 8.92% | 7.45% |
| Sharpe ratio | 0.69 | 0.62 | 0.46 | 0.71 |
| Proportion of positive months | 70% | 68% | 67% | 70% |
| Correlation with hedge fund index | 84.8% | 85.3% | 85.7% | 91.3% |
| Maximum drawdown | 15.4% | 18.7% | 21.1% | 9.7% |
| **HFR FOF Index** | | | | |
| Average excess return (% per year) | 4.36% | 4.65% | 3.50% | 4.52% |
| Standard deviation (% per year) | 5.97% | 6.13% | 6.00% | 5.62% |
| Sharpe ratio | 0.73 | 0.76 | 0.58 | 0.80 |
| Proportion of positive months | 70% | 75% | 72% | 71% |
| Correlation with hedge fund index | 71.1% | 73.7% | 75.8% | 75.7% |
| Maximum drawdown | 11.3% | 9.5% | 7.9% | 5.2% |
| **CSFB-Tremont Index** | | | | |
| Average excess return (% per year) | 6.49% | 7.44% | 5.52% | 5.49% |
| Standard deviation (% per year) | 7.71% | 8.14% | 8.19% | 6.75 |
| Sharpe ratio | 0.84 | 0.91 | 0.67 | 0.81 |
| Proportion of positive months | 66% | 73% | 68% | 70% |
| Correlation with hedge fund index | 67.1% | 70.0% | 72.9% | 73.7% |
| Maximum drawdown | 15.0% | 9.2% | 9.9% | 6.3% |

Figure 7 – Descriptive statistics on simulated factor models

## Application to alternative beta

### Building a clone

To evaluate whether factor-models replicating funds are worth investing in, we report in Figure 8 the backtest of the strategy over the period 1997-2007 (with cash reinvested at the one-month Libor rate). In Figure 9, we compare the risk-return profile (in terms of mean-variance). More statistics are given in Figure 10.

Several comments are worth stressing. First, concerning composite indices of single funds, the trackers underperform the indices they are trying to replicate, despite the fairly high correlation. But perfect replication seems to be an unfair objective in the present case. It is well known that hedge fund indices are affected by various biases, such as survivorship and self-selection biases [Fung and Hsieh (2004), Lhabitant (2006)]. An indication of the validity of this explanation is that this result does not hold for the funds-of-funds replication. Indeed, it is widely acknowledged that the returns of funds-of-funds are less affected by those biases since, for example, they do not cease reporting to databases when an underlying single fund in which they have invested collapses, even though their NAVs are directly impacted. Obviously, one should keep in mind that the difference in performance between single funds and funds-of-funds also reflects the additional layer of fees. By and large, it seems that over a full economic and market cycle (that is, the last 10 years), trackers have posted performances that are characteristic of hedge fund performance with a slight underperformance, probably due to the lack of alpha generation (an issue to which we return later).

Second, an attractive feature visible in the results is that trackers present larger skewness than the indices. This positive asymmetric shift is also clear when one observes the extrema, since trackers exhibit lower minima and higher maxima. While it is beyond the scope of the present paper, an explanation for this result is that, by construction, factor-based models with standard assets are not specialized in reproducing strategies similar to writing puts on the market [Mitchell and Pulvino (2001), Naik et al. (2004)]. This suggests that one should be cautious about introducing option payoffs to factor-based trackers.

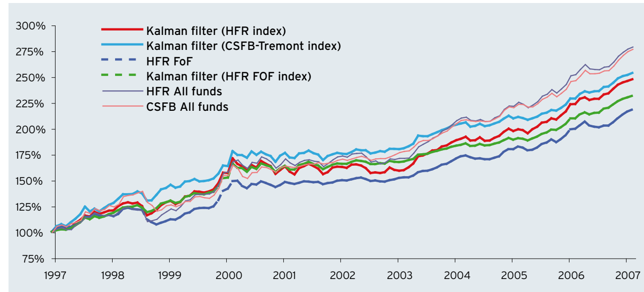


Figure 8 – Backtest of the strategy

| | HFR_KF | CSFB_HK | HFRFOF_HK | HFR_FOF | HFR | CSFB | S&P | U.S. Treasuries | LIBOR |
|---|---|---|---|---|---|---|---|---|---|
| Average | 9.2% | 9.5% | 8.5% | 7.9% | 10.5% | 10.4% | 8.2% | 5.7% | 4.0% |
| Std dev | 7.5% | 6.8% | 5.7% | 6.0% | 7.3% | 7.2% | 15.3% | 7.0% | 0.5% |

Figure 9 – Risk-return profile of hedge funds and standard assets

| | HFR Index | HFR Tracker | HFR FOF Index | HFR FOF Tracker | CSFB-Tremont Index | CSFB-Tremont Tracker |
|---|---|---|---|---|---|---|
| Average return (% per year) | 10.5% | 9.2% | 7.9% | 8.5% | 10.4% | 9.5% |
| Standard deviation (% per year) | 7.3% | 7.5% | 6.0% | 5.7% | 7.2% | 6.8% |
| Sharpe (risk-free rate = 4%) | 0.90 | 0.71 | 0.66 | 0.81 | 0.90 | 0.81 |
| Proportion of positive months | 69.7% | 69.7% | 65.6% | 71.3% | 72.1% | 70.5% |
| Skewness | -0.43 | 0.03 | -0.25 | 1.01 | 0.18 | 0.52 |
| Excess kurtosis | 3.20 | 2.42 | 4.73 | 5.21 | 4.04 | 2.40 |
| Minimum | -8.70% | -7.12% | -7.47% | -3.57% | -7.55% | -4.98% |
| Maximum | 7.65% | 8.99% | 6.85% | 9.01% | 8.53% | 8.86% |
| Correlation with S&P 500 | 72.0% | 69.6% | 52.9% | 55.4% | 48.7% | 61.4% |
| Correlation with US Treasuries | -17.2% | -14.5% | -12.4% | -5.1% | -2.7% | 3.5% |

Figure 10 – A statistical comparison of hedge fund indices and their trackers

| (% per year) | HFR Composite | CFSB-Tremont | HFR FOF |
|---|---|---|---|
| Total performance | 10.39% (100.0%) | 10.30% (100.0%) | 7.87% (100.0%) |
| Cash | 3.92% (37.7%) | 3.92% (38.1%) | 3.92% (49.8%) |
| Traditional beta | 2.79% (26.8%) | 2.39% (23.2%) | 2.02% (25.6%) |
| Alternative beta | 2.49% (24.0%) | 3.09% (30.0%) | 2.51% (31.8%) |
| Alpha | 1.19% (11.4%) | 0.90% (8.7%) | -0.57% (-7.2%) |

Figure 11 – Decomposition of the total return of hedge fund indices (January 1997 – February 2007)

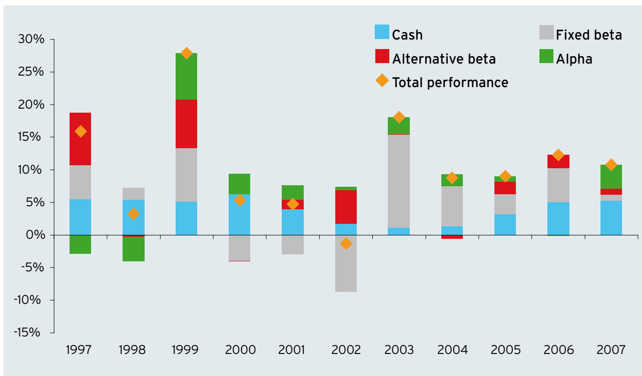


Figure 12 – Year-by-year decomposition of the performances for the HFR index

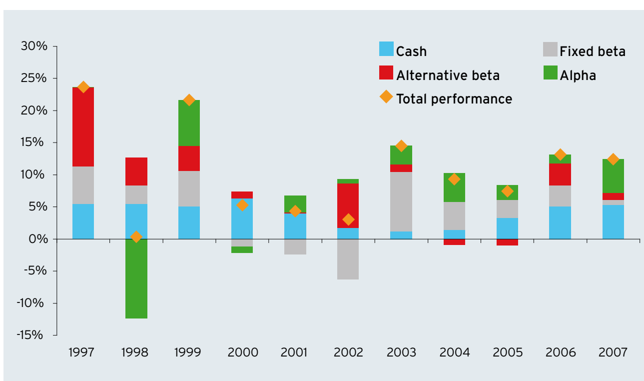


Figure 13 – Year-by-year decomposition of the performances for the CSFB-Tremont index

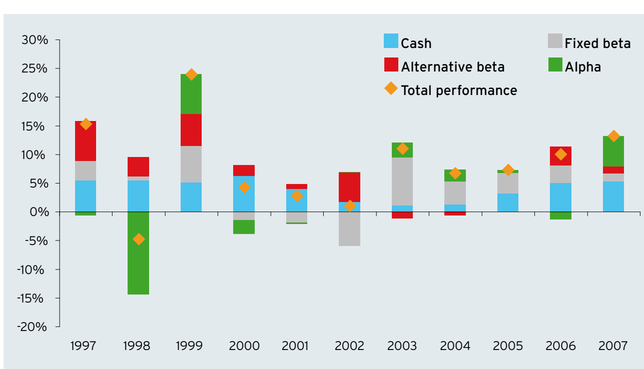


Figure 14 – Year-by-year decomposition of the performances for the HFR FoF

Third, we can pose the standard criticism directed at regression-based trackers, which is that their correlation with equities is high. This is undeniably true but the correlation seems to simply reflect the indices they are tracking. Having said that, it is interesting to note that the correlation between the CSFB-Tremont tracker and the S&P 500 index is far lower than with HFR, which is totally consistent with the respective correlation of the indices[10]. We also notice that this correlation is also observed for the U.S. Treasuries.

## Estimating the alpha

With the previous framework, it is easy to compute the various components of the hedge fund performance. Obviously, the calculations can be crude at best since, and in practice one should take into account the impact of fees and above all the biases affecting the first moment (average) of hedge fund indices. Let $R_t$ denote the index monthly return observed at time t. $\beta_t^{(i)}$ is the exposure to the i-th factor at time t as estimated through the Kalman filter and $\overline{\beta}^{(i)}$ is its average over the whole period, $\overline{\beta}^{(i)} = T^{-1}\Sigma_{t=1\rightarrow T}\ \beta_t^{(i)}$. Our aim is to decompose the total performance into fixed-traditional beta, alternative beta, and alpha. We adopt the following breakdown: Excess return: $R_t^* = R_t - Libor_t$; traditional beta: $\Sigma_{i=1\rightarrow m}\ \overline{\beta}^{(i)}\ F_t^{(i)}$; alternative beta: $\Sigma_{i=1\rightarrow m}\ (\beta_t^{(i)} - \overline{\beta}^{(i)})F_t^{(i)}$; alpha: $R_t^* - \Sigma_{i=1\rightarrow m}\ \beta_t^{(i)}\ F_t^{(i)}$

In Figure 11, we report the average values obtained for each of these components. For the various indices, alternative beta does represent between a quarter and a third of the total return and near one-half of the excess return. In Figures 12, 13, and 14, we report the same calculations but for every year. It is interesting to observe that alternative beta has contributions to performance which are most of the time positive and that their contribution was the most important during the bear equity market (2002 and to a lesser extent 2001).

## Conclusion

Hedge fund replication meets growing interest. As far as the motivation is to get liquid and transparent instruments, factor-models appear as an attractive approach. To date, though, available approaches are based on rolling regressions where the window is fixed in an ad-hoc way. In this paper, we have shown that more efficient econometric methods such as the Kalman filter can be used as substitutes to these regressions. They offer risk-return profiles which share several characteristics with the ones posted by hedge funds indices: Sharpe ratios above buy-and-hold strategies on standard assets, moderate correlation with standard assets, or limited drawdowns during equity downward trends. An interesting result is that the shortfall risk seems less important than with hedge fund indices. All in all, alternative beta appears to explain between a quarter and a third of the total performance of hedge funds.

10 The higher correlation of the HFR index is often attributed to the heavier weight of equity-linked strategies (notably equity hedge).

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## Appendix

**A – Quadratic programming form of linear regression**

We consider the linear model $y_t = x_t^T\beta + \varepsilon_t$. The matrix form is $Y = X\beta + \varepsilon$. The residual sum of squares is equal to $(Y-X\beta)^T (Y-X\beta) = Y^TY + \beta^T(X^TX)\beta - 2Y^TX\beta$. The OLS estimate $\hat{\beta}$ is also the solution of the quadratic programming problem: $\hat{\beta} = \arg\min {}^1/_2\, \beta^T (X^TX)\beta - (Y^TX)\beta$ (A1). Without any other constraints, the solution is well-know: $\hat{\beta} = (X^TX)^{-1}X^TY$. If we impose linear equality or inequality constraints, we may use a standard QP computer routine to find the solution. For example, we might want to impose some bounds on the coefficients. When there is no constant in the linear regression, the residuals are not centered. If we want to center the residuals, we impose the linear equality $1^TX\beta = 1^TY$.

**B – The state space model and the Kalman filter**

A state space model is defined by a transition equation and a measurement equation. In the measurement equation we postulate the relationship between an observable vector and a state vector, while the transition equation describes the generating process of the state variables. The state vector $\beta_t$ is generated by a first-order Markov process of the form: $\beta_t = M_t\beta_{t-1} + c_t + S_t\eta_t$, $t=1,\cdots,T$, where $\beta_t$ is the m-dimension state vector, $M_t$ is a m×m matrix, $c_t$ is a m×1 vector, and $S_t$ is a m×G matrix. The measurement equation of the state-space representation is: $y_t = Z_t\beta_t + d_t + \varepsilon_t$, $t=1,\cdots,T$, where $y_t$ is a N-dimension time series, $Z_t$ is a N×m matrix, and $d_t$ is a N×1 vector. $\eta_t$ and $\varepsilon_t$ are assumed to be white noise processes of dimensions G×1 and N×1 respectively. These two last uncorrelated processes are Gaussian with zero mean and with respective covariances matrices $Q_t$ and $H_t$, that is: $E(\eta_t) = 0$ and $var(\eta_t) = Q_t$; $E(\varepsilon_t) = 0$ and $var(\varepsilon_t) = H_t$.

Let us consider the linear model $y_t = x_t\beta + \varepsilon_t$. The model is in state-space form with $Z_t = x_t$, $c_t = d_t = 0$ and $Q_t = 0$. $\{y_t = x_t\beta_t + \varepsilon_t;\ \beta_t = \beta_{t-1}\}$ (A2). The state vector is constant and not stochastic. Others examples of SSM are structural models or models with unobservable variables. For example, the local level model which is given by: $\{y_t = \mu_t + \varepsilon_t;\ \mu_t = \mu_{t-1} + \eta_t\}$ (A3) is in state-space form with $Z_t = 1$, $c_t = d_t = 0$ and $\beta_t = \mu_t$. Note that we may use the idea of the stochastic trend of the local level model in the linear model to obtain a linear model with stochastic beta: $\{y_t = x_t\beta_t + \varepsilon_t;\ \beta_t = \beta_{t-1} + \eta_t\}$ (A4). In this case, the beta are time-varying and are unobservable variables.

One of the challenges in SSM is to estimate the state variable $\beta_t$. It may be done using the Kalman filter. If we now consider $b_t$ as the optimal estimator of $\beta_t$, based on all the relevant and available observations at time t, we have $b_t = E_t[\beta_t]$, where $E_t$ indicates the conditional expectation operator. The covariance $P_t$ of this estimator is defined by $P_t = E_t[(b_t - \beta_t)(b_t - \beta_t)^T]$. We also denote $b_{t|t-1} = E_{t-1}[\beta_t]$ the optimal estimator of $\beta_t$ based on all the relevant and available observations at time t-1 and $P_{t|t-1}$ the covariance of this estimator. The Kalman filter consists of the following set of recursive equations: $\{b_{t|t-1} = M_tb_{t-1} + c_t;\ P_{t|t-1} = M_tP_{t-1}M_t^T + S_tQ_tS_t^T;\ y_{t|t-1} = Z_tb_{t|t-1} + d_t;\ v_t = y_t - y_{t|t-1};\ F_t = Z_tP_{t|t-1}Z_t^T + H_t;\ b_t = b_{t|t-1} + P_{t|t-1}Z_t^TF_t^{-1}v_t;\ P_t = (I_m - P_{t|t-1}Z_t^TF_t^{-1}Z_t)P_{t|t-1}\}$. It should be noted that we first need to initialize the Kalman filter. This may be done by assuming that the initial position is a Gaussian variable such that: $E(\beta_0) = a_0$ and $var(\beta_0) = P_0$.

To obtain the estimator of $\beta_t$ based on all the relevant and available observations at time T, we use the following smoothing filter (called the Kalman smoother): $P_t^* = P_tM_{t+1}^TP_{t+1|t}^{-1};\ b_{t|T} = b_t + P_t^*(b_{t+1|T} - b_{t+1|t});\ P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*T}$. To illustrate the difference between the three estimators, $b_{t|t-1}$, $b_t$, and $b_{t|T}$, we consider the linear model example. Recursive least squares regression corresponds to the estimator $b_t$ whereas the ordinary least squares coefficients are given by $b_{t|T}$. We remark also that the one-step forecast of the recursive least squares regression is exactly $b_{t|t-1}$.

Generally, the state space model contains some unknown parameters other than the state vector. In this case, we may estimate them by maximum likelihood. Let $\theta$ be the vector of parameters. The log-likelihood can be expressed in terms of the innovation process. It is then equal to: $\ell_t = \log L_t = -N/2\log 2\pi - 1/2\log|F_t| - {}^1/_2\, v_t^TF_t^{-1}v_t$. For example, we have to estimate the volatility of the residuals $\varepsilon_t$ in the models (A2) and (A3), and the volatilities of the stochastic components $\eta_t$ in the model (A3).