

# Introduction to Risk Parity and Budgeting

## Appendix A – Technical Appendix

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Instructors may find the description of the book at the following addresses:

<http://www.crcpress.com/product/isbn/9781482207156>

<http://www.thierry-roncalli.com/RiskParityBook.html>

May 22, 2013

Figure: Example of building a bivariate probability distribution with a copula function

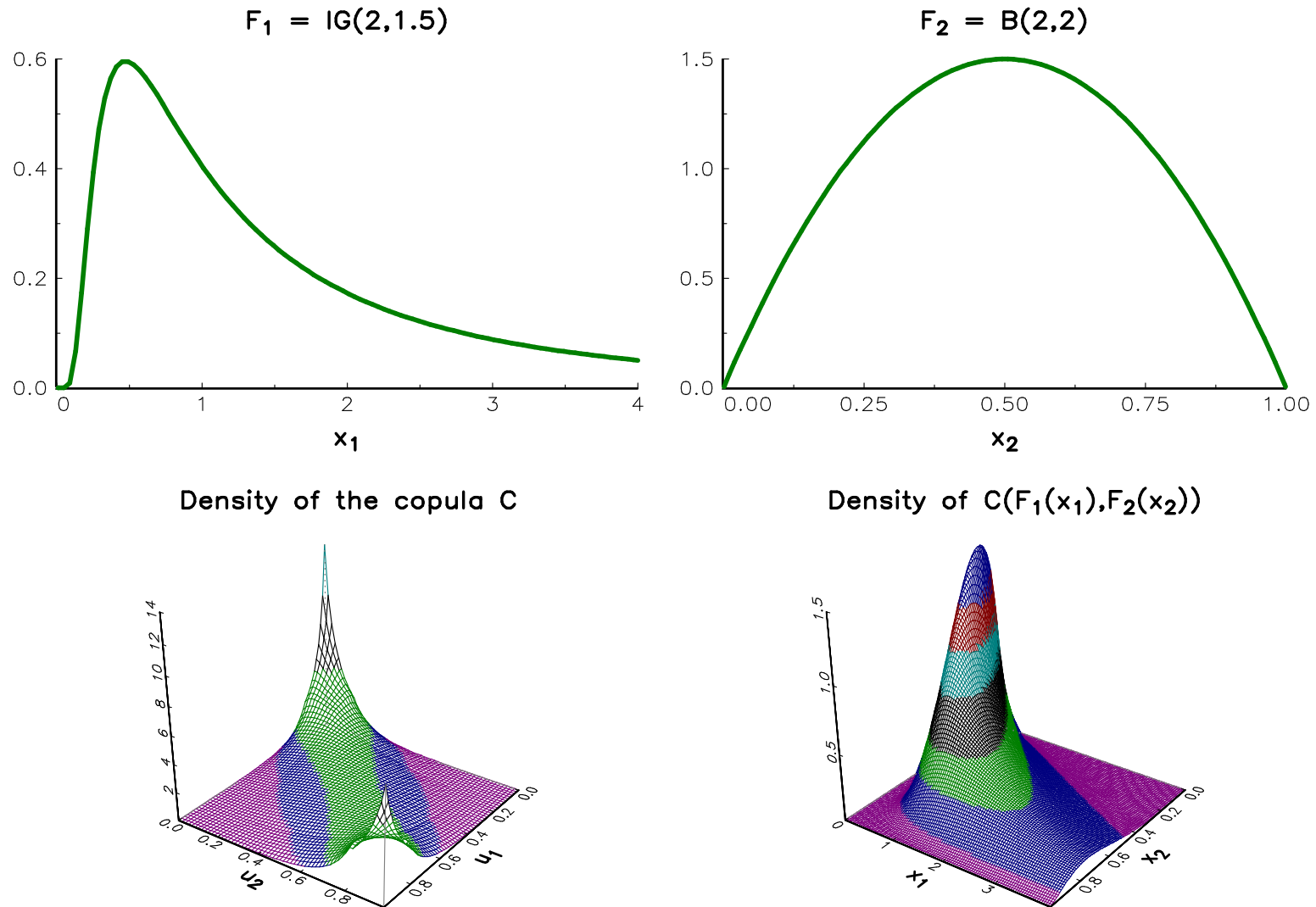


Figure: Level curves of bivariate distributions (Frank copula)

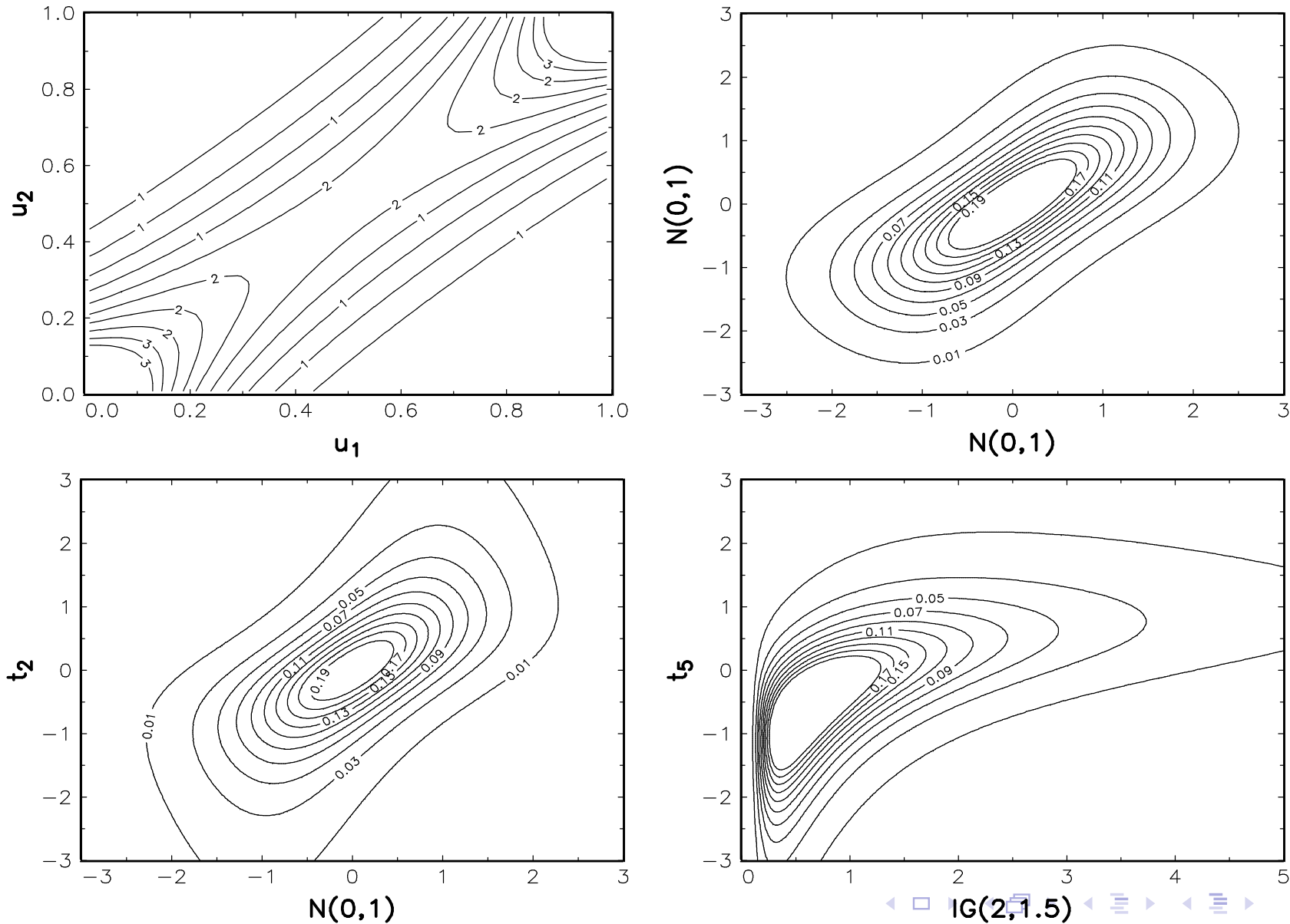
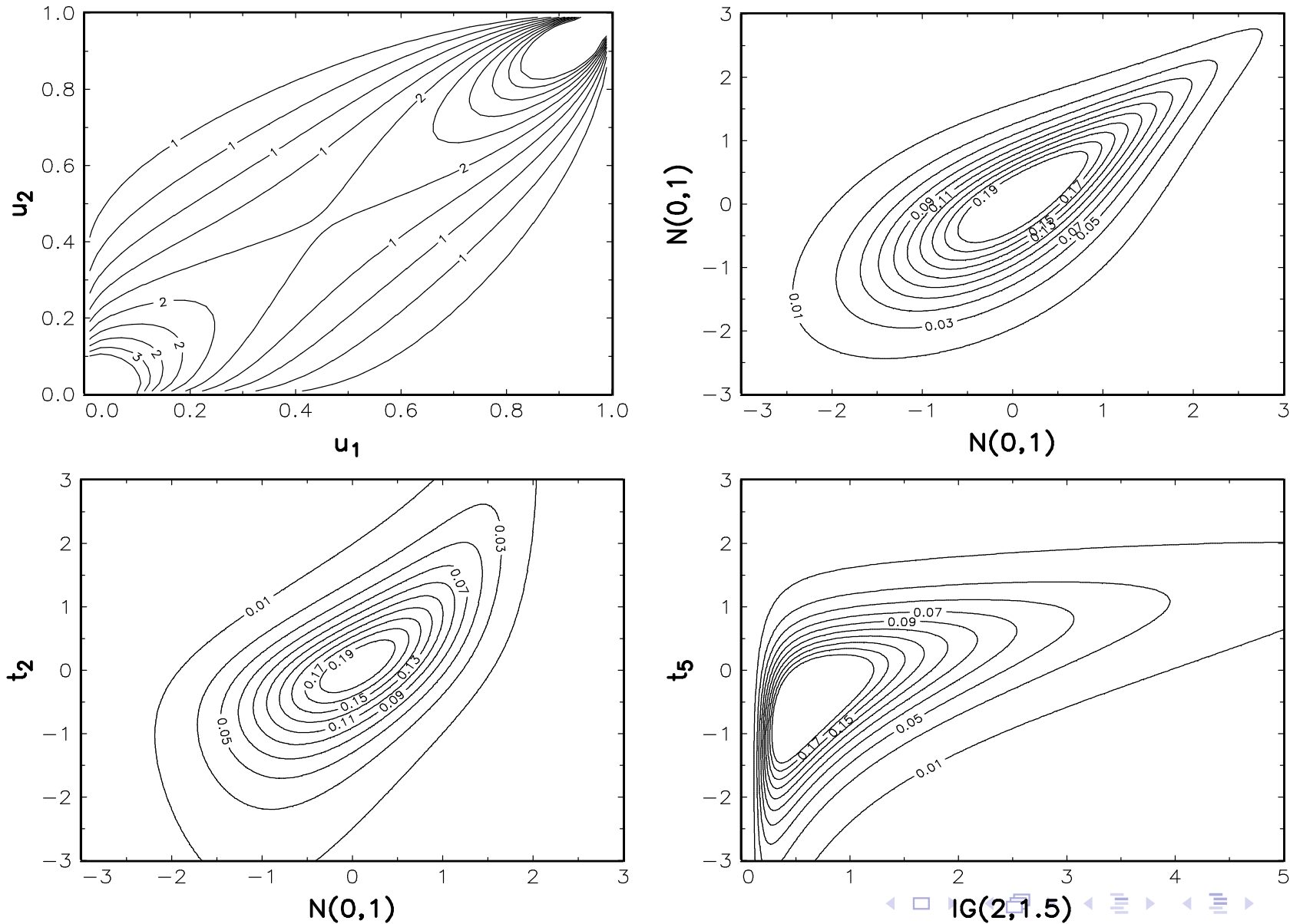


Figure: Level curves of bivariate distributions (Gumbel copula)

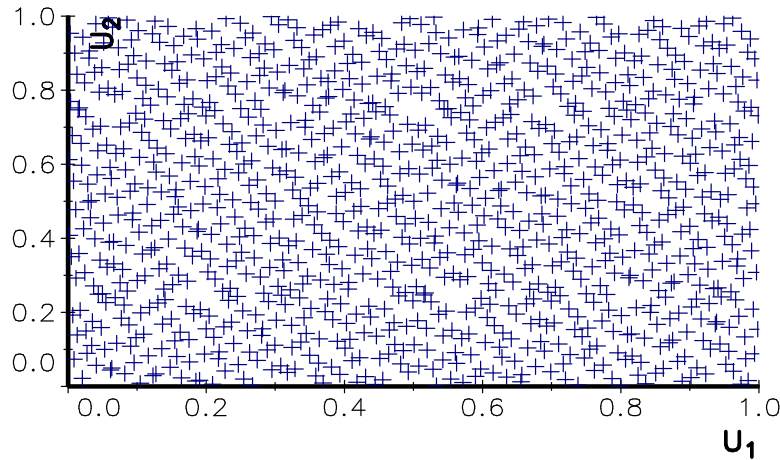


**Table:** Examples of Archimedean copula functions

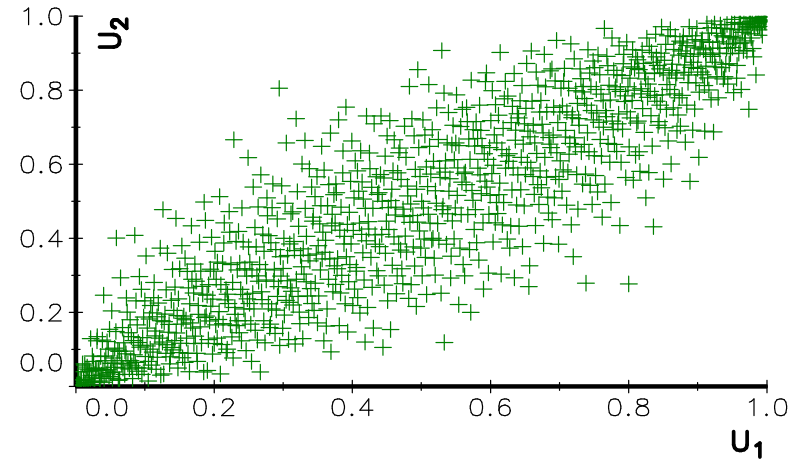
Copula	$\varphi(u)$	$\mathbf{C}(u_1, u_2)$
$C^\perp$	$-\ln u$	$u_1 u_2$
GumbelGumbel copula	$(-\ln u)^\theta$	$\exp\left(-(\tilde{u}_1^\theta + \tilde{u}_2^\theta)^{1/\theta}\right)$
FrankFrank copula	$-\ln \frac{e^{-\theta u} - 1}{e^{-\theta} - 1}$	$-\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\right)$
Joe	$-\ln \left(1 - (1 - u)^\theta\right)$	$1 - (\bar{u}_1^\theta + \bar{u}_2^\theta - \bar{u}_1^\theta \bar{u}_2^\theta)^{1/\theta}$
ClaytonClayton copula	$u^{-\theta} - 1$	$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$

Figure: Comparison of normal and  $t$  copulas

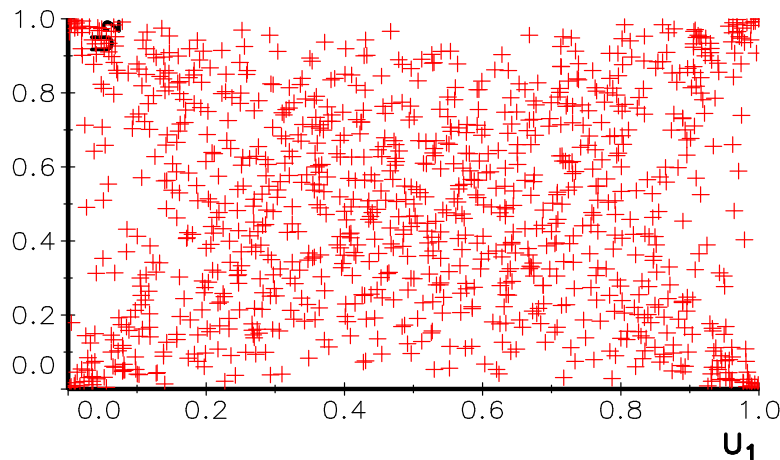
Normal copula  
 $\rho = 0$



Normal copula  
 $\rho = 90\%$



Student's  $t$  copula  
 $\rho = 0 \quad \nu = 1$



Student's  $t$  copula  
 $\rho = 90\% \quad \nu = 1$

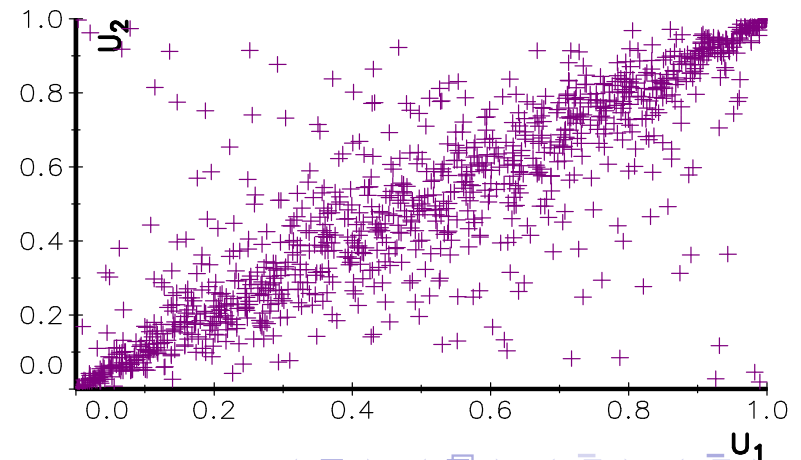


Figure: Quantile-quantile dependence measure for the normal copula

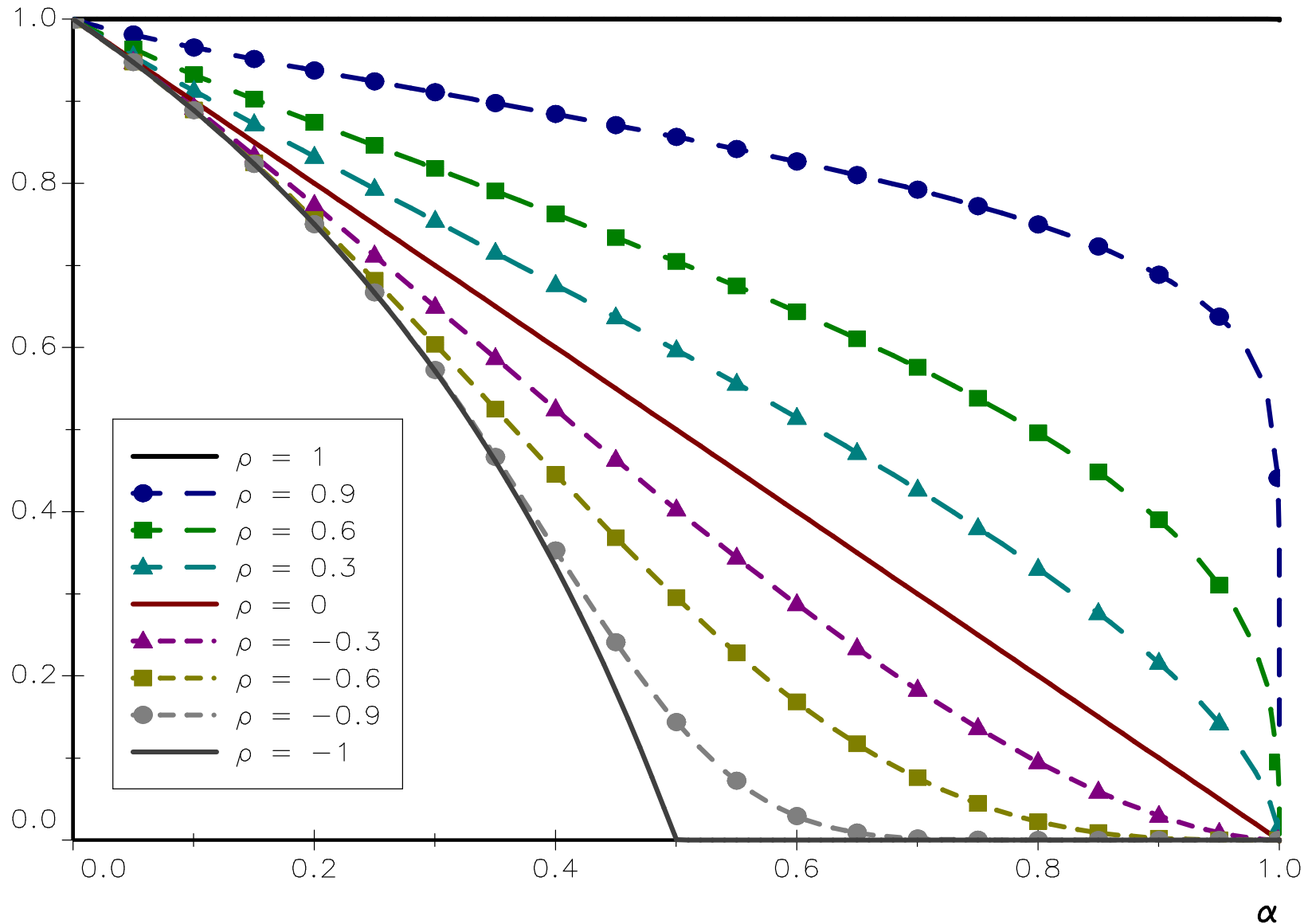
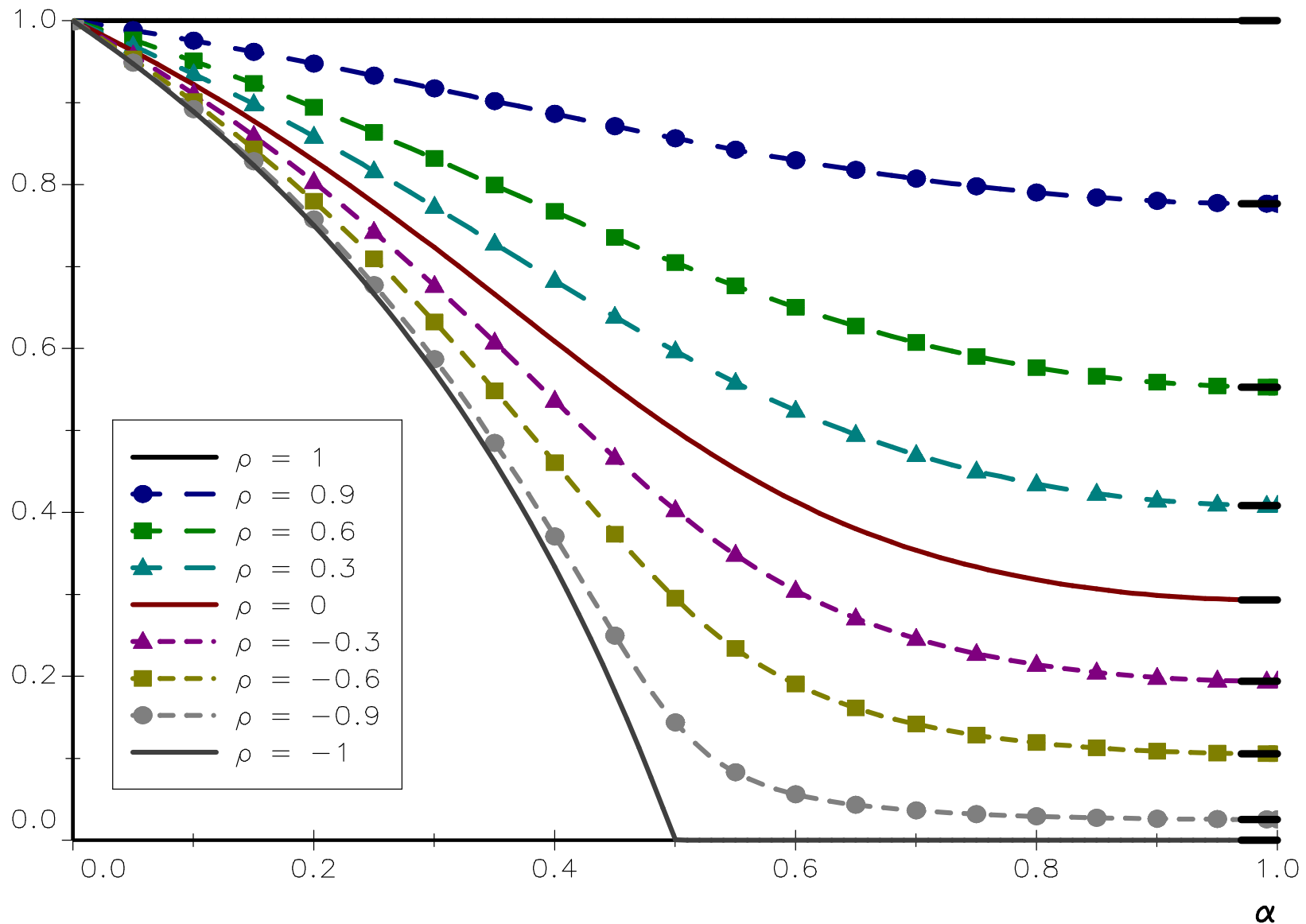


Figure: Quantile-quantile dependence measure for the  $t_1$  copula





**Table:** Calibration of the lifestyle fund profiles ( $T = 10$  years,  $\rho_{S,B} = 20\%$ )

Profile	$\alpha_C^*$	$\alpha_B^*$	$\alpha_S^*$	$\hat{\gamma}$
Safety	-0.03	99.88	0.15	867
Defensive	-3.02	85.52	17.50	7.20
Balanced	-8.55	59.07	49.48	2.55
Dynamic	-14.07	32.61	81.46	1.55

**Table:** Calibration of the lifestyle fund profiles ( $T = 10$  years,  $\rho_{S,B} = -20\%$ )

Profile	$\alpha_C^*$	$\alpha_B^*$	$\alpha_S^*$	$\hat{\gamma}$
Safety	-0.08	99.93	0.15	867
Defensive	-10.59	90.62	19.98	6.72
Balanced	-26.16	76.82	49.34	2.72
Dynamic	-41.72	63.03	78.70	1.71

Figure: Sensitivity of the equity allocation  $\alpha_S^*$  (in %) in lifestyle funds

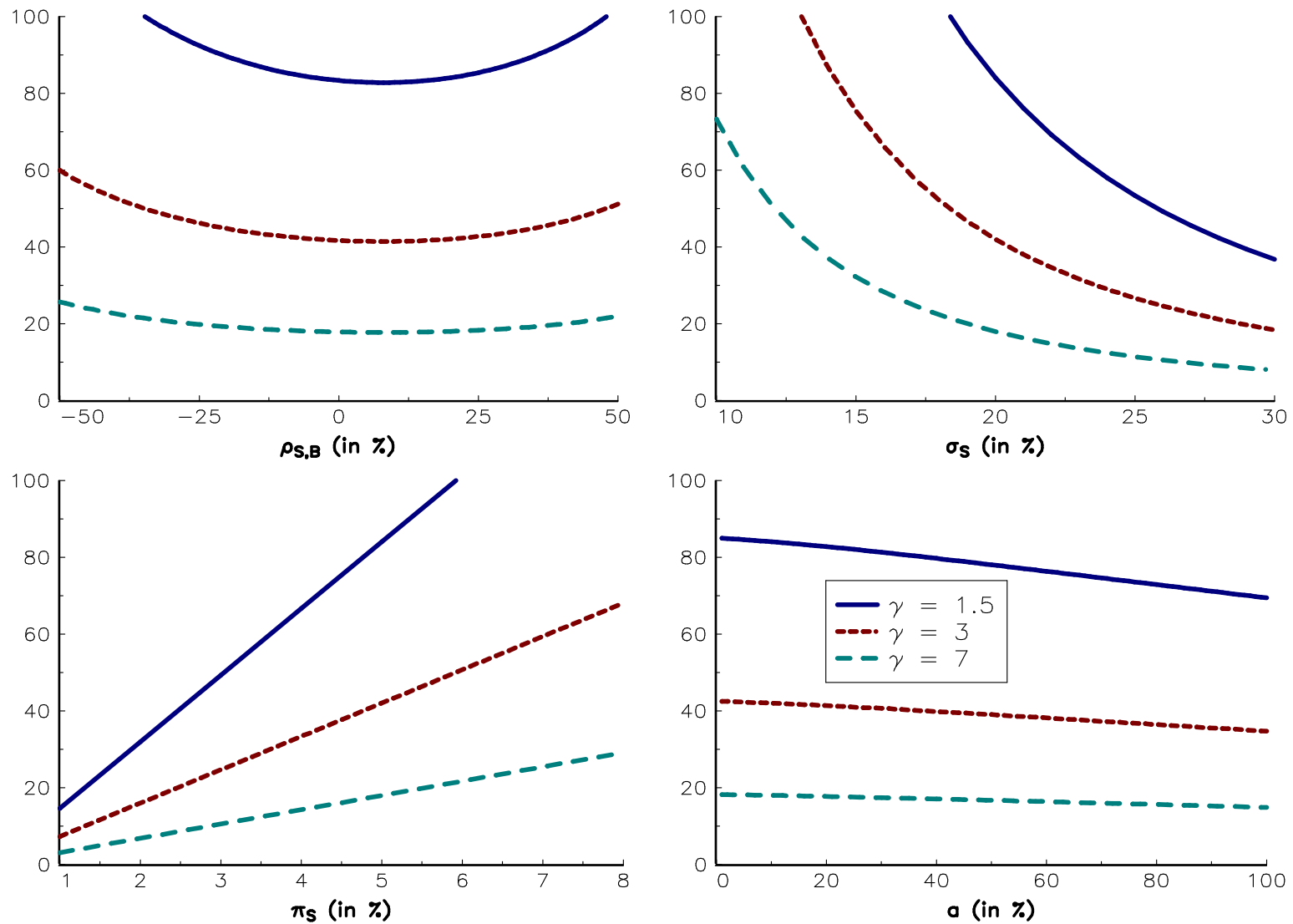


Figure: Influence of the parameters on the glide path of target-date funds

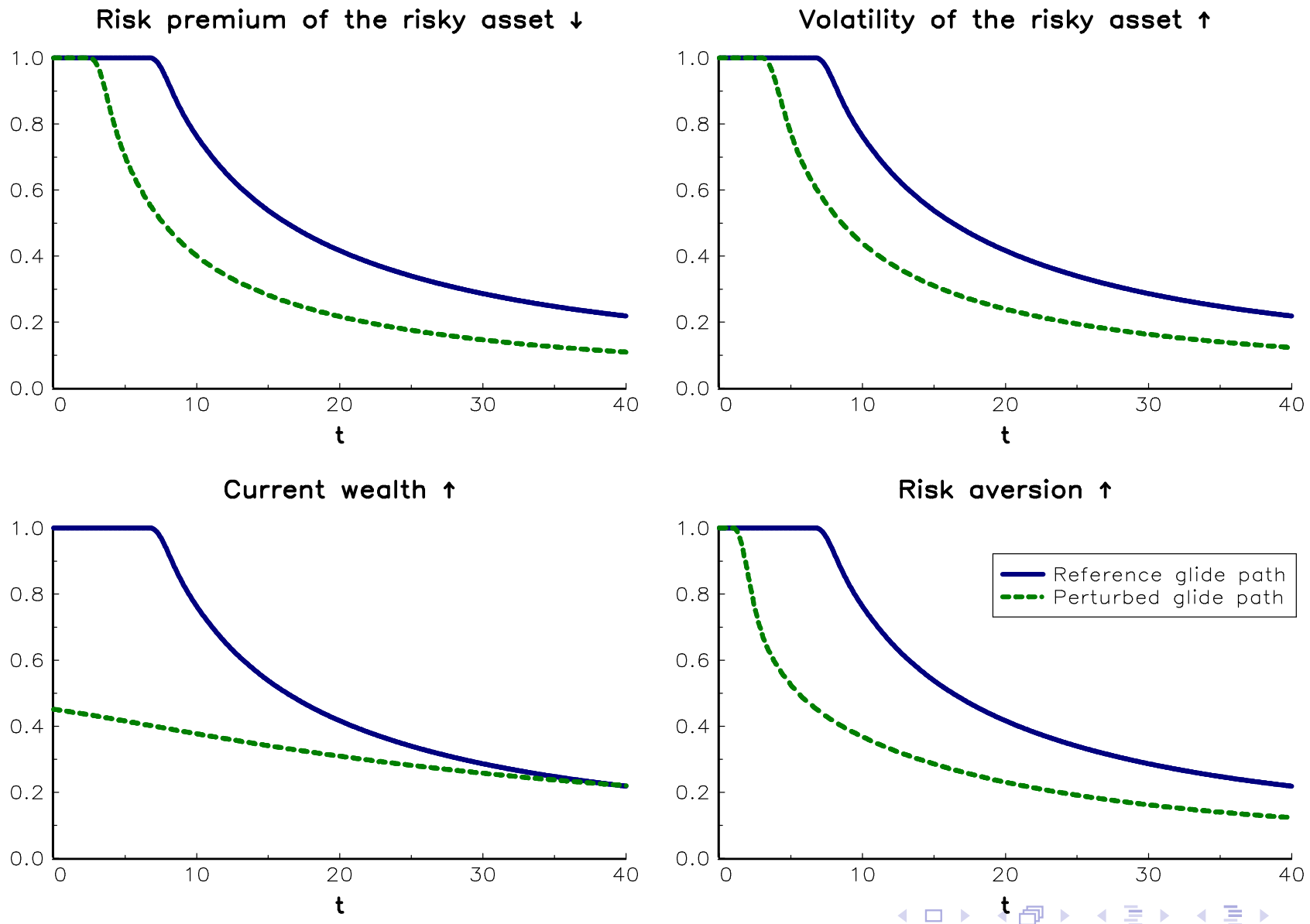


Figure: Example of the LDI utility function

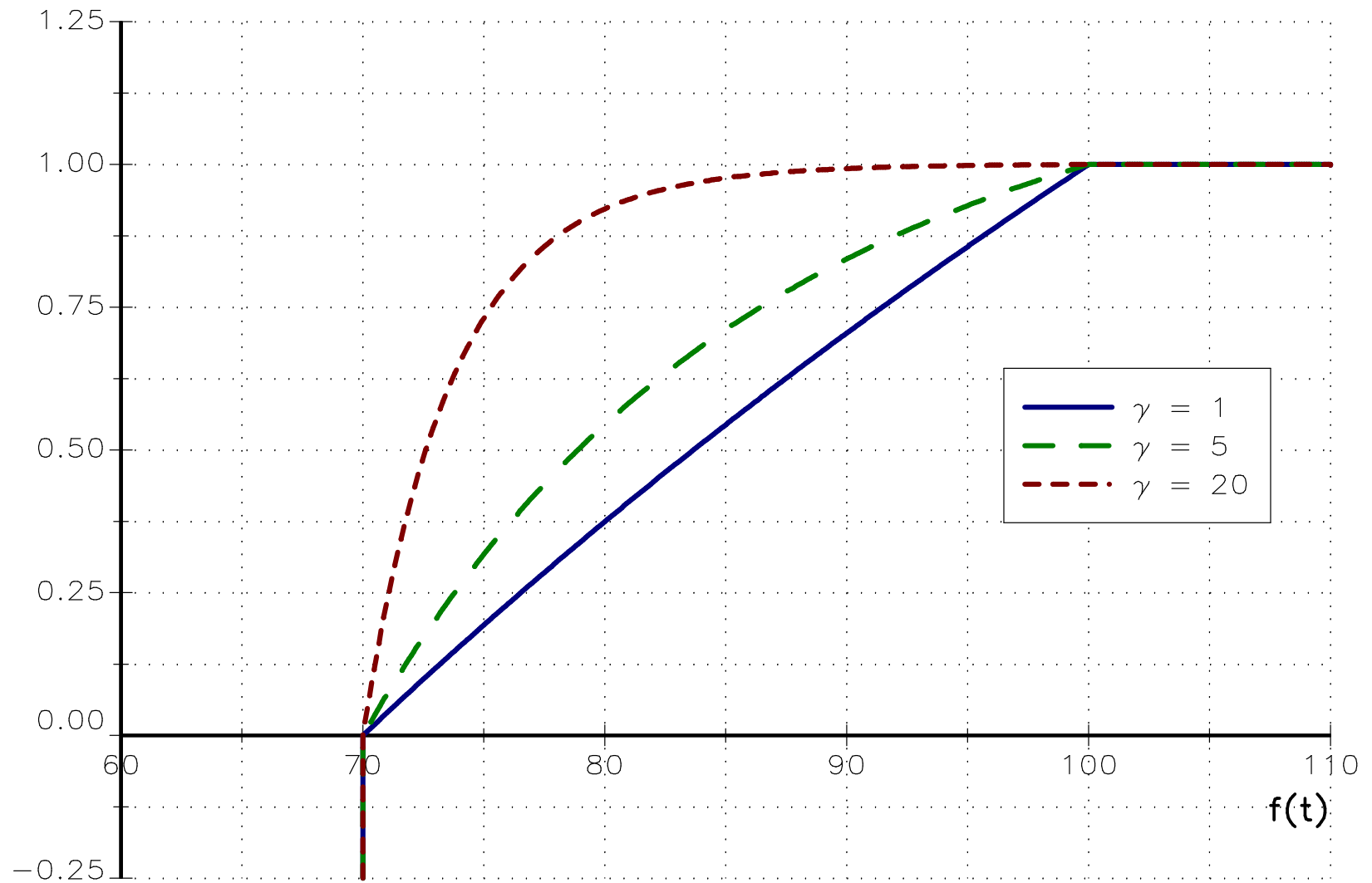


Figure: Optimal exposure  $\alpha^*(t)$  (in %) in the LDI portfolio

