

# Introduction to Risk Parity and Budgeting

## Chapter 2 – Risk Budgeting Approach

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<sup>†</sup>Evry University & Lyxor Asset Management, France

Instructors may find the description of the book at the following addresses:

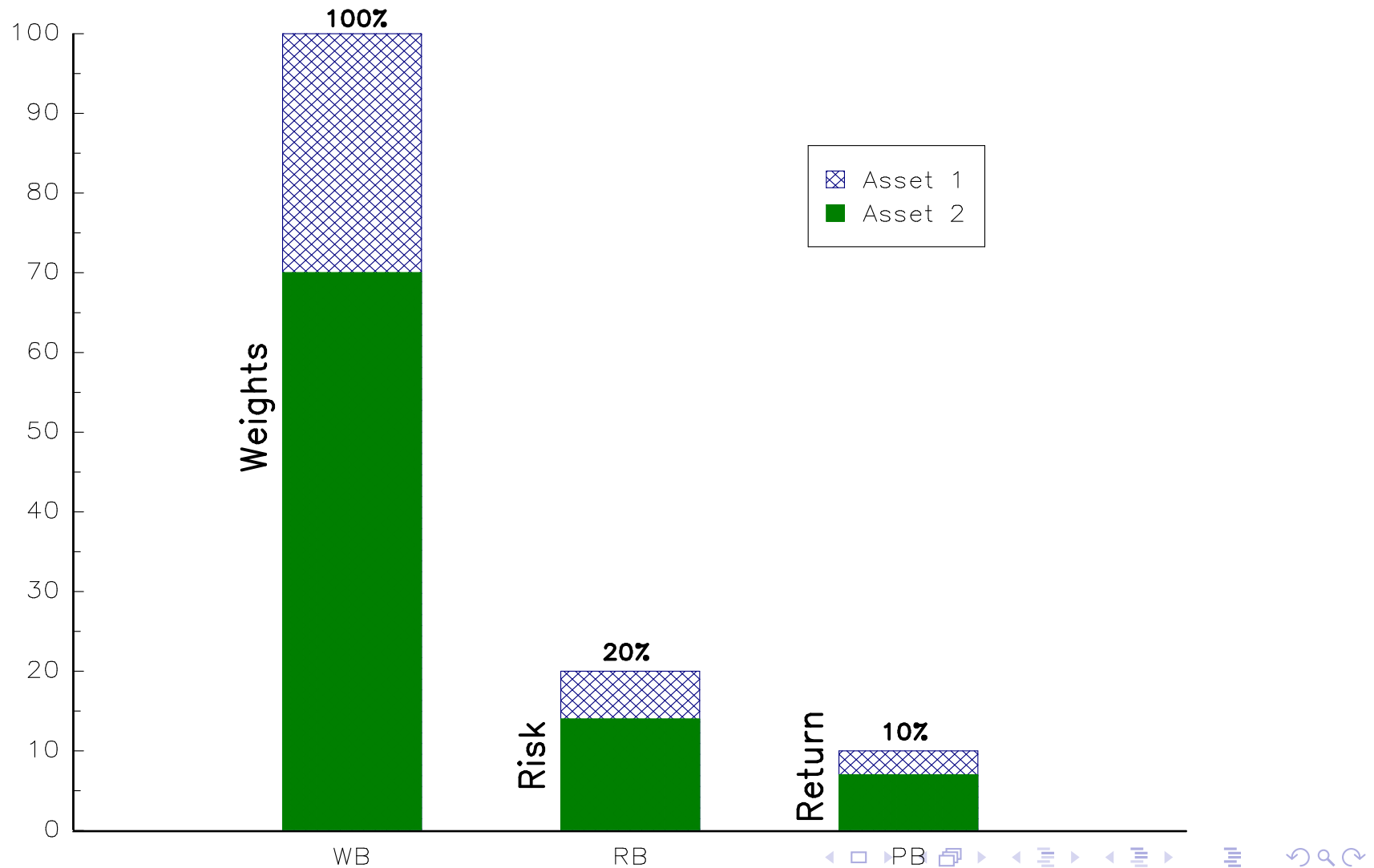
<http://www.crcpress.com/product/isbn/9781482207156>

<http://www.thierry-roncalli.com/RiskParityBook.html>

May 22, 2013

# Figure 2.1, Page 72

Figure: Three budgeting methods with a 30/70 policy rule



# Table 2.1, Page 76

Table: Computation of risk measures  $\text{VaR}_\alpha(x)$  and  $\text{ES}_\alpha(x)$

| Portfolio | $\mathcal{R}(x)$ |         | $\alpha$ |       |       |       |
|-----------|------------------|---------|----------|-------|-------|-------|
|           |                  |         | 90%      | 95%   | 99%   | 99.5% |
| #1        | VaR              | (in %)  | 1.52     | 2.06  | 3.06  | 3.43  |
|           |                  | (in \$) | 14.27    | 19.30 | 28.74 | 32.20 |
|           | ES               | (in %)  | 2.22     | 2.67  | 3.56  | 3.90  |
|           |                  | (in \$) | 20.83    | 25.09 | 33.44 | 36.58 |
| #2        | VaR              | (in %)  | 5.68     | 7.45  | 10.76 | 11.98 |
|           |                  | (in \$) | 14.94    | 19.59 | 28.31 | 31.50 |
|           | ES               | (in %)  | 7.98     | 9.48  | 12.41 | 13.52 |
|           |                  | (in \$) | 21.00    | 24.94 | 32.64 | 35.54 |

Table: Risk decomposition of the volatility

| Asset            | $x_i$ | $\mathcal{M}R_i$ | $\mathcal{R}C_i$ | $\mathcal{R}C_i^*$ |
|------------------|-------|------------------|------------------|--------------------|
| 1                | 50.00 | 29.40            | 14.70            | 70.43              |
| 2                | 20.00 | 16.63            | 3.33             | 15.93              |
| 3                | 30.00 | 9.49             | 2.85             | 13.64              |
| $\mathcal{R}(x)$ |       |                  | 20.87            |                    |

# Tables 2.3 & 2.4, Page 82

Table: Risk decomposition of the value-at-risk

| Asset            | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|------------------|-------|--------|--------|----------|
| 1                | 50.00 | 68.39  | 34.19  | 70.43    |
| 2                | 20.00 | 38.68  | 7.74   | 15.93    |
| 3                | 30.00 | 22.07  | 6.62   | 13.64    |
| $\mathcal{R}(x)$ |       |        | 48.55  |          |

Table: Risk decomposition of the expected shortfall

| Asset            | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|------------------|-------|--------|--------|----------|
| 1                | 50.00 | 78.35  | 39.17  | 70.43    |
| 2                | 20.00 | 44.31  | 8.86   | 15.93    |
| 3                | 30.00 | 25.29  | 7.59   | 13.64    |
| $\mathcal{R}(x)$ |       |        | 55.62  |          |

# Tables 2.5 & 2.6, Page 84

**Table:** Sensitivity analysis of the volatility with respect to the factor  $h$

| Asset | 1 bp    | 10 bp   | 1%      | 10%     | 50%     | $-x_i$  |
|-------|---------|---------|---------|---------|---------|---------|
| 1     | 20.8728 | 20.8992 | 21.1639 | 23.8170 | 35.6938 | 6.8593  |
| 2     | 20.8715 | 20.8865 | 21.0364 | 22.5599 | 29.7077 | 17.6847 |
| 3     | 20.8708 | 20.8793 | 20.9650 | 21.8495 | 26.2640 | 18.3576 |

**Table:** Marginal analysis of the volatility with respect to the factor  $h$

| Asset | 1 bp    | 10 bp   | 1%      | 10%     | 50%     | $-x_i$  |
|-------|---------|---------|---------|---------|---------|---------|
| 1     | 20.8728 | 20.8992 | 21.1638 | 23.8095 | 35.5681 | 6.1716  |
| 2     | 20.8715 | 20.8865 | 21.0361 | 22.5325 | 29.1833 | 17.5445 |
| 3     | 20.8708 | 20.8793 | 20.9647 | 21.8186 | 25.6135 | 18.0236 |

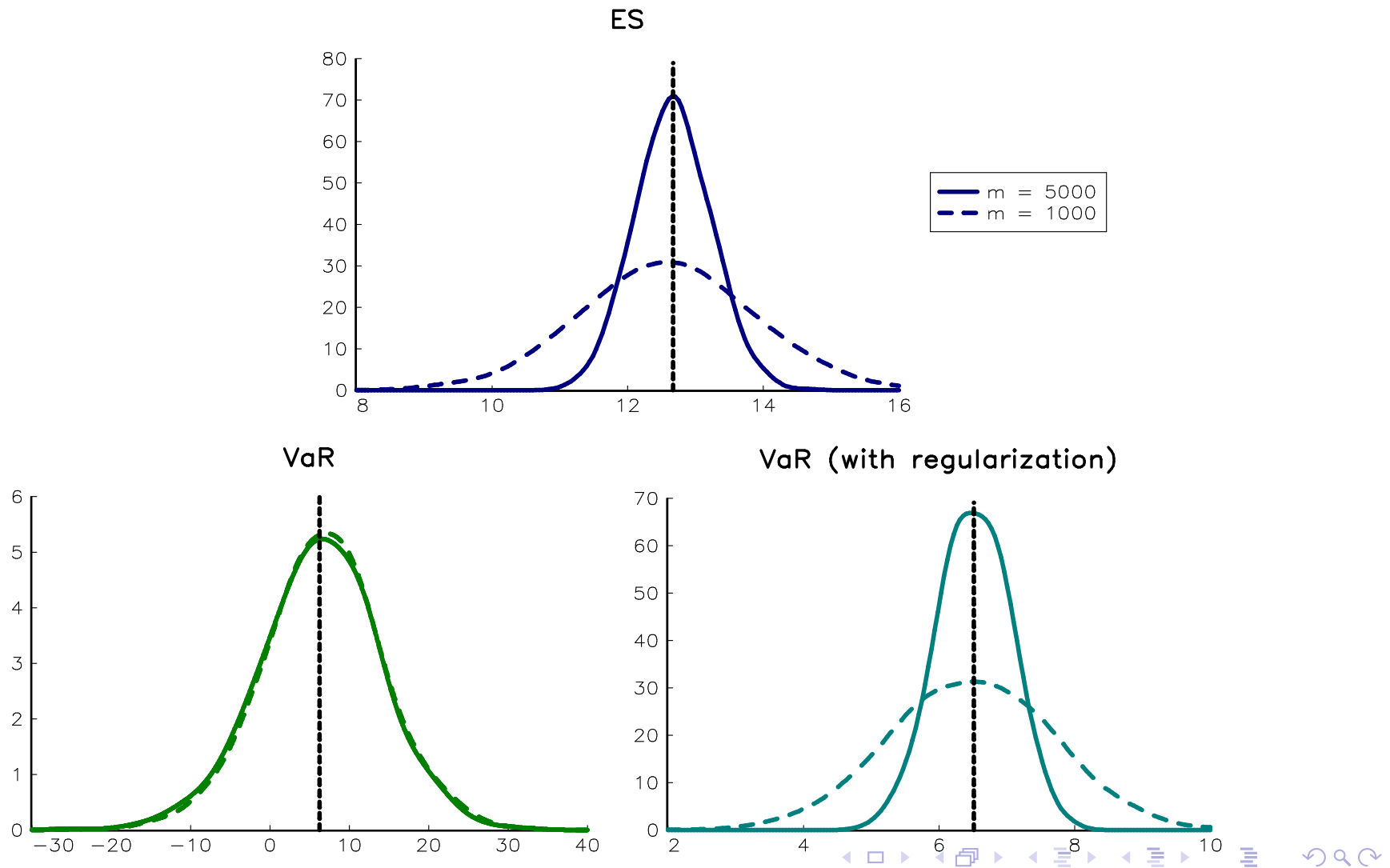
# Tables, Pages 98 & 99

|             |     |     |     |     |    |     |    |    |    |    |
|-------------|-----|-----|-----|-----|----|-----|----|----|----|----|
| $j$         | 1   | 2   | 3   | 4   | 5  | 6   | 7  | 8  | 9  | 10 |
| $L_1^{(j)}$ | 14  | 3   | -4  | 5   | 6  | 8   | 12 | 25 | 23 | -9 |
| $L_2^{(j)}$ | 10  | -3  | 8   | 7   | 2  | 17  | 14 | 22 | -8 | -2 |
| $j$         | 11  | 12  | 13  | 14  | 15 | 16  | 17 | 18 | 19 | 20 |
| $L_1^{(j)}$ | -50 | -17 | 18  | -9  | -6 | -2  | 0  | 17 | 19 | 1  |
| $L_2^{(j)}$ | -10 | 12  | -12 | -19 | 25 | -10 | 4  | 12 | 36 | -5 |

|               |     |     |     |    |     |    |    |    |    |     |
|---------------|-----|-----|-----|----|-----|----|----|----|----|-----|
| $j$           | 1   | 2   | 3   | 4  | 5   | 6  | 7  | 8  | 9  | 10  |
| $L_1^{(j:m)}$ | -50 | -9  | -2  | -9 | -17 | 1  | 3  | -4 | 0  | 18  |
| $L_2^{(j:m)}$ | -10 | -19 | -10 | -2 | 12  | -5 | -3 | 8  | 4  | -12 |
| $j$           | 11  | 12  | 13  | 14 | 15  | 16 | 17 | 18 | 19 | 20  |
| $L_1^{(j:m)}$ | 6   | 5   | 23  | -6 | 14  | 8  | 12 | 17 | 25 | 19  |
| $L_2^{(j:m)}$ | 2   | 7   | -8  | 25 | 10  | 17 | 14 | 12 | 22 | 36  |

# Figure 2.2, Page 90

Figure: Density of the risk contribution estimator  $RC_1$





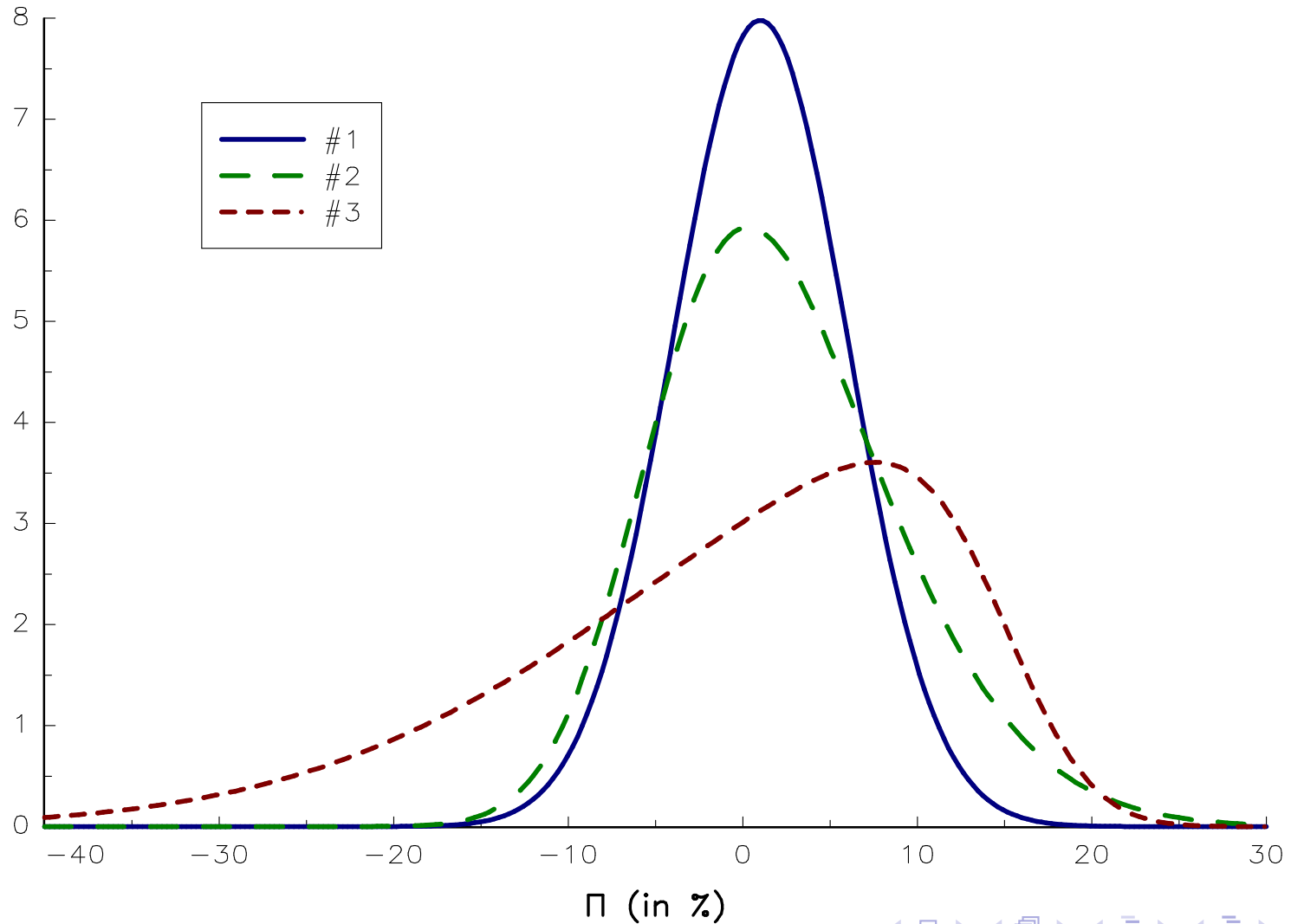
# Table 2.7, Page 96

Table: Value-at-risk (in %) when the P&L is skew normal distributed

| $\alpha$       |    | 80%   | 85%   | 90%   | 95%   | 99%   |
|----------------|----|-------|-------|-------|-------|-------|
| Normal         | #1 | 3.21  | 4.18  | 5.41  | 7.22  | 10.63 |
|                | #2 | 3.76  | 5.12  | 6.84  | 9.39  | 14.16 |
|                | #3 | 11.13 | 13.56 | 16.61 | 21.14 | 29.62 |
| Cornish-Fisher | #1 | 3.21  | 4.18  | 5.41  | 7.22  | 10.63 |
|                | #2 | 3.80  | 4.94  | 6.34  | 8.34  | 11.95 |
|                | #3 | 10.63 | 13.79 | 17.90 | 24.20 | 36.52 |
| Skew normal    | #1 | 3.21  | 4.18  | 5.41  | 7.22  | 10.63 |
|                | #2 | 3.86  | 5.03  | 6.43  | 8.41  | 11.78 |
|                | #3 | 10.67 | 13.70 | 17.66 | 23.80 | 36.08 |

# Figure 2.3, Page 97

Figure: Density of the P&L with a skew normal distribution



# Table 2.8, Page 98

Table: Statistics (in %) to compute the Cornish-Fisher risk contributions

| Portfolio                           | #1     | #2     | #3     | #4      | #5     | #6     |
|-------------------------------------|--------|--------|--------|---------|--------|--------|
| $x_1$                               | 0.00   | 10.00  | 25.00  | 50.00   | 75.00  | 100.00 |
| $x_2$                               | 100.00 | 90.00  | 75.00  | 50.00   | 25.00  | 0.00   |
| $\mathbb{E}[L]$                     | -0.10  | -0.11  | -0.13  | -0.16   | -0.19  | -0.23  |
| $\sigma^2(L)$                       | 0.01   | 0.01   | 0.01   | 0.02    | 0.04   | 0.07   |
| $\gamma_1(L)$                       | 2.67   | 7.73   | 8.42   | 19.20   | 29.18  | 35.13  |
| $\gamma_2(L)$                       | -13.05 | -21.50 | -16.08 | 58.12   | 103.97 | 124.21 |
| $\partial_{x_1} \gamma_1$           | 94.76  | 11.12  | 14.46  | 24.44   | 7.68   | 0.00   |
| $\partial_{x_2} \gamma_1$           | 0.00   | -1.24  | -4.82  | -24.44  | -23.05 | -18.14 |
| $\partial_{x_1} \gamma_2$           | -44.29 | -90.59 | 159.39 | 131.49  | 29.58  | 0.00   |
| $\partial_{x_2} \gamma_2$           | 0.00   | 10.07  | -53.13 | -131.49 | -88.74 | -53.28 |
| $z$                                 | 231.52 | 233.07 | 234.80 | 258.95  | 275.19 | 282.86 |
| $\partial_{x_1} z$                  | 57.41  | -13.65 | 46.98  | 45.18   | 10.88  | 0.00   |
| $\partial_{x_2} z$                  | 0.00   | 1.52   | -15.66 | -45.18  | -32.63 | -21.00 |
| $\text{VaR}_\alpha(L)$              | 1.92   | 1.90   | 2.19   | 3.59    | 5.39   | 7.28   |
| $\mathcal{R}\mathcal{C}_1$          | 0.00   | 0.23   | 1.23   | 3.49    | 5.44   | 7.28   |
| $\mathcal{R}\mathcal{C}_2$          | 1.92   | 1.67   | 0.96   | 0.10    | -0.05  | 0.00   |
| $\overline{\text{VaR}}_\alpha^*(L)$ | 1.93   | 1.89   | 2.17   | 3.21    | 4.53   | 5.94   |

# Tables 2.9 & 2.10, Pages 99 & 100

**Table:** Risk budgeting portfolio when the risk measure is the expected shortfall ( $\alpha = 95\%$ )

| Asset | $x_i$     | $w_i$  | $MR_i$ | $RC_i$  | $RC_i^*$ |
|-------|-----------|--------|--------|---------|----------|
| 1     | 534 430   | 28.21% | 46.78% | 250 000 | 50.00%   |
| 2     | 372 705   | 19.68% | 26.83% | 100 000 | 20.00%   |
| 3     | 987 007   | 52.11% | 15.20% | 150 000 | 30.00%   |
| sum   | 1 894 142 |        |        | 500 000 |          |

**Table:** Risk budgeting portfolio when the risk measure is the expected shortfall ( $\alpha = 99\%$ )

| Asset | $x_i$     | $w_i$  | $MR_i$ | $RC_i$  | $RC_i^*$ |
|-------|-----------|--------|--------|---------|----------|
| 1     | 391 926   | 29.00% | 63.79% | 250 000 | 50.00%   |
| 2     | 273 737   | 20.26% | 36.53% | 100 000 | 20.00%   |
| 3     | 685 779   | 50.74% | 21.87% | 150 000 | 30.00%   |
| sum   | 1 351 441 |        |        | 500 000 |          |

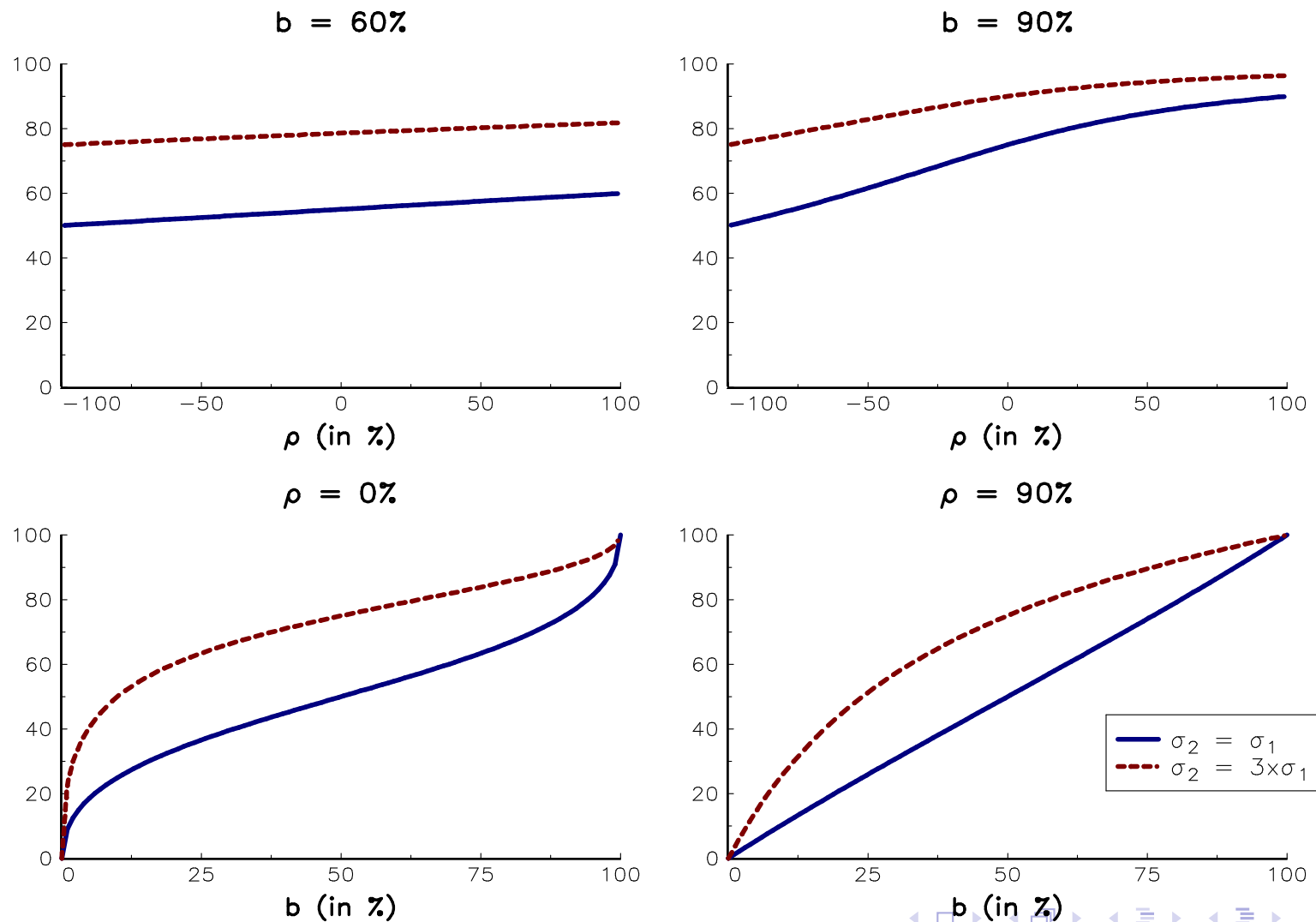
# Table 2.11, Page 103

Table: Weights  $w^*$  in the RB portfolio with respect to some values of  $b$  and  $\rho$

| $\rho/b$ | $\sigma_2 = \sigma_1$ |      |      |      | $\sigma_2 = 3 \times \sigma_1$ |      |      |      |
|----------|-----------------------|------|------|------|--------------------------------|------|------|------|
|          | 20%                   | 50%  | 70%  | 90%  | 20%                            | 50%  | 70%  | 90%  |
| -50%     | 41.9                  | 50.0 | 55.2 | 61.6 | 68.4                           | 75.0 | 78.7 | 82.8 |
| 0%       | 33.3                  | 50.0 | 60.4 | 75.0 | 60.0                           | 75.0 | 82.1 | 90.0 |
| 25%      | 29.3                  | 50.0 | 63.0 | 80.6 | 55.5                           | 75.0 | 83.6 | 92.6 |
| 50%      | 25.7                  | 50.0 | 65.5 | 84.9 | 51.0                           | 75.0 | 85.1 | 94.4 |
| 75%      | 22.6                  | 50.0 | 67.8 | 87.9 | 46.7                           | 75.0 | 86.3 | 95.6 |
| 90%      | 21.0                  | 50.0 | 69.1 | 89.2 | 44.4                           | 75.0 | 87.1 | 96.1 |

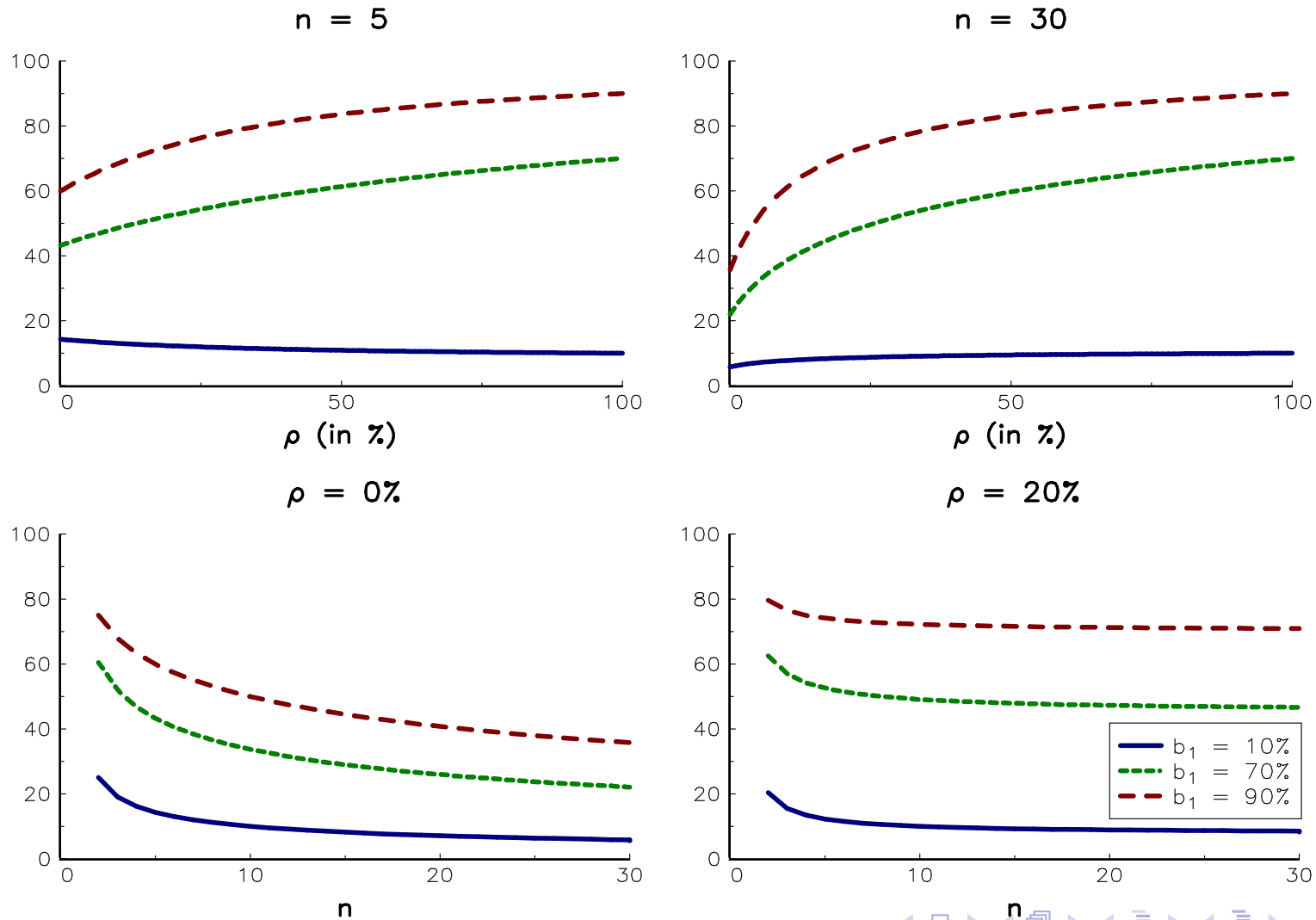
# Figure 2.4, Page 104

Figure: Evolution of the weight  $w^*$  in the RB portfolio with respect to  $b$  and  $\rho$



# Figure 2.5, Page 107

Figure: Simulation of the weight  $x_1$  when the correlation is constant



# Table 2.12, Page 112

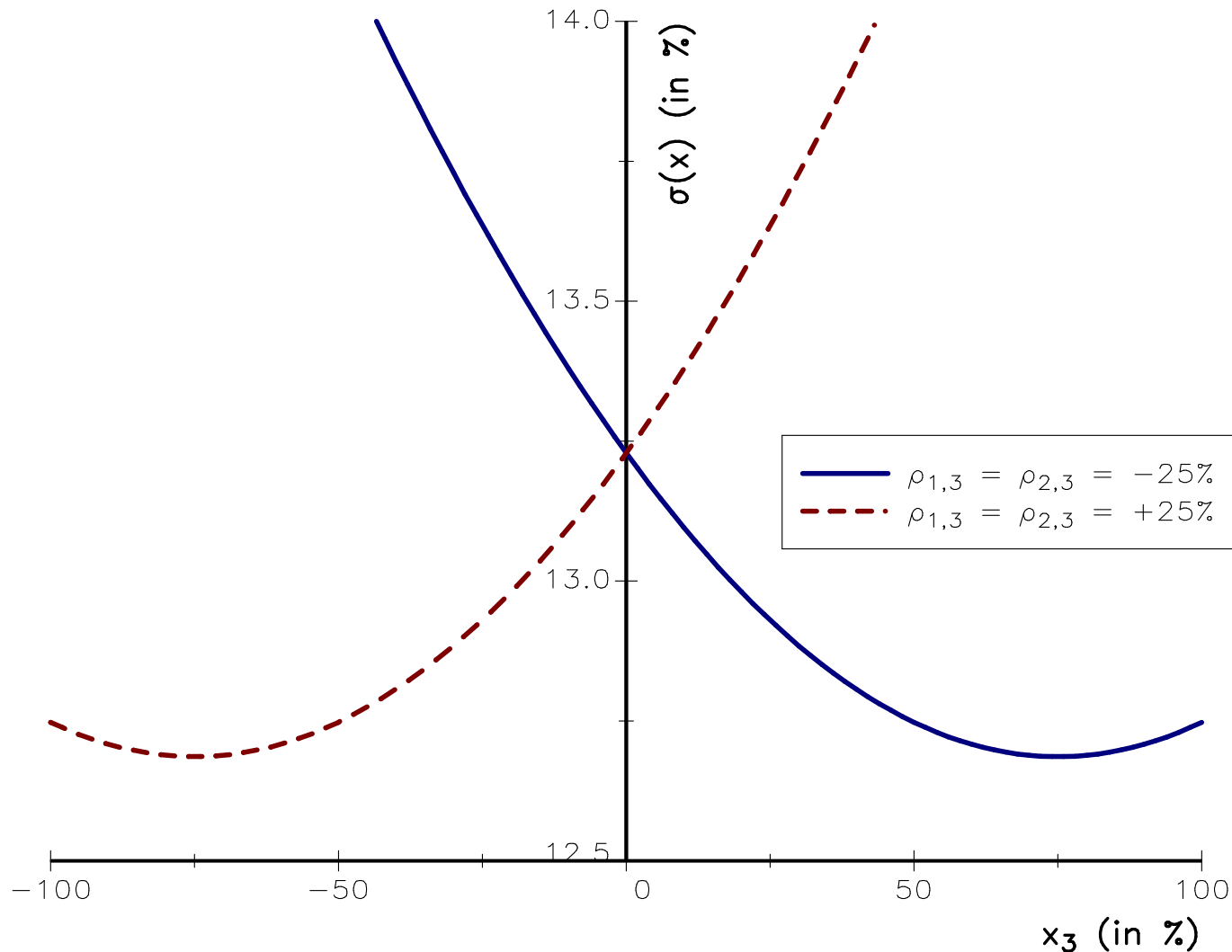
Table: RB solutions when the risk budget  $b_3$  is equal to 0

| $\rho_{1,3} = \rho_{2,3}$ | Solution                | 1      | 2      | 3      | $\sigma(x)$ |
|---------------------------|-------------------------|--------|--------|--------|-------------|
| -25%                      | $x_i$                   | 20.00% | 40.00% | 40.00% |             |
|                           | $\mathcal{S}_1$ $MR_i$  | 16.58% | 8.29%  | 0.00%  | 6.63%       |
|                           | $RC_i$                  | 50.00% | 50.00% | 0.00%  |             |
|                           | $x_i$                   | 33.33% | 66.67% | 0.00%  |             |
|                           | $\mathcal{S}_2$ $MR_i$  | 17.32% | 8.66%  | -1.44% | 11.55%      |
|                           | $RC_i$                  | 50.00% | 50.00% | 0.00%  |             |
| 25%                       | $x_i$                   | 19.23% | 38.46% | 42.31% |             |
|                           | $\mathcal{S}'_1$ $MR_i$ | 16.42% | 8.21%  | 0.15%  | 6.38%       |
|                           | $RC_i$                  | 49.50% | 49.50% | 1.00%  |             |
| 25%                       | $x_i$                   | 33.33% | 66.67% | 0.00%  |             |
|                           | $\mathcal{S}_1$ $MR_i$  | 17.32% | 8.66%  | 1.44%  | 11.55%      |
|                           | $RC_i$                  | 50.00% | 50.00% | 0.00%  |             |



# Figure 2.6, Page 112

Figure: Evolution of the portfolio's volatility with respect to  $x_3$



# Table 2.13, Page 113

Table: RB solutions when the risk budgets  $b_3$  and  $b_4$  are equal to 0

| Solution        |        | 1      | 2      | 3      | 4      | $\sigma(x)$ |
|-----------------|--------|--------|--------|--------|--------|-------------|
| $\mathcal{S}_1$ | $x_i$  | 20.00% | 40.00% | 26.67% | 13.33% | 6.53%       |
|                 | $MR_i$ | 16.33% | 8.16%  | 0.00%  | 0.00%  |             |
|                 | $RC_i$ | 50.00% | 50.00% | 0.00%  | 0.00%  |             |
| $\mathcal{S}_2$ | $x_i$  | 33.33% | 66.67% | 0.00%  | 0.00%  | 11.55%      |
|                 | $MR_i$ | 17.32% | 8.66%  | -1.44% | -2.89% |             |
|                 | $RC_i$ | 50.00% | 50.00% | 0.00%  | 0.00%  |             |
| $\mathcal{S}_3$ | $x_i$  | 20.00% | 40.00% | 40.00% | 0.00%  | 6.63%       |
|                 | $MR_i$ | 16.58% | 8.29%  | 0.00%  | -1.51% |             |
|                 | $RC_i$ | 50.00% | 50.00% | 0.00%  | 0.00%  |             |
| $\mathcal{S}_4$ | $x_i$  | 25.00% | 50.00% | 0.00%  | 25.00% | 8.29%       |
|                 | $MR_i$ | 16.58% | 8.29%  | -0.75% | 0.00%  |             |
|                 | $RC_i$ | 50.00% | 50.00% | 0.00%  | 0.00%  |             |

# Tables 2.14 & 2.15, Page 116

Table: Implied risk premia when  $b = (20\%, 25\%, 40\%, 15\%)$

| Asset           | $x_i$ | $\mathcal{M}R_i$ | $\tilde{\mu}_i$ | $\mathcal{P}C_i$ | $\mathcal{P}C_i^*$ |
|-----------------|-------|------------------|-----------------|------------------|--------------------|
| 1               | 40.91 | 7.10             | 3.55            | 1.45             | 20.00              |
| 2               | 25.12 | 14.46            | 7.23            | 1.82             | 25.00              |
| 3               | 25.26 | 23.01            | 11.50           | 2.91             | 40.00              |
| 4               | 8.71  | 25.04            | 12.52           | 1.09             | 15.00              |
| Expected return |       |                  |                 | 7.27             |                    |

Table: Implied risk premia when  $b = (10\%, 10\%, 10\%, 70\%)$

| Asset           | $x_i$ | $\mathcal{M}R_i$ | $\tilde{\mu}_i$ | $\mathcal{P}C_i$ | $\mathcal{P}C_i^*$ |
|-----------------|-------|------------------|-----------------|------------------|--------------------|
| 1               | 35.88 | 5.27             | 2.63            | 0.94             | 10.00              |
| 2               | 17.94 | 10.53            | 5.27            | 0.94             | 10.00              |
| 3               | 10.18 | 18.56            | 9.28            | 0.94             | 10.00              |
| 4               | 35.99 | 36.75            | 18.37           | 6.61             | 70.00              |
| Expected return |       |                  |                 | 9.45             |                    |

# Tables 2.16 & 2.17, Page 117

Table: Sensitivity of the MVO portfolio to input parameters

| $\rho$     |      | 70%  | 90%  |      | 90%  |      |
|------------|------|------|------|------|------|------|
| $\sigma_2$ |      |      |      | 18%  | 18%  |      |
| $\mu_1$    |      |      |      |      |      | 9%   |
| $x_1$      | 38.3 | 38.3 | 44.6 | 13.7 | 0.0  | 56.4 |
| $x_2$      | 20.2 | 25.9 | 8.9  | 56.1 | 65.8 | 0.0  |
| $x_3$      | 41.5 | 35.8 | 46.5 | 30.2 | 34.2 | 43.6 |

Table: Sensitivity of the RB portfolio to input parameters

| $\rho$     |      | 70%  | 90%  |      | 90%  |      |
|------------|------|------|------|------|------|------|
| $\sigma_2$ |      |      |      | 18%  | 18%  |      |
| $\mu_1$    |      |      |      |      |      | 9%   |
| $x_1$      | 38.3 | 37.7 | 38.9 | 37.1 | 37.7 | 38.3 |
| $x_2$      | 20.2 | 20.4 | 20.0 | 22.8 | 22.6 | 20.2 |
| $x_3$      | 41.5 | 41.9 | 41.1 | 40.1 | 39.7 | 41.5 |

# Tables 2.18 & 2.19, Pages 118 & 119

Table: Shrinkage covariance matrix  $\tilde{\Sigma}^{(1)}$  associated to the RB portfolio

| Asset | $\tilde{\sigma}_i$ | $\tilde{\rho}_{i,j}$ |         |        |
|-------|--------------------|----------------------|---------|--------|
| 1     | 19.13%             | 100.00%              |         |        |
| 2     | 18.92%             | 82.54%               | 100.00% |        |
| 3     | 22.93%             | 57.69%               | 68.08%  | 100.00 |

Table: Shrinkage covariance matrix  $\tilde{\Sigma}^{(3)}$  associated to the RB portfolio

| Asset | $\tilde{\sigma}_i$ | $\tilde{\rho}_{i,j}$ |         |        |
|-------|--------------------|----------------------|---------|--------|
| 1     | 18.26%             | 100.00%              |         |        |
| 2     | 17.93%             | 67.67%               | 100.00% |        |
| 3     | 24.40%             | 33.25%               | 49.39%  | 100.00 |

# Tables 2.20 & 2.21, Pages 121 & 123

Table: Risk contributions of EW, ERC and MV portfolios

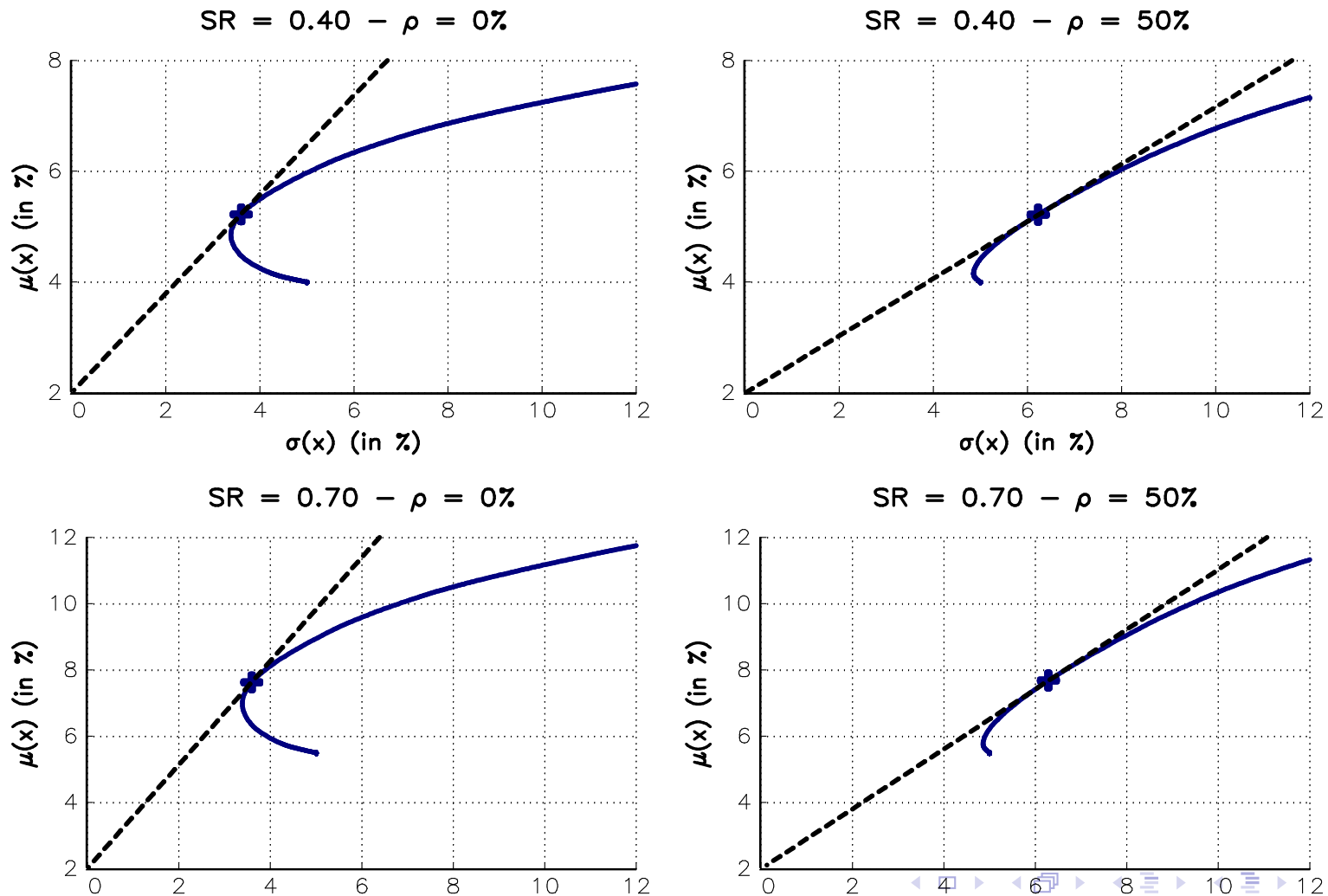
| Portfolio | Asset | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|-----------|-------|-------|--------|--------|----------|
| EW        | 1     | 50.00 | 16.06  | 8.03   | 36.84    |
|           | 2     | 50.00 | 27.53  | 13.76  | 63.16    |
| ERC       | 1     | 60.00 | 17.32  | 10.39  | 50.00    |
|           | 2     | 40.00 | 25.98  | 10.39  | 50.00    |
| MV        | 1     | 85.71 | 19.64  | 16.83  | 85.71    |
|           | 2     | 14.29 | 19.64  | 2.81   | 14.29    |

Table: Composition of the ERC portfolio

| Asset      | $x_i$ | $MR_i$ | $\beta_i$ | $RC_i$ | $RC_i^*$ |
|------------|-------|--------|-----------|--------|----------|
| 1          | 31.34 | 8.52   | 0.80      | 2.67   | 25.00    |
| 2          | 17.49 | 15.27  | 1.43      | 2.67   | 25.00    |
| 3          | 13.05 | 20.46  | 1.92      | 2.67   | 25.00    |
| 4          | 38.12 | 7.00   | 0.66      | 2.67   | 25.00    |
| Volatility |       |        |           | 10.68  |          |

# Figure 2.7, Page 124

Figure: Location of the ERC portfolio in the mean-variance diagram when the Sharpe ratios are the same and the asset correlations are uniform



# Figure 2.8, Page 125

Figure: Location of the ERC portfolio in the mean-variance diagram when the Sharpe ratios are identical and the asset correlations are not uniform

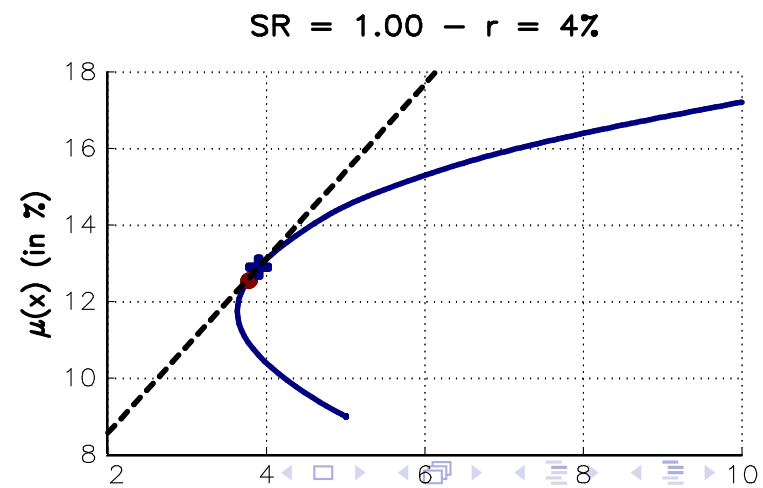
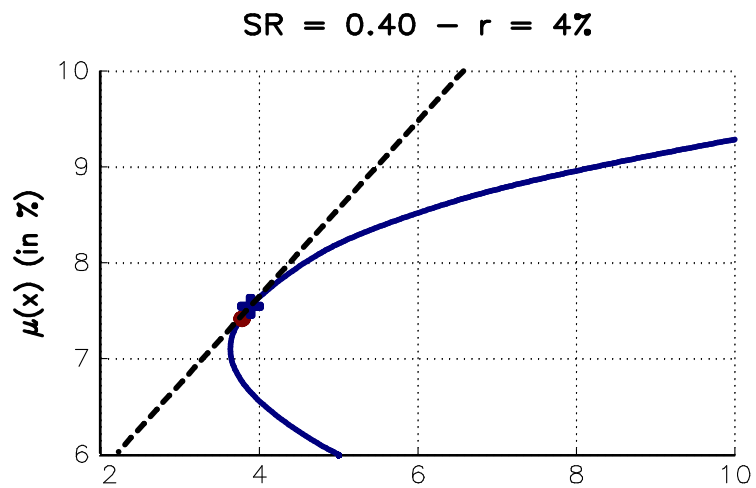
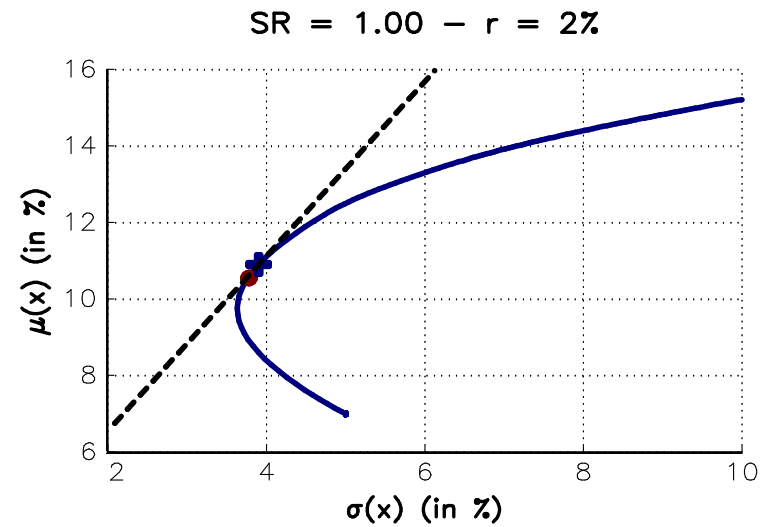
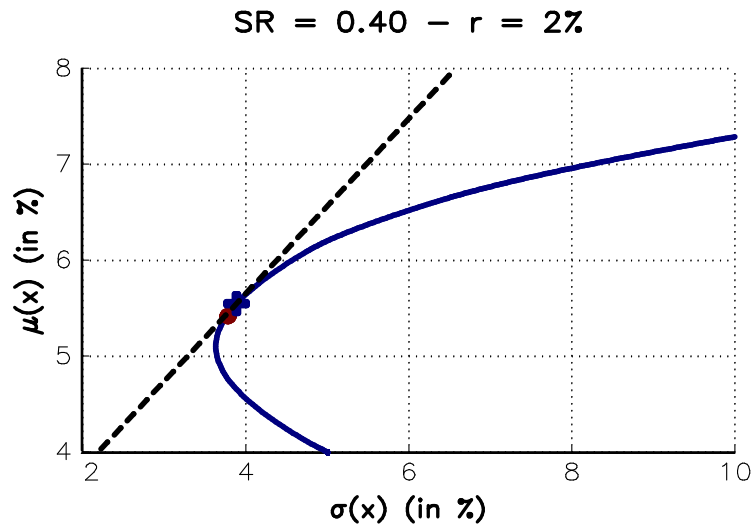
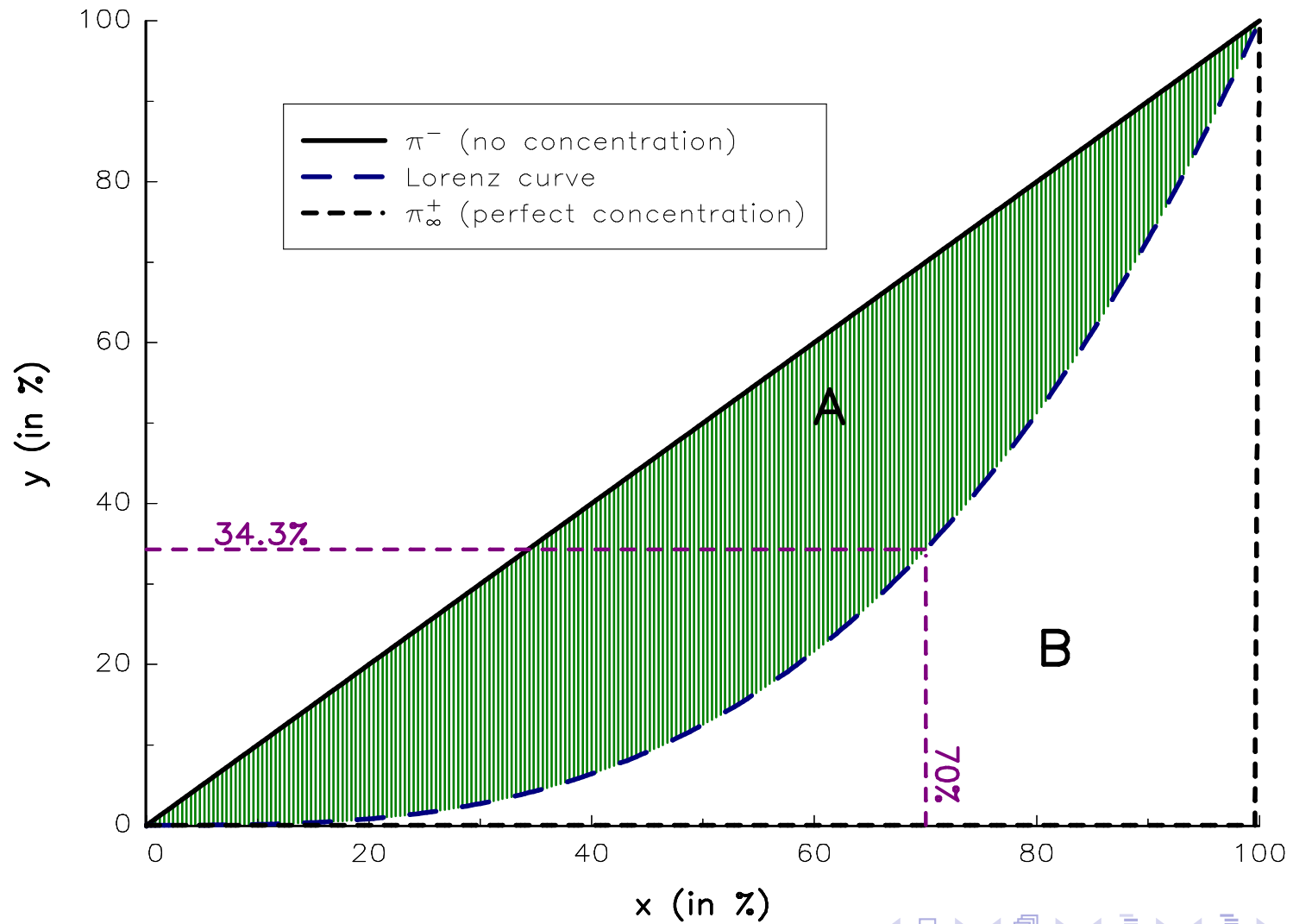




Figure: Geometry of the Lorenz curve



# Table 2.22, Page 129

Table: Diversification measures of MV, ERC, MDP and EW portfolios

| Asset            | MV    |          | ERC   |          | MDP   |          | EW    |          |
|------------------|-------|----------|-------|----------|-------|----------|-------|----------|
|                  | $x_i$ | $RC_i^*$ | $x_i$ | $RC_i^*$ | $x_i$ | $RC_i^*$ | $x_i$ | $RC_i^*$ |
| 1                | 0.00  | 0.00     | 15.70 | 16.67    | 0.00  | 0.00     | 16.67 | 16.18    |
| 2                | 3.61  | 3.61     | 17.84 | 16.67    | 0.00  | 0.00     | 16.67 | 14.08    |
| 3                | 96.39 | 96.39    | 28.03 | 16.67    | 0.00  | 0.00     | 16.67 | 8.68     |
| 4                | 0.00  | 0.00     | 13.08 | 16.67    | 0.00  | 0.00     | 16.67 | 19.78    |
| 5                | 0.00  | 0.00     | 10.86 | 16.67    | 42.86 | 50.00    | 16.67 | 24.43    |
| 6                | 0.00  | 0.00     | 14.49 | 16.67    | 57.14 | 50.00    | 16.67 | 16.86    |
| $\sigma(x)$      | 13.99 |          | 19.53 |          | 26.56 |          | 21.39 |          |
| $\mathcal{D}(x)$ | 0.98  |          | 0.80  |          | 0.77  |          | 0.80  |          |
| $\mathcal{H}^*$  | 0.92  | 0.92     | 0.02  | 0.00     | 0.41  | 0.40     | 0.00  | 0.02     |
| $\mathcal{G}$    | 0.82  | 0.82     | 0.17  | 0.00     | 0.69  | 0.67     | 0.00  | 0.16     |
| $\mathcal{I}^*$  | 1.17  | 1.17     | 5.71  | 6.00     | 1.98  | 2.00     | 6.00  | 5.74     |

# Table 2.23, Page 132

Table: Risk decomposition of WB, RB and MR portfolios

| Portfolio          | Asset | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|--------------------|-------|-------|--------|--------|----------|
| WB                 | 1     | 50.00 | 41.59  | 20.80  | 71.40    |
|                    | 2     | 20.00 | 24.18  | 4.84   | 16.60    |
|                    | 3     | 30.00 | 11.65  | 3.50   | 12.00    |
| Expected shortfall |       |       |        | 29.13  |          |
| RB                 | 1     | 30.65 | 39.07  | 11.97  | 50.00    |
|                    | 2     | 21.04 | 22.76  | 4.79   | 20.00    |
|                    | 3     | 48.32 | 14.87  | 7.18   | 30.00    |
| Expected shortfall |       |       |        | 23.94  |          |
| MR                 | 1     | 0.00  | 29.11  | 0.00   | 0.00     |
|                    | 2     | 30.34 | 18.81  | 5.71   | 30.34    |
|                    | 3     | 69.66 | 18.81  | 13.10  | 69.66    |
| Expected shortfall |       |       |        | 18.81  |          |

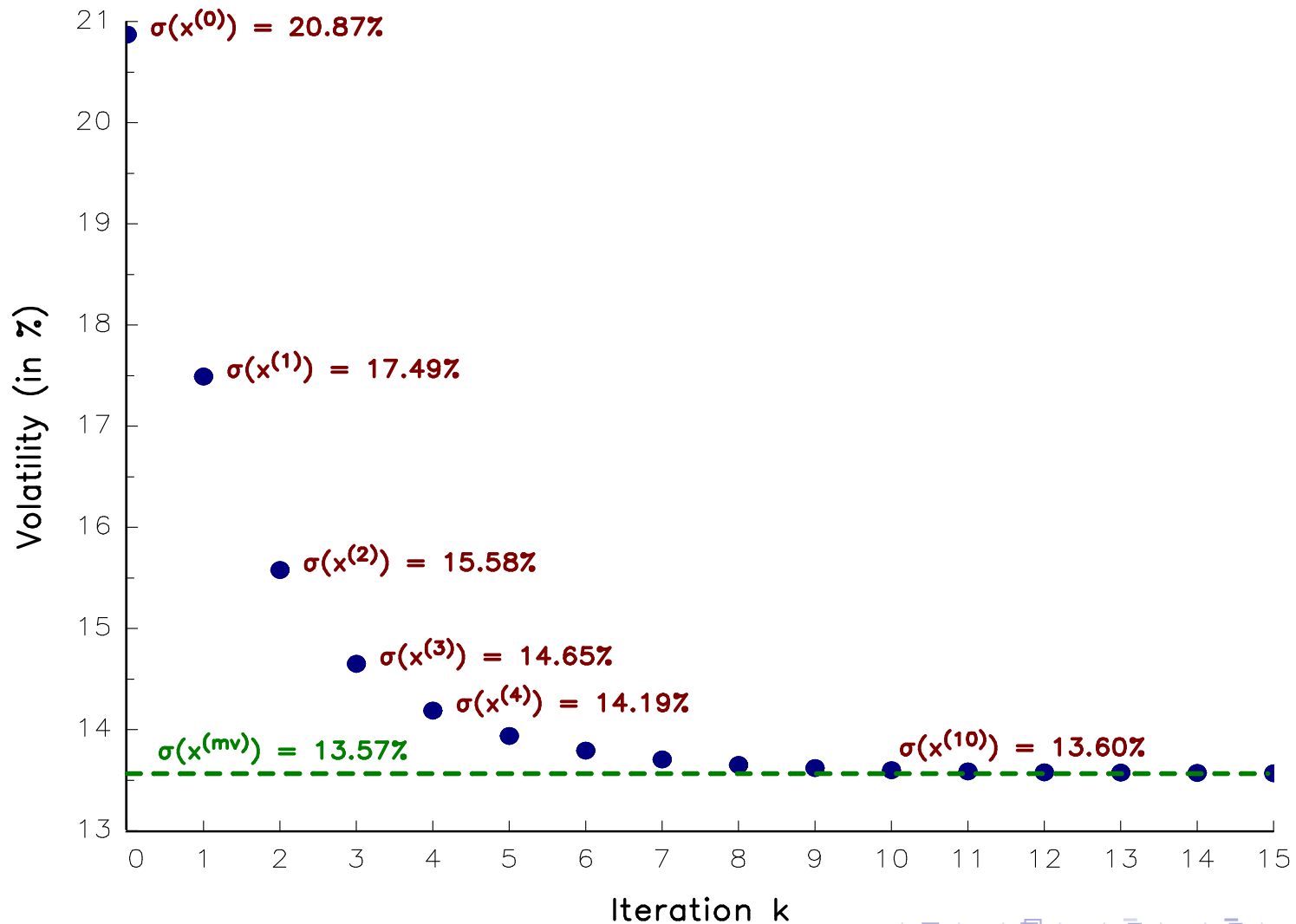
# Table 2.24, Page 134

Table: Weights and risk contributions of the iterative RB portfolio  $x^{(k)}$

| Portfolio | Asset      | $x_i$ | $RC_i^*$ | Portfolio | Asset      | $x_i$ | $RC_i^*$ |
|-----------|------------|-------|----------|-----------|------------|-------|----------|
| $x^{(0)}$ | 1          | 50.00 | 70.43    | $x^{(1)}$ | 1          | 31.15 | 50.00    |
|           | 2          | 20.00 | 15.93    |           | 2          | 21.90 | 20.00    |
|           | 3          | 30.00 | 13.64    |           | 3          | 46.96 | 30.00    |
|           | Volatility |       | 20.87    |           | Volatility |       | 17.49    |
| $x^{(2)}$ | 1          | 18.52 | 31.15    | $x^{(3)}$ | 1          | 11.04 | 18.52    |
|           | 2          | 22.81 | 21.90    |           | 2          | 23.71 | 22.81    |
|           | 3          | 58.67 | 46.96    |           | 3          | 65.25 | 58.67    |
|           | Volatility |       | 15.58    |           | Volatility |       | 14.65    |
| $x^{(4)}$ | 1          | 6.67  | 11.04    | $x^{(5)}$ | 1          | 4.07  | 6.67     |
|           | 2          | 24.76 | 23.71    |           | 2          | 25.86 | 24.76    |
|           | 3          | 68.57 | 65.25    |           | 3          | 70.07 | 68.57    |
|           | Volatility |       | 14.19    |           | Volatility |       | 13.94    |
| $x^{(6)}$ | 1          | 2.49  | 4.07     | $x_{mv}$  | 1          | 0.00  | 0.00     |
|           | 2          | 26.87 | 25.86    |           | 2          | 30.34 | 30.34    |
|           | 3          | 70.63 | 70.07    |           | 3          | 69.66 | 69.66    |
|           | Volatility |       | 13.79    |           | Volatility |       | 13.57    |

# Figure 2.10, Page 134

Figure: Convergence of the iterative RB portfolio  $x^{(k)}$  to the MV portfolio



# Tables 2.25 & 2.26, Page 139

Table: Risk decomposition of Portfolio #1 with respect to the synthetic assets

| Asset $i$       | $x_i$ | $\mathcal{M}R_i$ | $\mathcal{R}C_i$ | $\mathcal{R}C_i^*$ |
|-----------------|-------|------------------|------------------|--------------------|
| $\mathcal{A}_1$ | 36.00 | 9.44             | 3.40             | 33.33              |
| $\mathcal{A}_2$ | 38.00 | 8.90             | 3.38             | 33.17              |
| $\mathcal{A}_3$ | 26.00 | 13.13            | 3.41             | 33.50              |

Table: Risk decomposition of Portfolio #1 with respect to the primary assets

| Asset $j$        | $y_j$ | $\mathcal{M}R_j$ | $\mathcal{R}C_j$ | $\mathcal{R}C_j^*$ |
|------------------|-------|------------------|------------------|--------------------|
| $\mathcal{A}'_1$ | 9.00  | 3.53             | 0.32             | 3.12               |
| $\mathcal{A}'_2$ | 9.00  | 7.95             | 0.72             | 7.02               |
| $\mathcal{A}'_3$ | 31.50 | 19.31            | 6.08             | 59.69              |
| $\mathcal{A}'_4$ | 31.50 | 6.95             | 2.19             | 21.49              |
| $\mathcal{A}'_5$ | 9.50  | 0.93             | 0.09             | 0.87               |
| $\mathcal{A}'_6$ | 9.50  | 8.39             | 0.80             | 7.82               |

# Tables 2.27 & 2.28, Pages 139 & 140

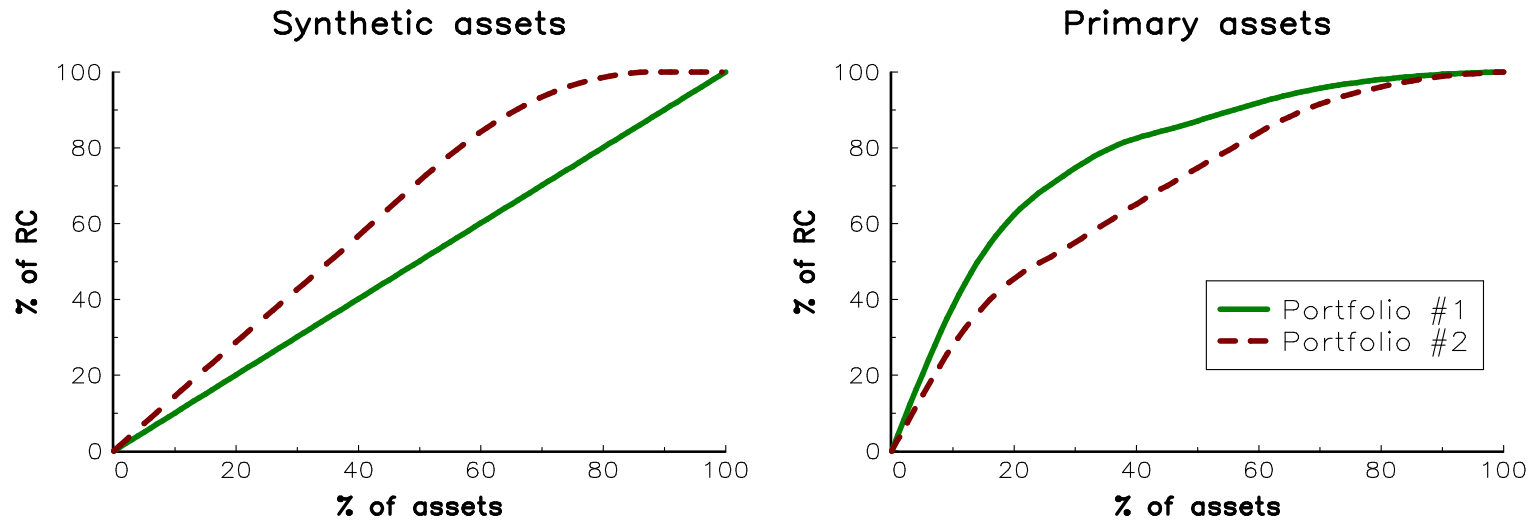
Table: Risk decomposition of Portfolio #2 with respect to the synthetic assets

| Asset $i$       | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|-----------------|-------|--------|--------|----------|
| $\mathcal{A}_1$ | 48.00 | 9.84   | 4.73   | 49.91    |
| $\mathcal{A}_2$ | 50.00 | 9.03   | 4.51   | 47.67    |
| $\mathcal{A}_3$ | 2.00  | 11.45  | 0.23   | 2.42     |

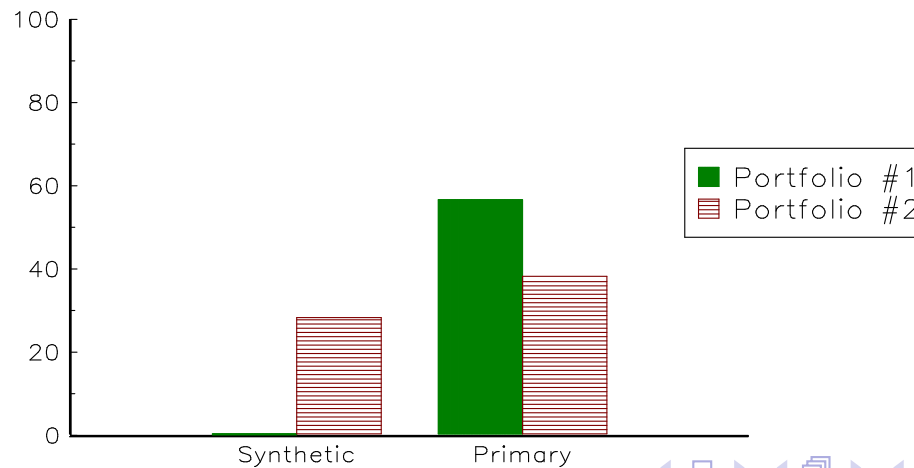
Table: Risk decomposition of Portfolio #2 with respect to the primary assets

| Asset $j$        | $y_j$ | $MR_j$ | $RC_j$ | $RC_j^*$ |
|------------------|-------|--------|--------|----------|
| $\mathcal{A}'_1$ | 12.00 | 5.07   | 0.61   | 6.43     |
| $\mathcal{A}'_2$ | 12.00 | 11.41  | 1.37   | 14.46    |
| $\mathcal{A}'_3$ | 25.50 | 16.84  | 4.29   | 45.35    |
| $\mathcal{A}'_4$ | 25.50 | 6.06   | 1.55   | 16.33    |
| $\mathcal{A}'_5$ | 12.50 | 1.32   | 0.17   | 1.74     |
| $\mathcal{A}'_6$ | 12.50 | 11.88  | 1.49   | 15.69    |

Figure: Lorenz curve of risk contributions



Gini coefficient (in %)





# Tables 2.29 & 2.30, Page 143

Table: Risk decomposition of the EW portfolio with respect to the assets

| Asset      | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|------------|-------|--------|--------|----------|
| 1          | 25.00 | 18.81  | 4.70   | 21.97    |
| 2          | 25.00 | 23.72  | 5.93   | 27.71    |
| 3          | 25.00 | 24.24  | 6.06   | 28.32    |
| 4          | 25.00 | 18.83  | 4.71   | 22.00    |
| Volatility |       |        | 21.40  |          |

Table: Risk decomposition of the EW portfolio with respect to the risk factors

| Factor                  | $y_j$  | $MR_j$ | $RC_j$ | $RC_j^*$ |
|-------------------------|--------|--------|--------|----------|
| $\mathcal{F}_1$         | 100.00 | 17.22  | 17.22  | 80.49    |
| $\mathcal{F}_2$         | 22.50  | 9.07   | 2.04   | 9.53     |
| $\mathcal{F}_3$         | 35.00  | 6.06   | 2.12   | 9.91     |
| $\tilde{\mathcal{F}}_1$ | 2.75   | 0.52   | 0.01   | 0.07     |
| Volatility              |        |        | 21.40  |          |

# Tables 2.31 & 2.32, Pages 144 & 145

Table: Risk decomposition of the RFP portfolio with respect to the risk factors

| Factor                  | $y_j$ | $MR_j$ | $RC_j$ | $RC_j^*$ |
|-------------------------|-------|--------|--------|----------|
| $\mathcal{F}_1$         | 93.38 | 11.16  | 10.42  | 49.00    |
| $\mathcal{F}_2$         | 24.02 | 22.14  | 5.32   | 25.00    |
| $\mathcal{F}_3$         | 39.67 | 13.41  | 5.32   | 25.00    |
| $\tilde{\mathcal{F}}_1$ | 16.39 | 1.30   | 0.21   | 1.00     |
| Volatility              |       |        |        | 21.27    |

Table: Risk decomposition of the RFP portfolio with respect to the assets

| Asset      | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|------------|-------|--------|--------|----------|
| 1          | 15.08 | 17.44  | 2.63   | 12.36    |
| 2          | 38.38 | 23.94  | 9.19   | 43.18    |
| 3          | 0.89  | 21.82  | 0.20   | 0.92     |
| 4          | 45.65 | 20.29  | 9.26   | 43.54    |
| Volatility |       |        |        | 21.27    |

# Tables 2.33 & 2.34, Pages 145 & 146

**Table:** Risk decomposition of the balanced RFP portfolio with respect to the risk factors

| Factor        | $y_j$ | $MR_j$ | $RC_j$ | $RC_j^*$ |
|---------------|-------|--------|--------|----------|
| $F_1$         | 91.97 | 7.91   | 7.28   | 33.26    |
| $F_2$         | 25.78 | 28.23  | 7.28   | 33.26    |
| $F_3$         | 42.22 | 17.24  | 7.28   | 33.26    |
| $\tilde{F}_1$ | 6.74  | 0.70   | 0.05   | 0.21     |
| Volatility    |       |        |        | 21.88    |

**Table:** Risk decomposition of the balanced RFP portfolio with respect to the assets

| Asset      | $x_i$ | $MR_i$ | $RC_i$ | $RC_i^*$ |
|------------|-------|--------|--------|----------|
| 1          | 0.30  | 16.11  | 0.05   | 0.22     |
| 2          | 39.37 | 23.13  | 9.11   | 41.63    |
| 3          | 0.31  | 20.93  | 0.07   | 0.30     |
| 4          | 60.01 | 21.09  | 12.66  | 57.85    |
| Volatility |       |        |        | 21.88    |

Table: Balanced RFP portfolios with  $x_i \geq 10\%$

| Criterion       | $\mathcal{H}(x)$ | $\mathcal{G}(x)$ | $\mathcal{I}(x)$ |
|-----------------|------------------|------------------|------------------|
| $x_1$           | 10.00            | 10.00            | 10.00            |
| $x_2$           | 22.08            | 18.24            | 24.91            |
| $x_3$           | 10.00            | 10.00            | 10.00            |
| $x_4$           | 57.92            | 61.76            | 55.09            |
| $\mathcal{H}^*$ | 0.0436           | 0.0490           | 0.0453           |
| $\mathcal{G}$   | 0.1570           | 0.1476           | 0.1639           |
| $\mathcal{I}^*$ | 2.8636           | 2.8416           | 2.8643           |