

Introduction to Risk Parity and Budgeting

Chapter 3 – Risk-Based Indexation

© Thierry Roncalli[†] & CRC Press

[†]Evry University & Lyxor Asset Management, France

Instructors may find the description of the book at the following addresses:

<http://www.crcpress.com/product/isbn/9781482207156>

<http://www.thierry-roncalli.com/RiskParityBook.html>

May 22, 2013

Table 3.1, Page 158

Table: Weight and risk concentration of several equity indices (June 29, 2012)

Ticker	Weights				Risk contributions			
	$\mathcal{G}(x)$	10%	$\mathbb{L}(x)$ 25%	50%	$\mathcal{G}(x)$	10%	$\mathbb{L}(x)$ 25%	50%
SX5P	30.8	24.1	48.1	71.3	26.3	19.0	40.4	68.6
SX5E	31.2	23.0	46.5	72.1	31.2	20.5	44.7	73.3
INDU	33.2	23.0	45.0	73.5	35.8	25.0	49.6	75.9
BEL20	39.1	25.8	49.4	79.1	45.1	25.6	56.8	82.5
DAX	44.0	27.5	56.0	81.8	47.3	27.2	59.8	84.8
CAC	47.4	34.3	58.3	82.4	44.1	31.9	57.3	79.7
AEX	52.2	37.2	61.3	86.0	51.4	35.3	62.0	84.7
HSI	53.7	40.9	64.7	84.6	56.5	41.4	67.6	87.4
HSCEI	54.8	39.7	69.3	85.9	53.8	36.5	67.2	85.9
SMI	58.1	44.2	70.0	87.8	49.1	30.3	60.2	85.1
NKY	60.2	47.9	70.4	87.7	61.4	49.6	70.9	88.1
UKX	60.8	47.5	73.1	88.6	60.4	46.1	72.8	88.7
SXXE	61.7	49.2	73.5	88.7	63.9	51.6	75.3	90.1
SPX	61.8	52.1	72.0	87.8	59.3	48.7	69.9	86.7
MEXBOL	64.6	48.2	75.1	91.8	65.9	45.7	78.6	92.9
IBEX	64.9	51.7	77.3	90.2	68.3	58.2	80.3	91.4
SXXP	65.6	55.0	76.4	90.1	64.2	52.0	75.5	90.0
SPTSX	65.9	54.3	76.1	90.8	69.6	58.5	80.3	92.0
NDX	66.3	58.6	77.0	89.2	64.6	56.9	74.9	88.6
TWSE	79.7	73.4	86.8	95.2	79.7	72.6	87.3	95.7
TPX	80.8	72.8	88.8	96.3	83.9	77.1	91.0	97.3
KOSPI	86.5	80.6	93.9	98.0	89.3	85.1	95.8	98.8

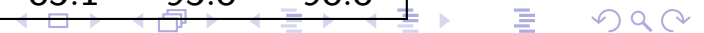


Figure 3.1, Page 158

Figure: Lorenz curve of several equity indices (June 29, 2012)

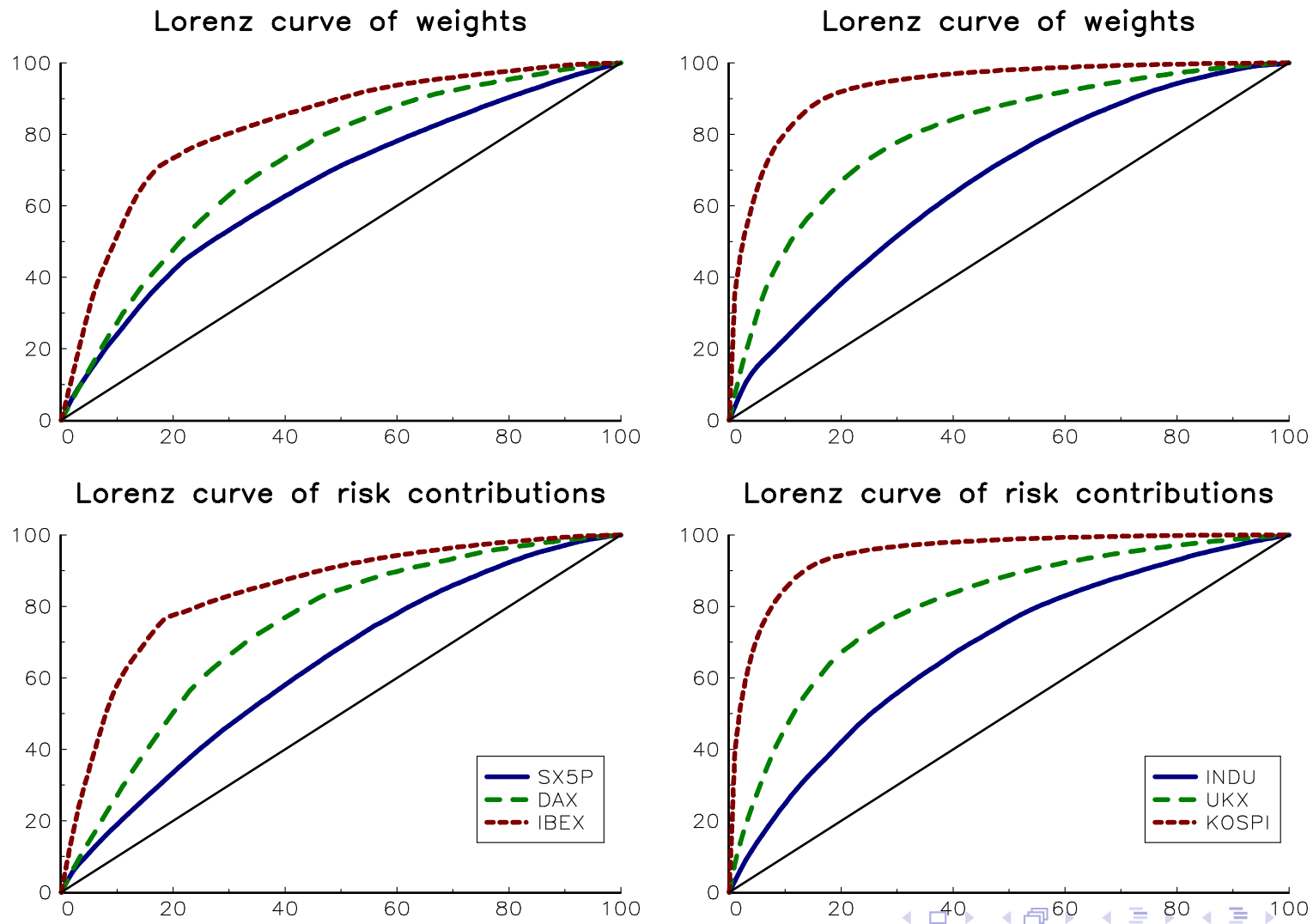


Figure 3.2, Page 162

Figure: Performance of the RAFI index since January 2000

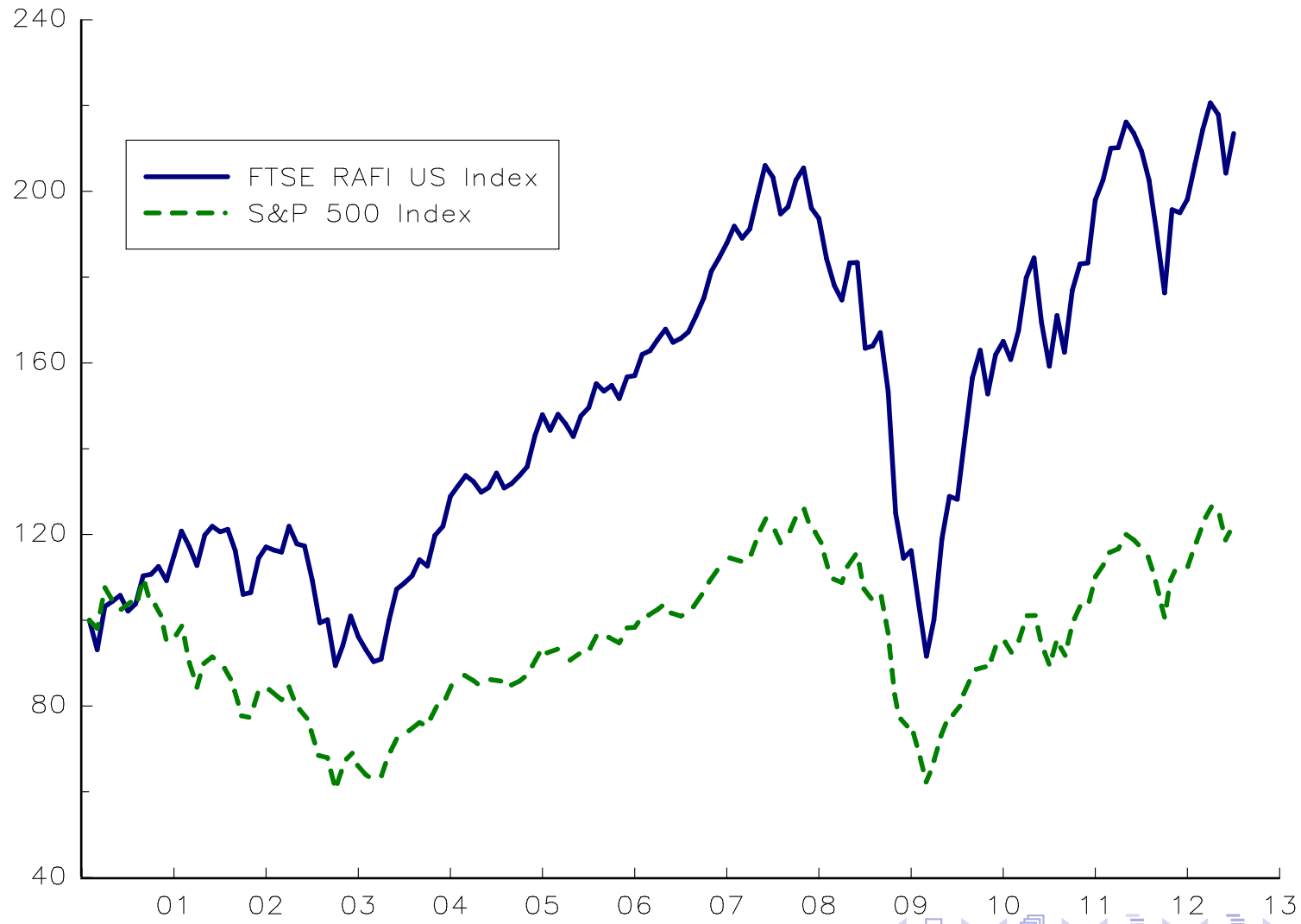


Figure 3.3, Page 163

Figure: Illustration of the diversification effect of AW indices

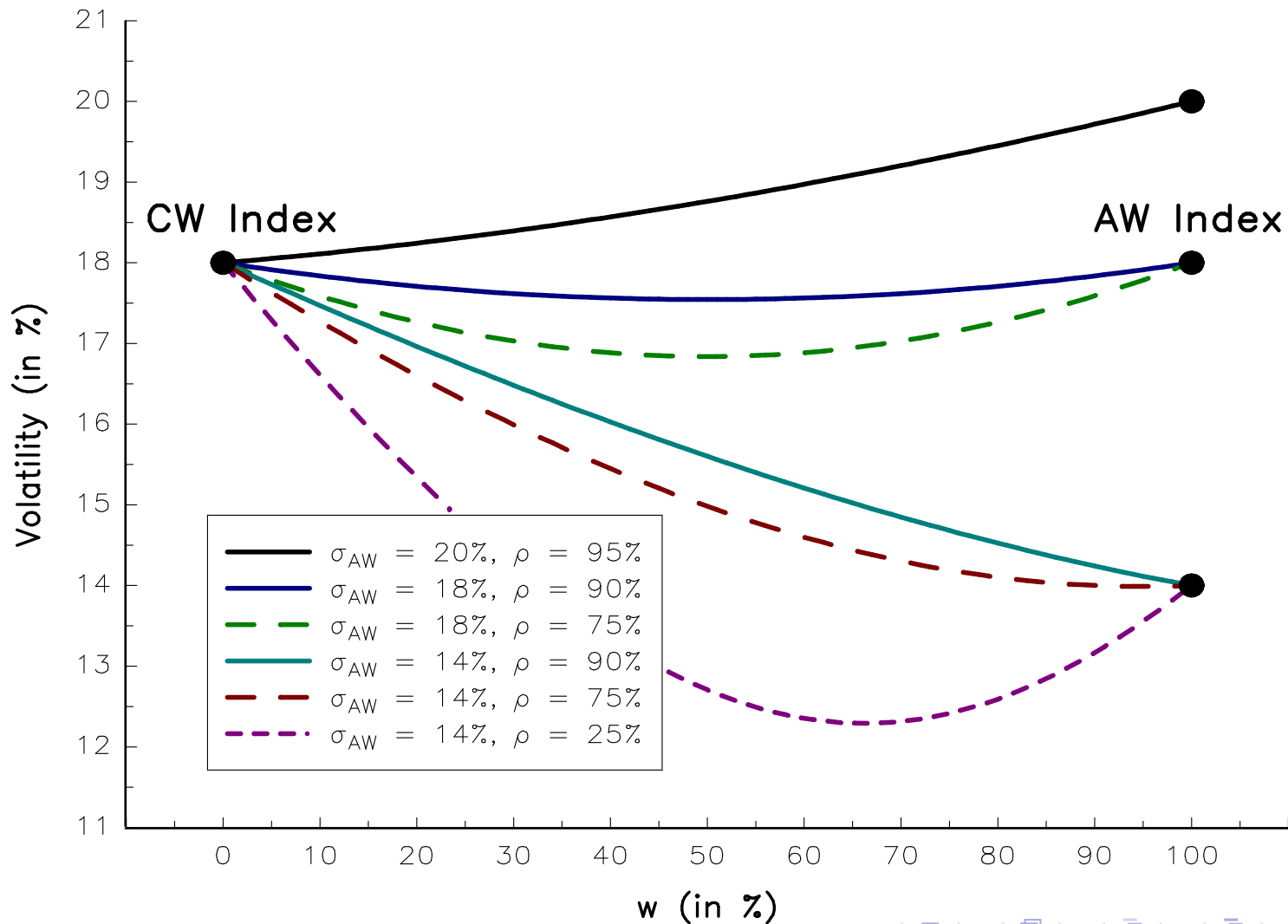
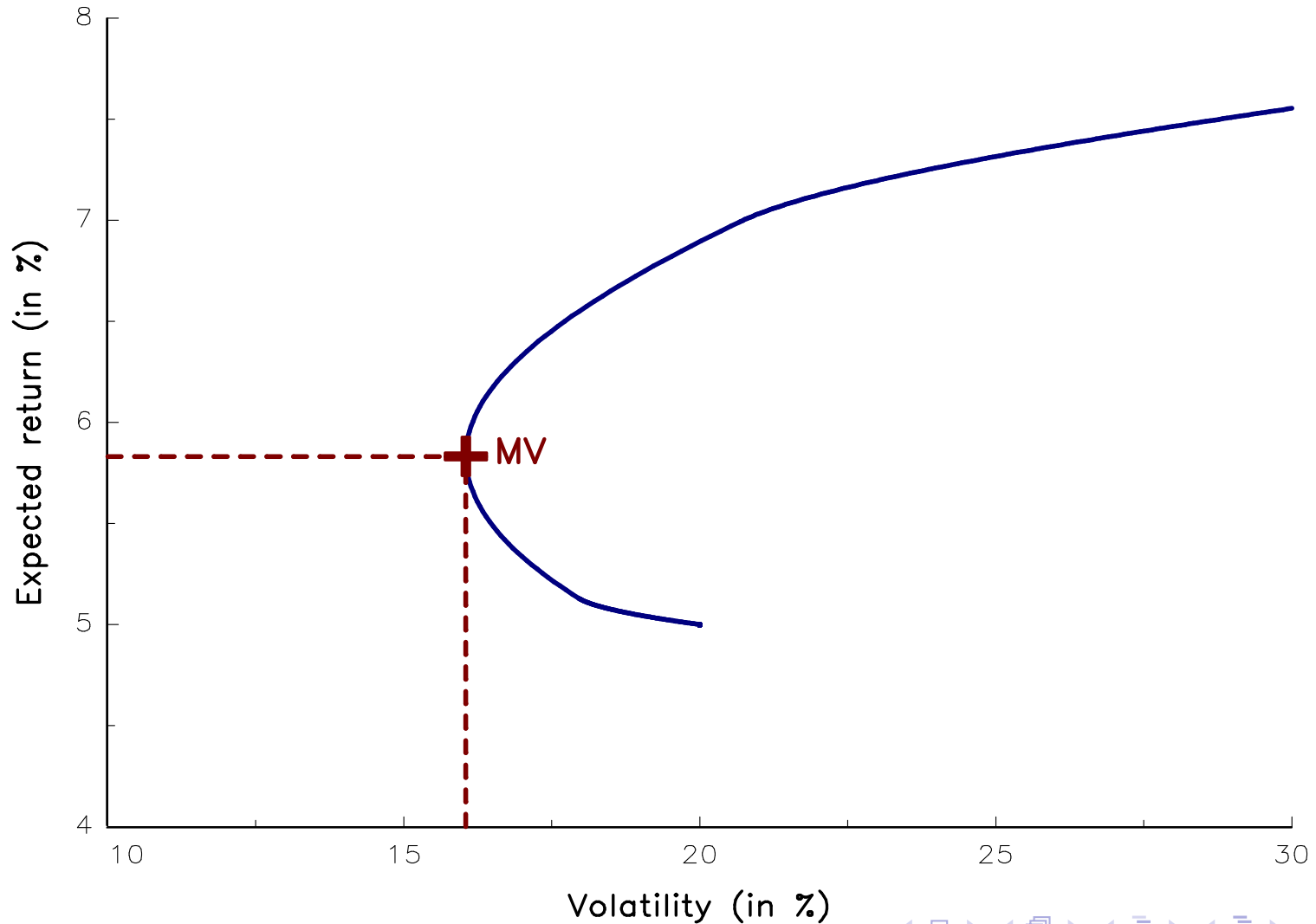


Figure 3.4, Page 165

Figure: Location of the minimum variance portfolio in the efficient frontier



Tables 3.2 & 3.3, Page 166

Table: Unconstrained minimum variance portfolios

Asset	ρ						
	-20%	0%	20%	50%	70%	90%	99%
1	44.35	53.92	65.88	90.65	114.60	149.07	170.07
2	25.25	23.97	22.36	19.04	15.83	11.20	8.38
3	17.32	13.48	8.69	-1.24	-10.84	-24.67	-33.09
4	13.08	8.63	3.07	-8.44	-19.58	-35.61	-45.37
$\sigma(x^*)$	1.93	2.94	3.52	3.86	3.62	2.52	0.87

Table: Long-only minimum variance portfolios

Asset	ρ						
	-20%	0%	20%	50%	70%	90%	99%
1	44.35	53.92	65.88	85.71	100.00	100.00	100.00
2	25.25	23.97	22.36	14.29	0.00	0.00	0.00
3	17.32	13.48	8.69	0.00	0.00	0.00	0.00
4	13.08	8.63	3.07	0.00	0.00	0.00	0.00
$\sigma(x^*)$	1.93	2.94	3.52	3.93	4.00	4.00	4.00

Tables 3.4 & 3.5, Page 168 & 171

Table: Composition of the MV portfolio

Asset	β_i	$\tilde{\beta}_i$	x_i^*	
			Unconstrained	Long-only
1	0.90	562.50	147.33	0.00
2	0.80	55.56	24.67	9.45
3	1.20	480.00	-49.19	0.00
4	0.70	109.37	74.20	90.55
5	1.30	520.00	-97.01	0.00
Volatility			11.45	19.19

Table: Composition of the MDP

Asset	Unconstrained		Long-only	
	x_i^*	$\rho(e_i, x^*)$	x_i^*	$\rho(e_i, x^*)$
1	-18.15	61.10	0.00	73.20
2	61.21	61.10	41.70	62.40
3	29.89	61.10	30.71	62.40
4	27.05	61.10	27.60	62.40
$\sigma(x^*)$		9.31		10.74
$\mathcal{DR}(x^*)$		1.64		1.60

Table 3.6, Page 175

Table: Weights and risk contributions (Example 26)

Asset	EW		MV		MDP		ERC	
	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i
1	25.00	4.20	10.87	0.96	10.87	0.96	17.26	2.32
2	25.00	4.20	10.87	0.96	10.87	0.96	17.26	2.32
3	25.00	1.17	39.13	3.46	39.13	3.46	32.74	2.32
4	25.00	1.17	39.13	3.46	39.13	3.46	32.74	2.32
$\mathcal{H}^*(x)$	0.00		10.65		10.65		3.20	
$\sigma(x)$	10.72		8.85		8.85		9.26	
$\mathcal{DR}(x)$	1.87		2.26		2.26		2.16	
$\mathcal{H}^*(\mathcal{RC})$	10.65		10.65		10.65		0.00	

Table 3.7, Page 176

Table: Weights and risk contributions (Example 27)

Asset	EW		MV		MDP		ERC	
	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i
1	25.00	1.41	74.48	6.43	27.78	1.23	38.36	2.57
2	25.00	3.04	0.00	0.00	13.89	1.23	19.18	2.57
3	25.00	1.63	15.17	1.31	33.33	4.42	24.26	2.57
4	25.00	5.43	10.34	0.89	25.00	4.42	18.20	2.57
$\mathcal{H}^*(x)$	0.00		45.13		2.68		3.46	
$\sigma(x)$	11.51		8.63		11.30		10.29	
$\mathcal{DR}(x)$	2.17		1.87		2.26		2.16	
$\mathcal{H}^*(\mathcal{RC})$	10.31		45.13		10.65		0.00	

Table 3.8, Page 177

Table: Weights and risk contributions (Example 28)

Asset	EW		MV		MDP		ERC	
	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i
1	25.00	9.32	0.00	0.00	4.18	0.74	7.29	1.96
2	25.00	6.77	4.55	0.29	5.57	0.74	9.72	1.96
3	25.00	1.09	27.27	1.74	30.08	2.66	27.66	1.96
4	25.00	0.00	68.18	4.36	60.17	2.66	55.33	1.96
$\mathcal{H}^*(x)$	0.00		38.84		27.65		19.65	
$\sigma(x)$	17.18		6.40		6.80		7.82	
$\mathcal{DR}(x)$	1.46		2.13		2.26		2.16	
$\mathcal{H}^*(\mathcal{RC})$	27.13		38.84		10.65		0.00	

Table 3.9, Page 178

Table: Weights and risk contributions (Example 29)

Asset	EW		MV		MDP		ERC	
	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i
1	25.00	2.52	82.61	11.50	0.00	0.00	40.53	4.52
2	25.00	5.19	17.39	2.42	0.00	0.00	22.46	4.52
3	25.00	3.89	0.00	0.00	57.14	12.86	21.12	4.52
4	25.00	9.01	0.00	0.00	42.86	12.86	15.88	4.52
$\mathcal{H}^*(x)$	0.00		61.69		34.69		4.61	
$\sigma(x)$	20.61		13.92		25.71		18.06	
$\mathcal{DR}(x)$	1.82		1.27		2.00		1.76	
$\mathcal{H}^*(\mathcal{RC})$	7.33		61.69		33.33		0.00	

Table 3.10, Page 179

Table: Weights and risk contributions (Example 30)

Asset	EW		MV		MDP		ERC	
	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i	x_i	\mathcal{RC}_i
1	16.67	3.19	0.00	0.00	44.44	8.61	14.51	2.72
2	16.67	2.42	6.11	0.88	55.56	8.61	18.14	2.72
3	16.67	2.01	65.16	9.33	0.00	0.00	21.84	2.72
4	16.67	2.45	22.62	3.24	0.00	0.00	18.20	2.72
5	16.67	4.32	0.00	0.00	0.00	0.00	10.92	2.72
6	16.67	2.75	6.11	0.88	0.00	0.00	16.38	2.72
$\mathcal{H}^*(x)$	0.00		37.99		40.74		0.83	
$\sigma(x)$	17.14		14.33		17.21		16.31	
$\mathcal{DR}(x)$	1.24		1.14		1.29		1.25	
$\mathcal{H}^*(\mathcal{RC})$	1.36		37.99		40.00		0.00	

Figure 3.5, Page 180

Figure: Weight of the first two assets in AW portfolios (Example 31)

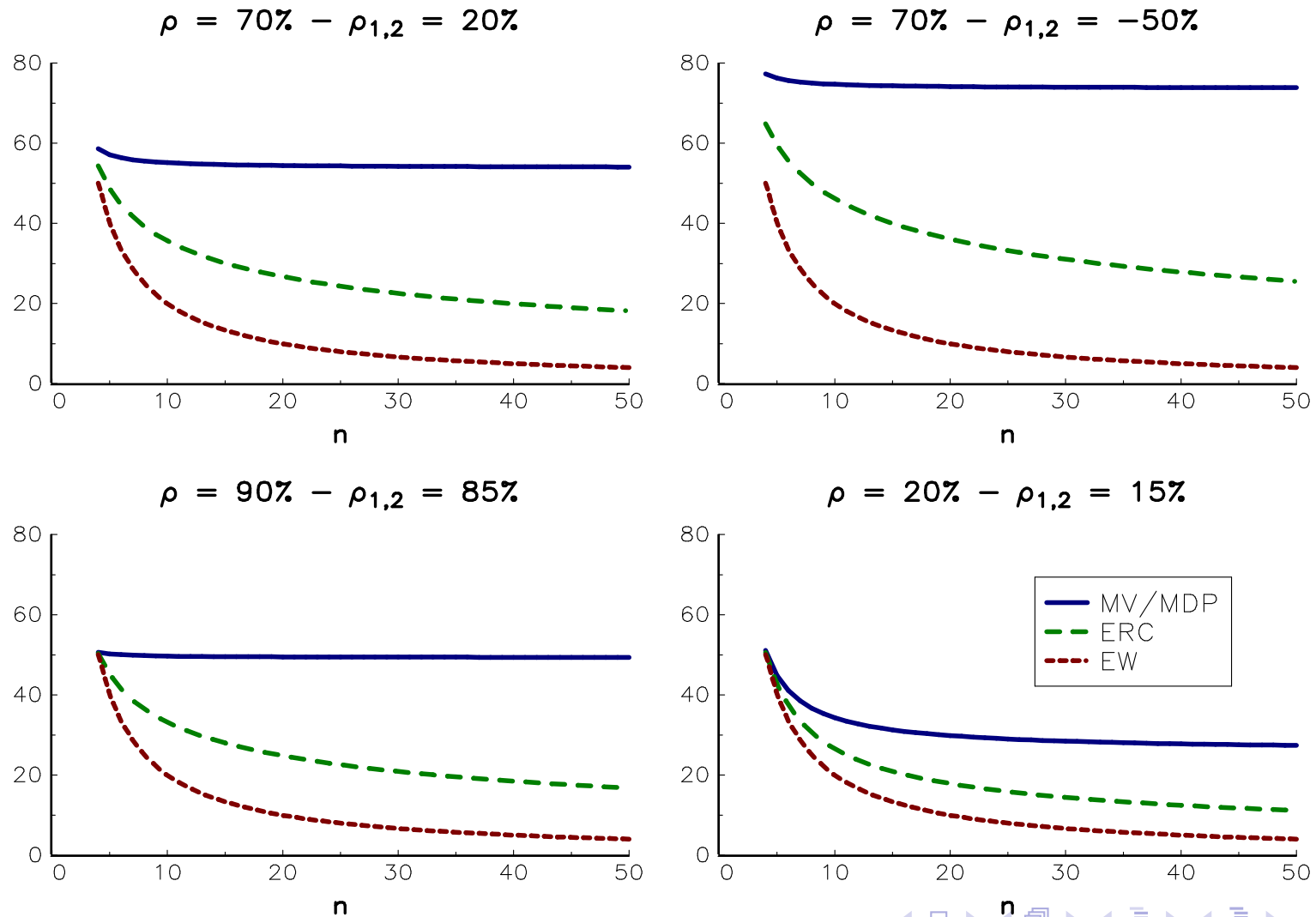


Figure 3.6, Page 180

Figure: Weight with respect to the asset beta β_i (Example 32)

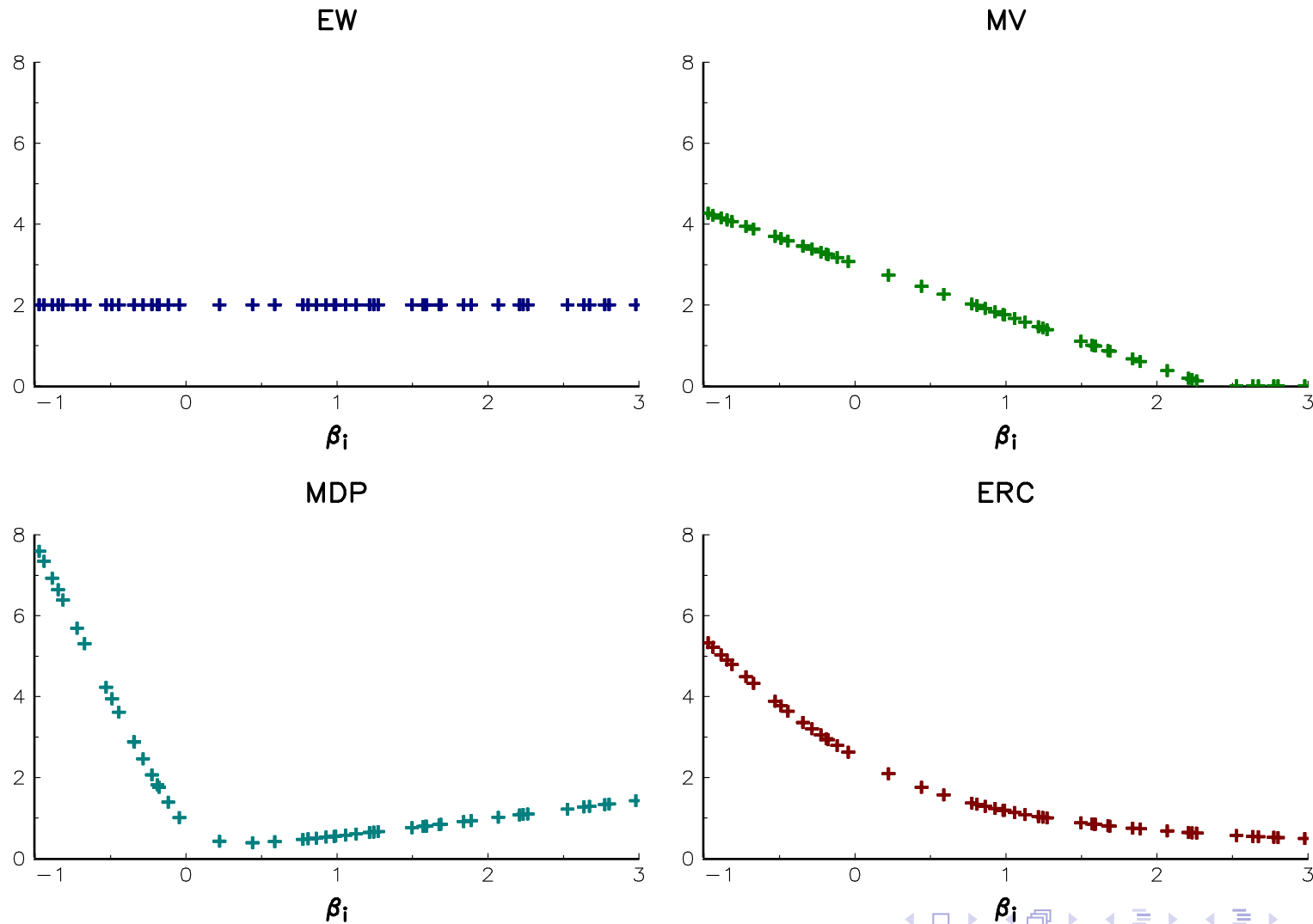


Table 3.11, Page 182

Table: Main statistics of AW indexations (Jan. 1993 – Sep. 2012)

	CW	EW	MV	MDP	ERC
$\mu(x)$	4.47	6.92	7.36	10.15	8.13
$\sigma(x)$	22.86	23.05	17.57	20.12	21.13
$SR(x r)$	0.05	0.16	0.23	0.34	0.23
$\sigma(x b)$		4.18	14.85	12.79	5.65
$IR(x b)$		0.56	0.19	0.42	0.62
$\overline{\mathcal{MDD}}(x)$	-66.88	-61.67	-56.04	-50.21	-56.85
$\gamma_1(x)$	0.10	0.11	1.83	2.91	0.23
$\gamma_2(x)$	5.28	6.06	49.88	74.13	7.13
$\gamma_1^*(x)$	-0.46	-0.41	-1.00	-0.54	-0.50
$\gamma_2^*(x)$	0.63	1.33	2.21	0.97	1.09
$\rho(x b)$		98.35	76.03	83.02	97.00
$\beta(x b)$		0.99	0.58	0.73	0.90

Table 3.12, Page 182

Table: Simulated performance of AW portfolios by year (in %)

Year	CW	EW	MV	MDP	ERC
1993	38.7	44.5	38.3	45.5	43.5
1994	-7.9	-2.6	-3.9	7.7	-1.9
1995	14.1	13.2	16.8	19.2	14.7
1996	22.8	30.3	28.1	34.9	30.0
1997	36.8	44.5	38.8	45.7	44.9
1998	32.0	34.2	47.5	50.6	35.1
1999	46.7	41.0	20.8	25.6	35.9
2000	-2.7	2.4	4.3	3.8	5.5
2001	-20.2	-17.6	-11.5	-10.7	-13.8
2002	-37.3	-34.8	-34.7	-29.2	-32.9
2003	15.7	23.3	4.3	25.0	18.8
2004	6.9	8.0	15.6	8.3	10.0
2005	21.3	20.4	16.9	16.1	20.0
2006	15.1	18.3	17.1	16.5	18.6
2007	6.8	5.2	-2.3	-2.5	5.0
2008	-44.4	-44.3	-15.7	-20.1	-36.3
2009	21.1	29.4	-5.2	16.9	25.5
2010	-5.8	-3.3	-5.8	0.0	-2.5
2011	-17.1	-18.0	1.6	-15.5	-15.3
2012	5.9	5.4	11.1	5.1	6.6



Figure 3.7, Page 183

Figure: Concentration statistics of AW portfolios

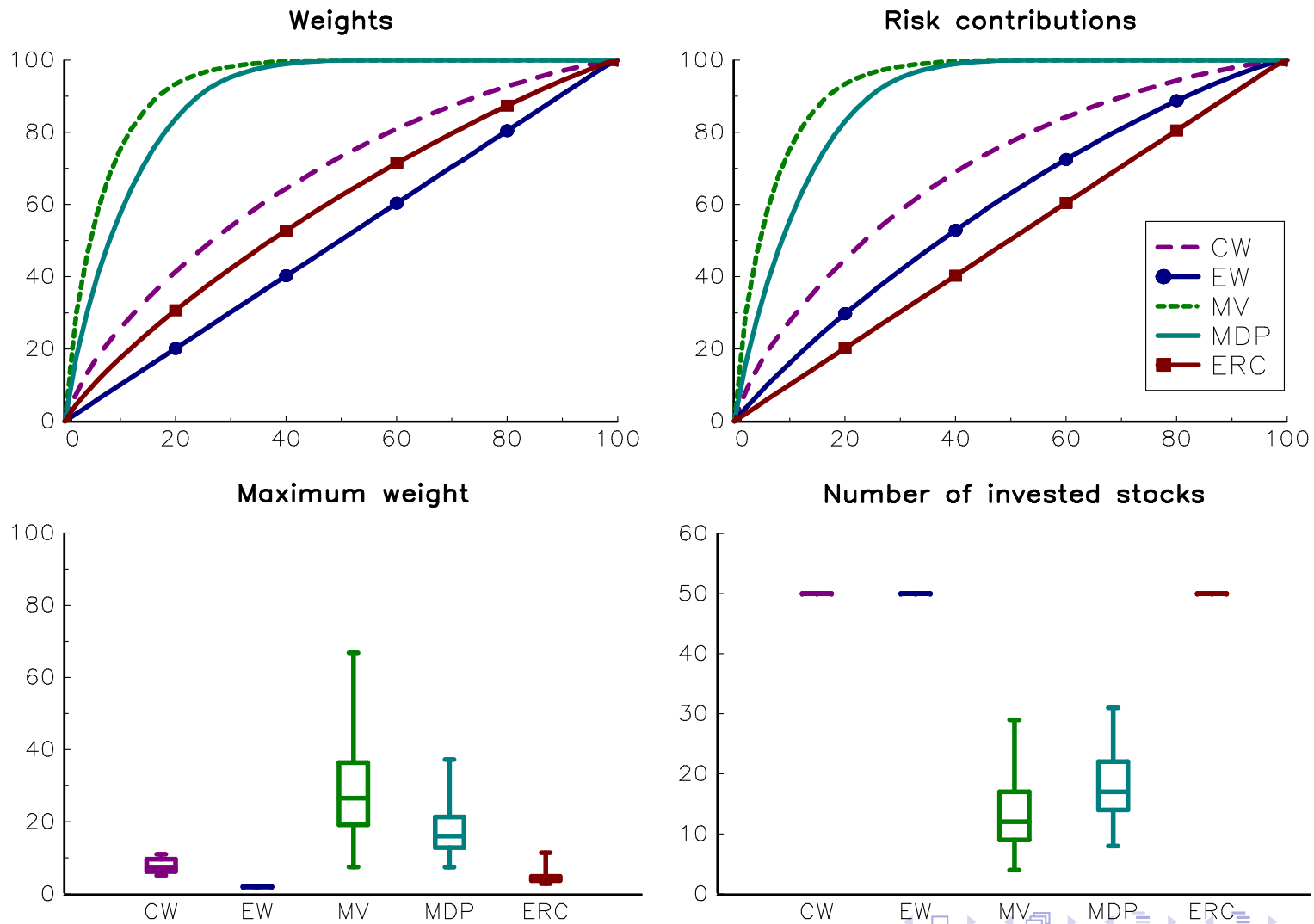


Table 3.13, Page 184

Table: Annualized monthly turnover of AW portfolios (in %)

Lag	EW	ERC	MV			MDP		
			10%	5%		10%	5%	
1M	19	551	1765	1401	969	1779	1571	1098
2M	19	290	1234	911	620	1313	1061	718
3M	20	195	909	681	462	985	806	539
1Y	19	63	330	248	161	343	290	197
2Y	19	43	202	152	101	223	196	127
3Y	18	34	149	113	73	164	147	95

Figure 3.8, Page 185

Figure: Concentration statistics of constrained MV and MDP indexations

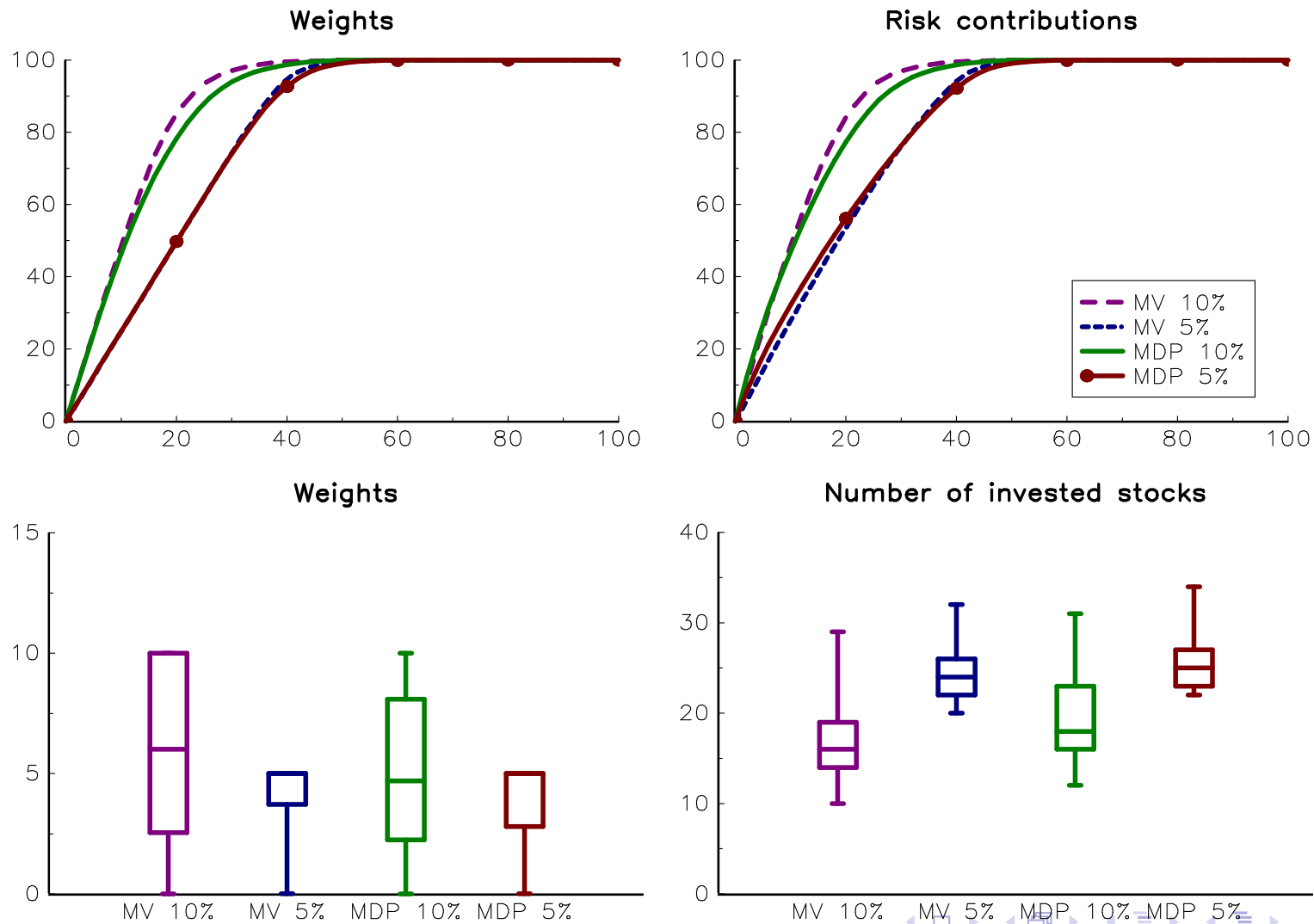


Table 3.14, Page 185

Table: Main statistics of constrained MV and MDP indexations (Jan. 1993 – Sep. 2012)

	MV			MDP		
		10%	5%		10%	5%
$\mu(x)$	7.36	8.49	8.57	10.15	9.72	9.32
$\sigma(x)$	17.57	17.34	17.79	20.12	19.03	19.52
$SR(x r)$	0.23	0.30	0.30	0.34	0.34	0.31
$\sigma(x b)$	14.85	11.67	8.91	12.79	9.40	7.52
$IR(x b)$	0.19	0.33	0.44	0.42	0.53	0.62
$\overline{\mathcal{MDD}}(x)$	-56.04	-55.10	-50.35	-50.21	-49.73	-51.04
$\gamma_1(x)$	1.83	0.79	0.14	2.91	0.11	0.02
$\gamma_2(x)$	49.88	17.32	7.00	74.13	5.50	5.43
$\gamma_1^*(x)$	-1.00	-0.71	-0.67	-0.54	-0.42	-0.52
$\gamma_2^*(x)$	2.21	1.25	0.95	0.97	0.64	1.06
$\rho(x b)$	76.03	86.65	93.41	83.02	91.52	94.91
$\beta(x b)$	0.58	0.66	0.73	0.73	0.76	0.81

Table 3.15, Page 187

Table: Influence of the covariance estimator

	Statistic	ERC	MV		MDP			
			10%	5%	10%	5%		
EMP	$\tau(x)$	63	330	248	161	343	290	197
	$\mathcal{G}(x)$	0.18	0.86	0.77	0.59	0.79	0.74	0.59
	$\mathcal{G}(\mathcal{R}\mathcal{C})$	0.01	0.86	0.77	0.61	0.78	0.74	0.62
	$\text{IR}(x b)$	0.62	0.19	0.33	0.44	0.42	0.45	0.41
CCM	$\tau(x)$	47	289	210	132	47	47	47
	$\mathcal{G}(x)$	0.14	0.87	0.77	0.59	0.14	0.14	0.14
	$\mathcal{G}(\mathcal{R}\mathcal{C})$	0.01	0.87	0.76	0.60	0.01	0.01	0.01
	$\text{IR}(x b)$	0.60	0.08	0.26	0.32	0.60	0.60	0.60
RMT	$\tau(x)$	64	262	198	144	260	228	168
	$\mathcal{G}(x)$	0.19	0.86	0.77	0.59	0.77	0.74	0.58
	$\mathcal{G}(\mathcal{R}\mathcal{C})$	0.01	0.86	0.77	0.61	0.76	0.74	0.61
	$\text{IR}(x b)$	0.65	0.23	0.32	0.46	0.54	0.54	0.55
CCS	$\tau(x)$	57	314	233	148	304	266	194
	$\mathcal{G}(x)$	0.16	0.86	0.77	0.59	0.74	0.71	0.57
	$\mathcal{G}(\mathcal{R}\mathcal{C})$	0.01	0.86	0.77	0.60	0.73	0.70	0.59
	$\text{IR}(x b)$	0.68	0.16	0.32	0.39	0.46	0.49	0.43
FSE	$\tau(x)$	63	306	231	155	309	267	187
	$\mathcal{G}(x)$	0.18	0.86	0.77	0.59	0.77	0.74	0.58
	$\mathcal{G}(\mathcal{R}\mathcal{C})$	0.01	0.86	0.77	0.61	0.76	0.73	0.62
	$\text{IR}(x b)$	0.62	0.21	0.32	0.43	0.48	0.47	0.42