# It is Time for Multi-Indexing[1]

Thierry Roncalli*

*Lyxor Asset Management, France

Quant Touch, June 18, 2012

[1] This presentation is based on the paper "*On the Market Portfolio for Multi-Asset Classes*" written by R. Louis and T. Roncalli.

# Outline

# Defining the active bets

## Absolute view

- Do not need a reference portfolio

| Asset | Weight |
|---|---|
| #1 | 30% |
| #2 | 50% |
| #3 | 20% |

The preference order is then:

$$\#2 \succ \#1 \succ \#3$$

## Relative view

- Requires a reference portfolio (a benchmark)

| Asset | Weight | Benchmark | Bet |
|---|---|---|---|
| #1 | 30% | 25% | +5% |
| #2 | 50% | 55% | −5% |
| #3 | 20% | 20% | +0% |

The preference order is then:

$$\#1 \succ \#3 \succ \#2$$

- Absolute view $\Rightarrow$ long/short portfolio
- Relative view $\Rightarrow$ long-only portfolio

# Defining the active bets

## An equity portfolio example

We consider an equity portfolio with fixed weights.

Figure: Benchmark based on the market capitalization

| Year | Microsoft | Exxon | Apple | Coca Cola | Wal Mart | Pfizer |
|---|---|---|---|---|---|---|
| 1998 | 33.3% | 17.1% | 0.5% | 15.9% | 17.5% | 15.6% |
| 1999 | 40.8% | 18.9% | 1.1% | 9.8% | 20.9% | 8.5% |
| 2000 | 19.0% | 24.8% | 0.4% | 12.4% | 19.5% | 23.8% |
| 2001 | 28.4% | 21.6% | 0.6% | 9.3% | 20.4% | 19.6% |
| 2002 | 26.7% | 22.7% | 0.5% | 10.5% | 21.5% | 18.2% |
| 2003 | 24.7% | 22.6% | 0.7% | 10.4% | 19.2% | 22.5% |
| 2004 | 24.8% | 28.2% | 2.2% | 8.6% | 19.0% | 17.2% |
| 2005 | 24.2% | 30.4% | 5.3% | 8.3% | 16.9% | 14.9% |
| 2006 | 22.5% | 34.2% | 5.6% | 8.7% | 14.7% | 14.3% |
| 2007 | 22.1% | 34.0% | 11.5% | 9.4% | 12.6% | 10.3% |
| 2008 | 15.7% | 37.0% | 6.9% | 9.5% | 20.0% | 10.9% |
| 2009 | 21.3% | 25.5% | 15.1% | 10.4% | 16.1% | 11.6% |
| 2010 | 17.2% | 26.6% | 21.3% | 11.0% | 13.8% | 10.1% |
| 2011 | 14.3% | 26.5% | 24.6% | 10.4% | 13.4% | 10.9% |
| Portfolio | 15.00% | 20.00% | 15.00% | 10.00% | 20.00% | 20.00% |

- 2011: MSFT ($\simeq$), XOM (-), AAPL (-), KO ($\simeq$), WMT (+), PFE (+)
- 2005: MSFT (-), XOM (-), AAPL (+), KO (+), WMT (+), PFE (+)

# Defining a reference portfolio

## The example of CPP Investment Board (Canada Pension Plan)

- The reference portfolio is : 65% Equities and 35% Bonds
  - 2012: 10% Canadian Equities, 55% Global Equities, 30% Canadian Nominal Bonds and 5% Foreign Sovereign Bonds
  - 2011: 15% Canadian Equities, 45% DM Equities, 5% EM Equities, 25% Canadian Nominal Bonds, 5% Foreign Sovereign Bonds and 5% Canadian Real Return Bonds
  - 2008: 25% Canadian Equities, 40% DM Equities, 25% Fixed Income and 10% Canadian Real Return Bonds
- The investment portfolio is:

| Year | 2012 | 2011 | 2010 | 2009 | 2008 | 2007 | 2000 |
|---|---|---|---|---|---|---|---|
| Equities | 50.4% | 53.5% | 55.8% | 57.4% | 62.8% | 64.6% | 5.4% |
| Fixed income | 31.2% | 30.1% | 30.8% | 27.8% | 25.5% | 25.4% | 94.6% |
| Inflation assets | 18.4% | 16.4% | 13.5% | 14.8% | 11.7% | 10.0% | 0.0% |

- Comparing the performance of the investment portfolio and the performance of the reference portfolio permits to define the alpha.

# Defining a strategic asset allocation

- Stichting Pensioenfonds ABP

| | 2010-12 SAA | 2012 Q1 | 2011 Q4 | 2011 Q2 | 2010 Q2 | 2009 Q2 |
|---|---|---|---|---|---|---|
| Real assets | 54.0% | 54.6% | 53.50% | 52.50% | 51.80% | 49.00% |
| Fixed income | 38.0% | 39.0% | 40.20% | 39.60% | 42.30% | 44.80% |
| HF / GTAA | 8.0% | 5.5% | 6.00% | 6.20% | 6.40% | 6.30% |
| Overlay | 0.0% | 1.0% | 0.30% | 1.70% | -0.50% | 0.00% |

- CalPERS

| | Equity | | Income | Real Assets | Inflation | Liquidity | ARS |
|---|---|---|---|---|---|---|---|
| | Public | Private | | | | | |
| SAA (Jul. 2011) | 49% | 14% | 16% | 13% | 4% | 4% | |
| Policy range | +/- 7% | +/- 4% | +/- 5% | +/- 5% | +/- 3% | +/- 3% | |
| SAA (Mar. 2009) | 66% | | 19% | 10% | 5% | 0% | |
| Feb. 2012 | 50.3% | 13.7% | 17.2% | 9.3% | 3.2% | 4.3% | 2.1% |
| Mar. 2009 | 53.5% | | 25.2% | 11.4% | 2.5% | 7.3% | |



Target of the active asset allocation tracking error = 0.75%.

- Performance of tactical asset allocation (TAA) with respect to the strategic asset allocation (SAA)

# Defining a benchmark for diversified funds

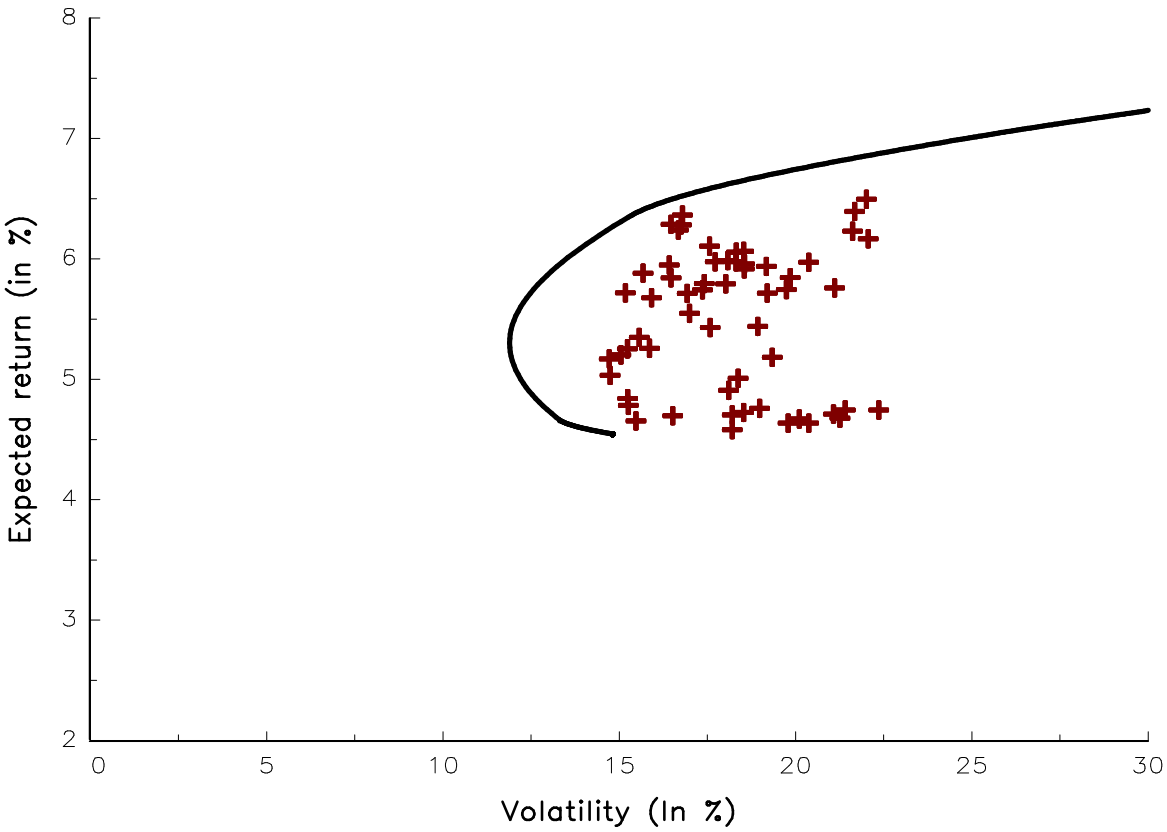
- Benchmark of some diversified funds:

| Fund | Benchmark |
|---|---|
| Carmignac Patrimoine | 50% MSCI AC World + 50% WGBI All Maturities |
| Invesco IBRA | 60% MSCI World + 40% JP Morgan Europe Government Bond |
| BNY Mellon Real Return | Euribor + 4% |
| BlackRock Global Allocation | 36% S&P 500 + 24% FTSE World (ex-US) + 24% ML US Treasury 5Y + 16% Non-USD WGBI ∼ 60% Equities + 40% Bonds |

- Investment portfolio of the fund / Benchmark ⇒ characterization of the bets of the fund manager
- **What are the bets of the benchmark?**
  - 50% Equities + 50% Bonds
  - Negative or positive bet in equities in 2000 ?
  - in 2005 ? in 2008 ? in 2012 ?

# The market portfolio theory

## The efficient frontier of Markowitz



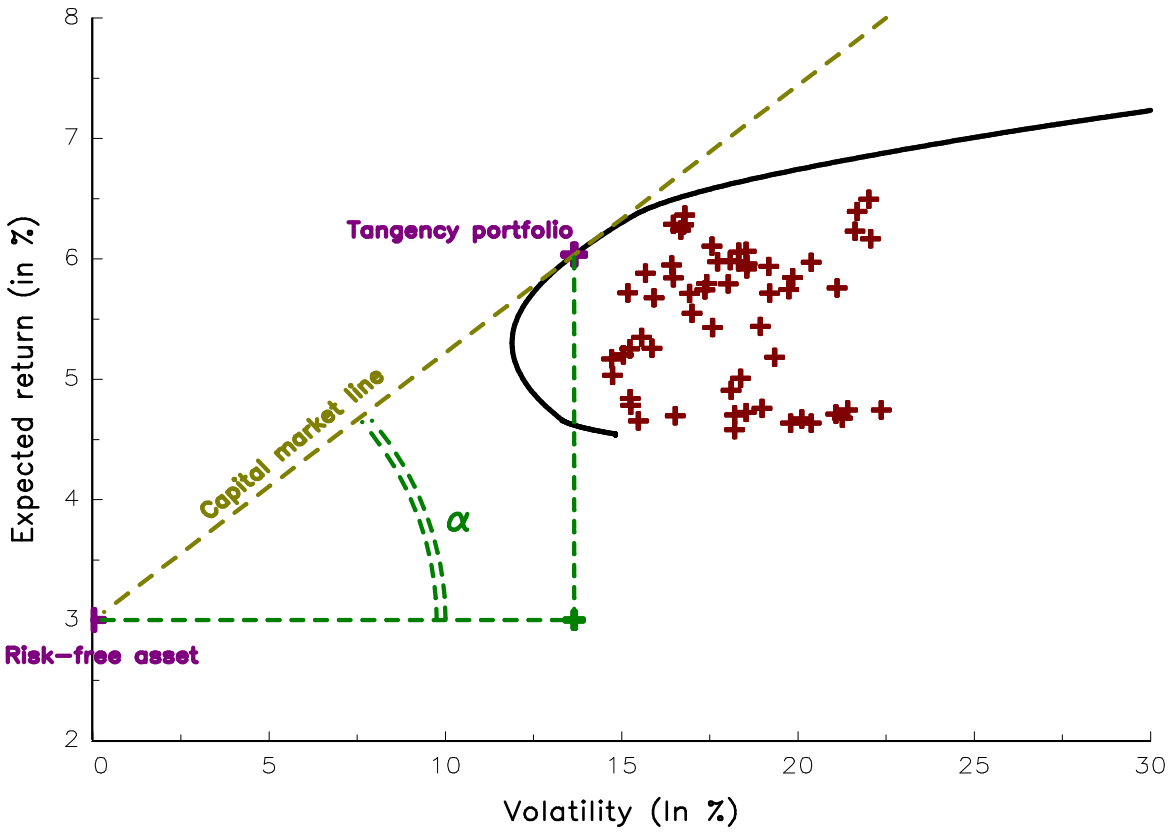

- "*the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing*"(Markowitz, 1952):

$$\max \mu(w) = \mu^\top w$$
$$\text{u.c.} \quad \sigma(w) = \sqrt{w^\top \Sigma w} = \sigma^\star$$

There isn't one optimal portfolio, but a set of optimal portfolios!

# The market portfolio theory

## Does one portfolio dominate all the other portfolios?




- Tobin (1958) introduces the risk-free rate and shows that the efficient frontier is a straight line.
- Optimal portfolios are a combination of the tangency portfolio and the risk-free asset.
- *Separation theorem* (Lintner, 1965).

There is one optimal (risky) portfolio!

# The market portfolio theory

## How to compute the tangency portfolio?

- Sharpe (1964) develops the CAPM theory.
- If the market is at the equilibrium, the prices of assets are such that the tangency portfolio is the market portfolio (or the market-cap portfolio).
- Avoids assumptions on expected returns, volatilities and correlations!
- It is the beginning of passive management:
  - Jensen (1969): no alpha in mutual equity funds
  - John McQuown (Wells Fargo Bank, 1971)
  - Rex Sinquefield (American National Bank, 1973)

# The market portfolio theory

## Which indexes correspond to market portfolios?

### Public equities

- Market portfolio
- MSCI, Stoxx, FTSE, Standard and Poor's, Datastream, etc.

### Private equities

- ~~Market portfolio~~
- LPX, Privex, etc.

### Bonds

- Market portfolio
- Citigroup, JP Morgan, Barclays, MTS, Iboxx, Datastream, etc.

### Commodities

- ~~Market portfolio~~
- GSCI, DJ UBS, RICI, CRB, etc.

### Hedge funds

- ~~Market portfolio~~
- HFR, Lyxor, Newedge, etc.

### Real Estate

- Market portfolio?
- EPRA NAREIT, etc.

# Characterization of the stock/bond market portfolio

## The single country case

$\mathscr{A}$ is the universe of the financial domestic assets. The market capitalization for this country is defined as follows:

$$\mathrm{MC}(t;\mathscr{A}) = \sum_{i\in\mathscr{A}} N_i(t)\,P_i(t)$$

where $N_i(t)$ and $P_i(t)$ are respectively the share number and the price of asset $i$. We divide the set $\mathscr{A}$ into $m$ disjoint subsets $\mathscr{A}_j$ such that $\mathscr{A} = \biguplus \mathscr{A}_j$.The market capitalization for the subset $\mathscr{A}_j$ is:

$$\mathrm{MC}(t;\mathscr{A}_j) = \sum_{i\in\mathscr{A}_j} N_i(t)\,P_i(t)$$

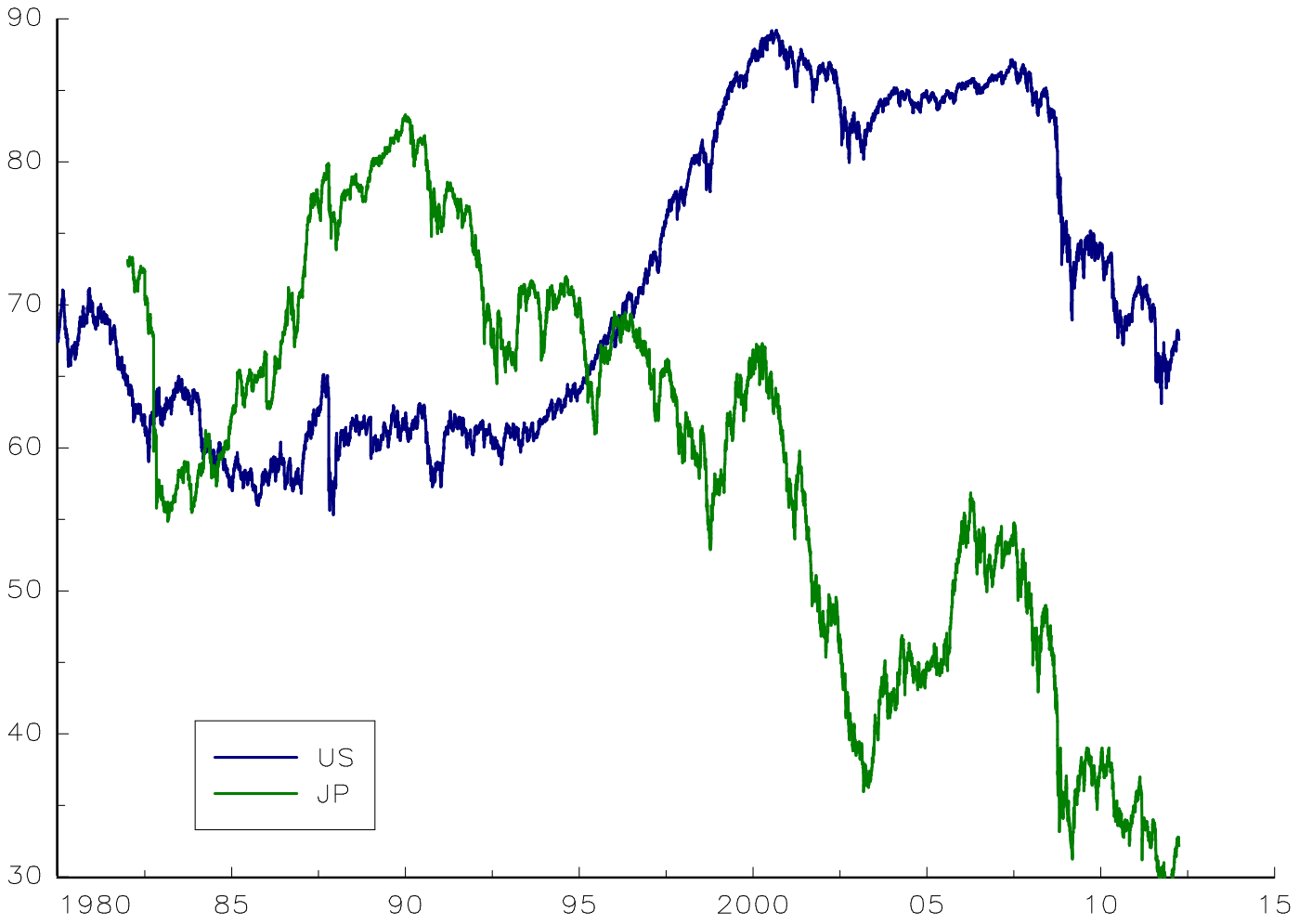
We could characterize the market portfolio by the weights $(w_1,\ldots,w_j,\ldots,w_m)$ associated to the different subsets with:

$$w_j = \frac{\mathrm{MC}(t;\mathscr{A}_j)}{\mathrm{MC}(t;\mathscr{A})}$$

# Characterization of the stock/bond market portfolio

## With public equities and sovereign bonds

Figure: Evolution of the equity weight for United States and Japan



# Characterization of the stock/bond market portfolio

## With public equities and sovereign bonds

Figure: Evolution of the equity weight for Germany, France and United Kingdom

# Characterization of the stock/bond market portfolio

## With public equities and sovereign bonds

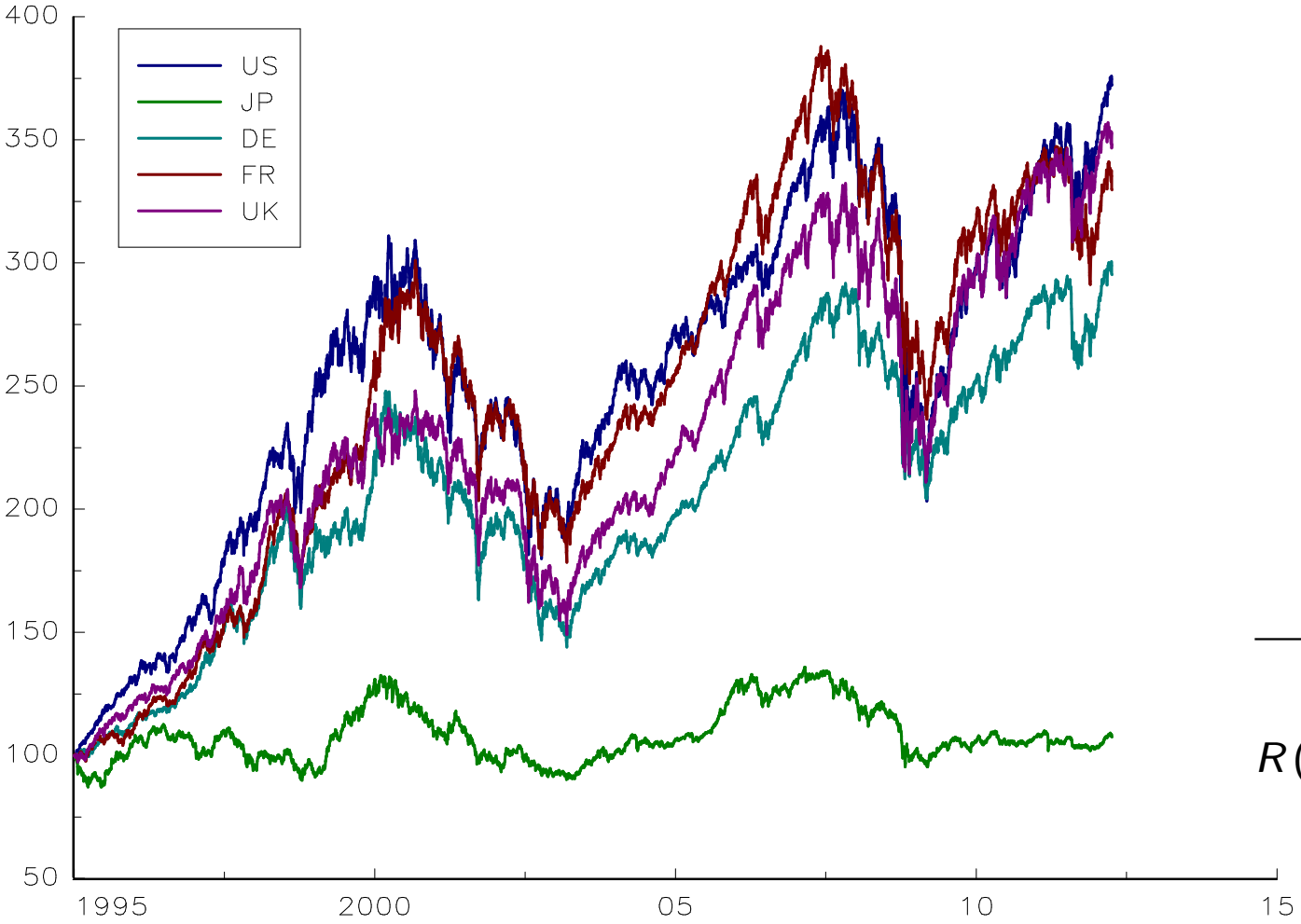
Table: Weight $w(t;\mathscr{E})$ of equities (in %) at the beginning of each year

| Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|
| US | 84.9 | 85.2 | 85.9 | 85.8 | 75.0 | 73.8 | 70.6 | 65.7 |
| JP | 44.9 | 53.8 | 52.9 | 49.8 | 36.3 | 37.6 | 34.6 | 29.4 |
| DE | 52.0 | 55.5 | 60.9 | 64.2 | 50.6 | 56.0 | 54.0 | 47.5 |
| FR | 62.5 | 67.7 | 72.8 | 74.1 | 59.7 | 62.7 | 59.9 | 53.0 |
| UK | 83.9 | 84.9 | 86.7 | 85.5 | 72.8 | 72.0 | 70.8 | 64.3 |
| IT | 45.3 | 48.4 | 53.6 | 51.7 | 35.3 | 34.6 | 32.0 | 28.5 |
| AU | 95.0 | 96.0 | 96.5 | 97.0 | 94.4 | 93.2 | 90.1 | 84.4 |
| NL | 70.7 | 70.4 | 75.5 | 75.4 | 55.6 | 61.1 | 59.8 | 51.2 |
| SE | 78.6 | 83.2 | 86.7 | 86.6 | 80.2 | 83.7 | 86.5 | 83.8 |

- Domestic differences of the market portfolio
- Home bias of SAA

# Characterization of the stock/bond market portfolio

## Performance of the market portfolio




- Daily rebalancing[a]
- Poor performance of the Japanese market portfolio
- Differences between France and Germany in terms of economic cycles & risk
- Eurozone crisis = Japanese scenario ?

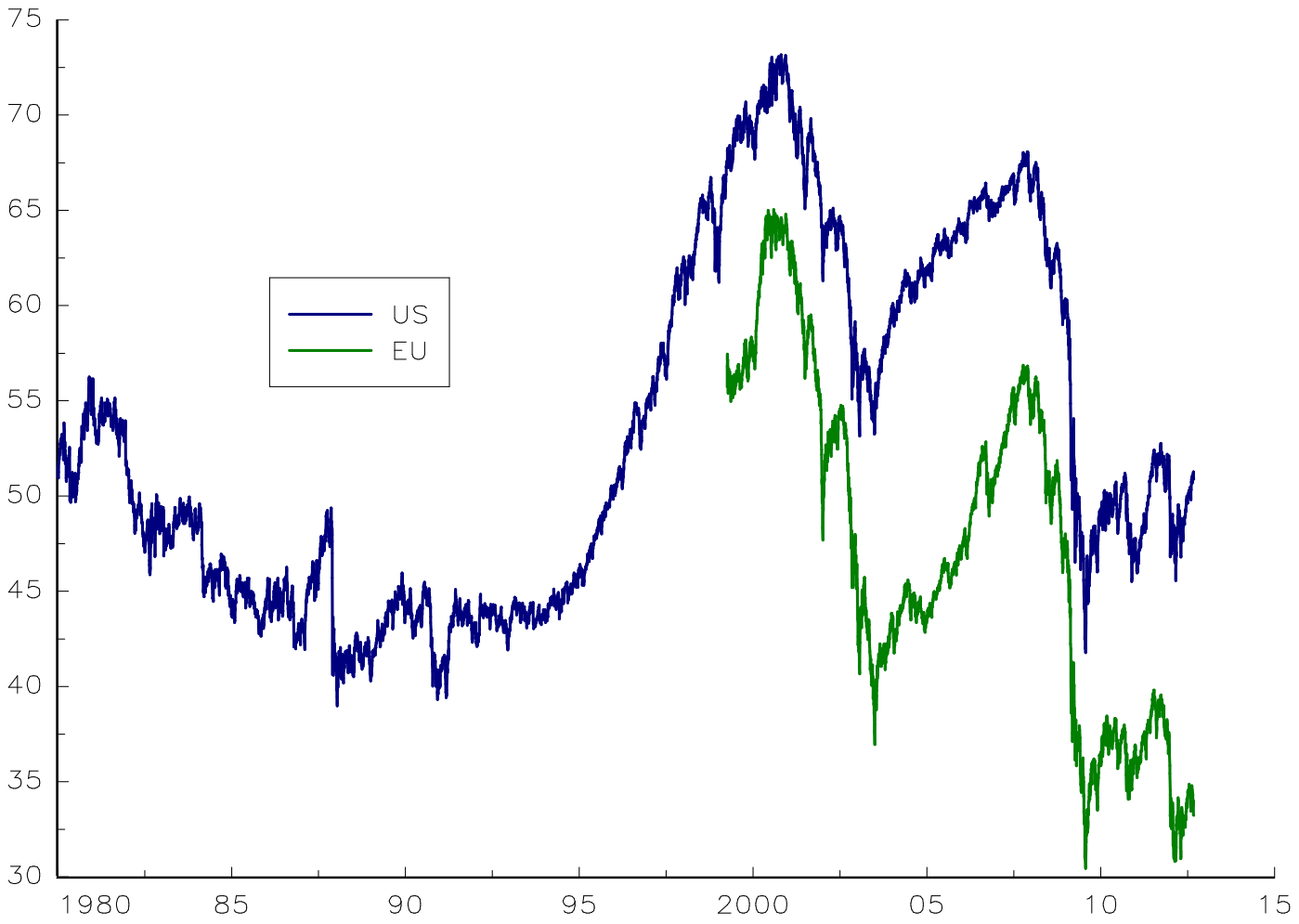
[a] We have:

$$R(t) = w(t-1;\mathscr{B}) \cdot R(t;\mathscr{B}) + w(t-1;\mathscr{E}) \cdot R(t;\mathscr{E})$$

# Characterization of the stock/bond market portfolio

## With public equities and investment grade bonds

Figure: Evolution of the equity weight for US and Eurozone



# Characterization of the stock/bond market portfolio

## With public equities and investment grade bonds

Table: Market portfolio allocation (in %) at the beginning of each year

| | Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|
| US | Equity | 63.7 | 65.0 | 66.2 | 65.2 | 49.9 | 49.6 | 50.0 | 48.7 |
| | Bond | 36.3 | 35.0 | 33.8 | 34.8 | 50.1 | 50.4 | 50.0 | 51.3 |
| | Sovereign | 14.0 | 13.8 | 12.9 | 12.5 | 18.3 | 18.8 | 21.5 | 22.2 |
| | Collateralized | 13.1 | 12.6 | 12.6 | 13.7 | 20.7 | 19.0 | 16.3 | 16.6 |
| | Corporate | 9.2 | 8.6 | 8.3 | 8.6 | 11.1 | 12.7 | 12.2 | 12.6 |
| Euro | Equity | 45.2 | 49.5 | 53.8 | 53.9 | 37.3 | 37.7 | 37.7 | 33.2 |
| | Bond | 54.8 | 50.5 | 46.2 | 46.1 | 62.7 | 62.3 | 62.3 | 66.8 |
| | Sovereign | 41.2 | 38.6 | 34.8 | 33.2 | 45.1 | 44.3 | 44.0 | 47.2 |
| | Collateralized | 7.0 | 6.4 | 6.6 | 7.0 | 9.0 | 7.1 | 7.3 | 8.5 |
| | Corporate | 6.6 | 5.6 | 4.9 | 5.9 | 8.6 | 10.9 | 11.0 | 11.2 |

Yearly turnover = 113.4% in the US and 113.3% in the Eurozone.

# The multi-currency market portfolio

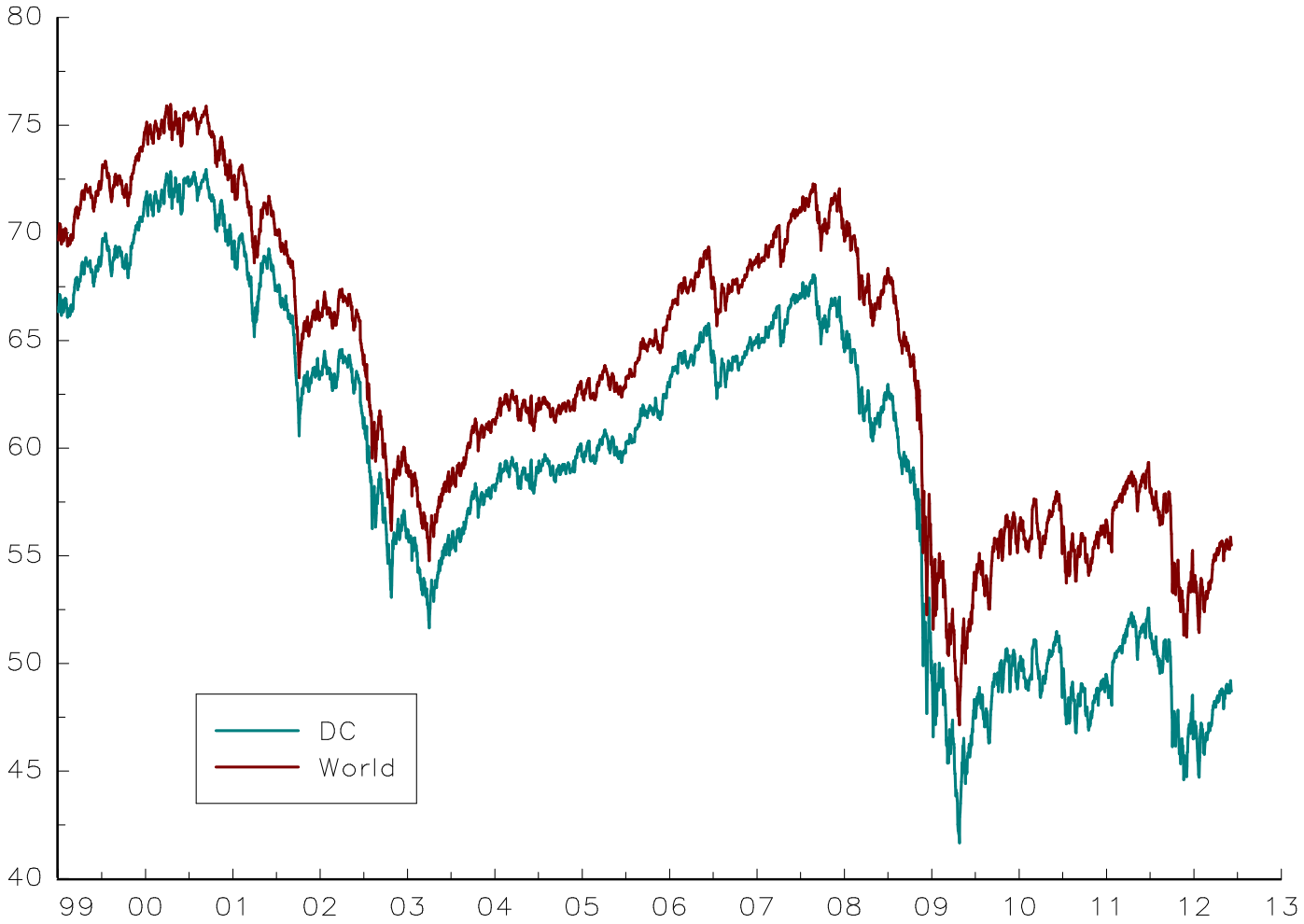
## The universe

- DC = Australia, Canada, Denmark, Eurozone, Japan, Norway, New Zealand, Sweden, Switzerland, UK and US
- World = DC + Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Egypt, Hong Kong, Hungary, India, Indonesia, Israel, Korea, Mexico, Peru, Philippine, Poland, Romania, Russia, South Africa, Singapore, Slovenia, Taiwan, Thailand, Turkey and Venezuela.
- Pivot currency = USD

# The multi-currency market portfolio

## DC and World market portfolios

Figure: Evolution of the equity weight for DC and World market portfolios



# The multi-currency market portfolio

## DC and World market portfolios

Table: Market portfolio allocation (in %) at the beginning of each year

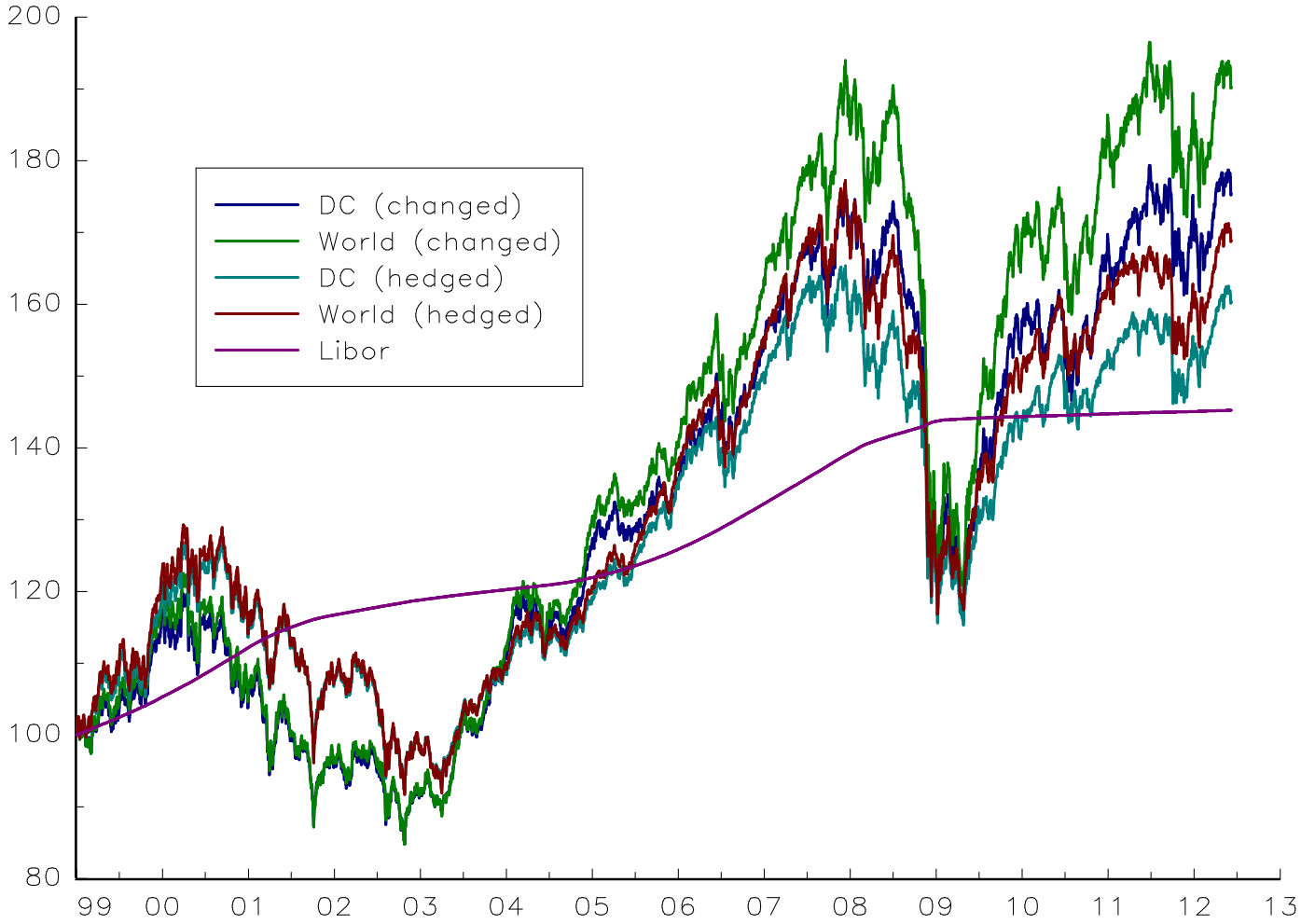
| | Year | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|
| DC | Equity | 60.3 | 63.6 | 65.4 | 64.6 | 48.1 | 50.2 | 50.8 | 46.8 |
| | Bond | 39.7 | 36.4 | 34.6 | 35.4 | 51.9 | 49.8 | 49.2 | 53.2 |
| | US | 45.2 | 44.4 | 42.6 | 41.7 | 43.2 | 43.4 | 45.3 | 46.1 |
| | Euro | 25.4 | 24.2 | 26.2 | 27.9 | 27.3 | 27.1 | 23.3 | 22.5 |
| | Others | 29.4 | 31.4 | 31.2 | 30.4 | 29.5 | 29.5 | 31.4 | 31.5 |
| World | Equity | 63.2 | 66.8 | 69.3 | 69.9 | 53.1 | 56.7 | 57.8 | 53.3 |
| | Bond | 36.8 | 33.2 | 30.7 | 30.1 | 46.9 | 43.3 | 42.2 | 46.7 |
| | DC | 93.6 | 91.8 | 89.2 | 85.8 | 90.9 | 87.6 | 86.1 | 88.3 |
| | EM | 6.4 | 8.2 | 10.8 | 14.2 | 9.1 | 12.4 | 13.9 | 11.7 |

- Since 2008, US ↗ and Euro ↘
- Since 2005, DC ↘ and EM ↗

# The multi-currency market portfolio

## DC and World market portfolios

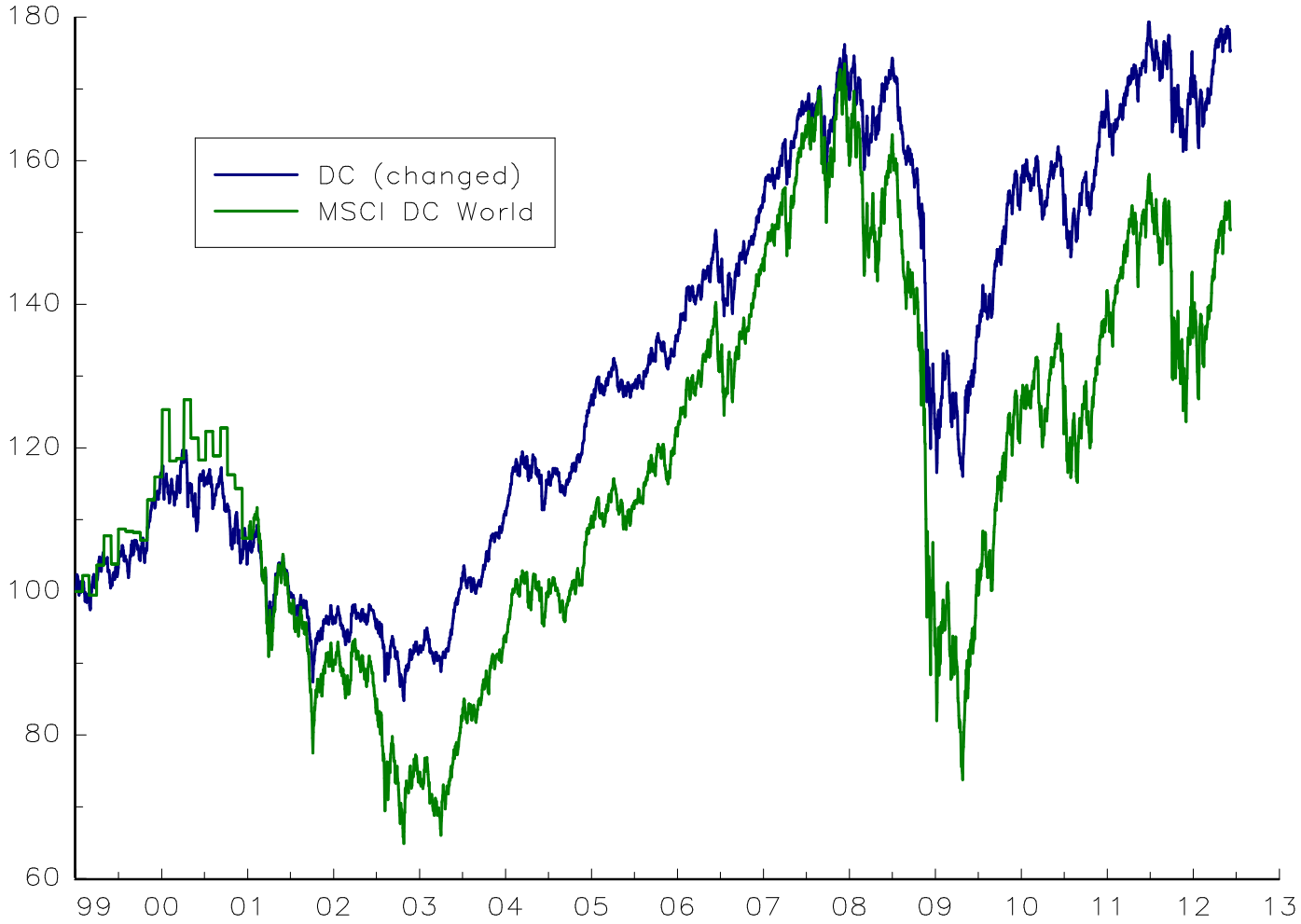
Figure: Performance of DC and World market portfolios



# The multi-currency market portfolio

## DC and World market portfolios

Figure: Comparison with the MSCI World index



# The multi-currency market portfolio

## DC and World market portfolios

Table: Some statistics

| | Return | Volatility | Sharpe ratio | MDD |
|---|---|---|---|---|
| DC (changed) | 4.33 | 10.04 | 14.69 | −34.14 |
| World (changed) | 4.97 | 10.54 | 20.05 | −38.24 |
| DC (hedged) | 3.62 | 8.99 | 8.58 | −30.27 |
| World (hedged) | 4.03 | 9.33 | 12.62 | −34.05 |

# From market portfolio to risk premia

In Markowitz, the quadratic utility function is $U(w) = \pi^\top w - \frac{\lambda}{2} w^\top \Sigma w$ with $\pi$ the risk premia vector and $\Sigma$ the covariance matrix of returns. The solution of this utility maximization program is:

$$w^\star = \frac{1}{\lambda} \Sigma^{-1} \pi$$

For a given portfolio $w^\star$, we could deduce the implied expected excess return:

$$\pi^\star = \lambda \Sigma w^\star$$

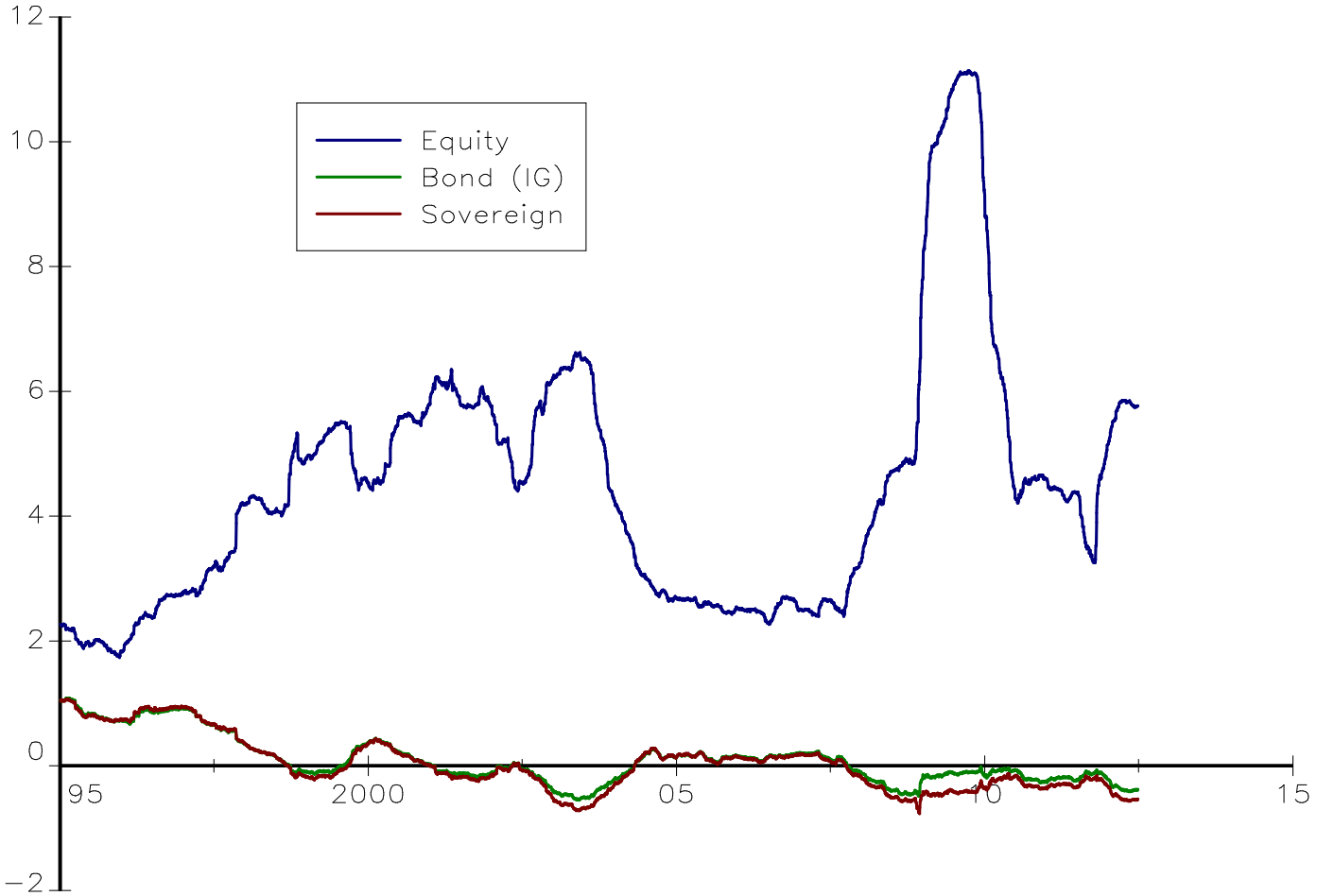
If we assume a constant Sharpe ratio, we obtain:

$$\pi^\star = \mathrm{sh}^\star \frac{\Sigma w^\star}{\sqrt{w^{\star\top} \Sigma w^\star}}$$

$\Rightarrow$ The risk premium is proportional to the marginal risk (micro-economic interpretation).

# Empirical findings
## Small risk premium for bonds

Figure: Ex-ante risk premia for the US market portfolio



# Empirical findings
## Why?

For the two-asset case, it comes that:

$$\pi_i^\star = \underbrace{c\sigma_i^2 w_i^\star}_{\text{variance}} + \underbrace{c\rho\sigma_i\sigma_j(1-w_i^\star)}_{\text{covariance}}$$

with $c = \text{sh}^\star / \sigma(w^\star)$. We notice that there are two components:

1. a variance component which is an increasing function of the volatility and the weight of the asset;
2. a covariance component with depends on the correlation between the asset returns (may be negative).

Equities are growth assets, bonds are hedging assets!

This result rejoins the Lucas model (1978) where required risk premia depend on the correlation between asset return and marginal utility of consumption.

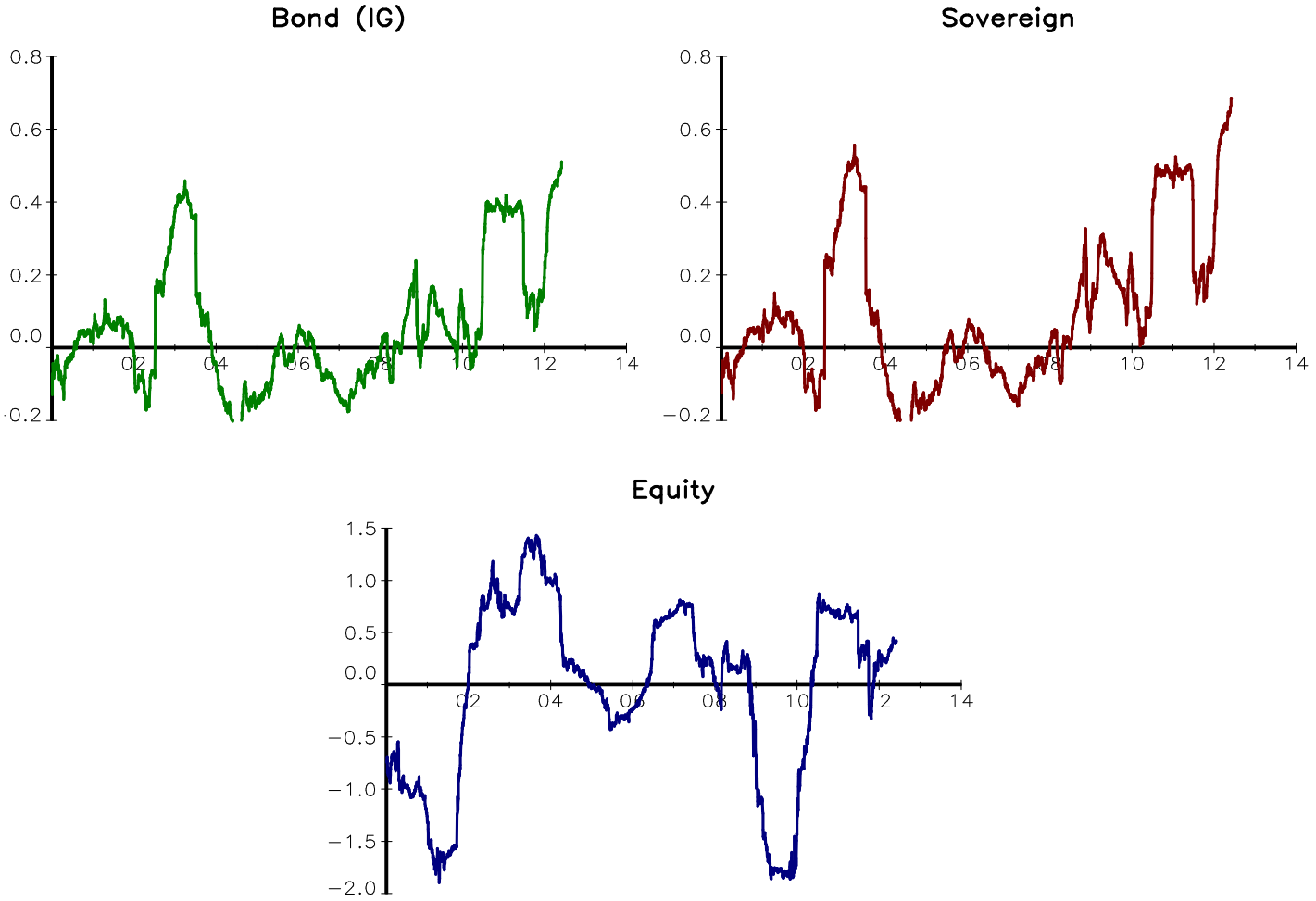
# Empirical findings
## Financial nature of assets

1. At the beginning of the 2000s, the distinction is between equities and bonds.
2. At the mid of the 2000s, the distinction is between growth assets and hedging assets.
3. Today, the distinction is between risky assets, hedging assets and real assets (including real estate, infrastructure, etc.).

Increasing part of the shadow banking assets

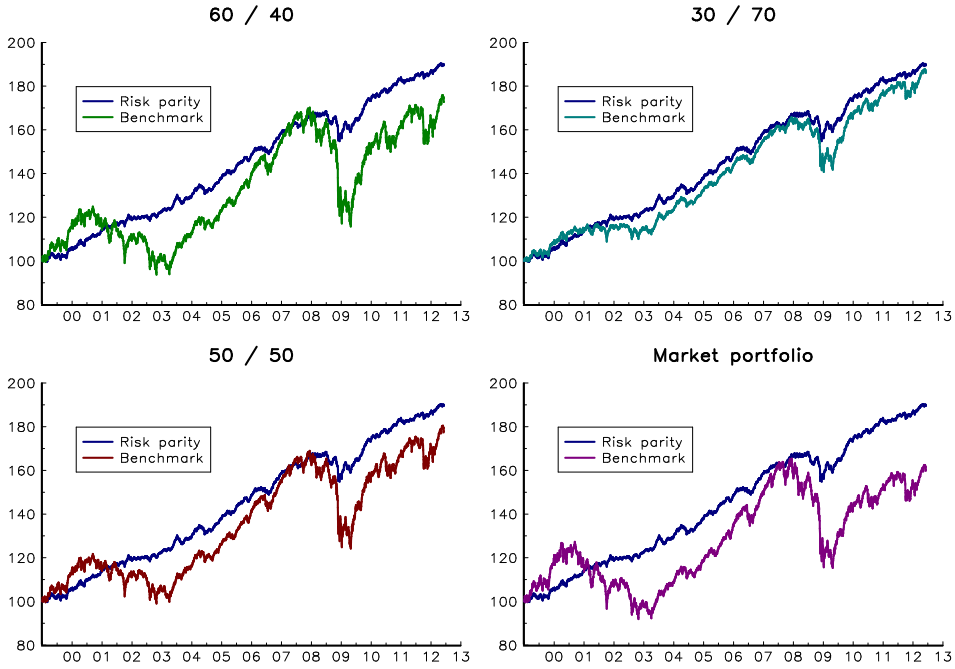
# Empirical findings
## Differences between US and Euro risk premia

Figure: Difference between ex-ante EURO and US risk premia



# Benchmarking diversified funds

- Universe = DC, Hedged.
- SR = 8.58% for MP, 14.34% for 60/40, 20.31% for 50/50, 41.78% for 30/70 and 73.77% for RP.
- MDD = −30.27% for MP, −32.02% for 60/40, −26.51% for 50/50, −15.12% for 30/70 and −8.10% for RP.



# Long-term investment policies

## The contrarian nature of long-term investors

*"The Government Pension Fund Global follows the investment strategy laid down by the Ministry of Finance in consultation with advisers including NBIM. The ministry has decided that 60 percent of the fund shall be invested in equities, 35-40 percent in fixed-income securities and as much as 5 percent in real estate. The most important decision for the fund's future returns and risk levels is how much capital will be invested in equities, which are expected to have both higher returns and risk over time than the other asset classes. The fund shall only invest outside Norway".*

# Long-term investment policies

## Constant mix investment policy

Illustration of the 60/40 asset mix policy of NBIM[2].

Table: Allocation (in %) of the Norwegian petroleum fund at the end of each quarter

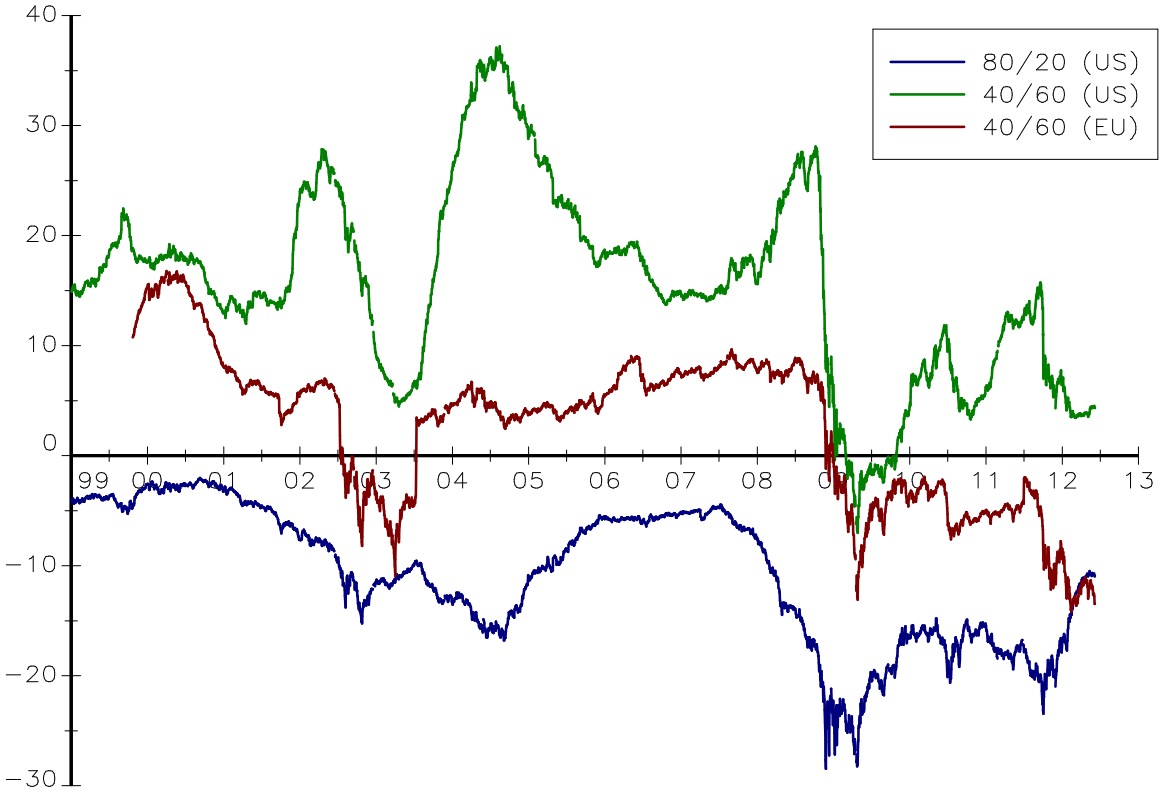
| Asset class | | 2007 1Q | 2007 4Q | 2008 4Q | 2009 4Q | 2010 4Q | 2011 1Q | 2011 2Q | 2011 3Q | 2011 4Q | 2012 1Q |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Equity | | 38.7 | 47.4 | 49.6 | 62.3 | 61.5 | 61.3 | 60.5 | 55.6 | 58.7 | 60.7 |
| Bond | | 57.8 | 52.6 | 50.4 | 37.7 | 38.5 | 38.6 | 39.4 | 44.1 | 40.9 | 39.0 |
| RE | | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | 0.1 | 0.3 | 0.3 | 0.3 |
| Equity | DC | 65.8 | 64.6 | 48.1 | 50.2 | 50.8 | 51.6 | 50.7 | 44.9 | 46.8 | 48.8 |
| | World | 69.6 | 69.9 | 53.1 | 56.7 | 57.8 | 58.5 | 57.4 | 51.5 | 53.3 | 55.5 |
| Bet | DC | −27.1 | −17.2 | 1.5 | 12.1 | 10.7 | 9.7 | 9.8 | 10.7 | 11.9 | 11.9 |
| | World | −30.9 | −22.5 | −3.5 | 5.6 | 3.7 | 2.8 | 3.1 | 4.1 | 5.4 | 5.2 |

$\Rightarrow$ Negative bet on equities until 2007 and positive bet on equities since 2008.

[2] The strategic allocation to equities was raised from 40% to 60% in summer 2007.

# Long-term investment policies
## Characterizing the bets of strategic asset allocation

Figure: Differences between the bond risk premia $\pi^{\star}_{\text{SAA}}$ and $\pi^{\star}_{\text{MP}}$ (in bps)



# Conclusion

- Market portfolio = a useful tool for long-term investors
- Difficulty to access to the market portfolio for multi-asset classes
- Multi-indexing may help long-term investors to better manage their asset mix policy and to understand their bets
- What is the impact of constant mix benchmarks?
- The business model of pension funds and long-term investors changes $\Rightarrow$ active members of the shadow banking system.

# For Further Reading


R. Louis, T. Roncalli.
On the Market Portfolio for Multi-Asset Classes.
SSRN, www.ssrn.com/abstract=2064807, April 2012.


Sharpe W.F.
Adaptive Asset Allocation Policies.
Financial Analysts Journal, 66(3), 2010.