Risk-Based Indexation (with a focus on the ERC method)[1]

Thierry Roncalli[2]

[2]Lyxor Asset Management and Évry University, France

Petits Déjeuners de la Finance, Maison des Polytechniciens, May 12, 2010

[1]The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.

Outline

Pros and Cons of Market-cap Indexation

Pros of market-cap indexation

- A convenient and **recognized approach** to participate to broad equity markets.
- **Management simplicity**: low turnover & transaction costs.

Cons of market-cap indexation

- Trend-following strategy: momentum bias leads to bubble risk exposure as weight of best performers ever increases.
  ⇒ Mid 2007, financial stocks represent 40% of the Eurostoxx 50 index.
- Growth biais as high valuation multiples stocks weight more than low-multiple stocks with equivalent realised earnings.
  ⇒ Mid 2000, the 8 stocks of the technology/telecom sectors represent 35% of the Eurostoxx 50 index.
  ⇒ 2½ years later after the dot.com bubble, these two sectors represent 12%.
- Concentrated portfolios.
  ⇒ The top 100 market caps of the S&P 500 account for around 70%.
- Lack of risk diversification and high drawdown risk: no portfolio construction rules leads to concentration issues (e.g. sectors, stocks).

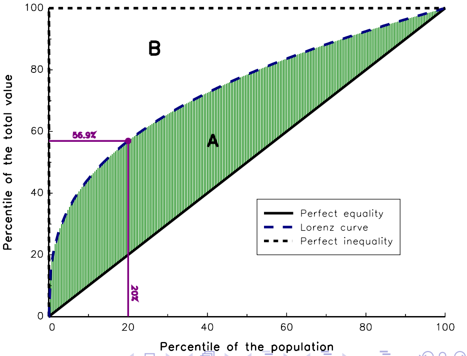
Statistical Measures of Concentration

- The Lorenz curve $\mathscr{L}(x)$
  It is a graphical representation of the concentration. It represents the cumulative weight of the first $x\%$ most representative stocks.

- The Gini coefficient
  It is a measure of dispersion using the Lorenz curve:

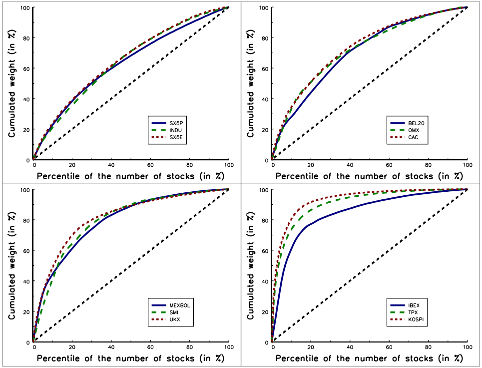
$$G = \frac{A}{A+B} = 2\int_0^1 \mathscr{L}(x)\,\mathrm{d}x - 1$$

  G takes the value 1 for a perfectly concentrated portfolio and 0 for the equally-weighted portfolio.



Concentration of Equity Indexes (December 31, 2009)

Index	Gini	$\mathscr{L}(x)$ 10	25	50
SX5P	0.27	23	45	68
INDU	0.29	21	42	71
SX5E	0.31	24	45	71
BEL20	0.41	28	51	79
OMX	0.44	33	57	79
CAC	0.47	34	58	82
DAX	0.47	29	58	84
HSI	0.51	39	63	83
AEX	0.51	34	62	85
NDX	0.53	47	66	82
NKY	0.59	47	69	87
MEXBOL	0.59	44	68	89
SMI	0.60	41	71	90
SPX	0.63	52	73	89
UKX	0.63	49	76	89
SXXE	0.64	52	76	90
HSCEI	0.64	53	77	90
SPTSX	0.66	55	77	90
SXXP	0.67	57	78	90
IBEX	0.69	61	81	91
TWSE	0.78	71	85	94
TPX	0.82	74	90	97
KOSPI	0.86	81	94	98



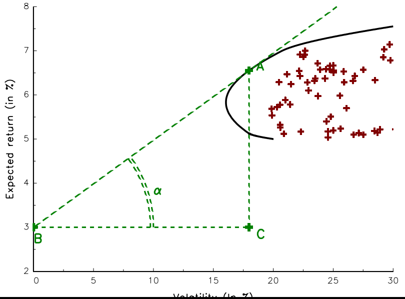
(*) In the case of the SX5P Index, 10% of stocks (respectively 25% and 50%) represent 23% of weight in the index (respectively 45% and 68%).

Main argument of passive management :

The Market Cap Index = The Tangency Portfolio

In the modern portfolio theory of Markowitz, we maximize the expected return for a given level of volatility:

$$\max \mu(w) = \mu^\top w \quad \text{u.c.} \quad \sigma(w) = \sqrt{w^\top \Sigma w} = \sigma^\star$$



- The optimal portfolio is the tangency portfolio.
- Main problem: the solution is very sensitive to the vector of expected returns ⇒ the solution is not robust.
- If the market cap index is the optimal portfolio, it means that expected returns are persistent.

- Academic research has illustrated that Capitalization-weighted indexes are not tangency portfolios.
- Dynamics of cap-weighted indexes = dynamics of price-weighted indexes (e.g. Nikkei and Topix indexes).

Alternative-Weighted Indexation

Alternative-weighted indexation aims at building passive indexes where the weights are not based on market capitalization.

Two sets of responses:

1. Fundamental indexation $\Rightarrow$ promising alpha.
2. Risk-based indexation $\Rightarrow$ promising diversification.

Two ways of using risk-based indexation:

1. Substitute as the capitalized-weighted index.
2. Complement to the capitalized-weighted index.

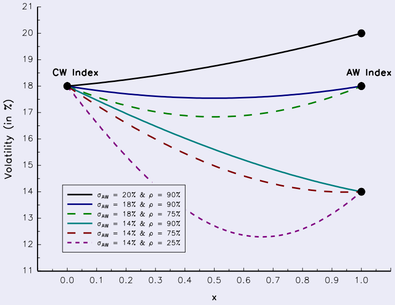
Substitution (Core Investment)

The beta of risk-based indexation is different than the beta of capitalized-weighted index.

Investors may prefer to have another beta and use risk-based indexes as a substitue of the capitalized-weighted indexes.

Complementary (Satellite Investment)

Investors want to diversify their passive equity exposure.



Example with $x\%$ invested in the CW index and $(1-x)\%$ invested in the AW index.

Portfolio Construction

Equally-weighted (1/n)

Most Diversified Portfolio (MDP)

Minimum-variance (MV)

Equal-Risk Contribution (ERC)

Notations

Let w be the vector of weights, μ the vector of risk premia (e.g. expected returns) and Σ the covariance matrix of returns. The volatility of the portfolio is:

$$\sigma(w) = \sqrt{w^\top \Sigma w}$$

wheras its expected return is:

$$\mu(w) = w^\top \mu$$

The 1/n Portfolio

We have:

$$w_i = \frac{1}{n}$$

Some properties

- It is the less concentrated portfolio:

$$G_w = 0$$

- It is a contrarian strategy.
- It has a take-profit scheme.

The Minimum-Variance Portfolio

We have:

$$\begin{aligned} w^\star &= \arg\min \sqrt{w^\top \Sigma w} \\ \text{u.c.} \quad & \mathbf{1}^\top x = 1 \text{ and } \mathbf{0} \leq x \leq \mathbf{1} \end{aligned}$$

In the short-selling case, the lagrangian function is:

$$f(w;\lambda_0) = \sigma(w) - \lambda_0 \left(\mathbf{1}^\top w - 1\right)$$

The solution $w^\star$ verifies the following system of first-order conditions:

$$\left\{ \begin{array}{r} \partial_x f(w;\lambda_0) = \frac{\partial\,\sigma(w)}{\partial\,w} - \lambda_0 \mathbf{1} = 0 \\ \partial_{\lambda_0} f(w;\lambda_0) = \mathbf{1}^\top w - 1 = 0 \end{array} \right.$$

We have:

$$\frac{\partial\,\sigma(w)}{\partial\,w_i} = \frac{\partial\,\sigma(w)}{\partial\,w_j} = \sigma(w) \quad \text{for all } i,j$$

In the case of no-short selling, write the Kühn-Tucker conditions and we have:

$$\boxed{\frac{\partial\,\sigma(w)}{\partial\,w_i} = \frac{\partial\,\sigma(w)}{\partial\,w_j} \quad \text{for all } w_i \neq 0, w_j \neq 0}$$

The MDP/MSR Portfolio

Let $D(w)$ be the diversification ratio:

$$D(w) = \frac{\sqrt{w^\top \tilde{\Sigma} w}}{\sqrt{w^\top \Sigma w}} = \frac{w^\top \sigma}{\sqrt{w^\top \Sigma w}}$$

where $\tilde{\Sigma}$ is the covariance matrix with $\tilde{\Sigma}_{i,j} = \sigma_i \sigma_j$ (all the correlations are equal to one). We have $D(w) \geq 1$. The MDP portfolio is defined by:

$$\begin{aligned} w^\star &= \arg\max D(w) \\ \text{u.c.} \quad & \mathbf{1}^\top x = 1 \text{ and } \mathbf{0} \leq x \leq \mathbf{1} \end{aligned}$$

Remark

If we assume that the Sharpe ratio is the same for all the assets – $\mu_i - r = s \times \sigma_i$, we obtain:

$$\text{sh}(w) = \frac{w^\top \mu - r}{\sqrt{w^\top \Sigma w}} = s \times D(w)$$

Maximizing $D(w)$ is equivalent to maximize $\text{sh}(w)$.

The ERC Allocation Method is a special case of Risk Budgeting

Because $\sigma(w) = \sqrt{w^\top \Sigma w}$, we have:

$$\frac{\partial\, \sigma(w)}{\partial\, w} = \frac{\Sigma w}{\sqrt{w^\top \Sigma w}}$$

We check that the volatility verifies the Euler decomposition:

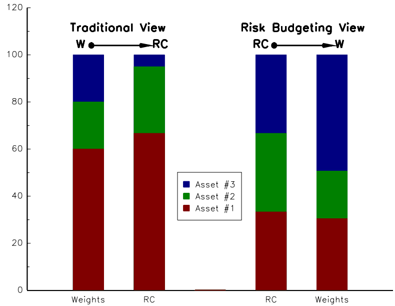
$$\sigma(w) = \sum_{i=1}^{n} w_i \times \frac{\partial\, \sigma(w)}{\partial\, w_i} = w^\top \frac{\Sigma w}{\sqrt{w^\top \Sigma w}} = \sqrt{w^\top \Sigma w}$$

The risk contribution of the i^{th} asset is then defined by:

$$\mathrm{RC}_i = w_i \times \frac{(\Sigma w)_i}{\sqrt{w^\top \Sigma w}}$$

An example

3 assets.
Volatilities are respectively 20%, 30% and 15%.
Correlations are set to 60% between the first and second asset, and 10% for the third assets.



Traditional View

w_i	MR_i	RC_i	in %
60.0%	18.8%	11.3%	66.7%
20.0%	23.9%	4.8%	28.3%
20.0%	4.3%	0.9%	5.0%
Volatility		16.9%	

Risk Budgeting View

w_i	MR_i	RC_i	in %
48.5%	17.7%	8.6%	**60.0%**
13.2%	21.7%	2.9%	**20.0%**
38.3%	7.5%	2.9%	**20.0%**
Volatility		14.3%	

ERC View

w_i	MR_i	RC_i	in %
30.4%	15.2%	4.6%	**33.3%**
20.3%	22.7%	4.6%	**33.3%**
49.3%	9.3%	4.6%	**33.3%**
Volatility		13.8%	

The Euler decomposition gives us:

$$\sigma(w) = \sum_{i=1}^{n} w_i \times \frac{\partial\,\sigma(w)}{\partial\,w_i} = \sum_{i=1}^{n} \mathrm{RC}_i$$

The idea of the ERC strategy is to find a risk-balanced portfolio such that the risk contribution is the same for all assets of the portfolio:

$$\mathrm{RC}_i = \mathrm{RC}_j \quad \text{for all } i,j$$

The ERC portfolio is then the solution of the following non-linear system:

$$\begin{cases} w_2 \times (\Sigma w)_2 = w_1 \times (\Sigma w)_1 \\ w_3 \times (\Sigma w)_3 = w_1 \times (\Sigma w)_1 \\ \vdots \\ w_n \times (\Sigma w)_n = w_1 \times (\Sigma w)_1 \\ w_1 + w_2 + \cdots + w_n = 1 \\ w_1 > 0, w_2 > 0, \ldots, w_n > 0 \end{cases}$$

Consider the following optimization problem:

$$w^\star(c) = \arg\min \sqrt{w^\top \Sigma w} \quad \text{u.c.} \quad \begin{cases} \sum_{i=1}^n \ln w_i \geq c \\ \mathbf{1}^\top x = 1 \\ \mathbf{0} \leq x \leq \mathbf{1} \end{cases}$$

We have $w^\star(-\infty) = w_{\mathrm{mv}}$ and $w^\star(-n\ln n) = w_{1/\mathrm{n}}$. The ERC portfolio corresponds to a particular value of c such that $-\infty \leq c \leq -n\ln n$.

1. The solution of the ERC problem exists and is **unique**.
2. We also obtain the following inequality:

   $$\sigma_{\mathrm{mv}} \leq \sigma_{\mathrm{erc}} \leq \sigma_{1/\mathrm{n}}$$

   because if $c_1 \leq c_2$, we have $\sigma(w^\star(c_1)) \leq \sigma(w^\star(c_2))$. The ERC portfolio may be viewed as a portfolio "between" the 1/n portfolio and the minimum-variance portfolio.
3. The ERC portfolio may be viewed as a form of variance-minimizing portfolio subject to a constraint of sufficient diversification in terms of component weights.

Some Properties

- **If the correlations are the same**, the solution is:

$$x_i = \frac{\sigma_i^{-1}}{\sum_{j=1}^n \sigma_j^{-1}}$$

The weight allocated to each component i is given by the ratio of the inverse of its volatility with the harmonic average of the volatilities.

- **If the volatilities are the same**, we have:

$$x_i \propto \left(\sum_{k=1}^n x_k \rho_{ik} \right)^{-1}$$

The weight of the asset i is proportional to the inverse of the weighted average of correlations of component i with other components.

- **In the general case**, we obtain:

$$x_i \propto \beta_i^{-1}$$

The weight of the asset i is proportional to the inverse of its beta.

Some Properties

- The ERC portfolio is the tangency portfolio if all the assets have the same Sharpe ratio and if the correlation is uniform (one-factor model).
- Let us consider the minimum variance portfolio with a constant correlation matrix $C_n(\rho)$. The solution is:

$$x_i = \frac{-((n-1)\rho+1)\sigma_i^{-2} + \rho\sum_{j=1}^{n}(\sigma_i\sigma_j)^{-1}}{\sum_{k=1}^{n}\left(-((n-1)\rho+1)\sigma_k^{-2} + \rho\sum_{j=1}^{n}(\sigma_k\sigma_j)^{-1}\right)}$$

The lower bound of $C_n(\rho)$ is achieved for $\rho = -(n-1)^{-1}$ and we have:

$$x_i = \frac{\sum_{j=1}^{n}(\sigma_i\sigma_j)^{-1}}{\sum_{k=1}^{n}\sum_{j=1}^{n}(\sigma_k\sigma_j)^{-1}} = \frac{\sigma_i^{-1}}{\sum_{k=1}^{n}\sigma_k^{-1}} \rightarrow \text{erc}$$

Comparison of the 4 Methods

Equally-weighted (1/n)

- Weights are equal
- Easy to understand
- Contrarian strategy with a take-profit scheme
- The least concentrated in terms of weights
- Do not depend on risks

Most Diversified Portfolio (MDP)

- Also known as the Max Sharpe Ratio (MSR) portfolio of EDHEC
- Based on the assumption that sharpe ratio is equal for all stocks
- It is the tangency portfolio if the previous assumption is verified
- Sensitive to the covariance matrix

Minimum-variance (MV)

- Low volatility portfolio
- The only optimal portfolio not depending on expected returns assumptions
- Good out of sample performance
- Concentrated portfolios
- Sensitive to the covariance matrix

Equal-Risk Contribution (ERC)

- Risk contributions are equal
- Highly diversified portfolios
- Less sensitive to the covariance matrix (than the MV and MDP portfolios)
- Not efficient for universe with a large number of stocks (equivalent to the 1/n portfolio)

Comparison of the 4 Methods

In terms of bets

$$\begin{array}{ll} \exists i : w_i = 0 & \text{(MV - MDP)} \\ \forall i : w_i \neq 0 & \text{(1/n - ERC)} \end{array}$$

In terms of risk factors

$$\begin{array}{rcll} w_i & = & w_j & \text{(1/n)} \\ \frac{\partial \sigma(w)}{\partial w_i} & = & \frac{\partial \sigma(w)}{\partial w_j} & \text{(MV)} \\ w_i \times \frac{\partial \sigma(w)}{\partial w_i} & = & w_j \times \frac{\partial \sigma(w)}{\partial w_j} & \text{(ERC)} \\ \frac{1}{\sigma_i} \times \frac{\partial \sigma(w)}{\partial w_i} & = & \frac{1}{\sigma_j} \times \frac{\partial \sigma(w)}{\partial w_j} & \text{(MDP)} \end{array}$$

Example 1

We consider an example with 4 assets. Volatilities are respectively 10%, 20%, 30% and 40%. All the cross-correlations are zero except a correlation of 80% between the first and second asset and a correlation of -50% between the third and fourth assets.

	MV			ERC			MDP/MSR			1/n		
Asset	w_i	MR_i	RC_i	w_i	MR_i	RC_i	w_i	MR_i	RC_i	w_i	MR_i	RC_i
1	74.5	8.6	6.4	38.4	6.7	2.6	27.8	4.4	1.2	25.0	5.6	1.4
2	0.0	13.8	0.0	19.2	13.4	2.6	13.9	8.8	1.2	25.0	12.2	3.0
3	15.2	8.6	1.3	24.3	10.6	2.6	33.3	13.3	4.4	25.0	6.5	1.6
4	10.3	8.6	0.9	18.2	14.1	2.6	25.0	17.7	4.4	25.0	21.7	5.4
$\sigma(w)$			8.6			10.3			11.3			11.5

Example 2

We consider another example with 6 assets. Volatilities are respectively 25%, 22%, 14%, 30%, 40% and 30%. All the cross-correlations are equal to 60% except a correlation of 20% between the fifth and sixth assets.

	MV			ERC			MDP/MSR			1/n		
Asset	w_i	MR_i	RC_i	w_i	MR_i	RC_i	w_i	MR_i	RC_i	w_i	MR_i	RC_i
1	0.0	15.3	0.0	15.7	20.7	3.3	0.0	19.4	0.0	16.7	20.8	3.5
2	3.6	14.0	0.5	17.8	18.2	3.3	0.0	17.0	0.0	16.7	18.1	3.0
3	96.4	14.0	13.5	28.0	11.6	3.3	0.0	10.8	0.0	16.7	11.1	1.9
4	0.0	18.4	0.0	13.1	24.9	3.3	0.0	23.2	0.0	16.7	25.4	4.2
5	0.0	24.5	0.0	10.9	30.0	3.3	42.9	31.0	13.3	16.7	31.4	5.2
6	0.0	18.4	0.0	14.5	22.5	3.3	57.1	23.2	13.3	16.7	21.6	3.6
$\sigma(w)$			14.0			19.5			26.6			21.4

Backtest with the DJ Eurostoxx 50 Universe

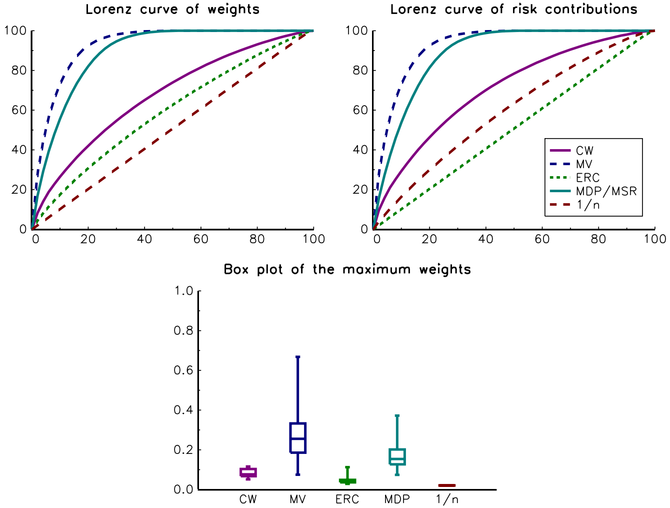
Backtesting rules

Monthly rebalancing of the weights.
The covariance matrix used for simulations is the empirical covariance matrix based on a rolling observation period of 1 year.
All indexes are price index (PI).
The study period is January 1993 – December 2009.

	CW	MV	ERC	MDP	1/n
Performance	6.39	8.08	10.30	12.63	9.22
Volatility	22.41	17.65	20.66	20.00	22.43
Sharpe	0.29	0.46	0.50	0.62	0.41
Volatility of TE		14.85	5.98	13.19	4.37
IR		0.11	0.65	0.47	0.65
Drawdown	66.88	55.89	56.84	49.95	61.79
Skewness (monthly)	−0.50	−1.06	−0.55	−0.58	−0.45
Kurtosis (monthly)	3.87	5.31	4.42	4.25	4.70
Skewness	0.06	2.12	0.24	3.44	0.08
Kurtosis	8.63	59.59	11.05	90.58	9.71
Correlation	100.00	75.00	94.66	81.24	98.10

On the Importance of Constraints

Concentration



On the Importance of Constraints

The impact of the lag window

- Turnover is calculated as the sum of sales and purchases of securities composing the index, based on monthly rebalancing dates.
- Takes into account changes in the universe.

Annualized monthly turnover (in %)

Lag	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
1M	1791	578	1932	20	1444	1597	991	1113
2M	1248	304	1321	20	939	1064	636	727
3M	913	205	984	20	705	818	472	548
1Y	327	65	340	20	249	292	162	202
2Y	194	43	212	20	149	190	99	129
3Y	145	36	157	20	112	144	74	95

On the Importance of Constraints

The impact of the CV estimator

EMP = empirical (or ML) estimate of the covariance matrix, CC = estimated covariance matrix built with empirical volatilities and a uniform correlation, PCA = principal component analysis with a number of factors computed according to Random Matrix Theory, CC-SHRINK and PCA-SHRINK = shrinkage estimators deduced from the previous CC and PCA estimators.

Stats	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
				Empirical covariance matrix (EMP)				
T_W	327	65	340	20	249	292	162	202
$\bar{G}_W$	0.85	0.18	0.78	0.00	0.77	0.74	0.59	0.58
$\bar{G}_{RC}$	0.85	0.00	0.77	0.19	0.76	0.74	0.60	0.62
IR	0.11	0.65	0.47	0.65	0.29	0.54	0.45	0.67
				Constant correlation matrix (CC)				
T_W	280	47	47	20	213	47	136	47
$\bar{G}_W$	0.86	0.14	0.14	0.00	0.76	0.14	0.59	0.14
$\bar{G}_{RC}$	0.86	0.00	0.01	0.16	0.76	0.01	0.60	0.01
IR	0.02	0.62	0.63	0.65	0.19	0.62	0.29	0.62
				Principal component analysis (PCA)				
T_W	264	65	259	20	206	230	144	173
$\bar{G}_W$	0.85	0.19	0.76	0.00	0.77	0.73	0.59	0.58
$\bar{G}_{RC}$	0.85	0.00	0.75	0.19	0.77	0.73	0.61	0.61
IR	0.14	0.68	0.55	0.65	0.25	0.54	0.48	0.69
				Shrinkage estimator with CC (CC-SHRINK)				
T_W	310	59	306	20	236	272	151	198
$\bar{G}_W$	0.86	0.17	0.73	0.00	0.76	0.70	0.59	0.57
$\bar{G}_{RC}$	0.86	0.00	0.72	0.18	0.76	0.70	0.60	0.59
IR	0.10	0.72	0.52	0.65	0.29	0.55	0.37	0.67
				Shrinkage estimator with PCA (PCA-SHRINK)				
T_W	303	65	307	20	233	271	155	192
$\bar{G}_W$	0.85	0.19	0.76	0.00	0.76	0.73	0.59	0.58
$\bar{G}_{RC}$	0.85	0.00	0.75	0.19	0.76	0.73	0.61	0.61
IR	0.13	0.66	0.54	0.65	0.28	0.54	0.44	0.67

(*) T_W is the annual turnover, T_W is the average of the Gini coefficients on weights, G_{RC} is the average of the Gini coefficients on Risk contributions, IR is the information ratio.

Composition in % (January 2010)

	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
TOTAL	6.1		2.1		2			5.0	
BANCO SANTANDER	5.8		1.3		2				
TELEFONICA SA	5.0	31.2	3.5		2	10.0		5.0	5.0
SANOFI-AVENTIS	3.6	12.1	4.5	15.5	2	10.0	10.0	5.0	5.0
E.ON AG	3.6		2.1		2				1.4
BNP PARIBAS	3.4		1.1		2				
SIEMENS AG	3.2		1.5		2				
BBVA(BILB-VIZ-ARG)	2.9		1.4		2				
BAYER AG	2.9		2.6	3.7	2	2.2	5.0	5.0	5.0
ENI	2.7		2.1		2				
GDF SUEZ	2.5		2.6	4.5	2		5.4	5.0	5.0
BASF SE	2.5		1.5		2				
ALLIANZ SE	2.4		1.4		2				
UNICREDIT SPA	2.3		1.1		2				
SOC GENERALE	2.2		1.2	3.9	2		3.7		5.0
UNILEVER NV	2.2	11.4	3.7	10.8	2	10.0	10.0	5.0	5.0
FRANCE TELECOM	2.1	14.9	4.1	10.2	2	10.0	10.0	5.0	5.0
NOKIA OYJ	2.1		1.8	4.5	2		4.8		5.0
DAIMLER AG	2.1		1.3		2				
DEUTSCHE BANK AG	1.9		1.0		2				
DEUTSCHE TELEKOM	1.9		3.2	2.6	2	5.7	3.7	5.0	5.0
INTESA SANPAOLO	1.9		1.3		2				
AXA	1.8		1.0		2				
ARCELORMITTAL	1.8		1.0		2				
SAP AG	1.8	21.0	3.4	11.2	2	10.0	10.0	5.0	5.0

	CW	MV	ERC	MDP	1/n	MV 10%	MDP 10%	MV 5%	MDP 5%
RWE AG (NEU)	1.7	2.7	2.7		2	7.0		5.0	
ING GROEP NV	1.6		0.8	0.4	2				
DANONE	1.6	1.9	3.4	1.8	2	8.7	3.3	5.0	5.0
IBERDROLA SA	1.6		2.5		2	5.1		5.0	1.2
ENEL	1.6		2.1		2			5.0	2.9
VIVENDI SA	1.6	2.8	3.1	4.5	2	10.0	5.9	5.0	5.0
ANHEUSER-BUSCH INB	1.6	0.2	2.7	10.9	2	2.1	10.0	5.0	5.0
ASSIC GENERALI SPA	1.6		1.8		2				
AIR LIQUIDE(L')	1.4		2.1		2			5.0	
MUENCHENER RUECKVE	1.3		2.1	2.1	2		3.1	5.0	5.0
SCHNEIDER ELECTRIC	1.3		1.5		2				
CARREFOUR	1.3	1.0	2.7	1.3	2	3.7	2.5	5.0	5.0
VINCI	1.3		1.6		2				
LVMH MOET HENNESSY	1.2		1.8		2				
PHILIPS ELEC(KON)	1.2		1.4		2				
L'OREAL	1.1	0.8	2.8		2	5.5		5.0	5.0
CIE DE ST-GOBAIN	1.0		1.1		2				
REPSOL YPF SA	0.9		2.0		2			5.0	
CRH	0.8		1.7	5.1	2		5.2		5.0
CREDIT AGRICOLE SA	0.8		1.1		2				
DEUTSCHE BOERSE AG	0.7		1.5		2				1.9
TELECOM ITALIA SPA	0.7		2.0		2				2.5
ALSTOM	0.6		1.5		2				
AEGON NV	0.4		0.7		2				
VOLKSWAGEN AG	0.2		1.8	7.1	2		7.4		5.0
Total of components	50	11	50	17	50	14	16	20	23

Conclusion

Alternative-Weighted Indexation

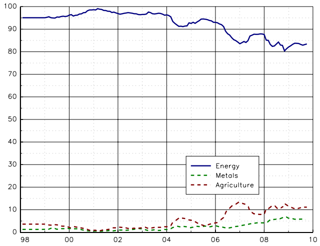
- Risk-based indexation historically posts better risk-adjusted performance than capitalization-weighted indexation.
- It is a promising way for investors to gain access to a well-diversified and diversifying exposure (or beta) to broad equity markets.
- Some practical issues:
  1. Turnover managing;
  2. Market price impact minimizing;
  3. Transparency (passive indexation or active strategy?);
  4. Understanding the style bias (small caps, growth, sectors, etc.);
  5. What is the impact of practical constraints on the strategy?
- Existence of professional solutions (indexes/mutual funds).

Conclusion

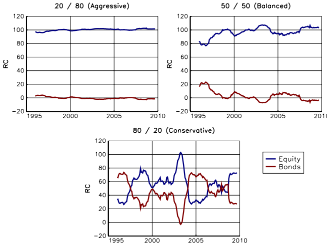
Application of the ERC Method to Portfolio Construction

- Equity
- Commodity
- Fixed-Income/Bonds
- Diversified portfolios
- Absolute Return
- Hedge funds

Risk contributions of the GSCI Index



Risk contributions of diversified/balanced funds



For Further Reading I

Rob Arnott, Vitali Kalesnik, Paul Moghtader, Craig Scholl.
Beyond Cap Weight – The Empirical Evidence for a Diversified Beta.
Journal of Indexes, January/February, pp. 16-29, 2010.

Yves Choueifaty, Yves Coignard.
Towards Maximum Diversification.
Journal of Portfolio Management, 35(1), pp. 40-51, 2008.

Paul Demey, Sébastien Maillard, Thierry Roncalli.
Risk-Based Indexation.
Working Paper, Available on http://www.lyxor.com/fr/lyxor-research/white-papers.html, 2008.

For Further Reading II

Victor DeMiguel, Lorenzo Garlappi, Raman Uppal.
Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?.
Review of Financial Studies, 22, pp. 1915-1953, 2009.

Eugene Fama, Kenneth French.
The Capital Asset Pricing Model: Theory and Evidence.
Journal of Economic Perspectives, 18(3), pp. 25-46, 2004.

Sébastien Maillard, Thierry Roncalli, Jérôme Teiletche.
On the Property of Equally-weighted Risk Contributions Portfolios.
Working Paper, available on SSRN, 2008.

Lionel Martellini.
Toward the Design of Better Equity Benchmarks.
Journal of Portfolio Management, 34(4), pp. 1-8, 2008.